Pedro Ferreira

Royal Brompton Hospital
London
United Kingdom

MRI DATA: K-SPACE AND SAMPLING ARTEFACTS

Imperial College London
Royal Brompton & Harefield NHS
NHS Foundation Trust

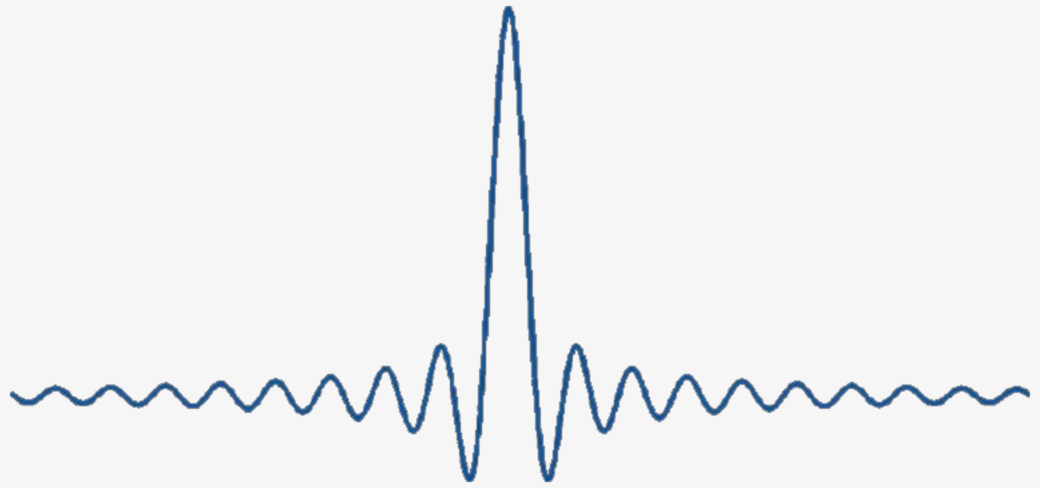
p.ferreira@rbht.nhs.uk

Contents

- k-space
- k-space properties
- k-space sampling artefacts
  - Wrap-around or aliasing
  - Gibbs ringing

p.ferreira@rbht.nhs.uk

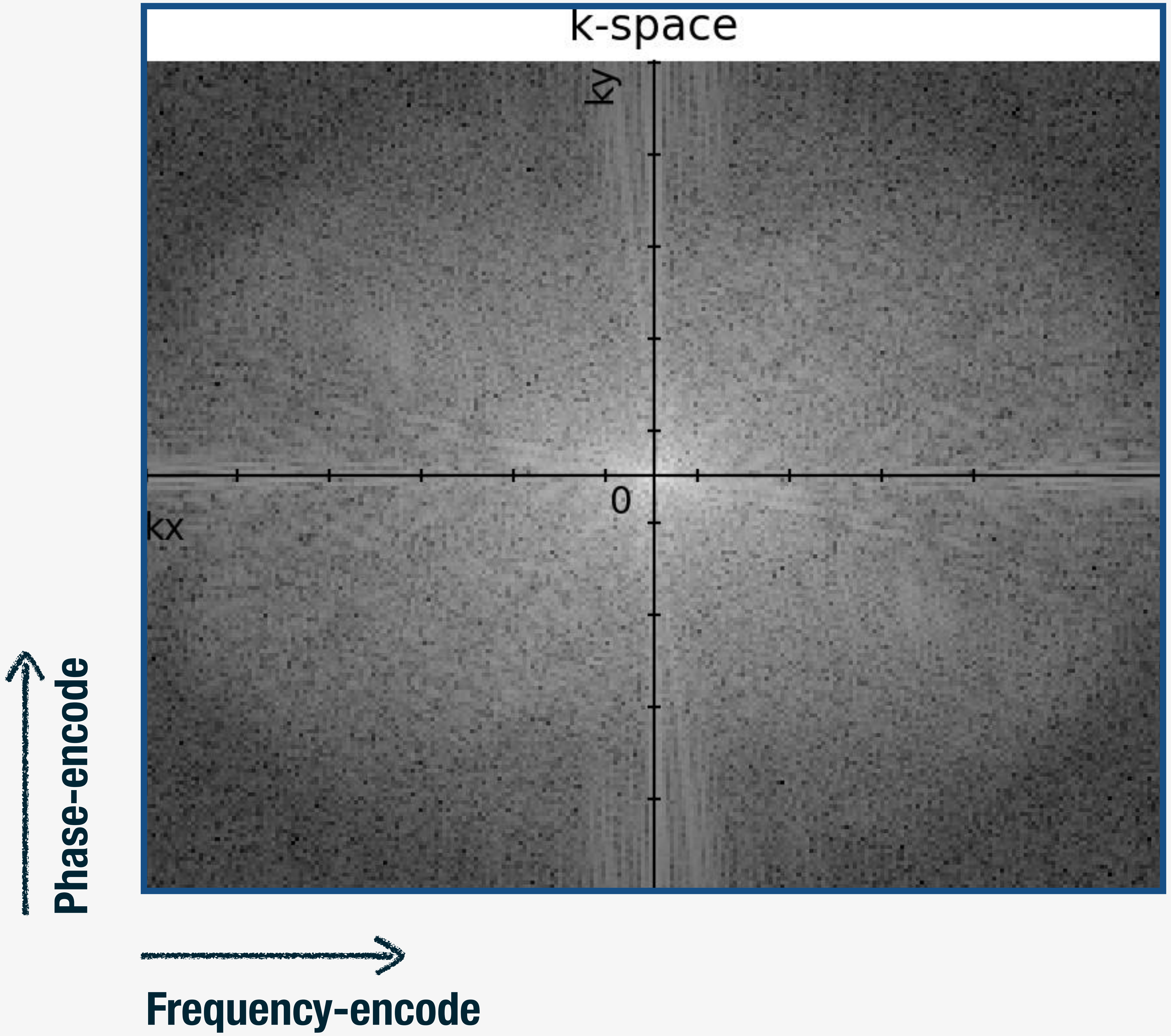
MRI



p.ferreira@rbht.nhs.uk

K-SPACE

k-space:
- MRI raw data.
- The imaged object is in the frequency domain.
- 0th frequency in the centre.
- k-space values are complex:
    - magnitude and phase.



p.ferreira@rbht.nhs.uk

WRAP-AROUND ARTEFACT

Image courtesy of Dr. Michael D. Noseworthy,
McMaster University, Toronto Canada

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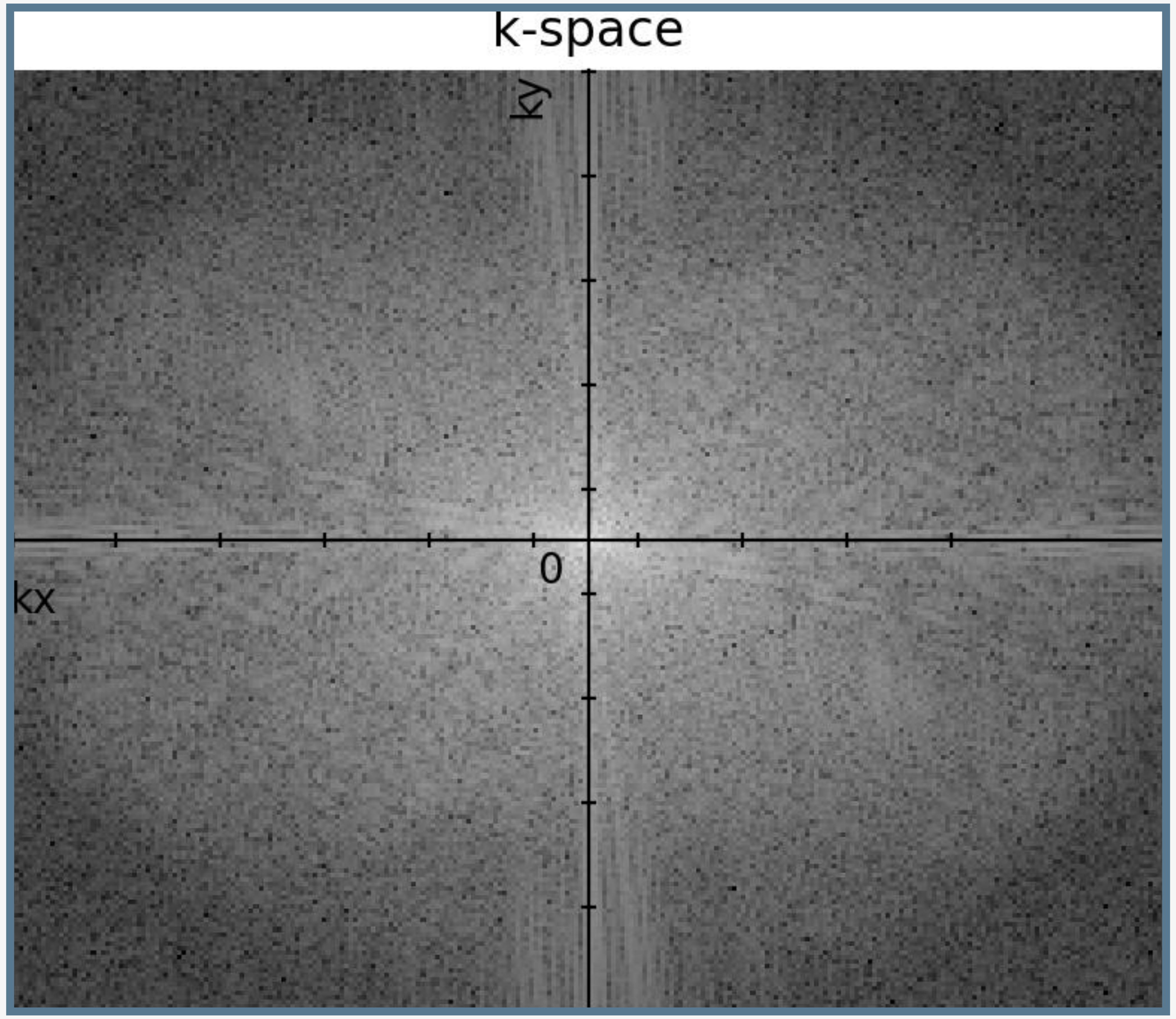
K-SPACE



2D inverse Fourier Transform

$$f(x,y) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(k_x,k_y)e^{i2\pi(k_x x + k_y y)}\,dk_x\,dk_y$$

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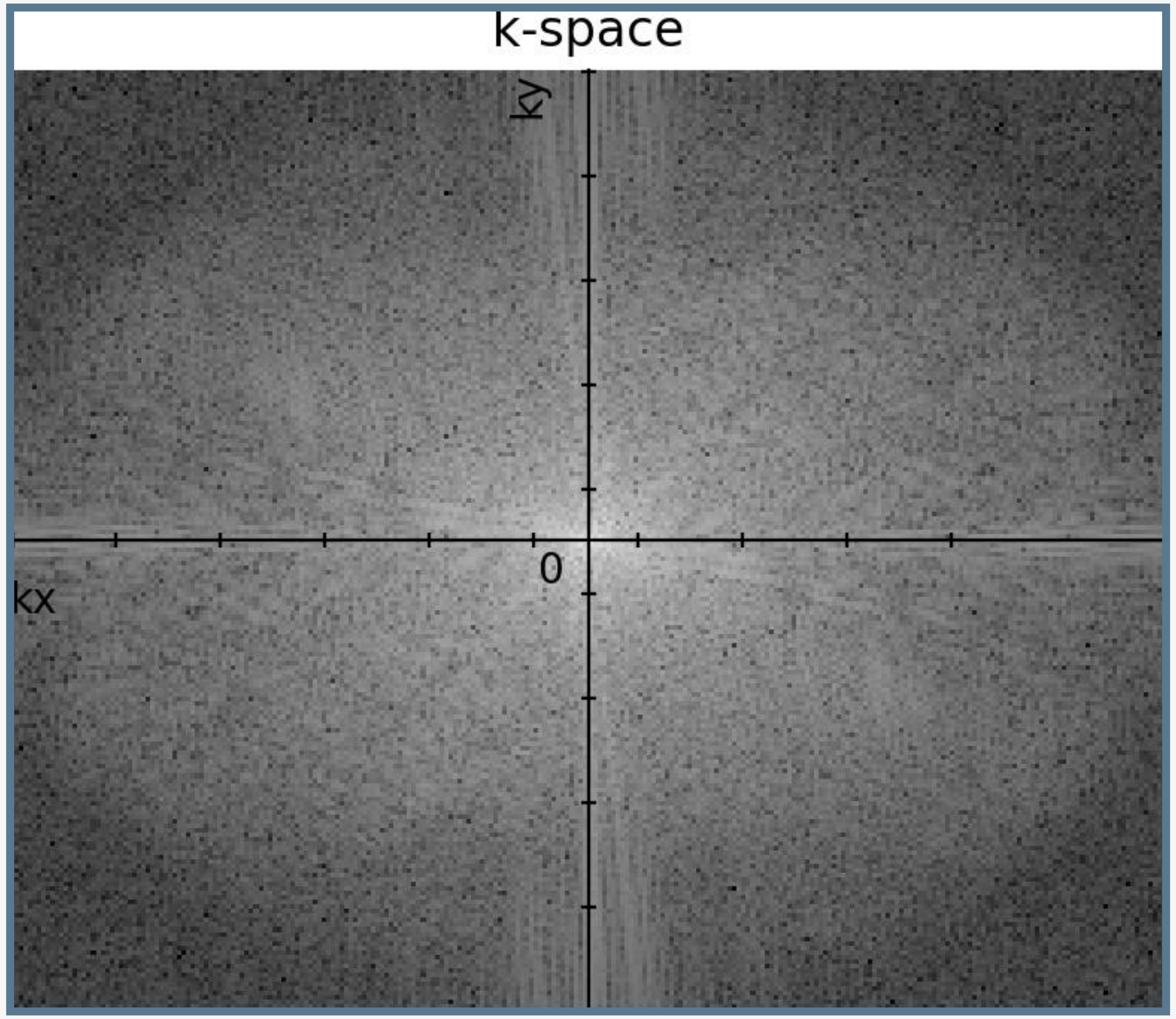
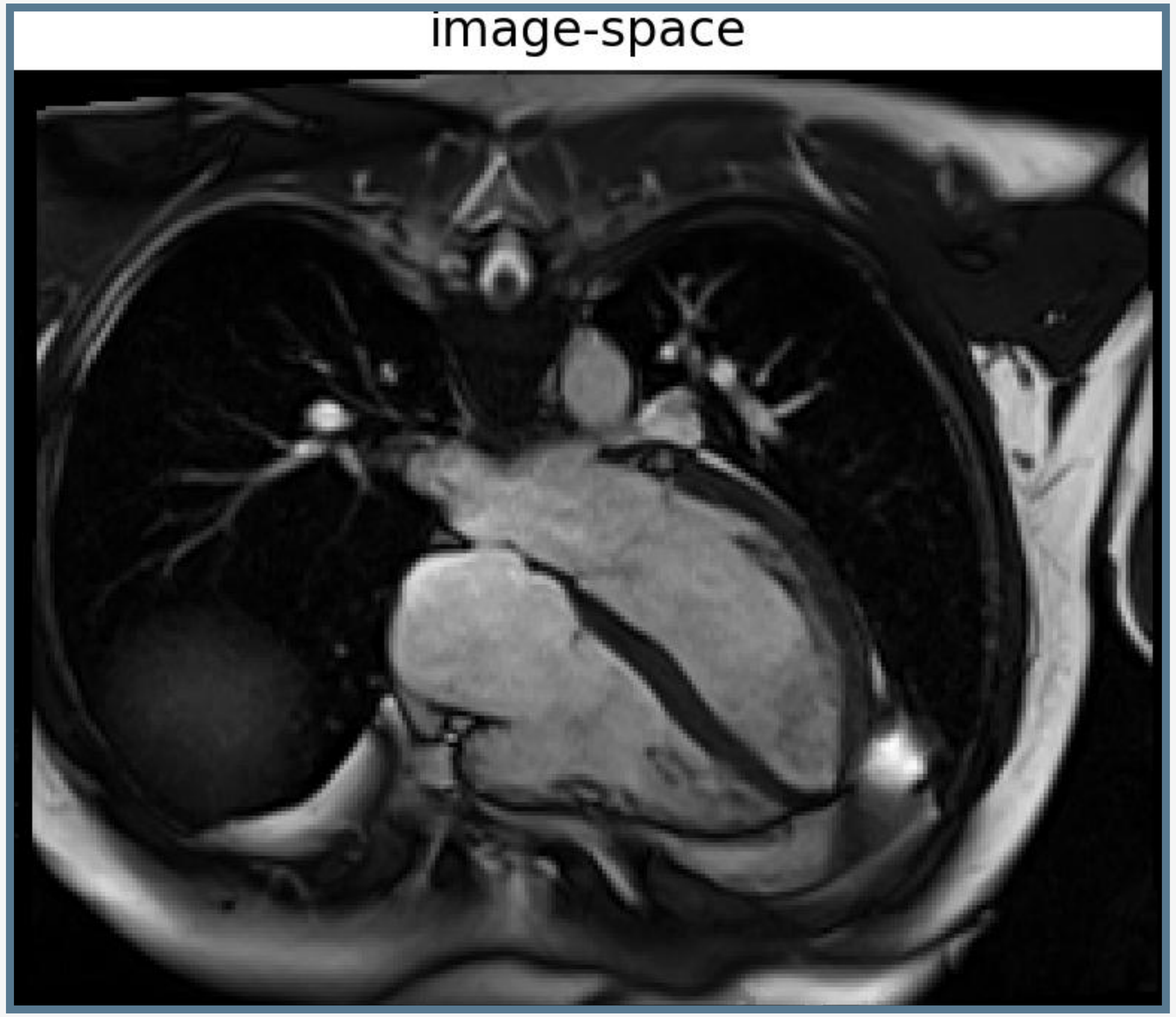
K-SPACE



2D inverse Fourier Transform



$$f(x,y) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_x, k_y) e^{i2\pi(k_x x + k_y y)} \, dk_x \, dk_y$$

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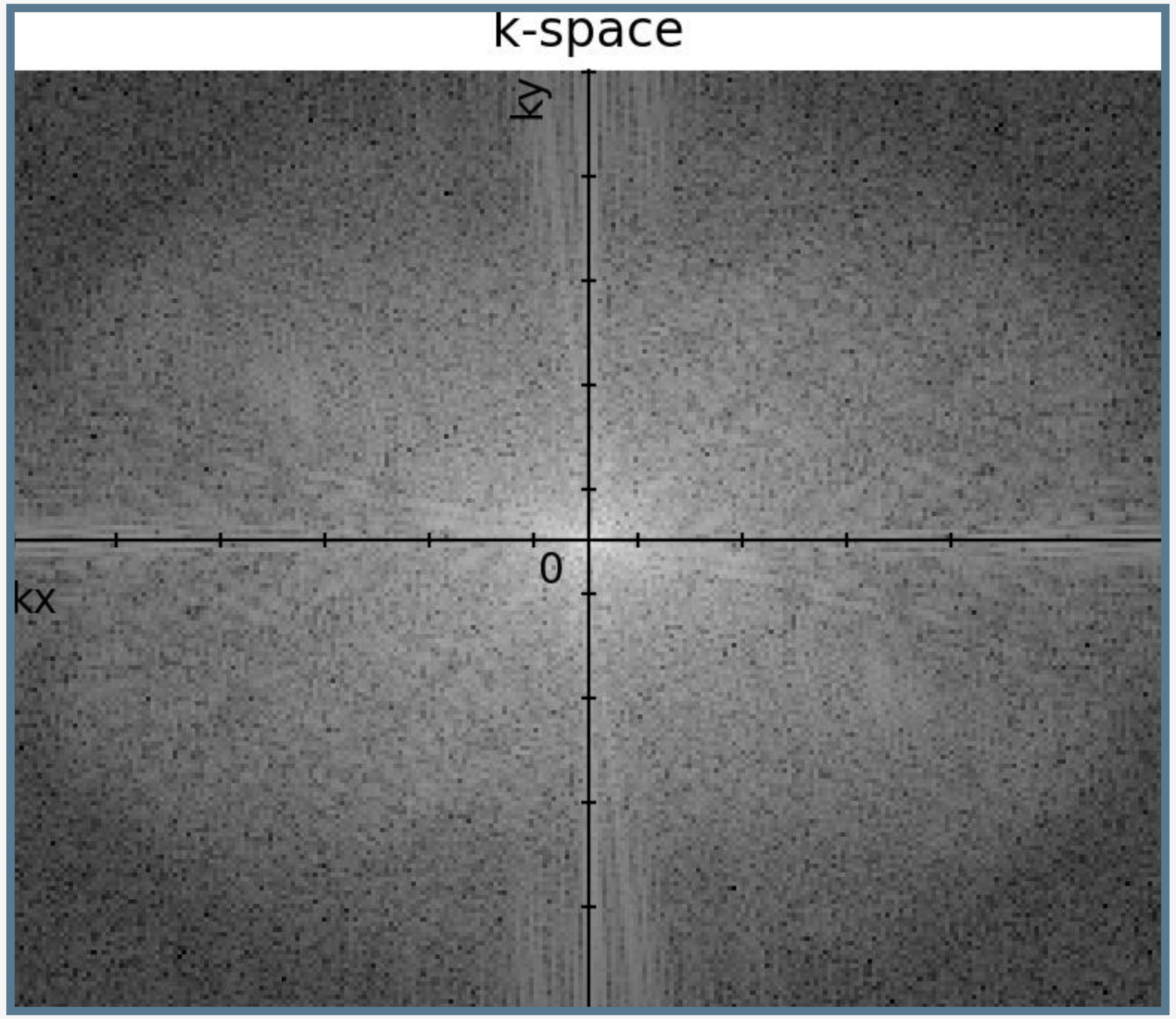
K-SPACE

k-space:

- **represents a large collection of many sinusoidal oscillations with weights given by the magnitude.**



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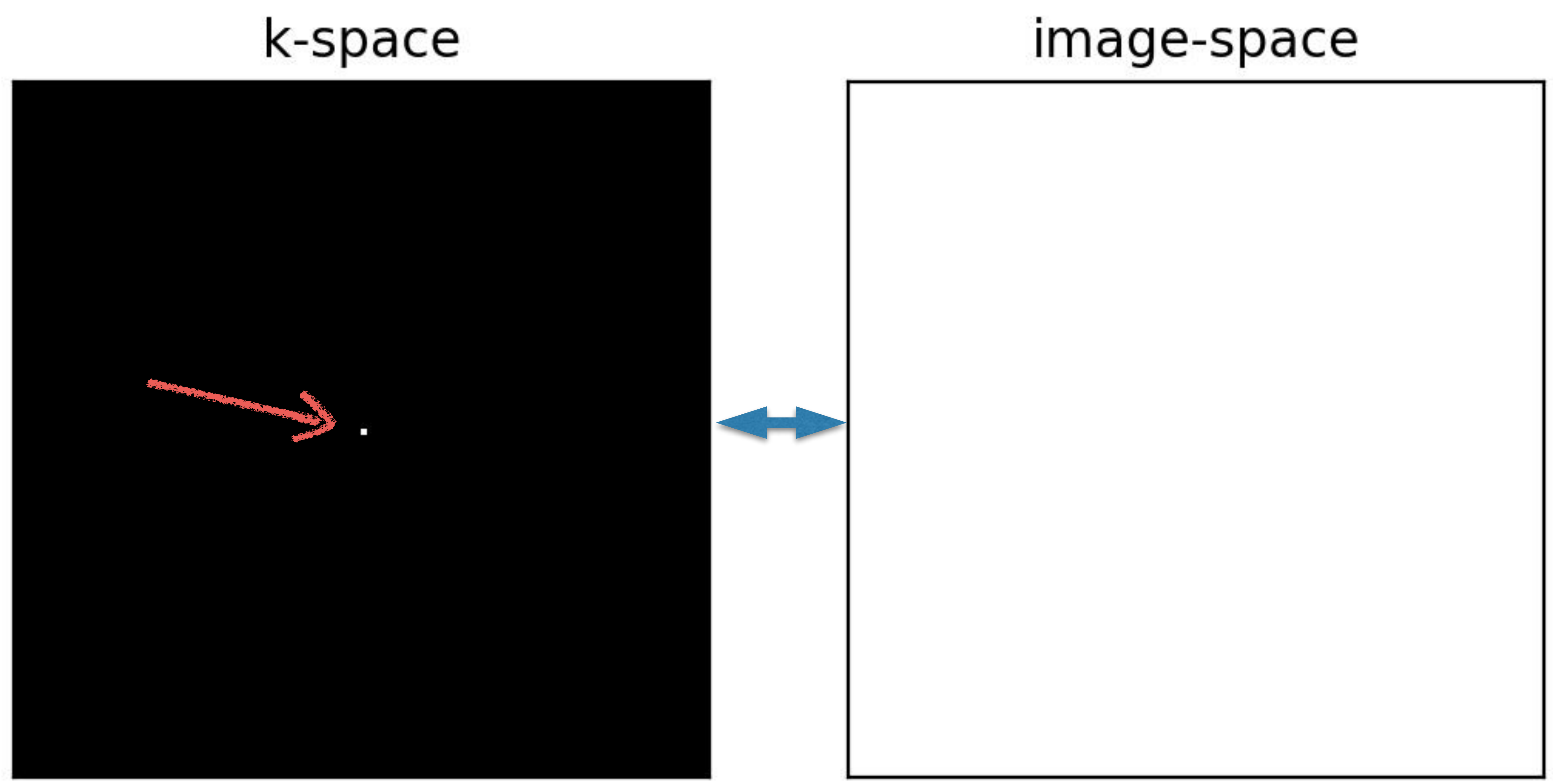

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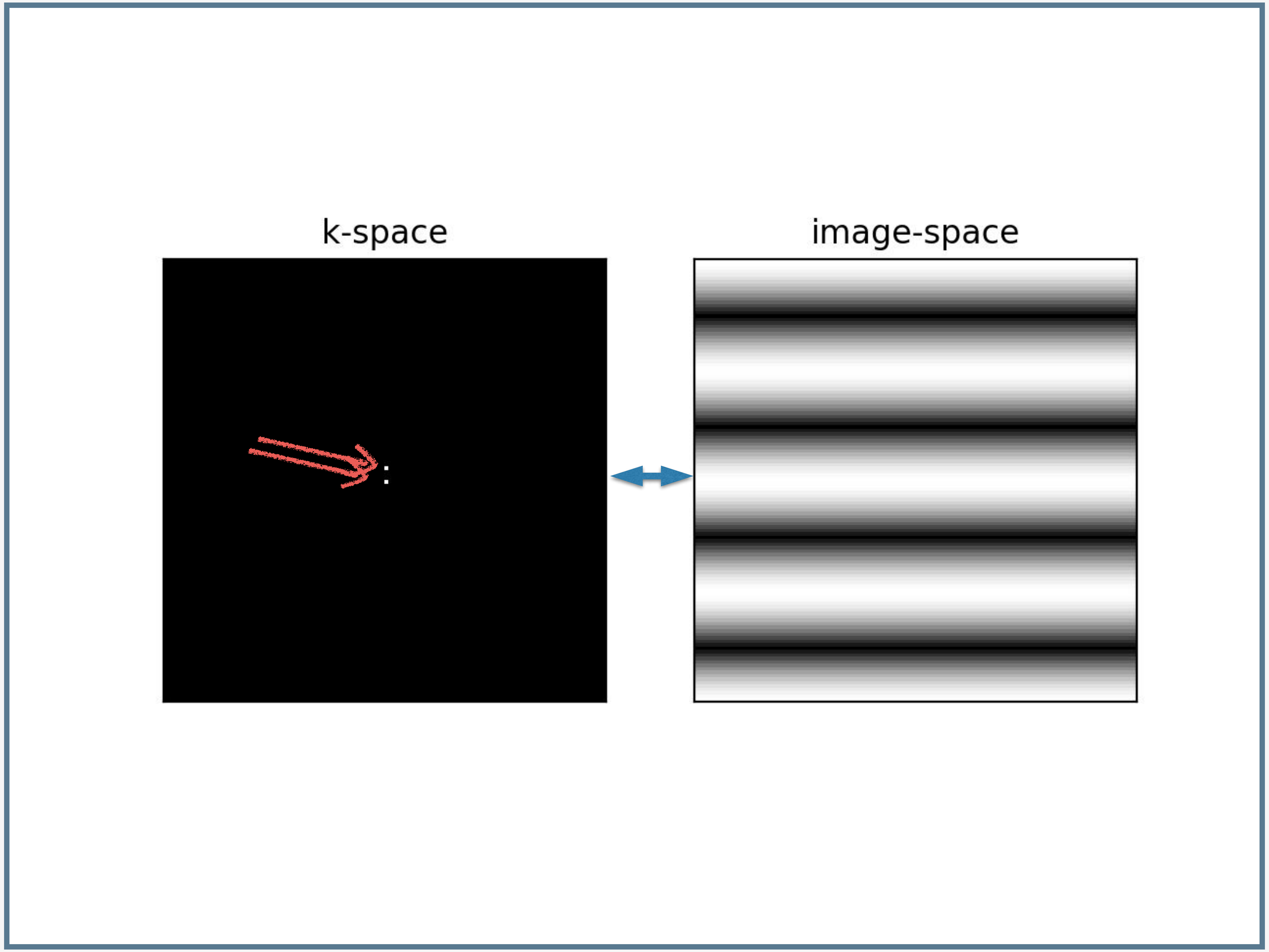

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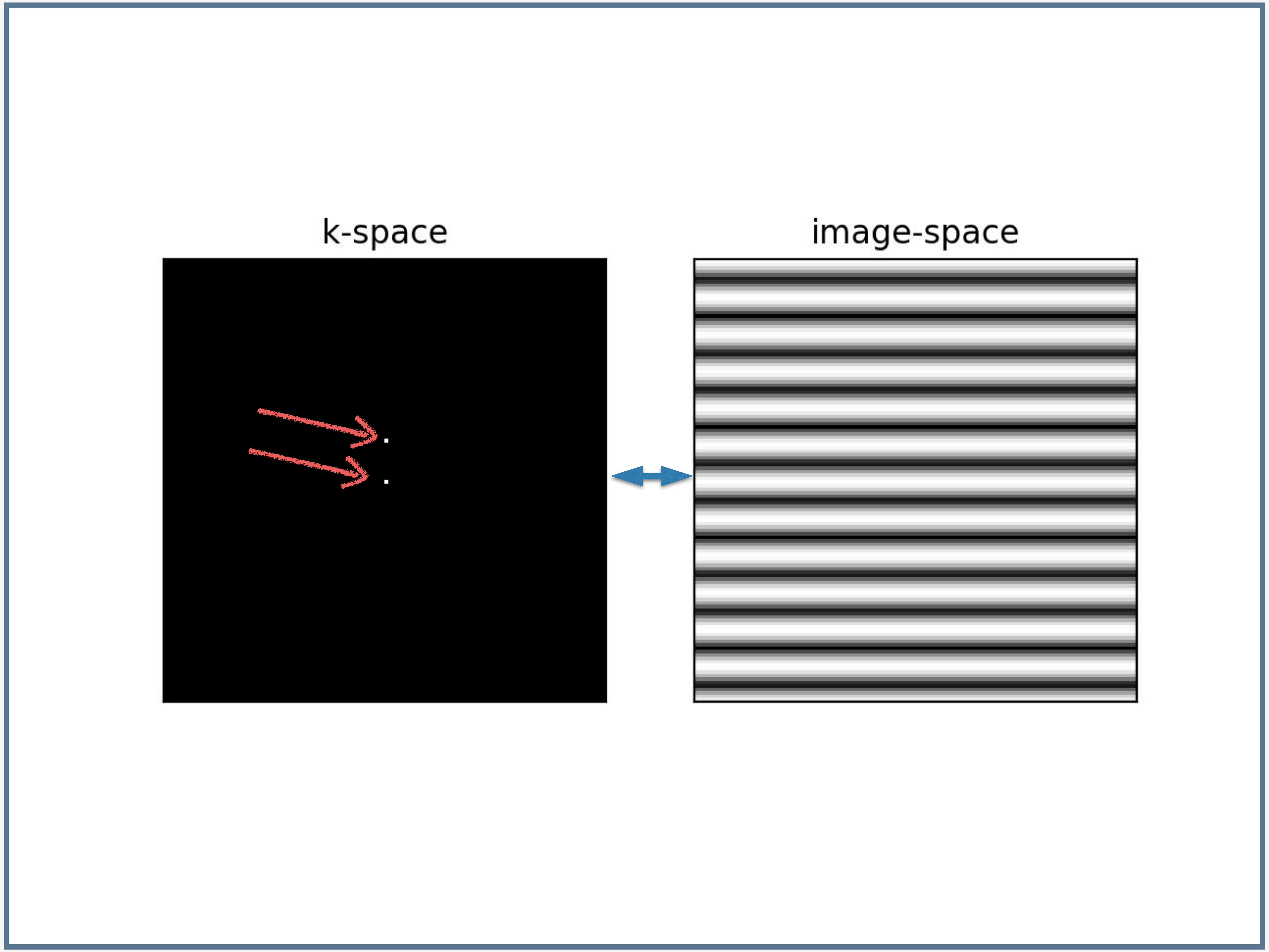

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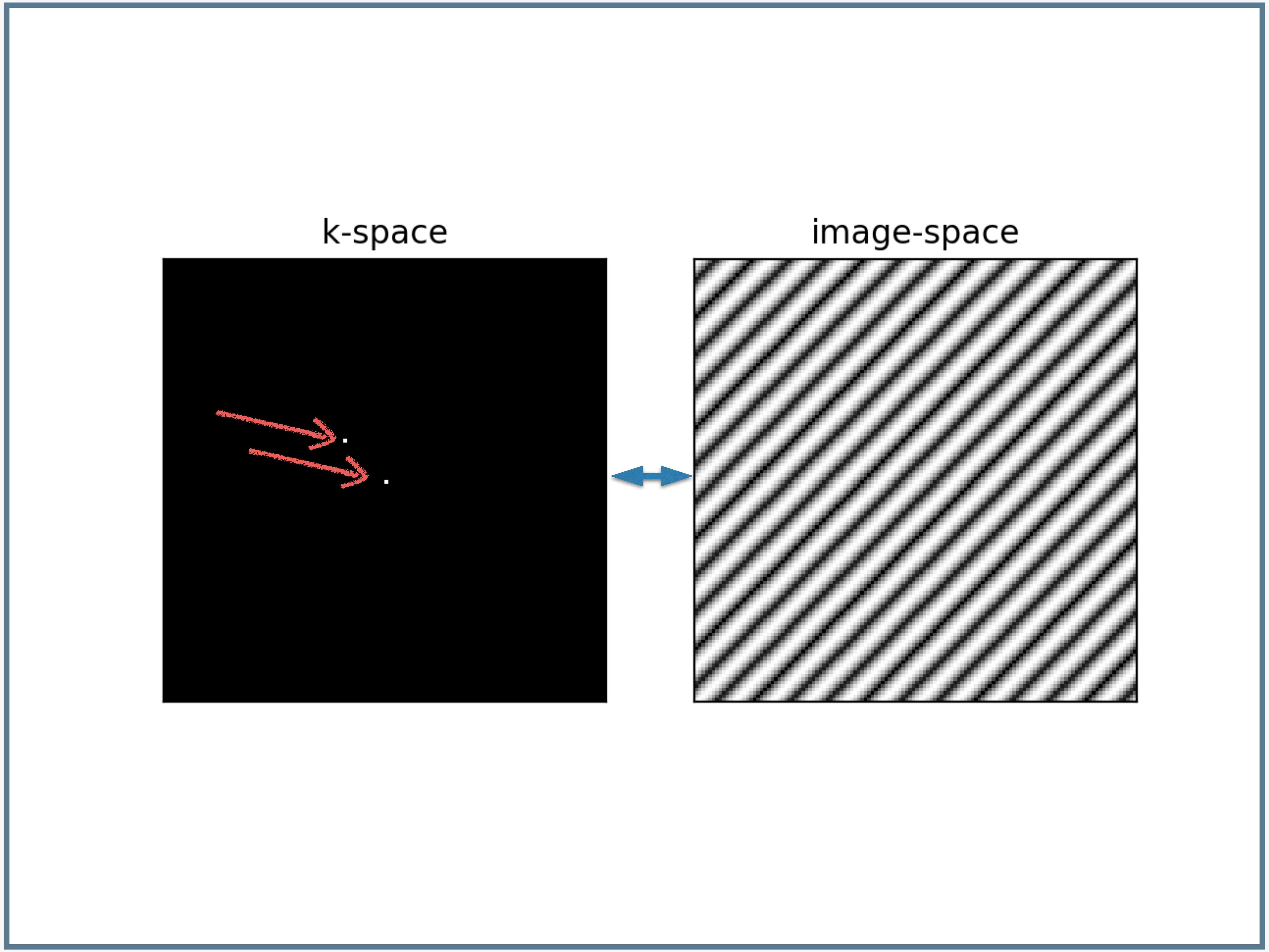

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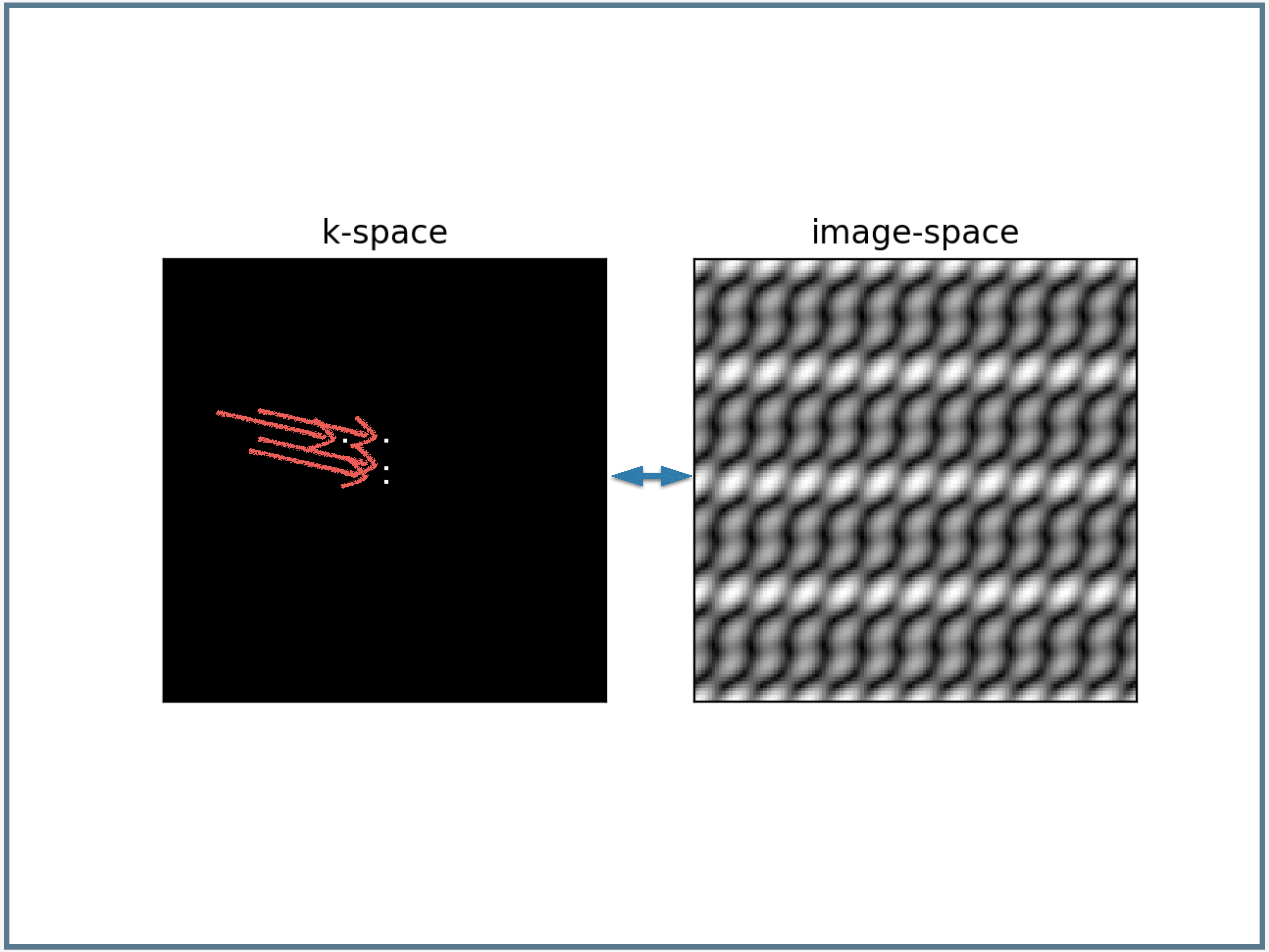

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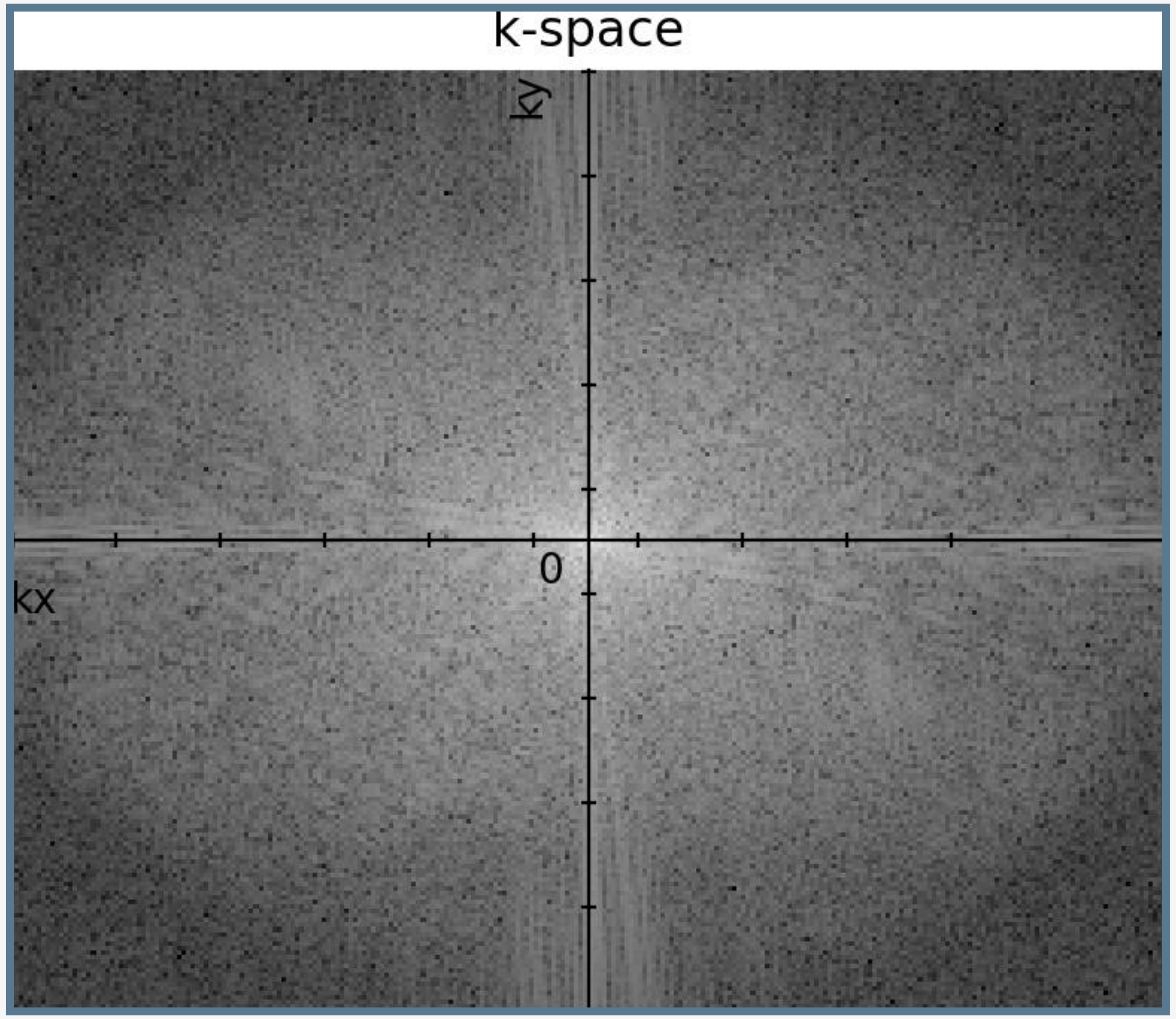
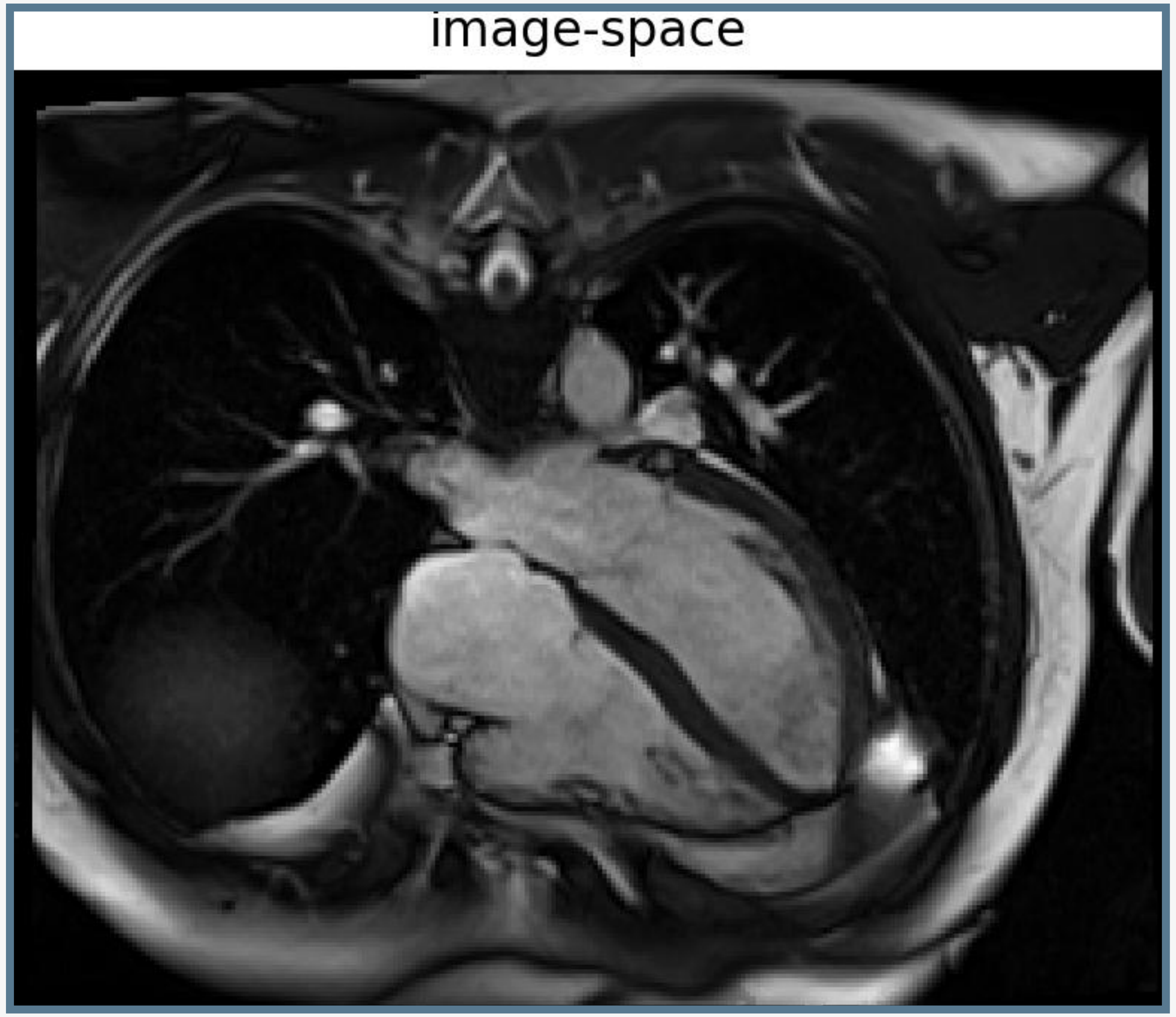


2D inverse Fourier Transform



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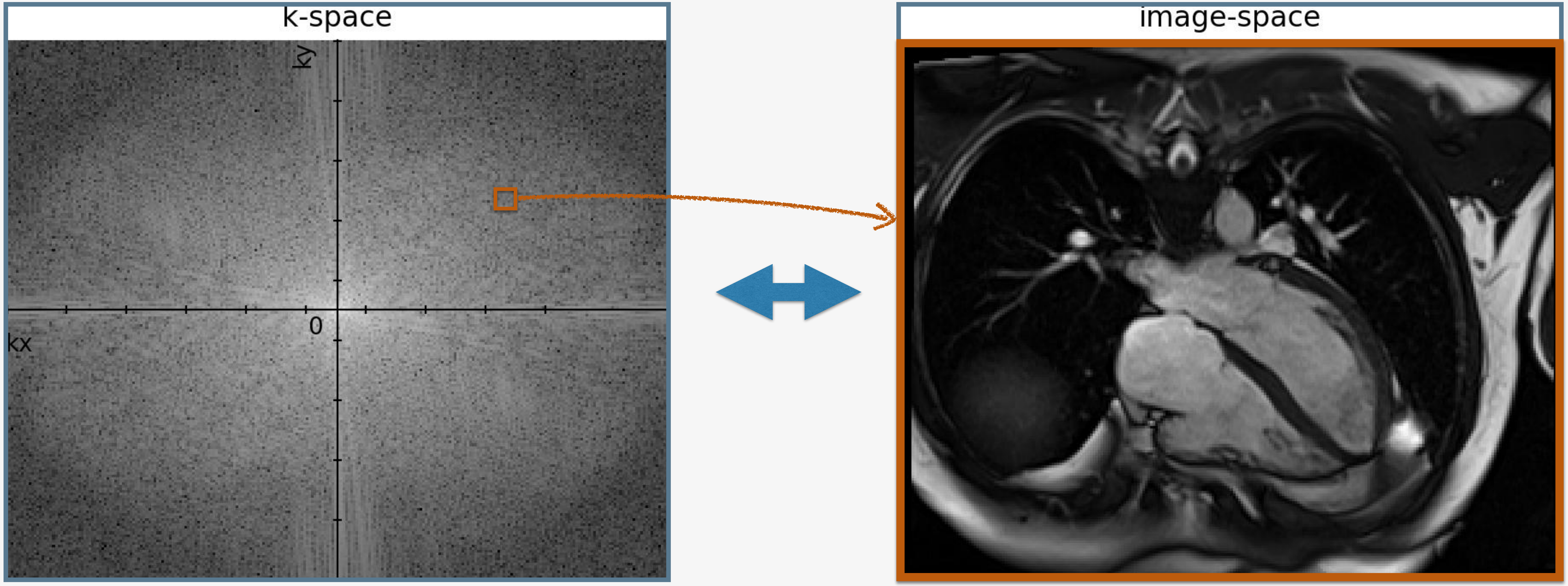

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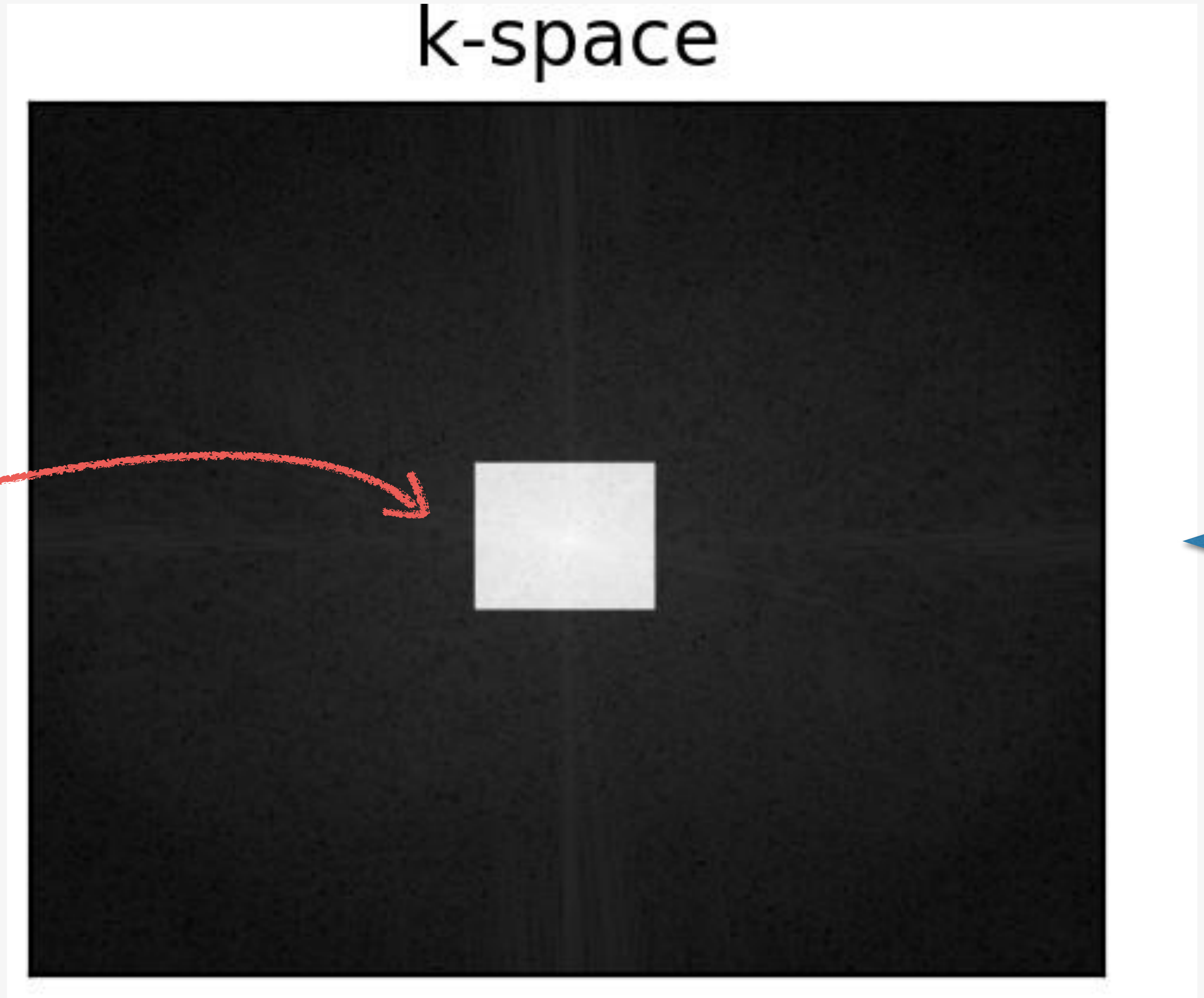
K-SPACE

central region of
k-space
(low frequencies)



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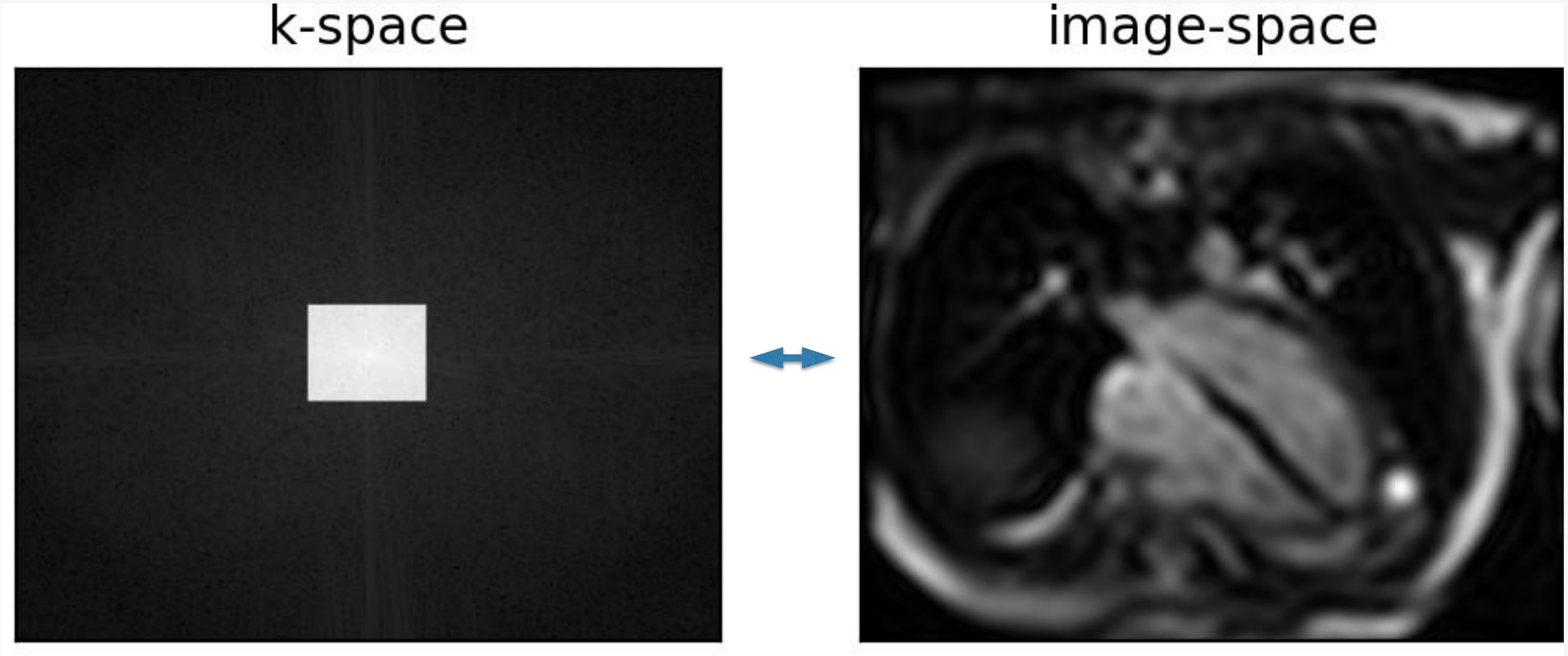

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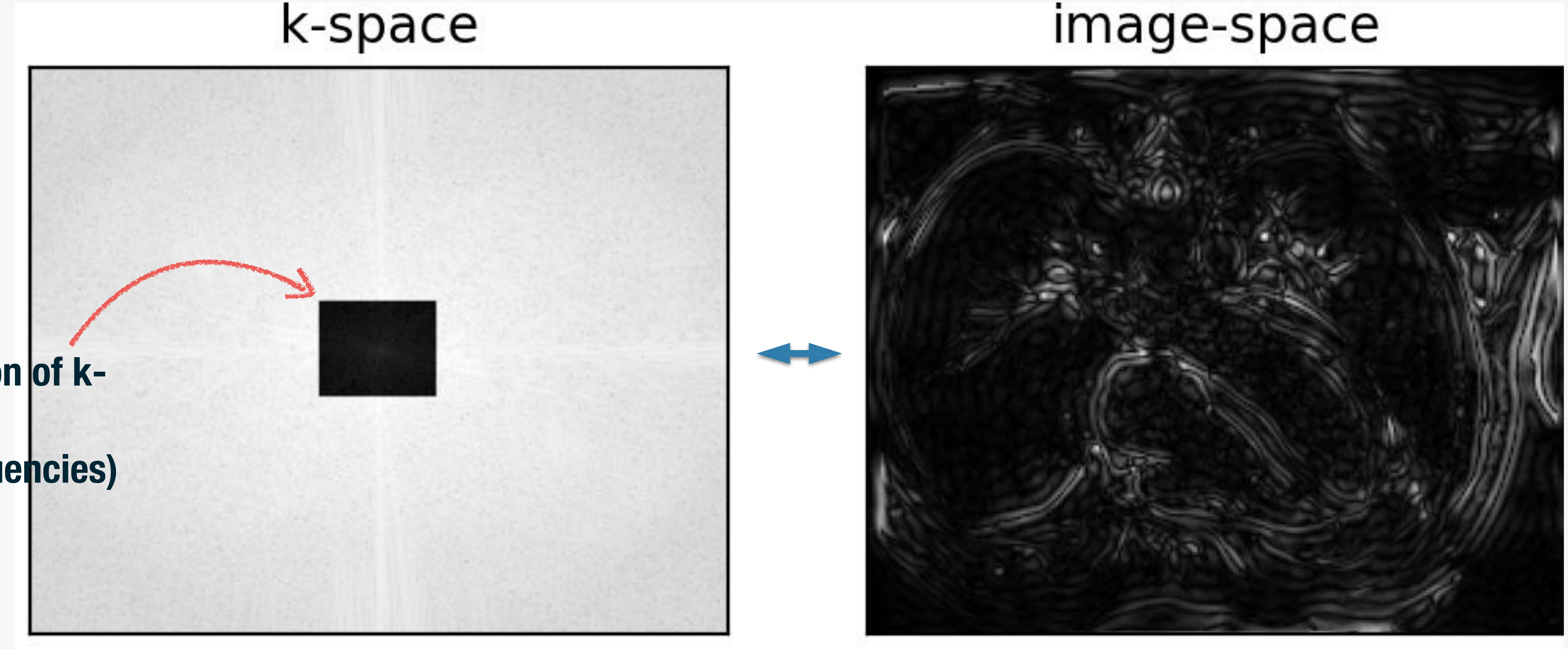

outer region of k-space
(high frequencies)

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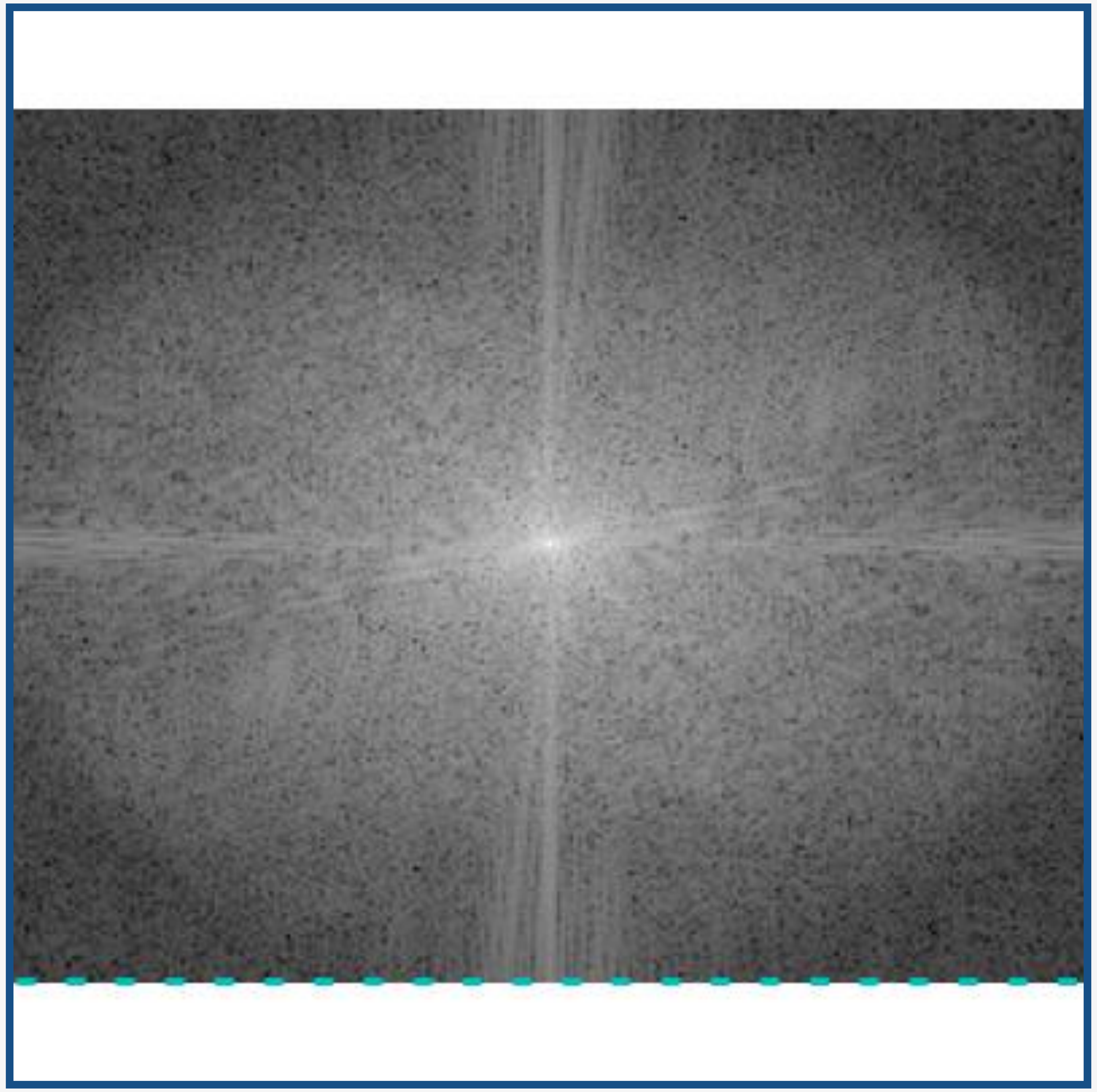
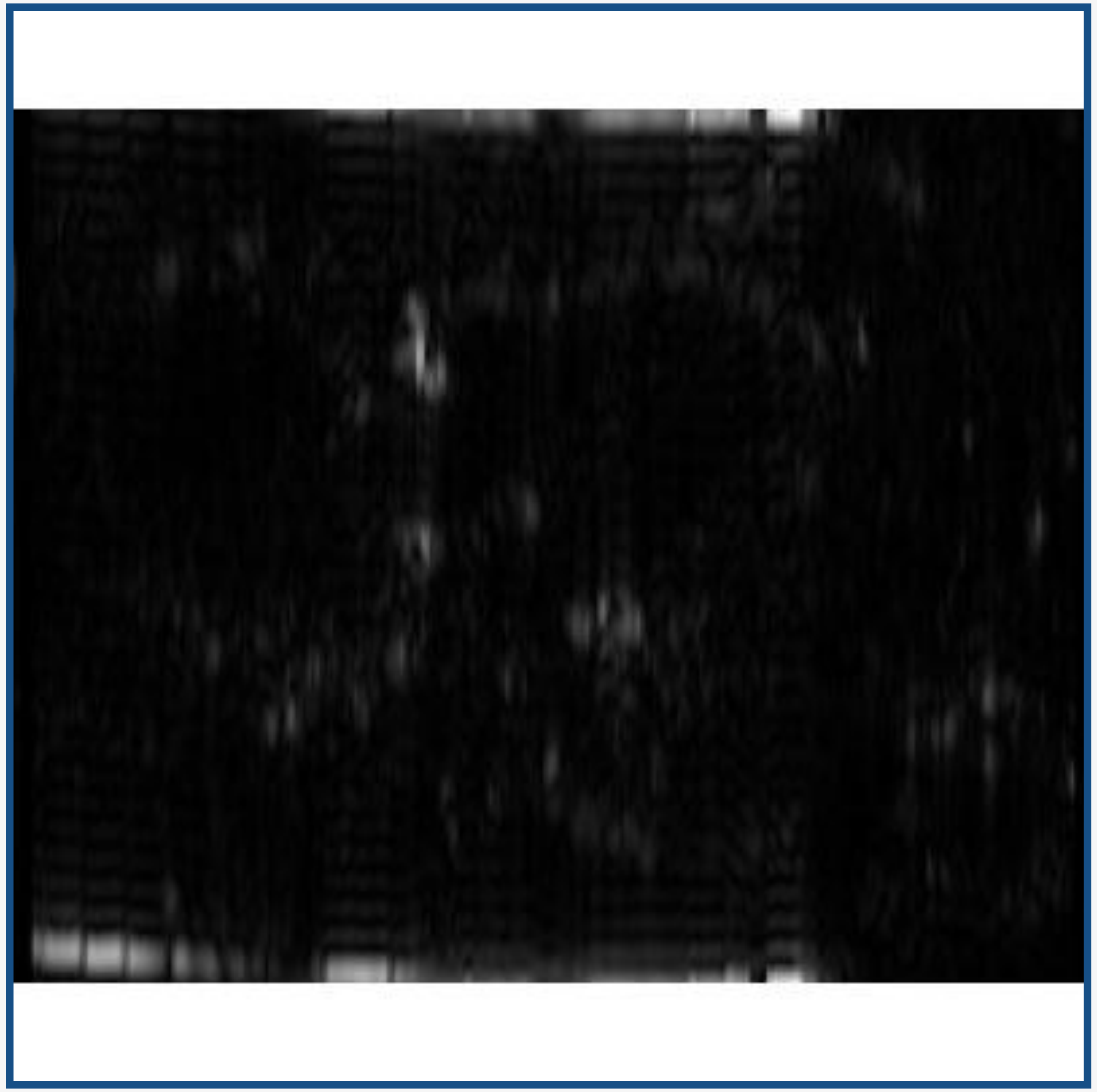
K-SPACE :: CARTESIAN SAMPLING



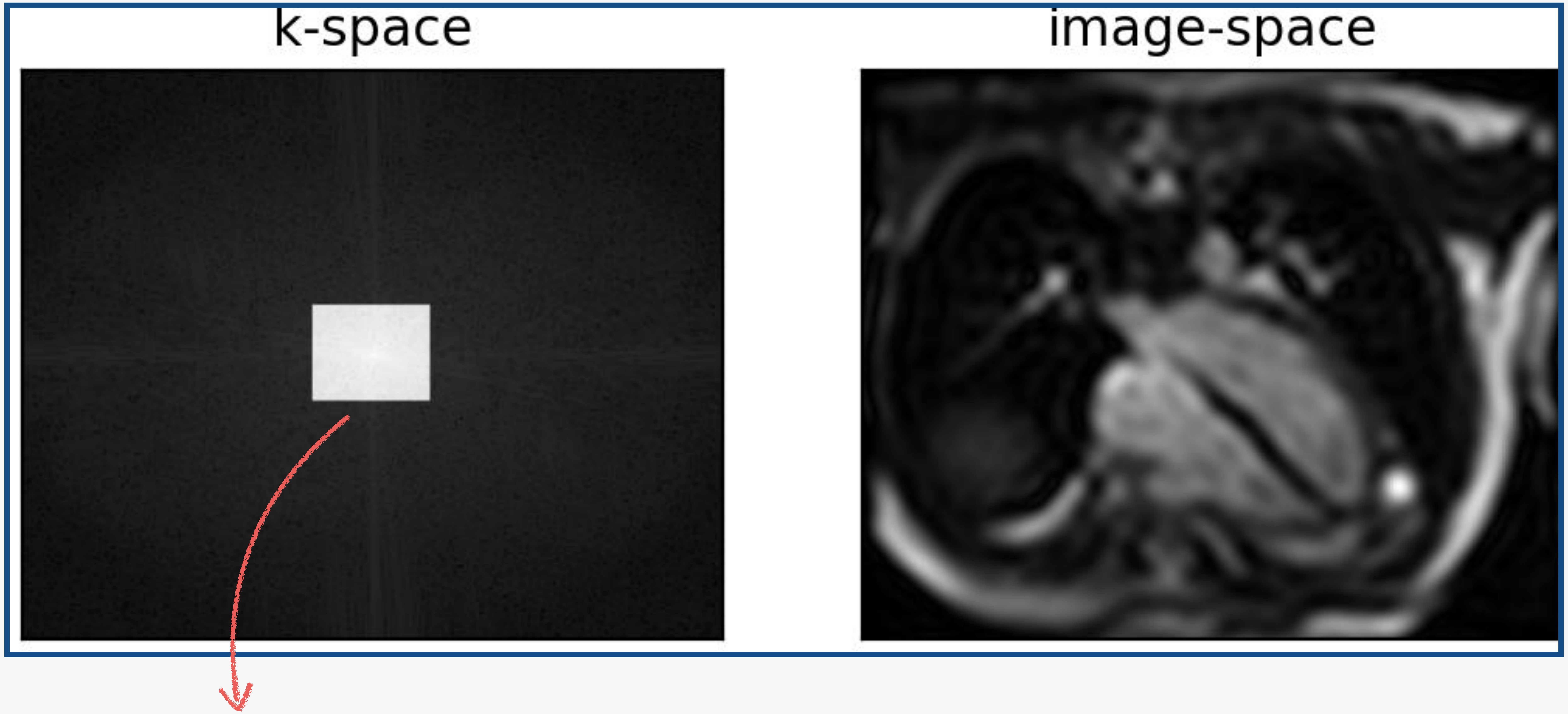
Phase-encode

Frequency-encode



p.ferreira@rbht.nhs.uk

K-SPACE



k-space region covered by mask: 2.8%
k-space signal covered by mask: 38.0%

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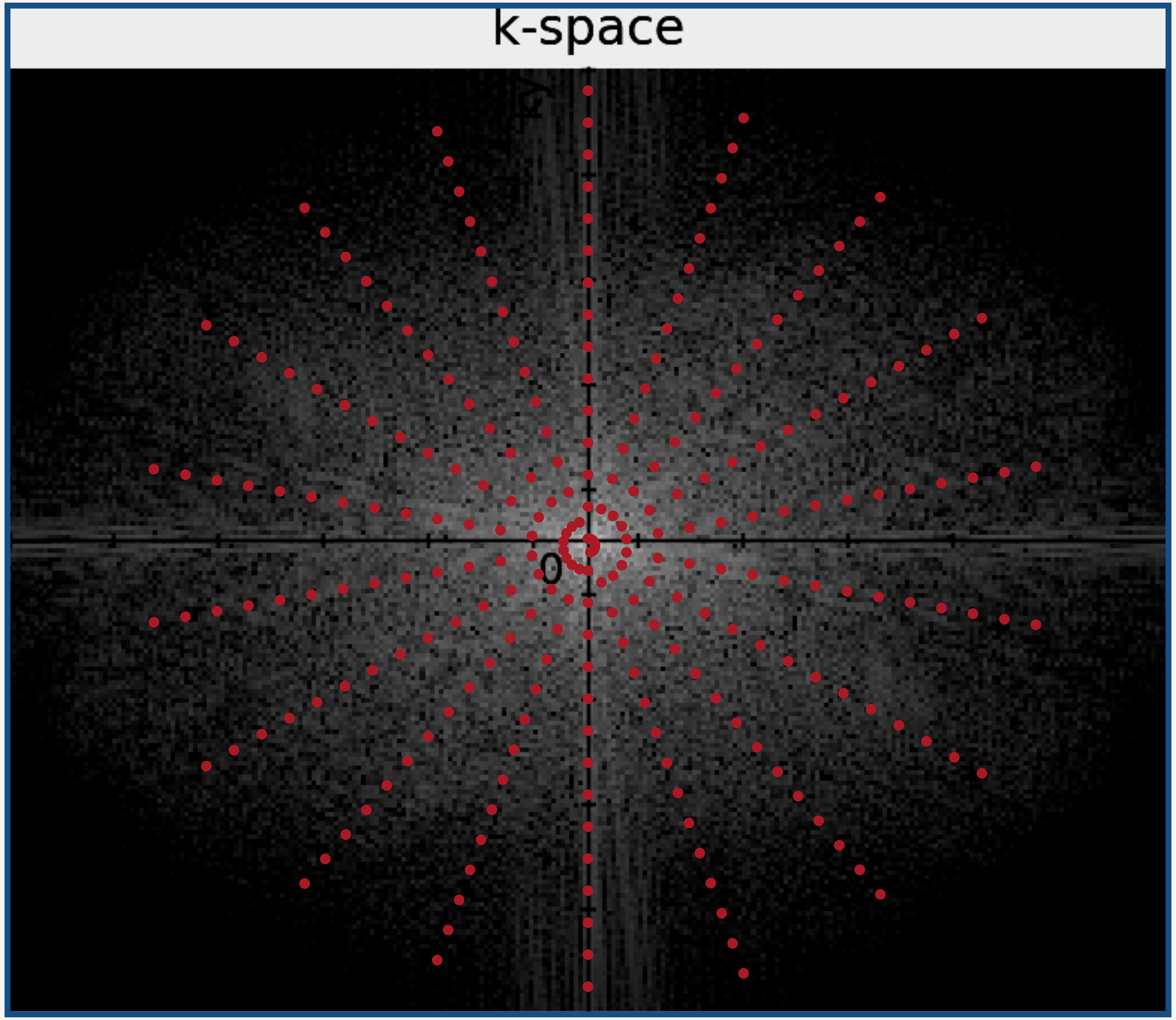
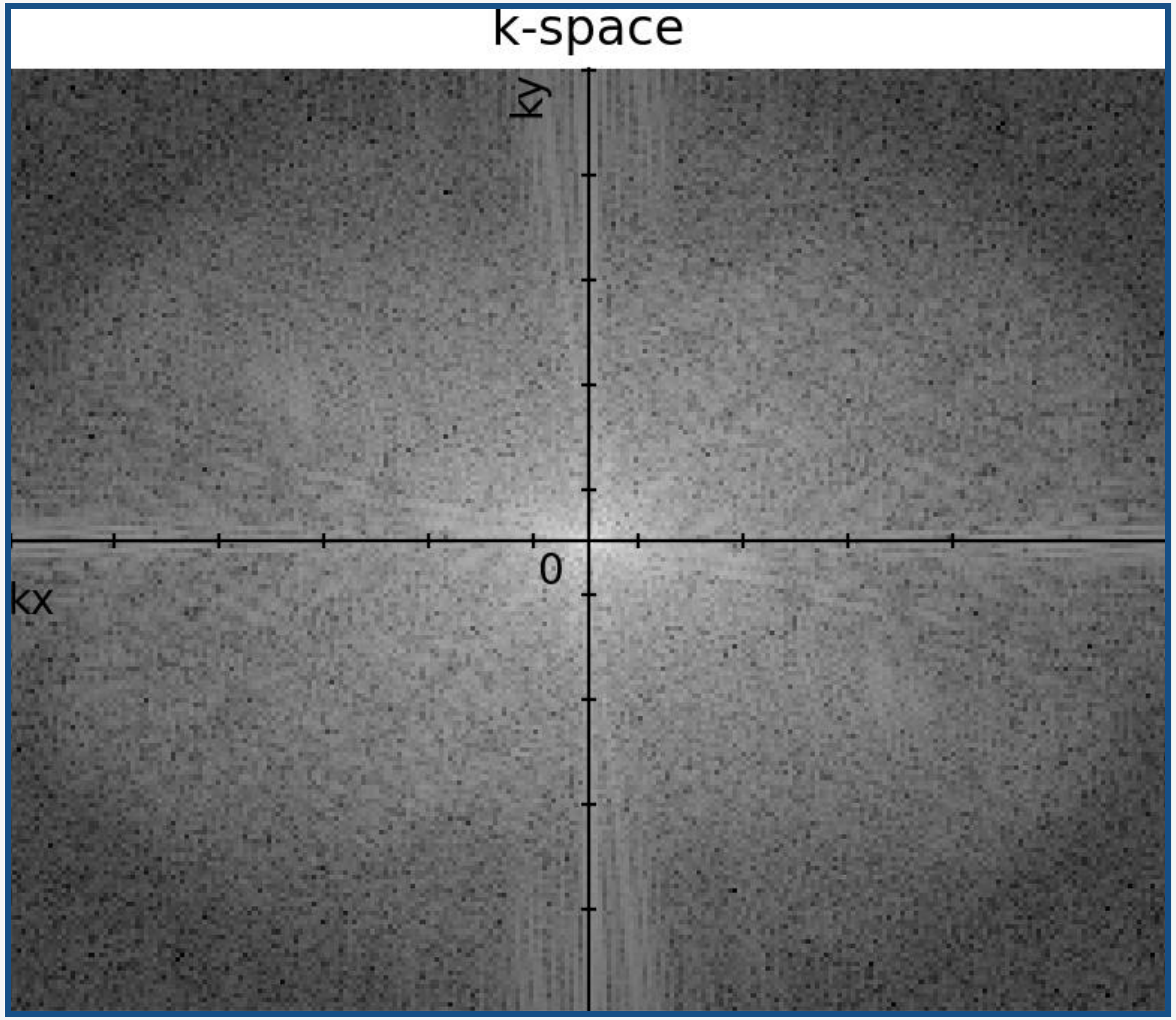
K-SPACE :: RADIAL ACQUISITION



p.ferreira@rbht.nhs.uk

K-SPACE :: SAMPLING ARTEFACTS



k-space sampling artefacts:

- finite sampling
- discrete sampling

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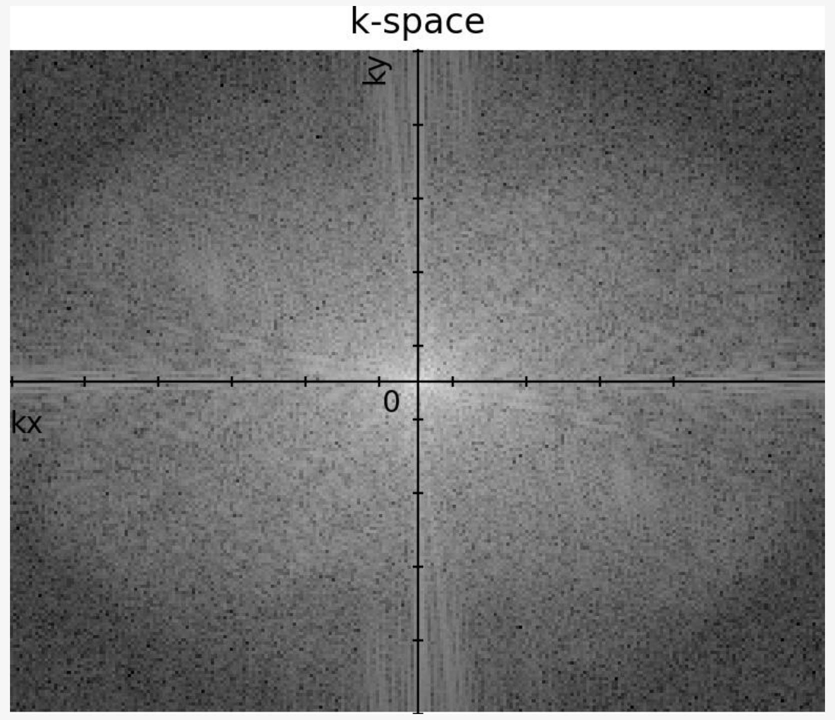
K-SPACE :: SAMPLING ARTEFACTS



...

ideal infinite continuous k-space

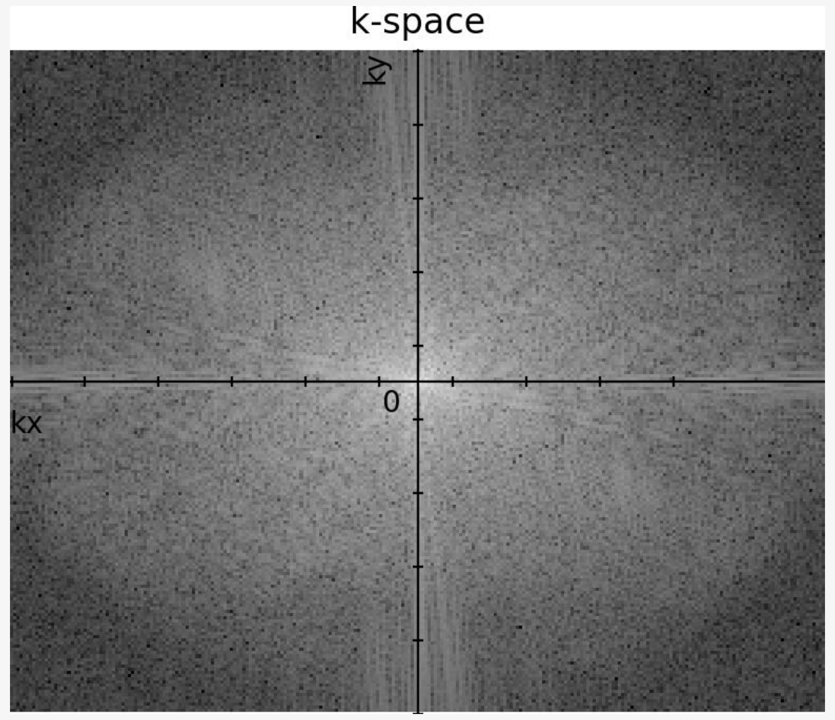
$\times$

$$H_{ws}(k) \equiv \mathrm{rect}\left(\frac{k + \frac{1}{2}\Delta k}{W}\right) \Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)$$

k-space filter

p.ferreira@rbht.nhs.uk

K-SPACE :: SAMPLING ARTEFACTS



$\times$

$$H_{ws}(k) \equiv \underbrace{\mathrm{rect}\left(\frac{k + \frac{1}{2}\Delta k}{W}\right)}_{\text{finite k-space}} \underbrace{\Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)}_{\text{discrete sampling}}$$

k-space filter

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FOURIER TRANSFORM MATHS

$$\mathscr{F}^{-1}(H(k) \times G(k)) = h(x) * g(x)$$

FT of the product of two functions is the convolution of the FT of each function

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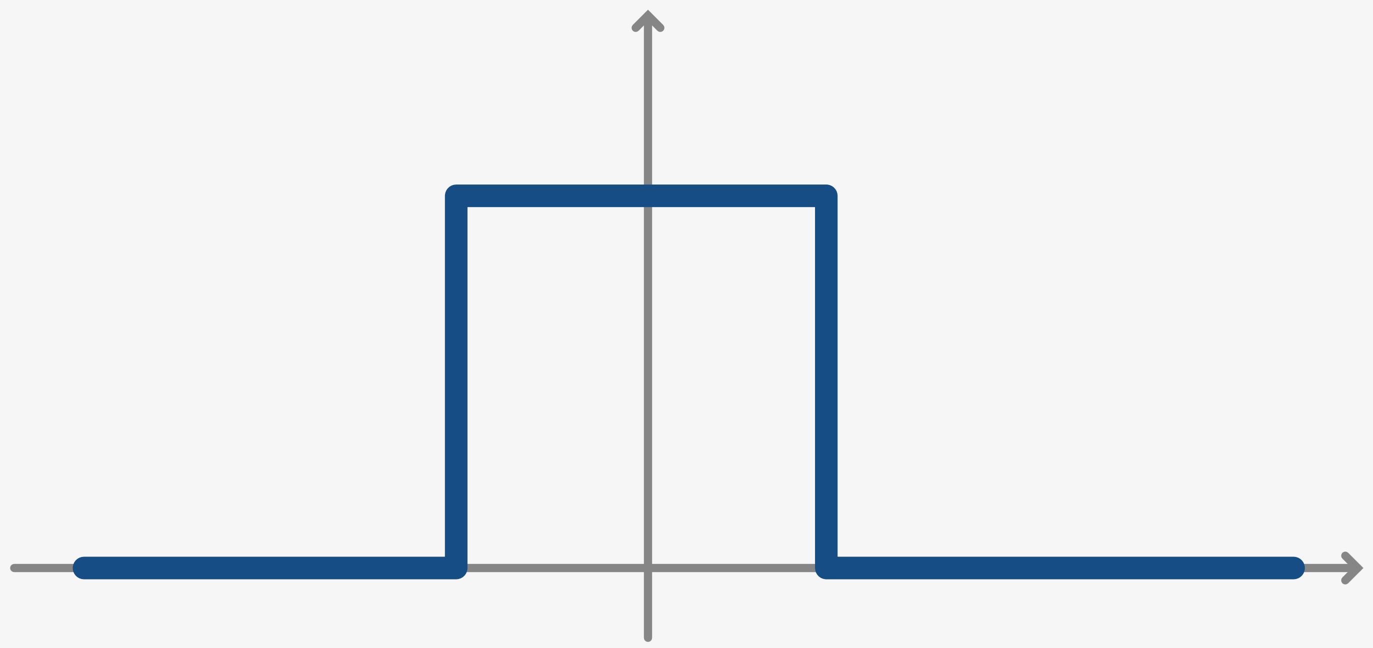
Fourier transform maths

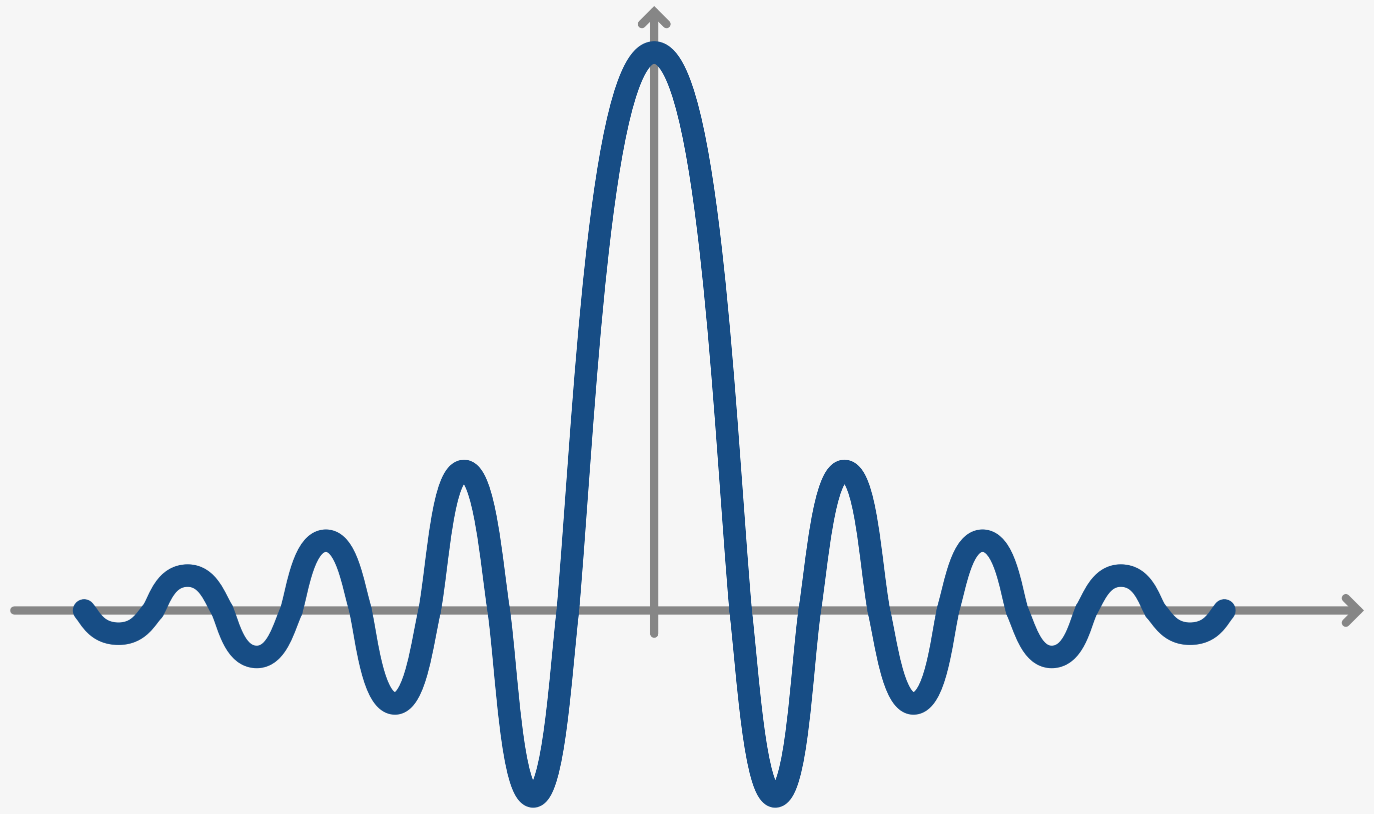
$$\mathscr{F}^{-1}(H(k) \times G(k)) = h(x) * g(x)$$

FT of the product of two functions is the convolution of the FT of each function

$$\text{rect}\left(\frac{x}{W}\right) <=\mathscr{F}=> W\frac{\sin(\pi W k)}{\pi W k}$$

Fourier transform pair: Rectangular function & sinc function





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Fourier transform maths

$$\mathscr{F}^{-1}(H(k) \times G(k)) = h(x) * g(x)$$

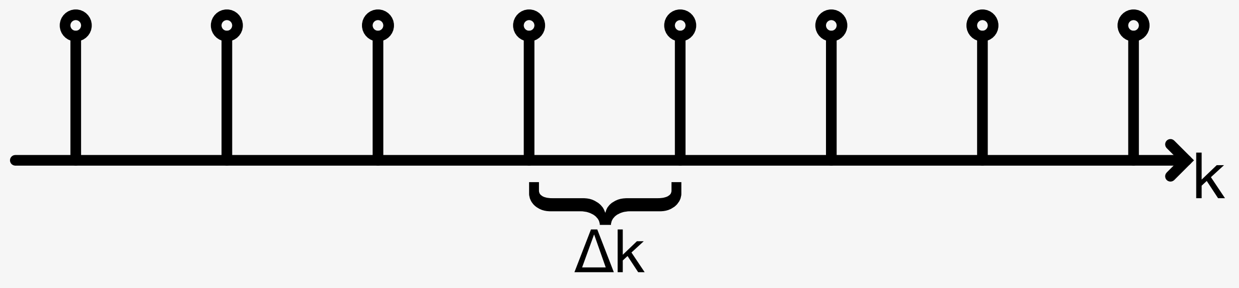
FT of the product of two functions is the convolution of the FT of each function

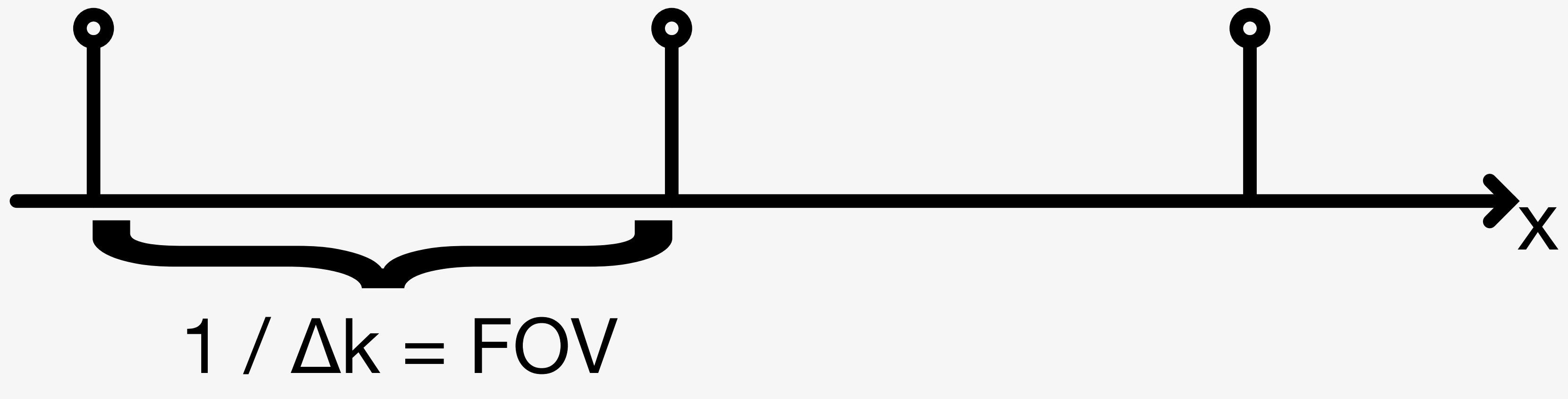
$$\mathrm{rect}\left(\frac{x}{W}\right) <=\mathscr{F}=> W\frac{\sin(\pi W k)}{\pi W k}$$

Fourier transform pair: Rectangular function & sinc function

$$\mathrm{comb}\left(\Delta k\right) <=\mathscr{F}=> \mathrm{comb}\left(\frac{1}{\Delta k}\right)$$

Fourier transform pair: comb function





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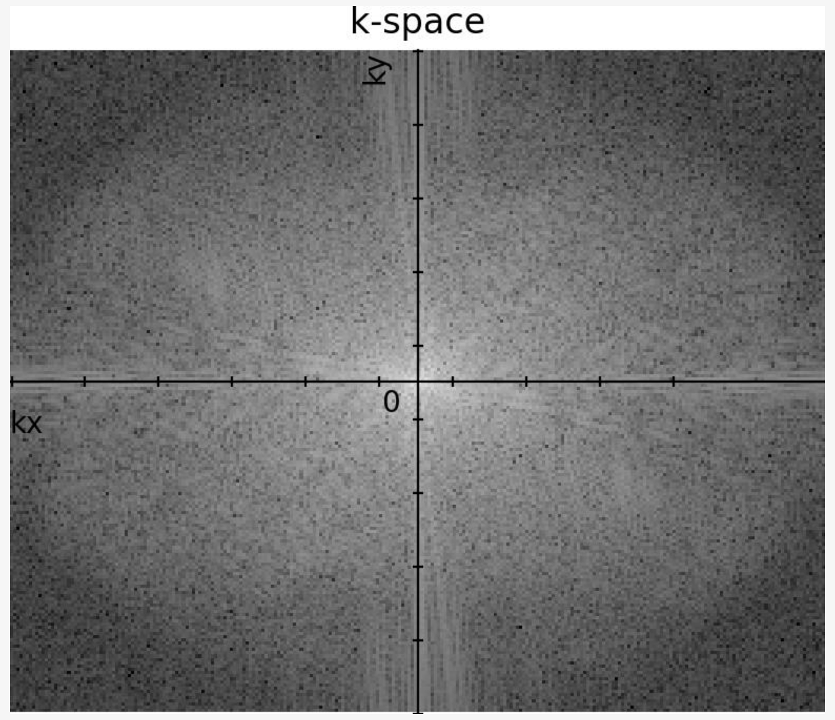
K-SPACE SAMPLING ARTEFACTS

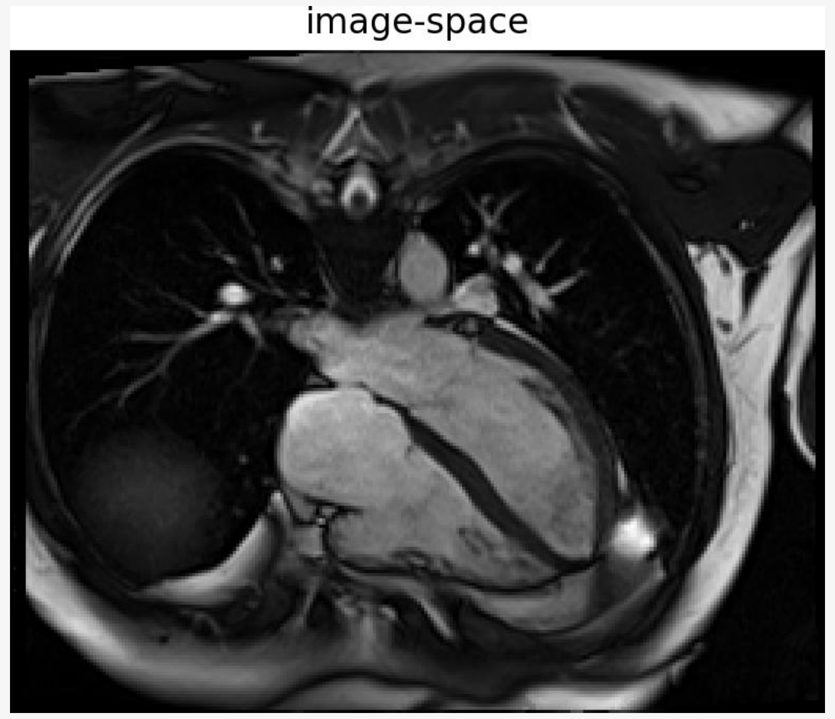


$\times$

$$H_{ws}(k) \equiv \underbrace{\mathrm{rect}\left(\frac{k+\frac{1}{2}\Delta k}{W}\right)}_{\textbf{finite k-space}} \underbrace{\Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)}_{\textbf{discrete sampling}}$$

k-space filter

$\mathcal{F}^{-1}$ $\qquad$ $\mathcal{F}^{-1}$



$*$ $\qquad h_{ws}(x)$

point spread function

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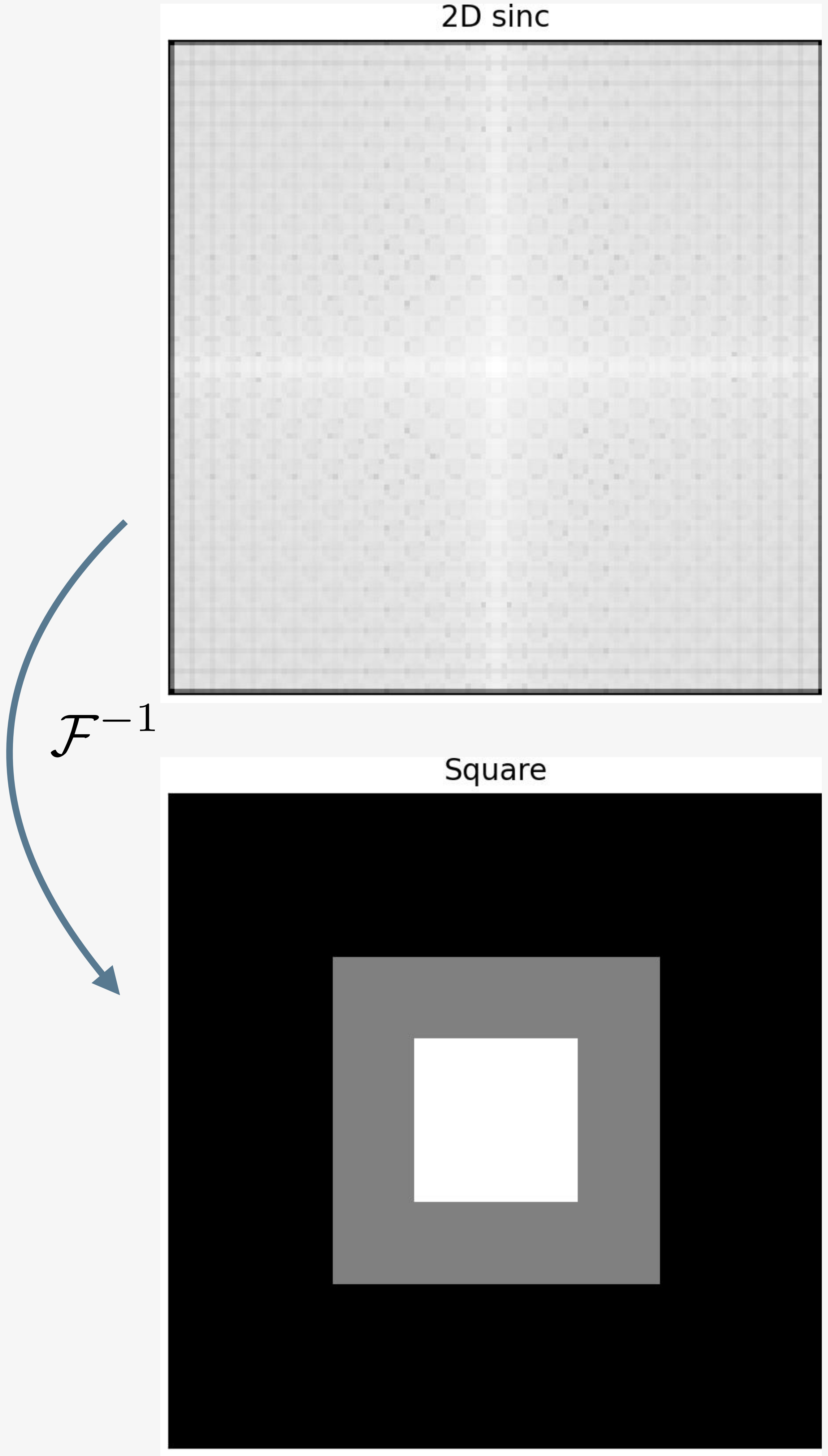
K-SPACE :: DATA TRUNCATION



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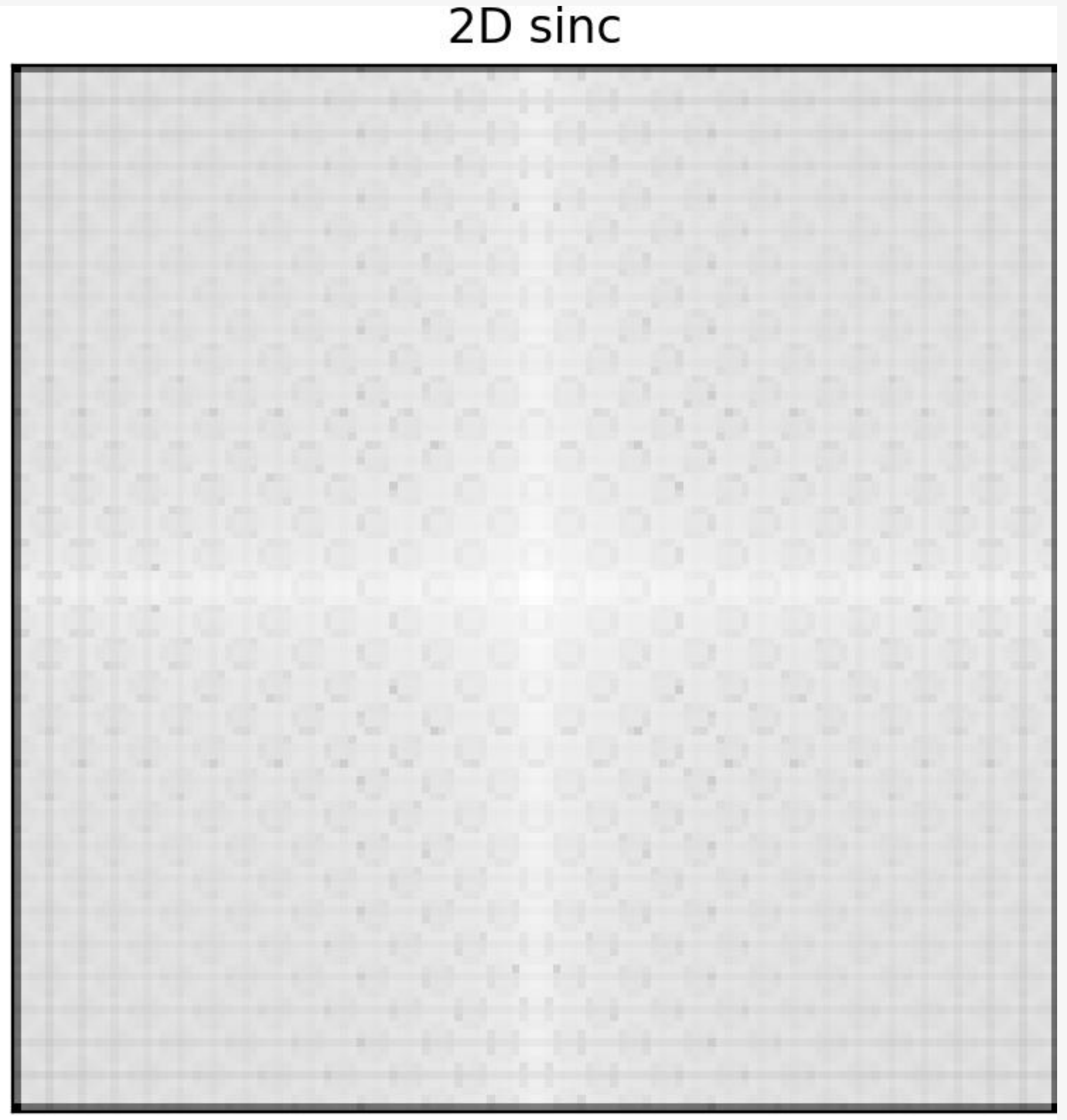
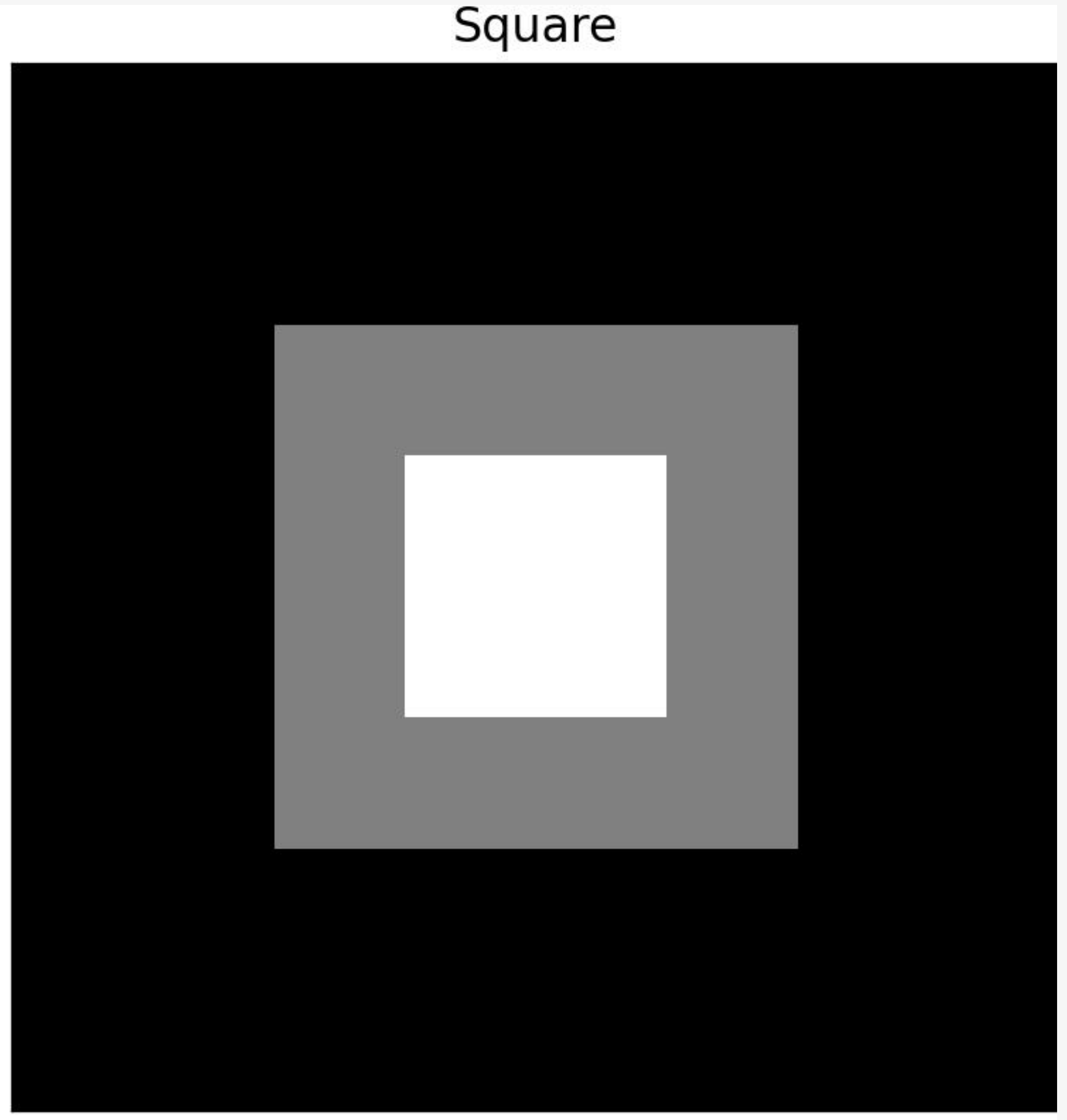


$\mathcal{F}^{-1}$



$$\times \operatorname{rect}\left(\frac{k+\frac{1}{2}\Delta k}{W}\right)$$

$$* \; W\operatorname{sinc}(\pi W x) \; =$$

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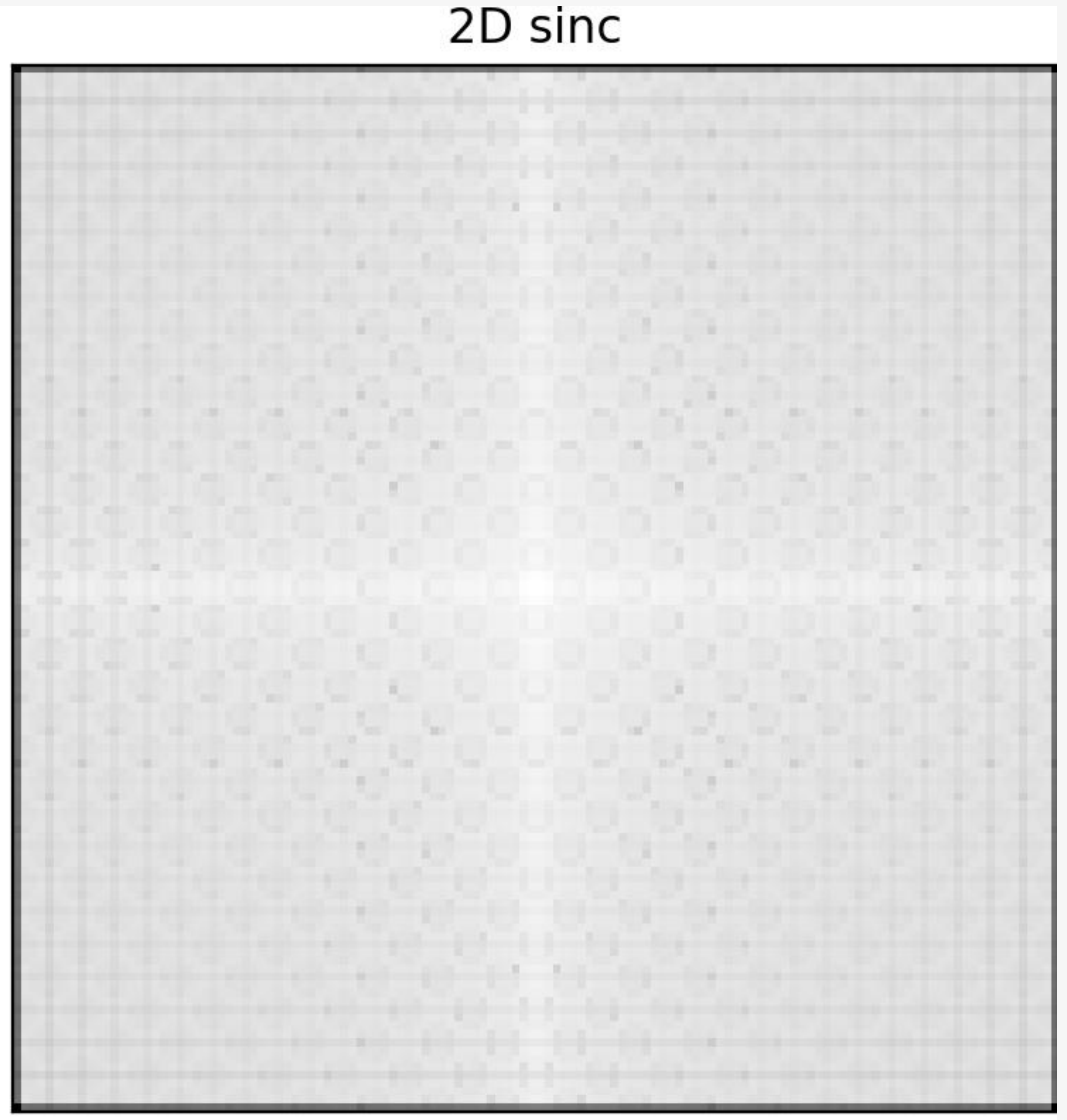
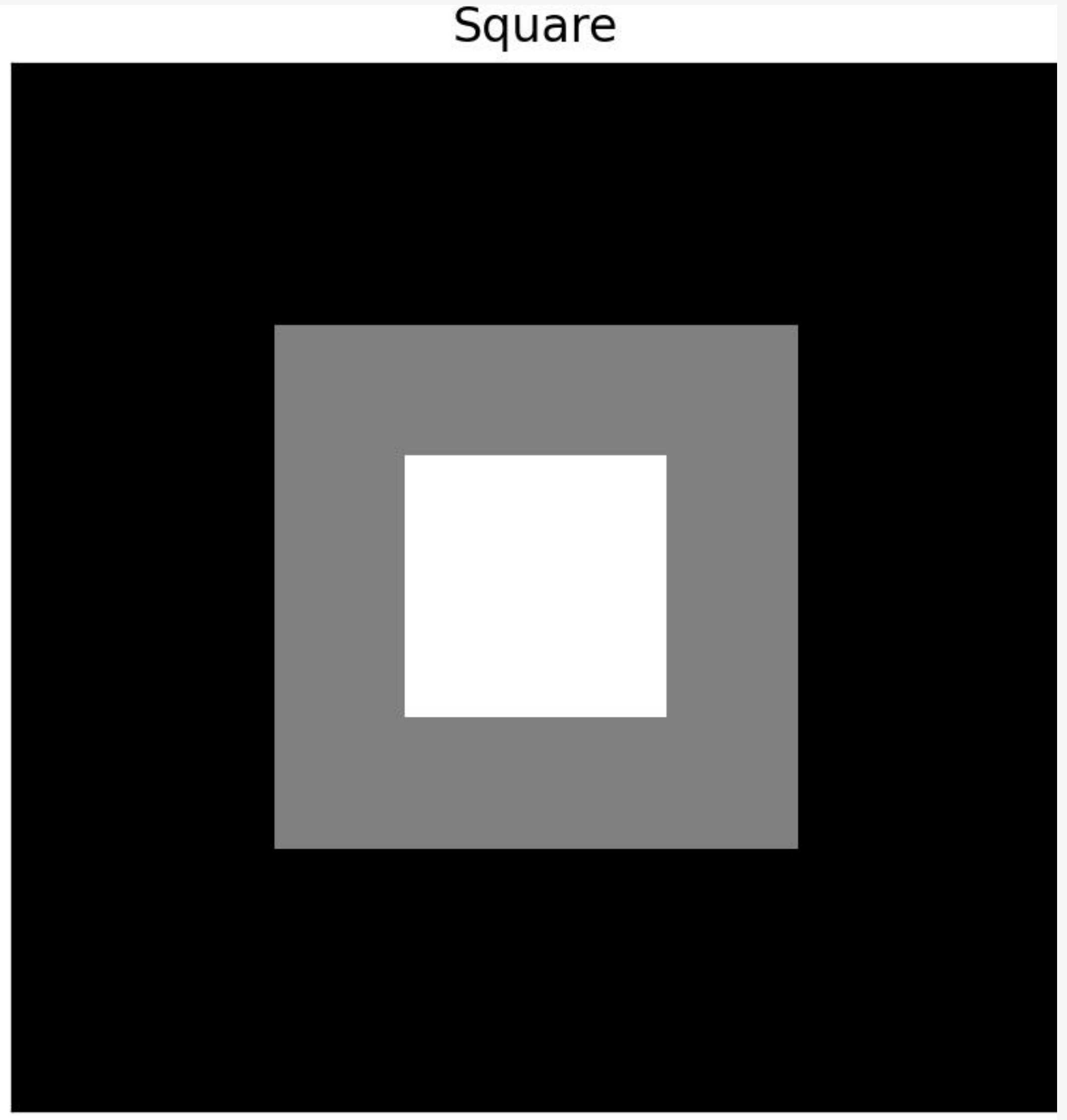
K-SPACE :: DATA TRUNCATION



$$\times \,\mathrm{rect}\left(\frac{k + \frac{1}{2}\Delta k}{W}\right)$$

$\mathcal{F}^{-1}$



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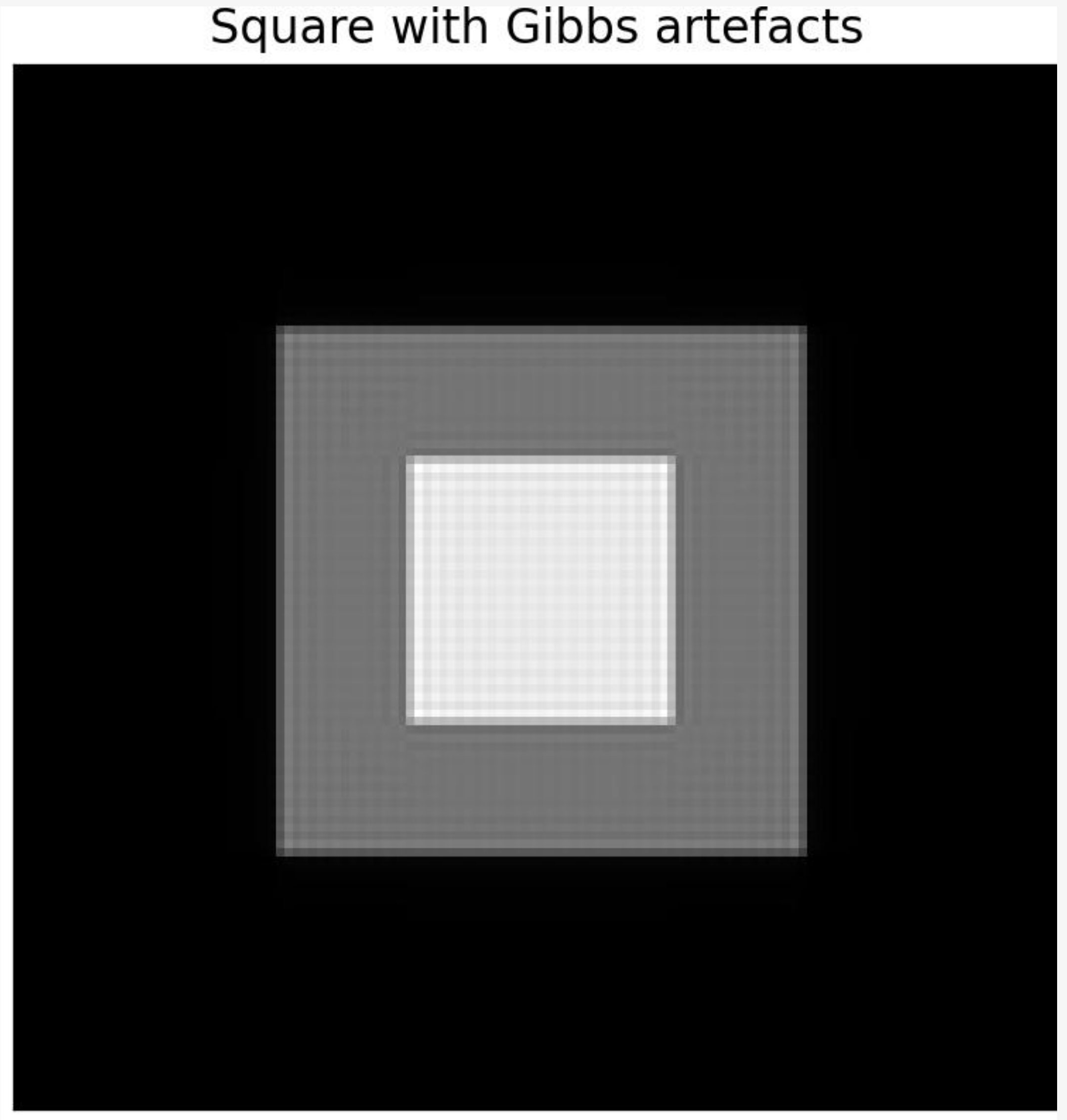

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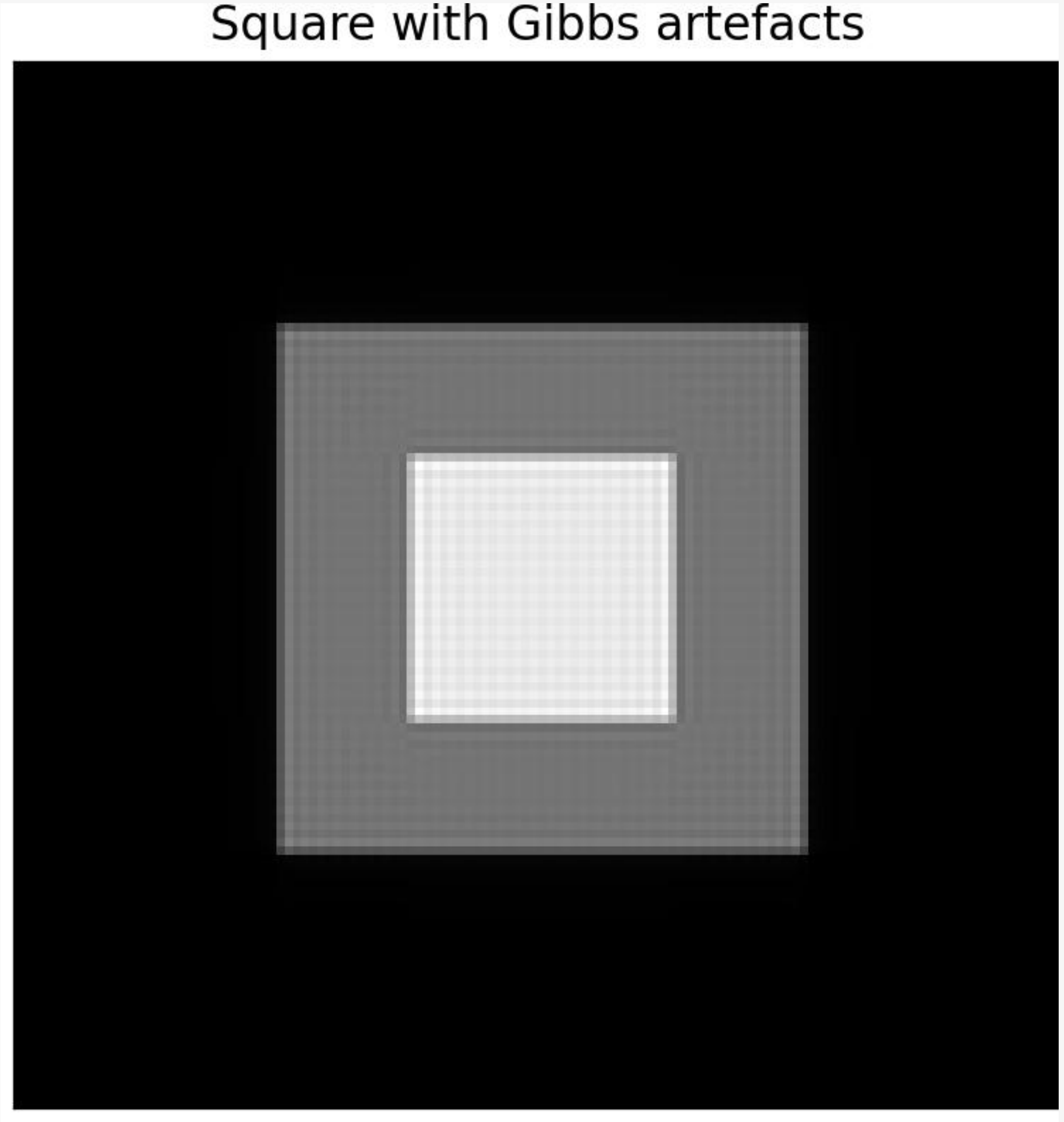

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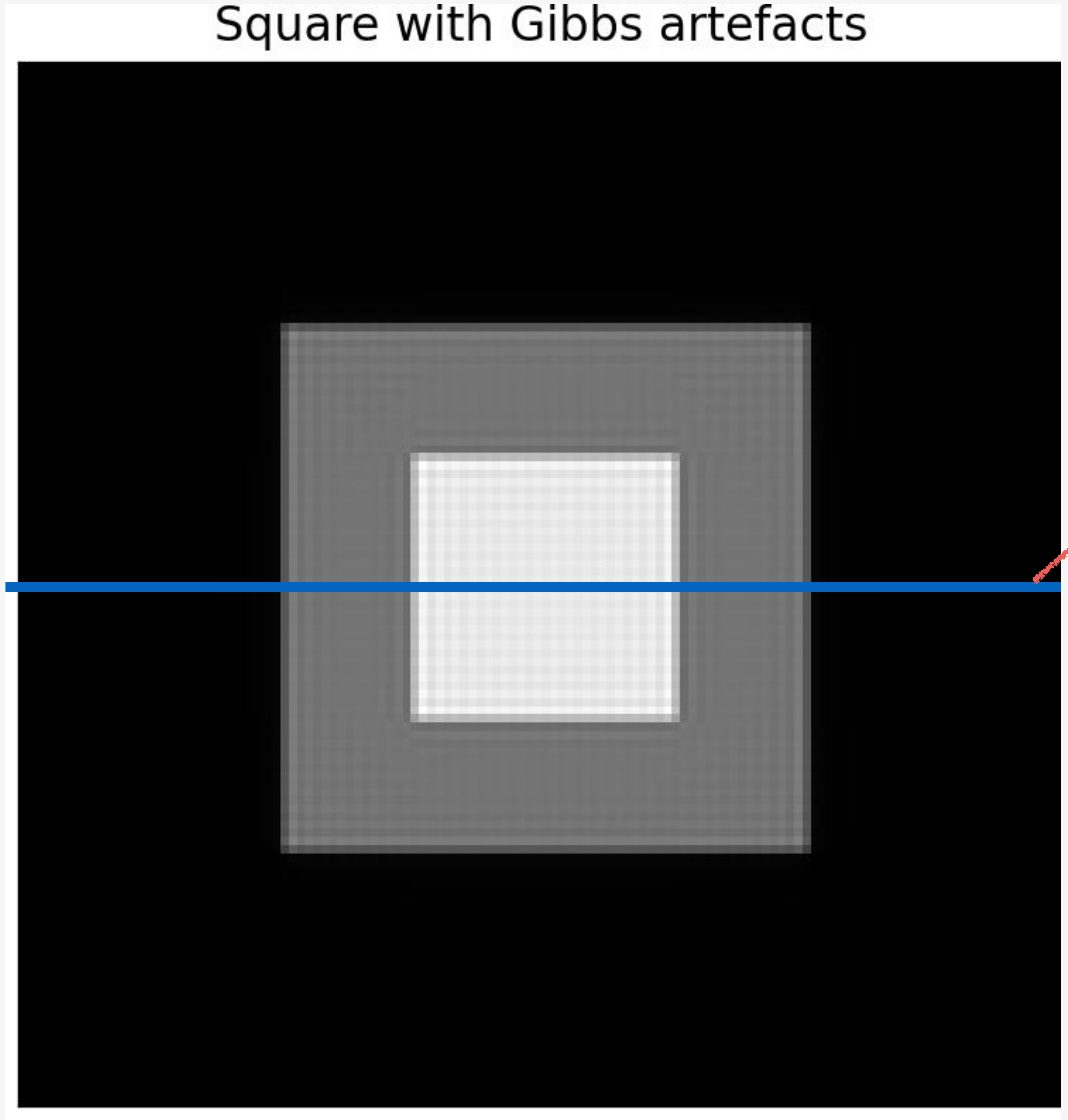
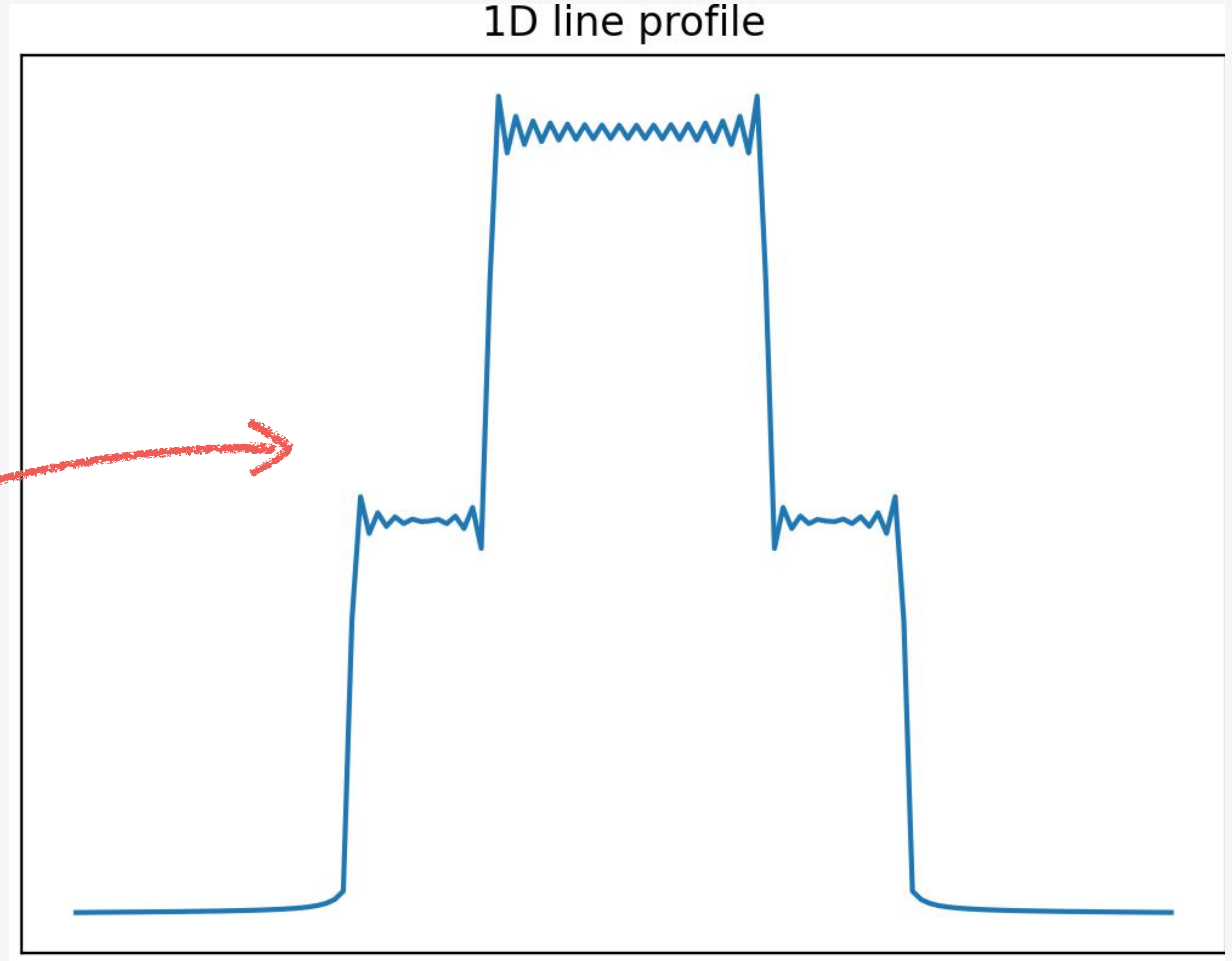
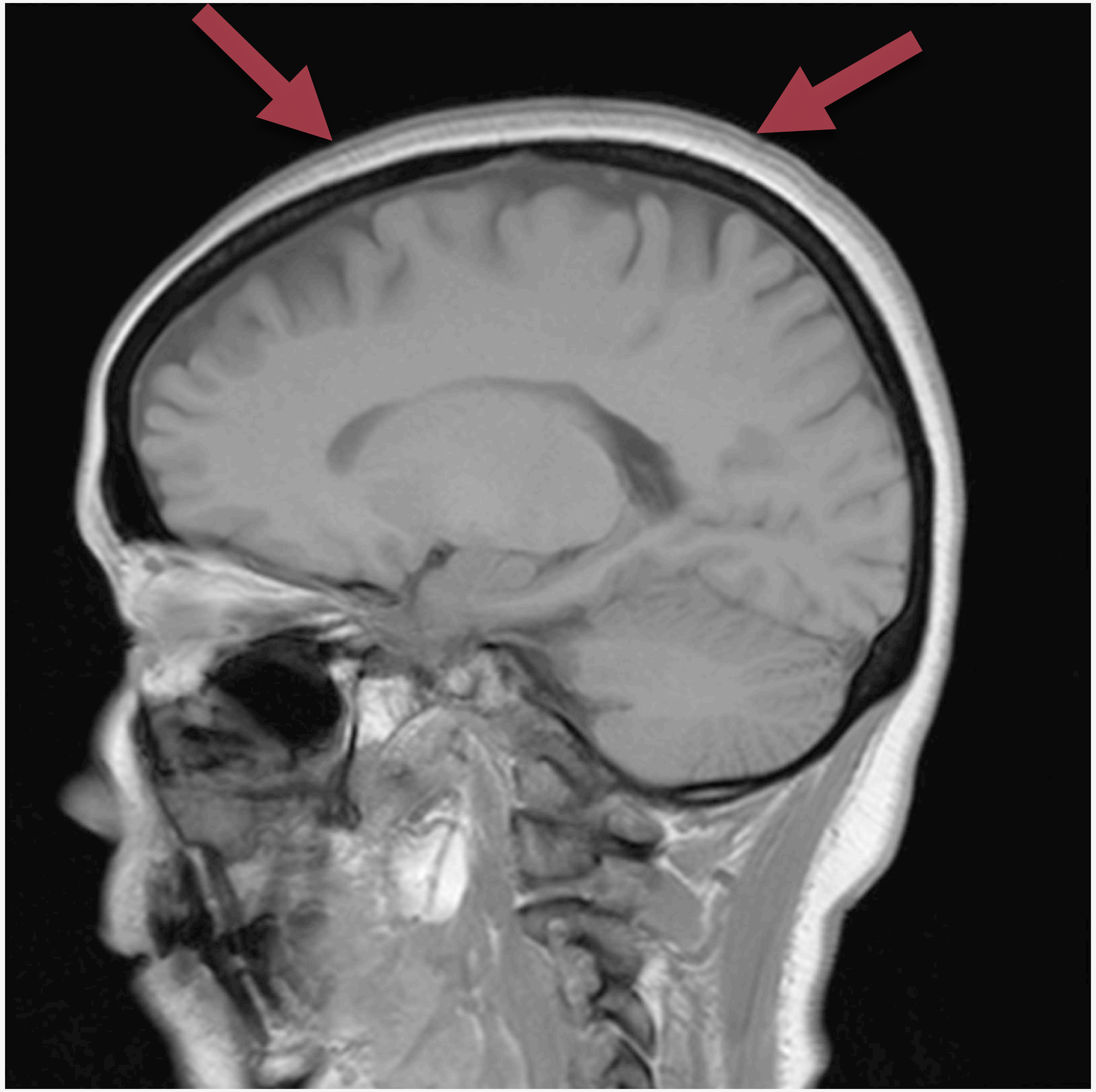




- **First overshoot and undershoot approx 9% of the signal jump**

p.ferreira@rbht.nhs.uk

K-SPACE SAMPLING ARTEFACTS :: GIBBS



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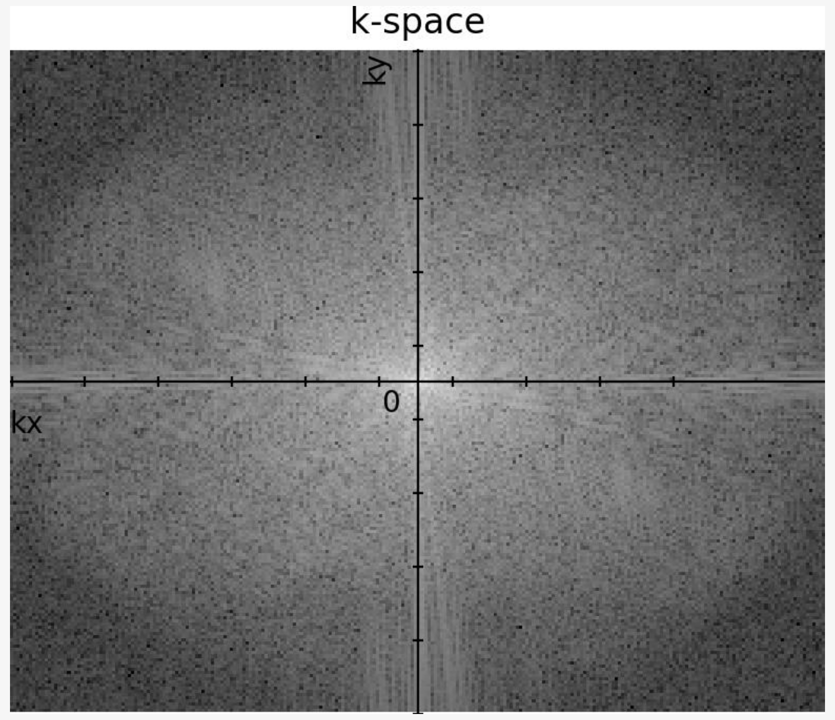


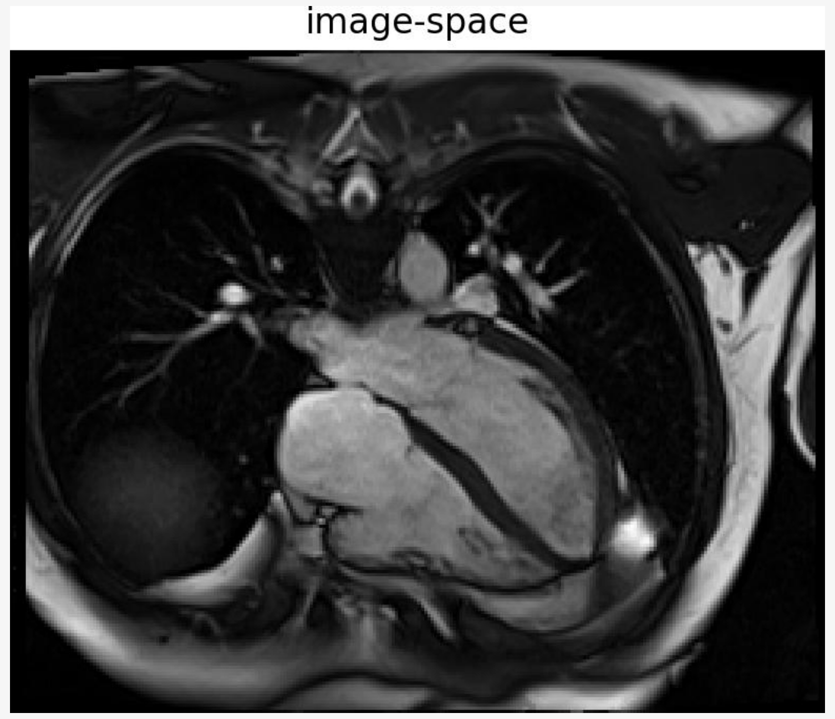
$$\times \quad H_{ws}(k) \equiv \underbrace{\mathrm{rect}\left(\frac{k+\frac{1}{2}\Delta k}{W}\right)}_{\text{finite k-space}} \underbrace{\Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)}_{\text{discrete sampling}}$$

k-space filter

Gibbs ringing

$\mathcal{F}^{-1}$

$\mathcal{F}^{-1}$



$$* \quad h_{ws}(x)$$

point spread function

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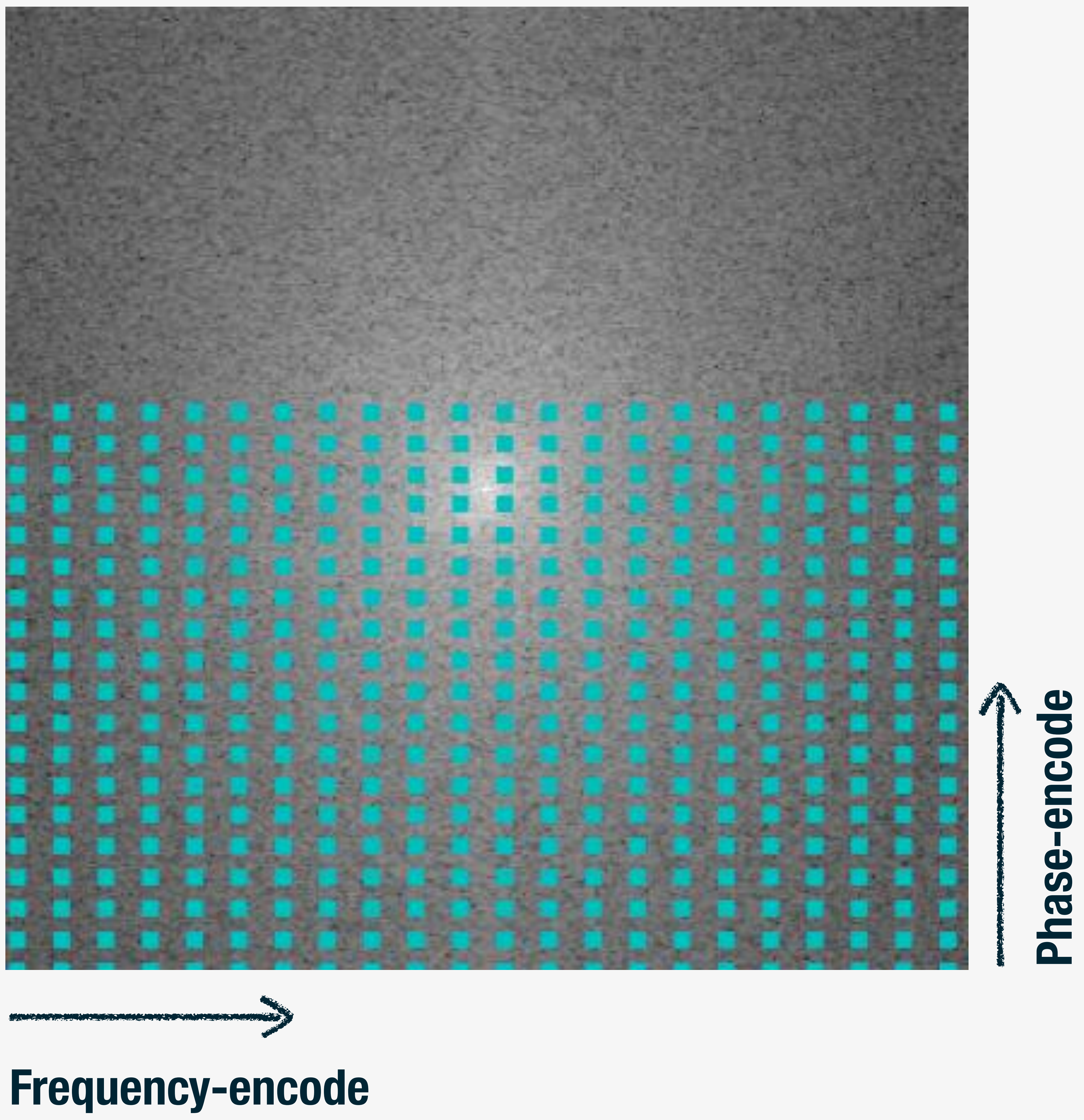
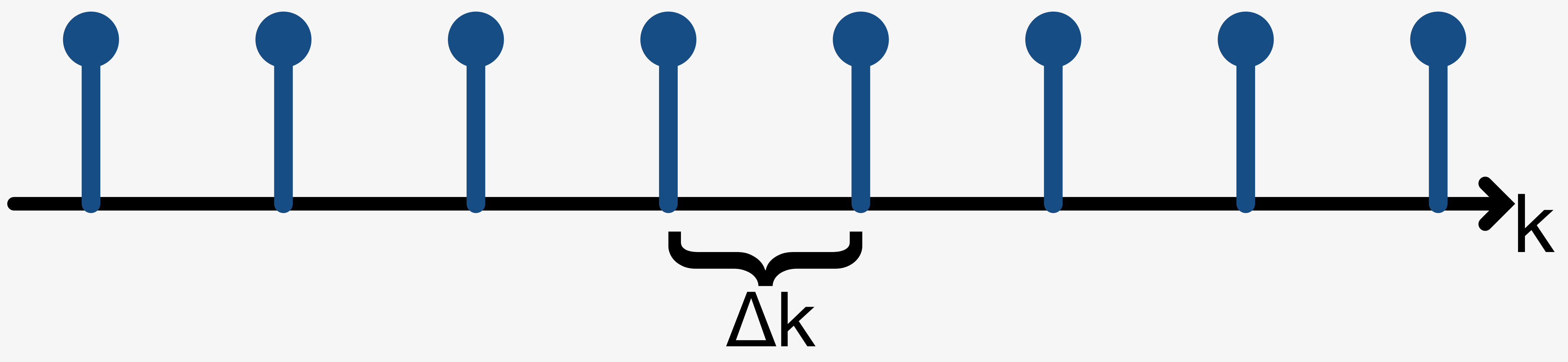
K-SPACE :: DISCRETE SAMPLING





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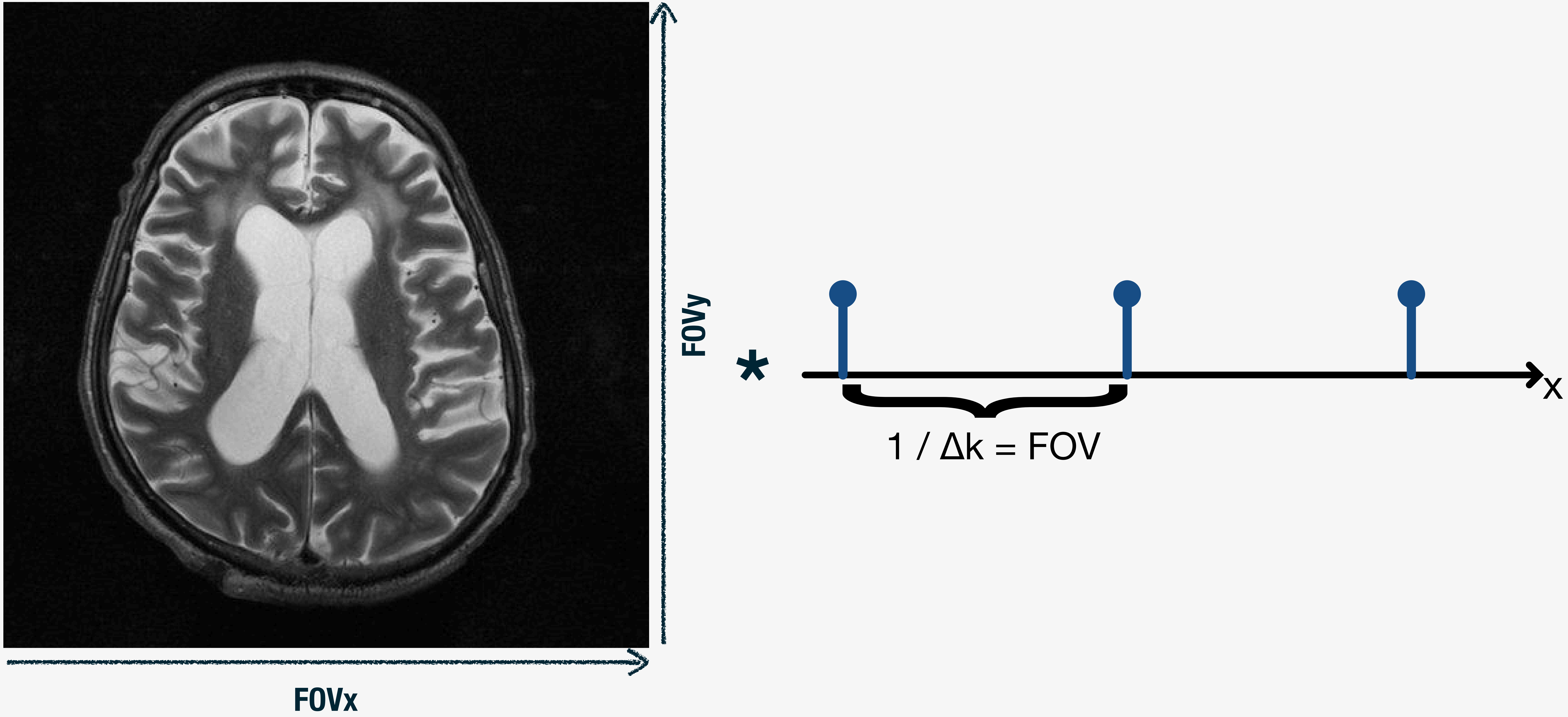

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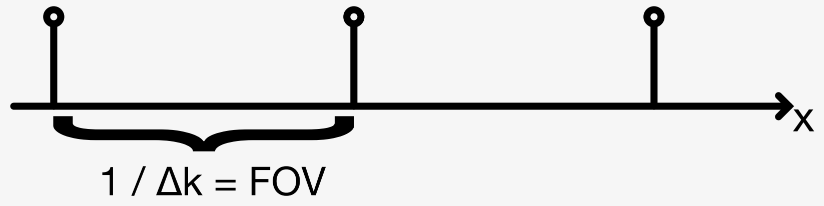
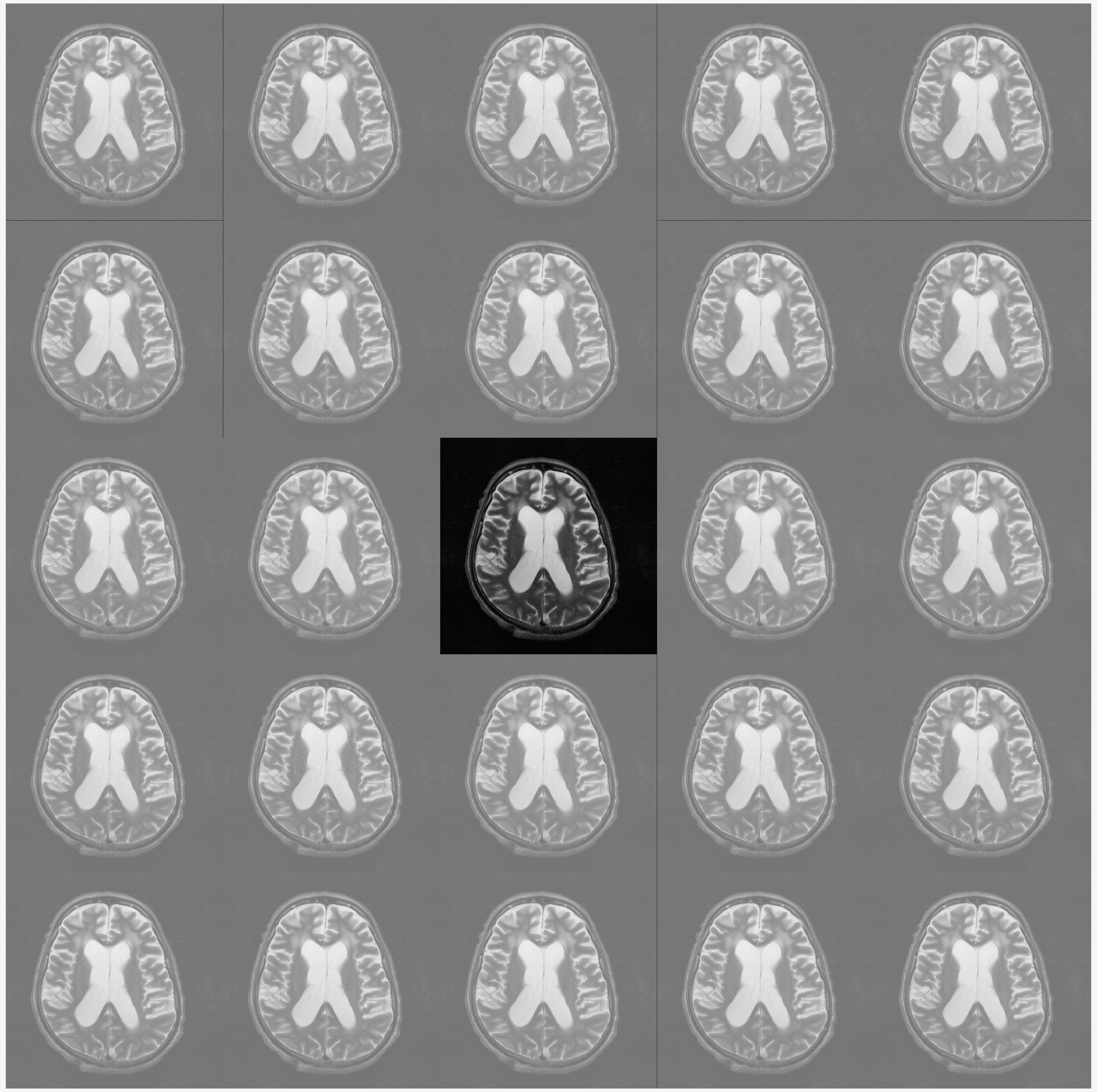
K-SPACE :: DISCRETE SAMPLING

*





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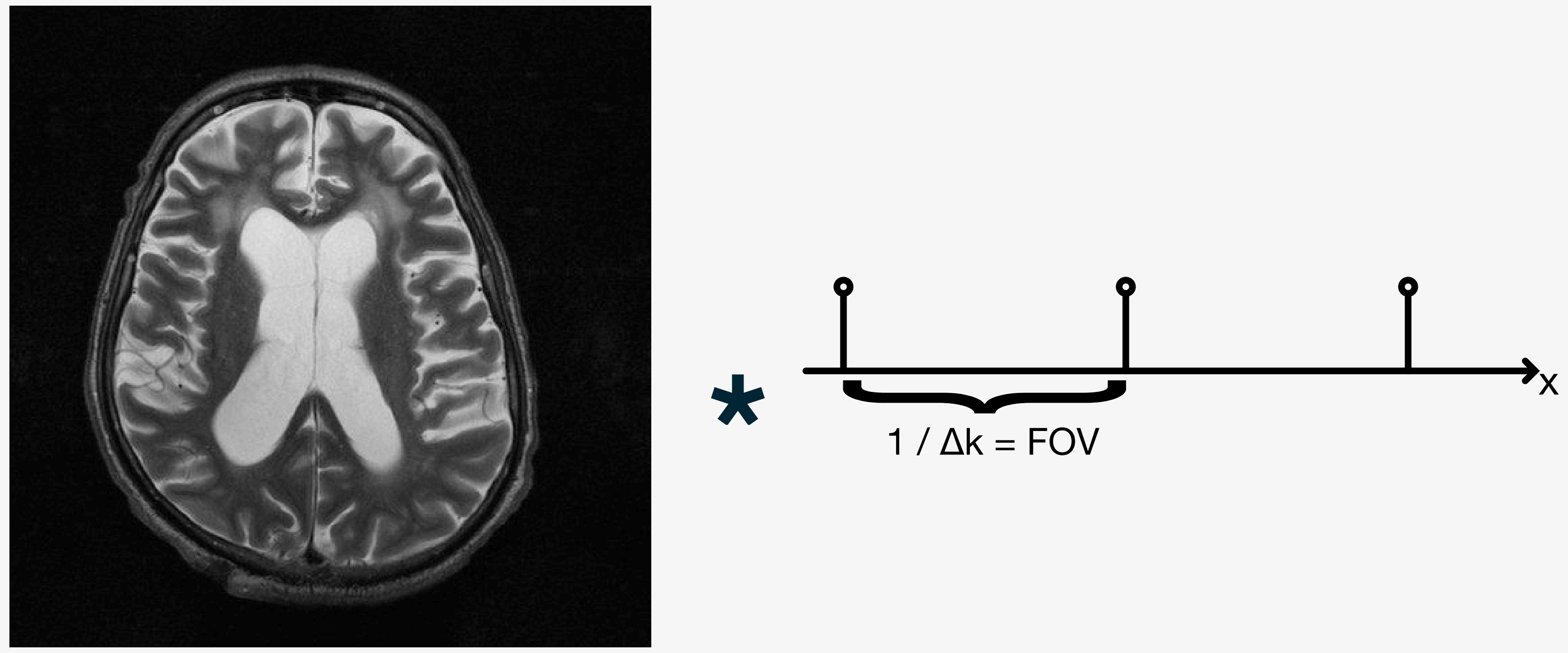

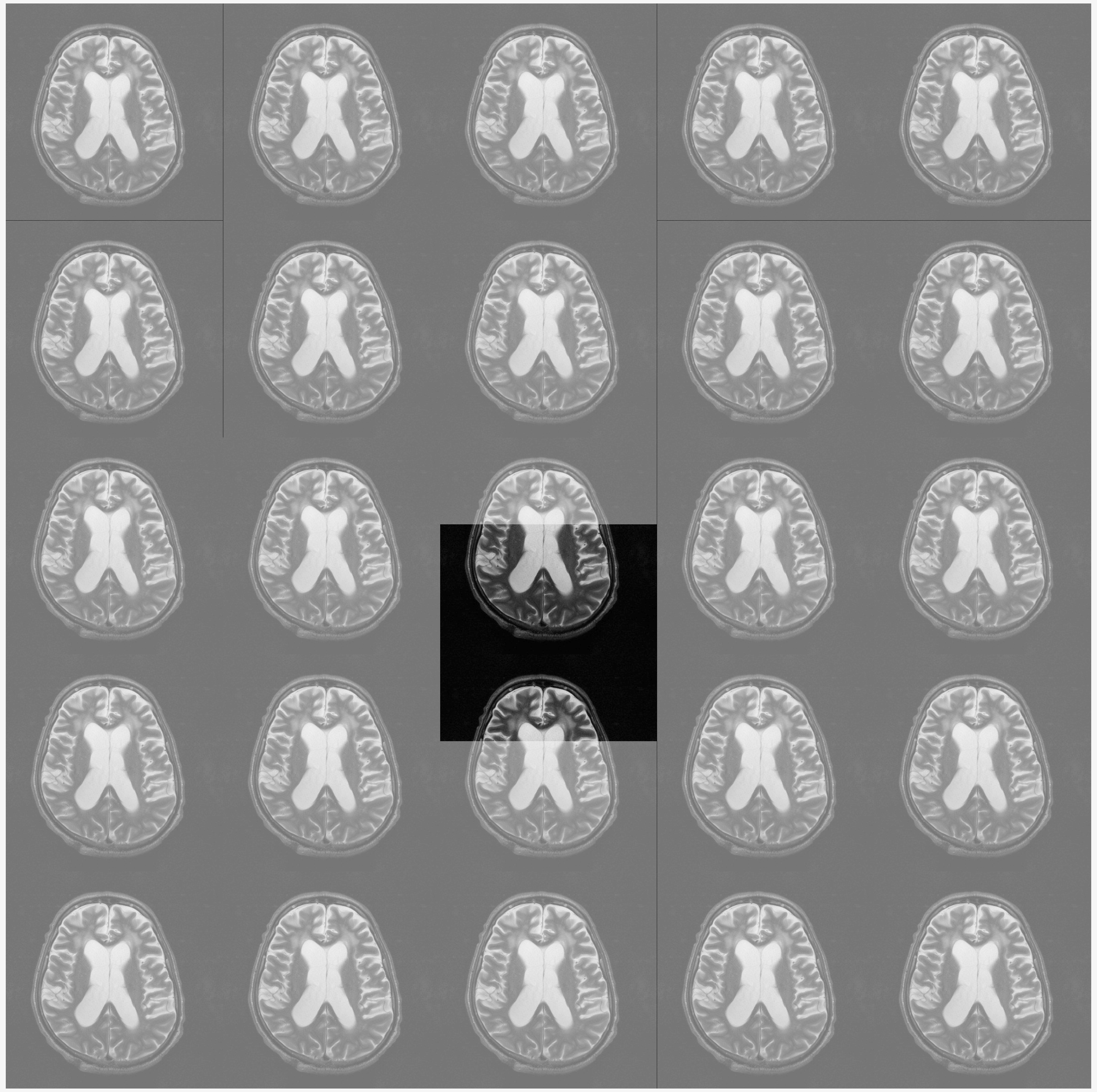

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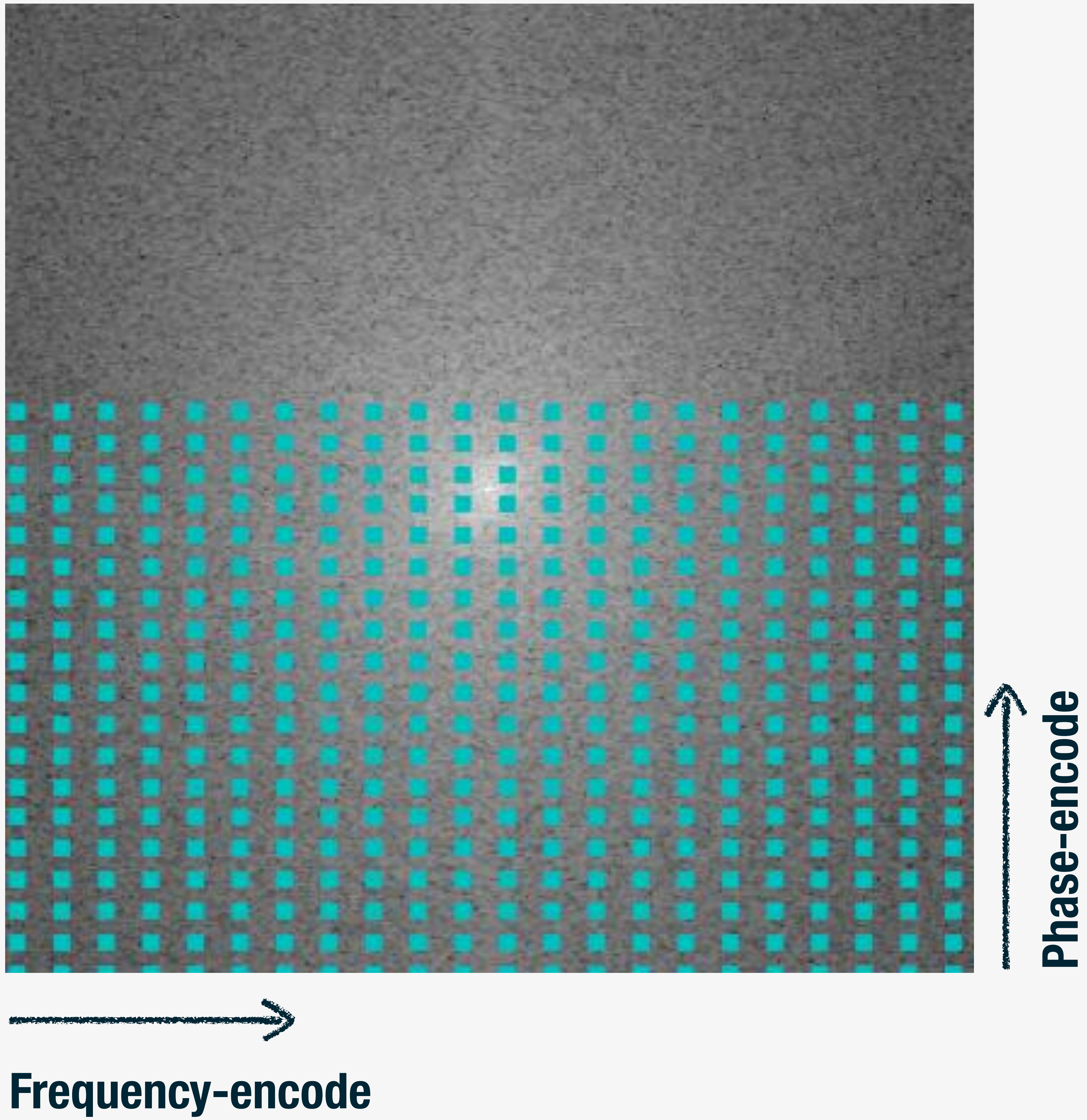

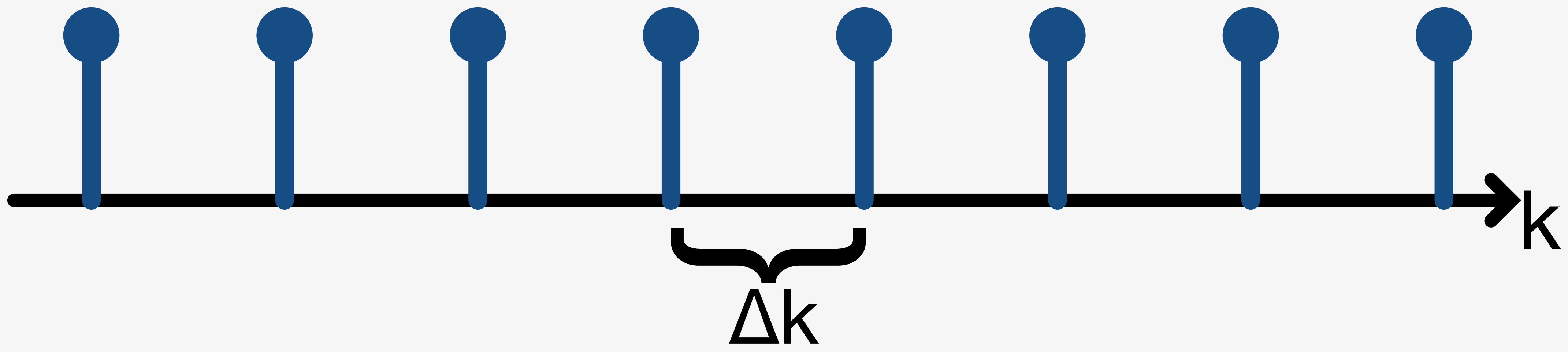

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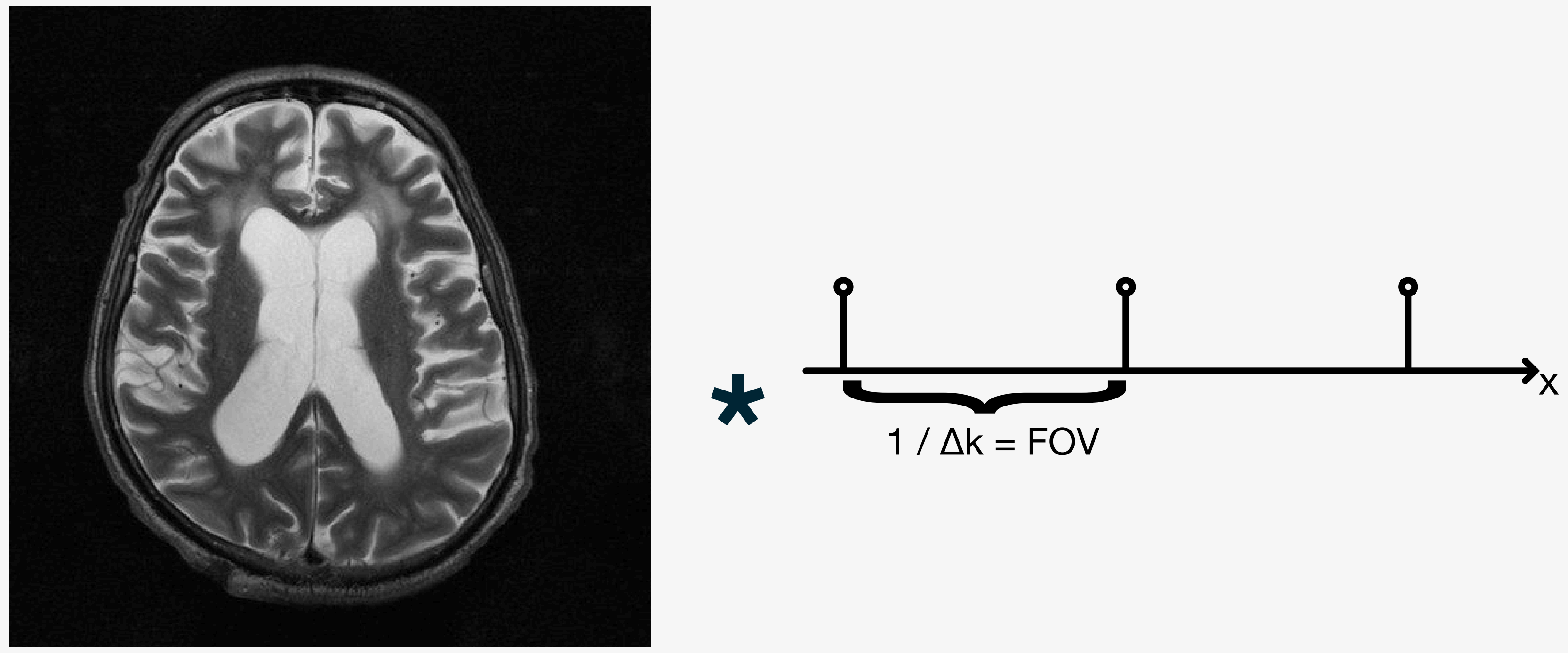

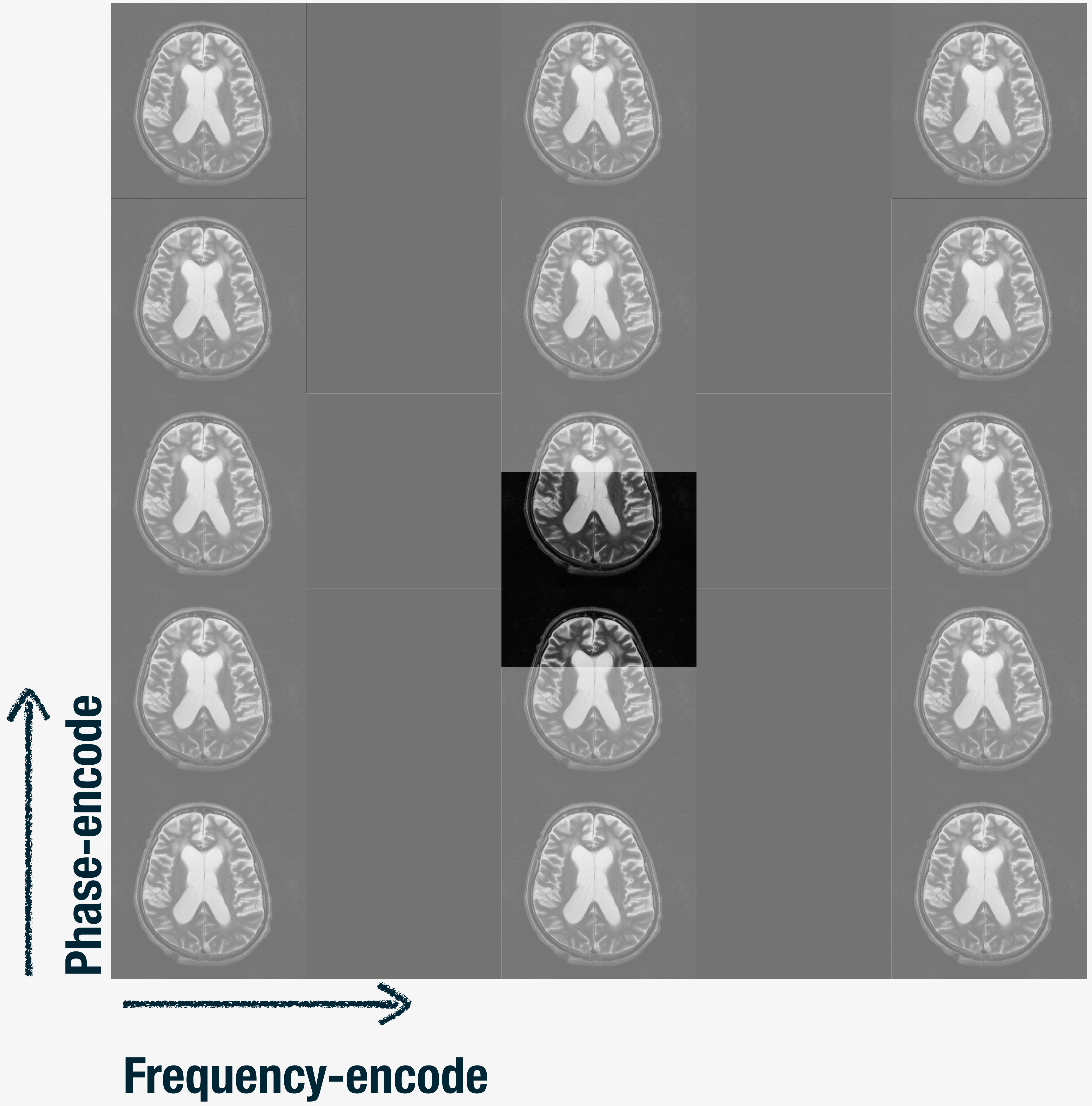

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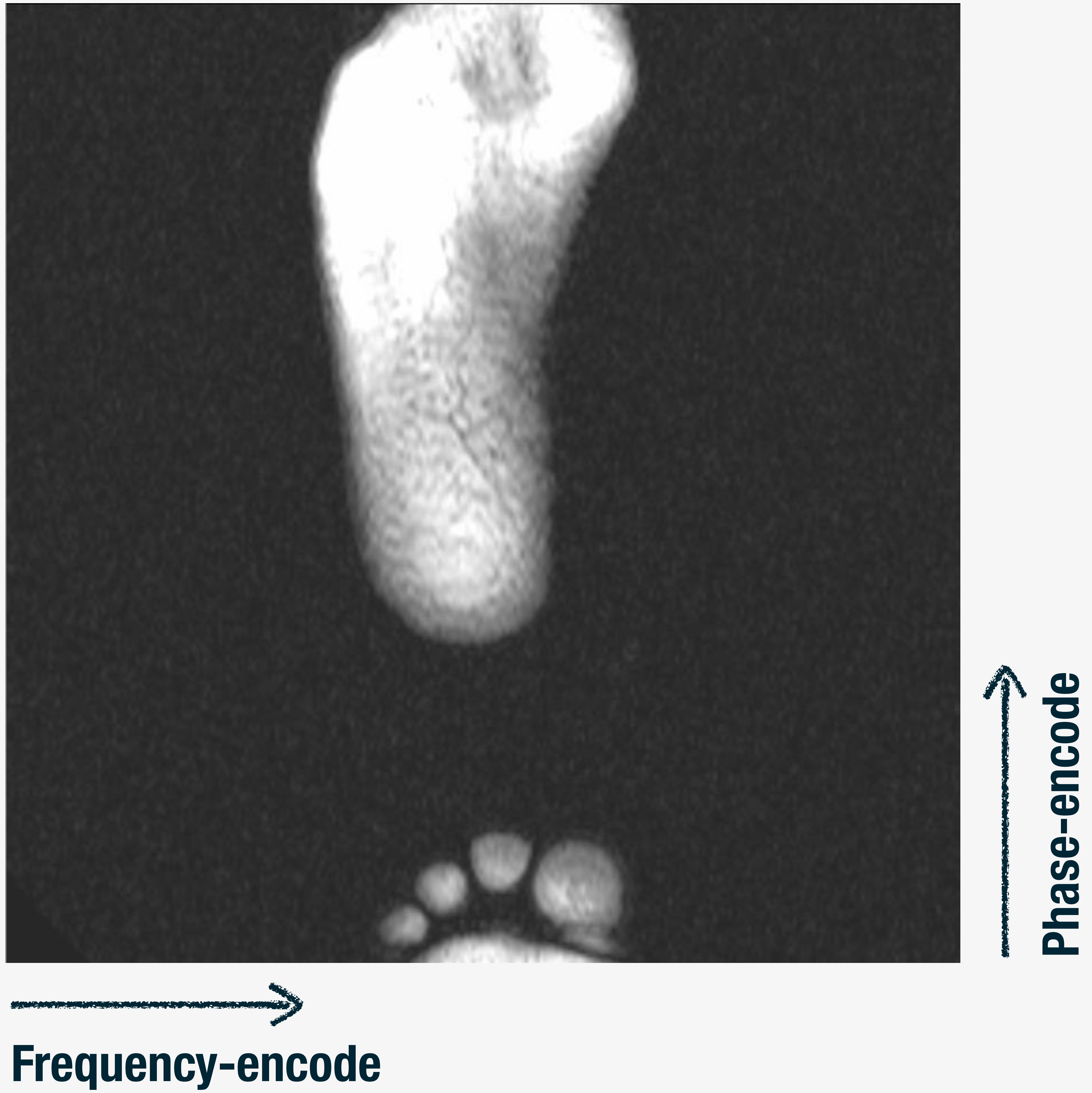
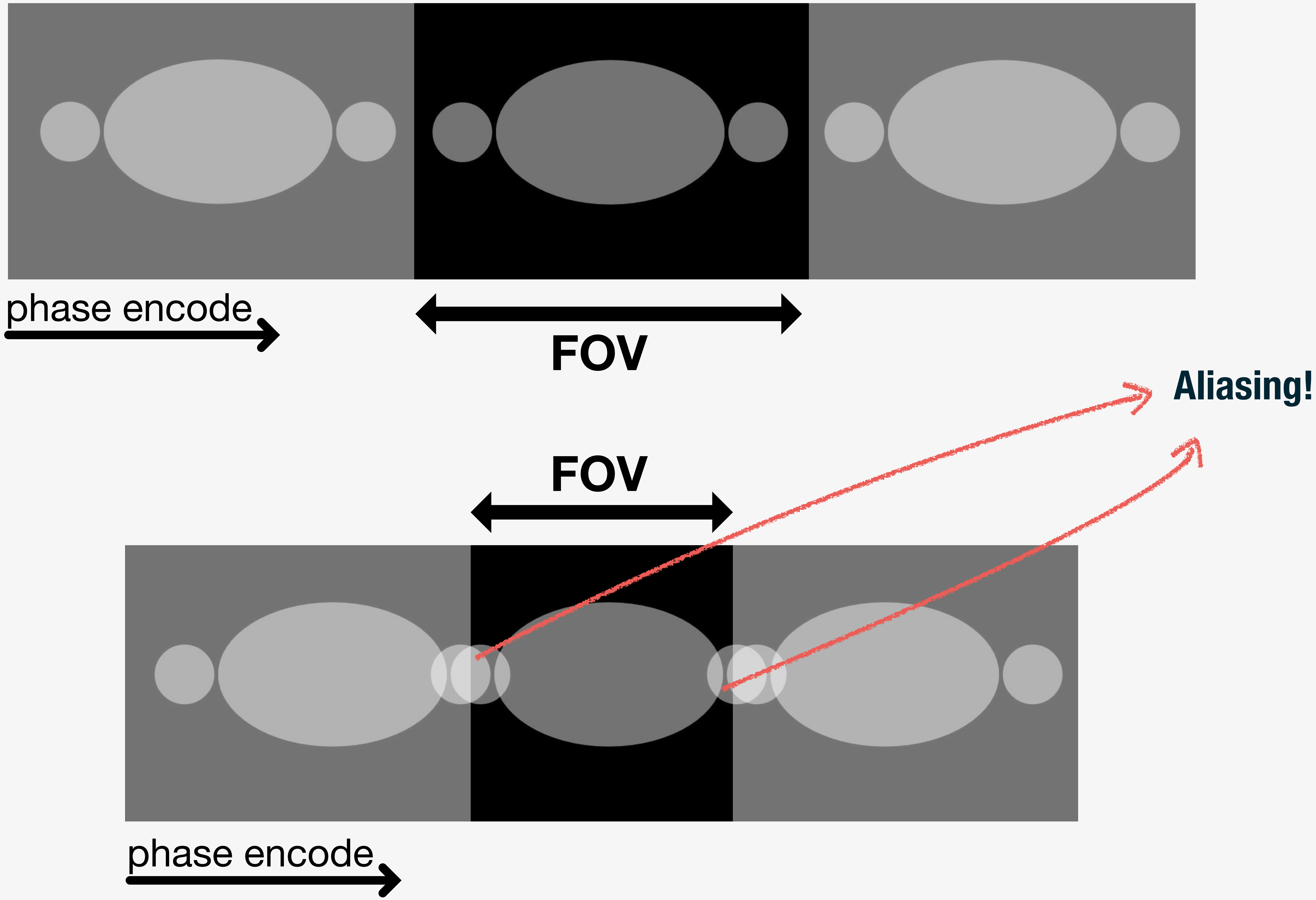
Wrap-around artefact



p.ferreira@rbht.nhs.uk

ALIASING ARTEFACT



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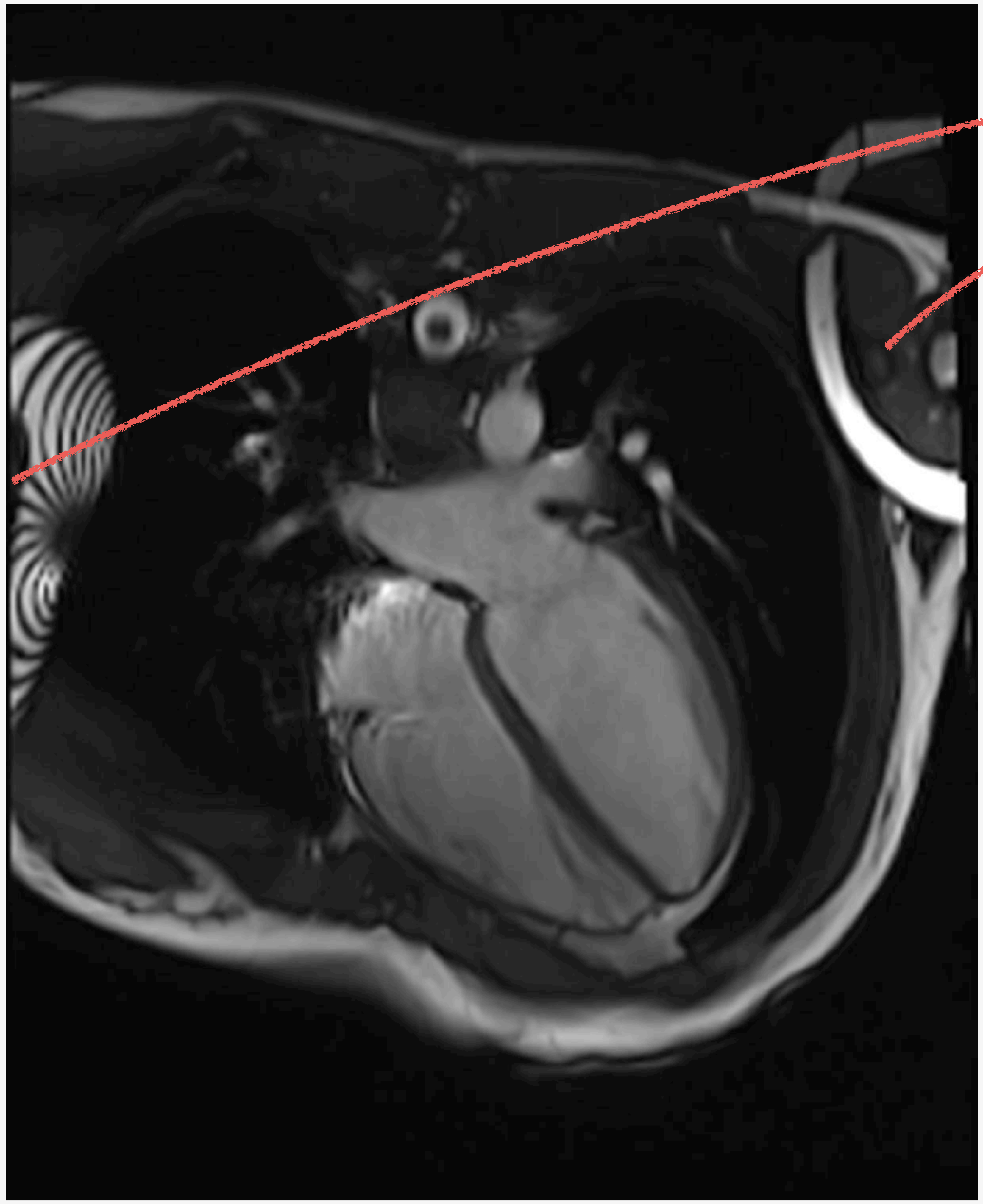
ALIASING ARTEFACT



Aliasing of the arms from other replicas

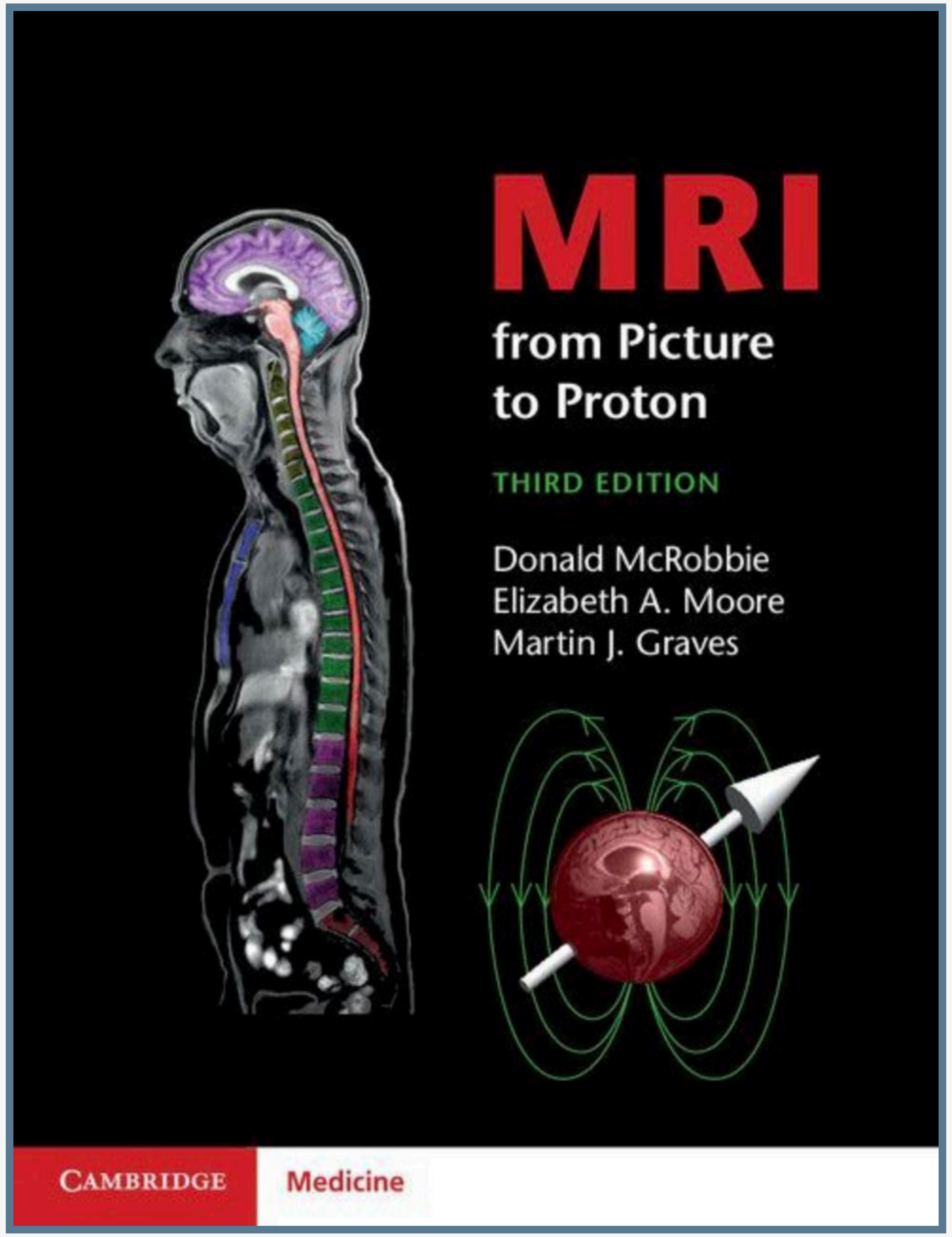
p.ferreira@rbht.nhs.uk

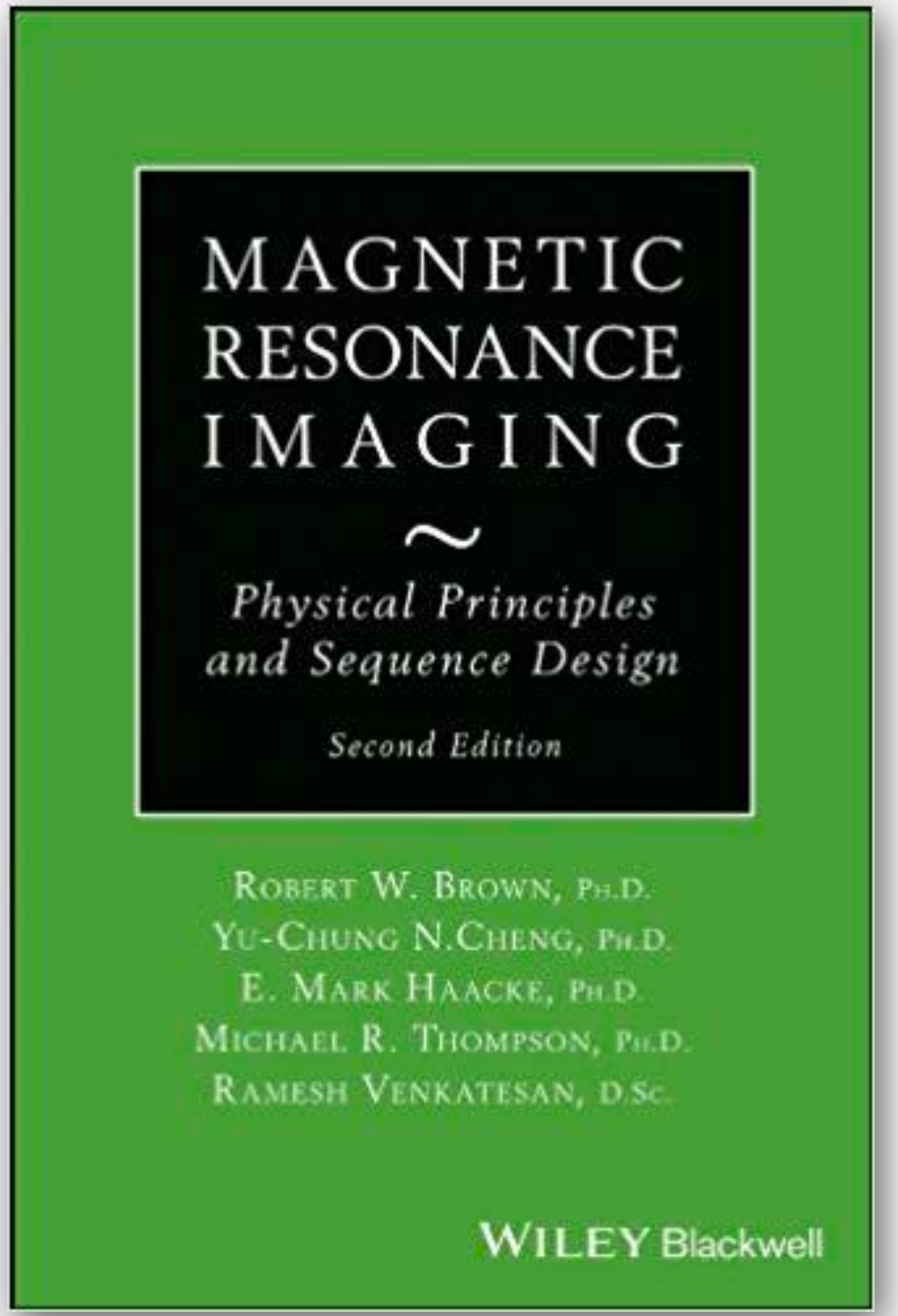
Summary

- MRI raw-signal is known as k-space
  - It is in the frequency domain.
  - It allows for clever k-space under sampling tricks.
  - But its also creates very characteristic image artefacts.

p.ferreira@rbht.nhs.uk

LITERATURE





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MATERIAL

- Github:
  - Jupyter notebook https://github.com/Pedro-Filipe/k-space_simulations
  - Slides

THANK YOU

p.ferreira@rbht.nhs.uk