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MRI DATA: K-SPACE AND SAMPLING ARTEFACTS

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Contents

- k-space
- k-space properties
- k-space sampling artefacts
  - Wrap-around or aliasing
  - Gibbs ringing

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MRI

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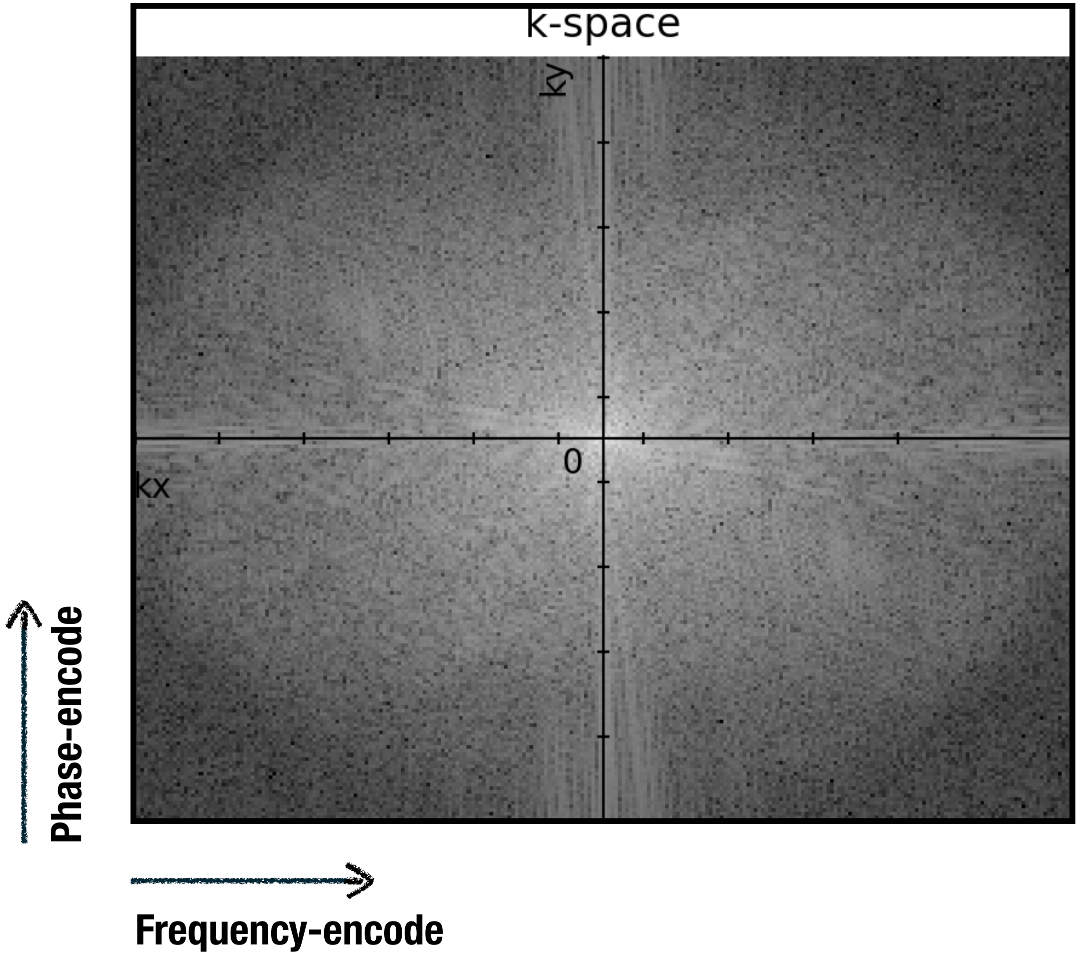
K-SPACE

k-space:

- MRI raw data.
- The imaged object is in the frequency domain.
- 0th frequency in the centre.
- k-space values are complex:
    - magnitude and phase.



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Wrap-around artefact

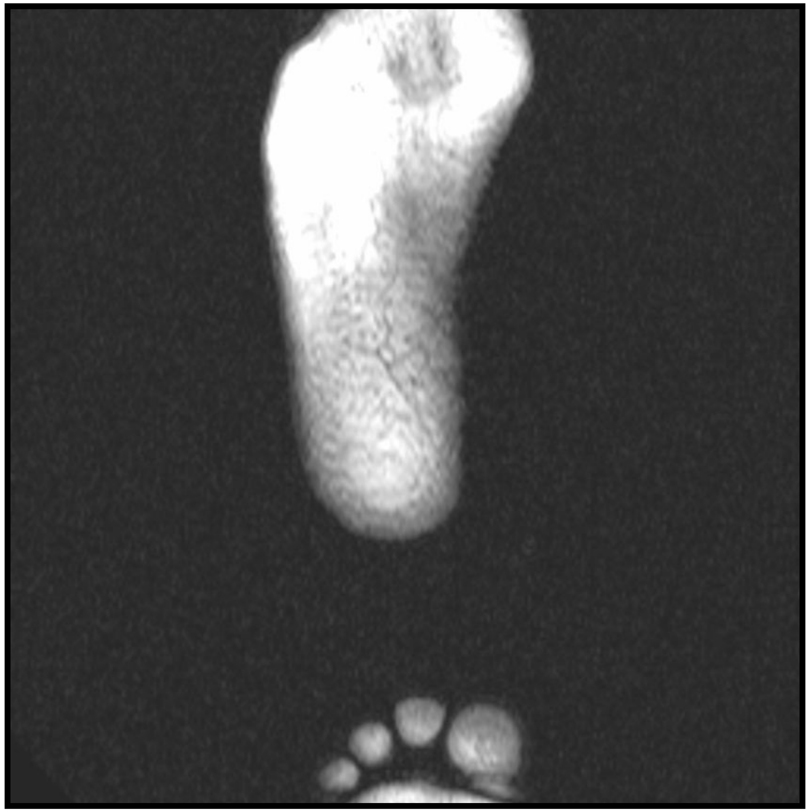

Image courtesy of Dr. Michael D. Noseworthy,
McMaster University, Toronto Canada

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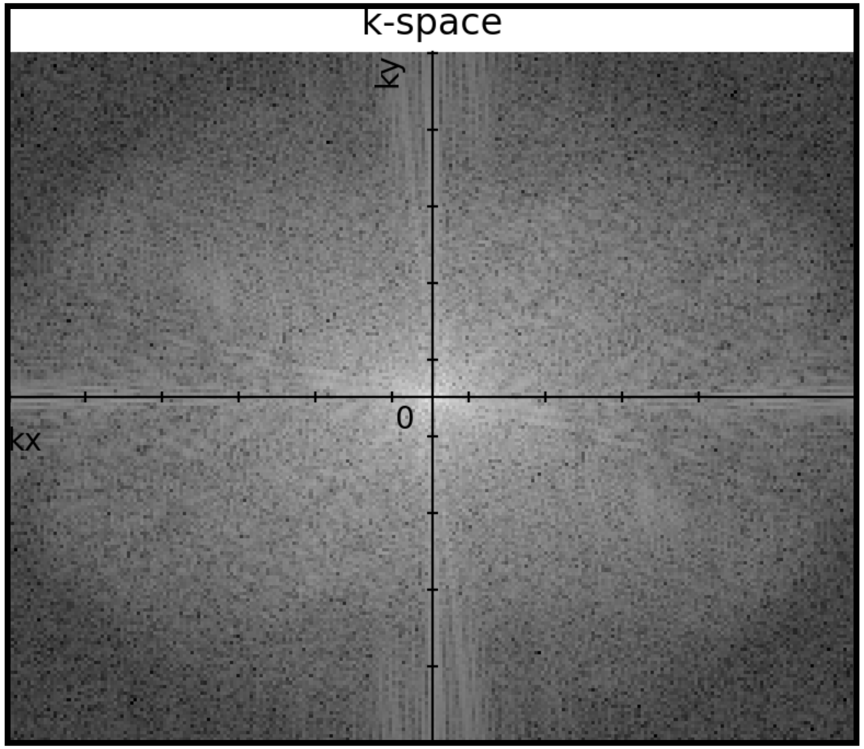
K-SPACE



2D inverse Fourier Transform

$$f(x, y) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_x, k_y) e^{i2\pi(k_x x + k_y y)} \, dk_x \, dk_y$$

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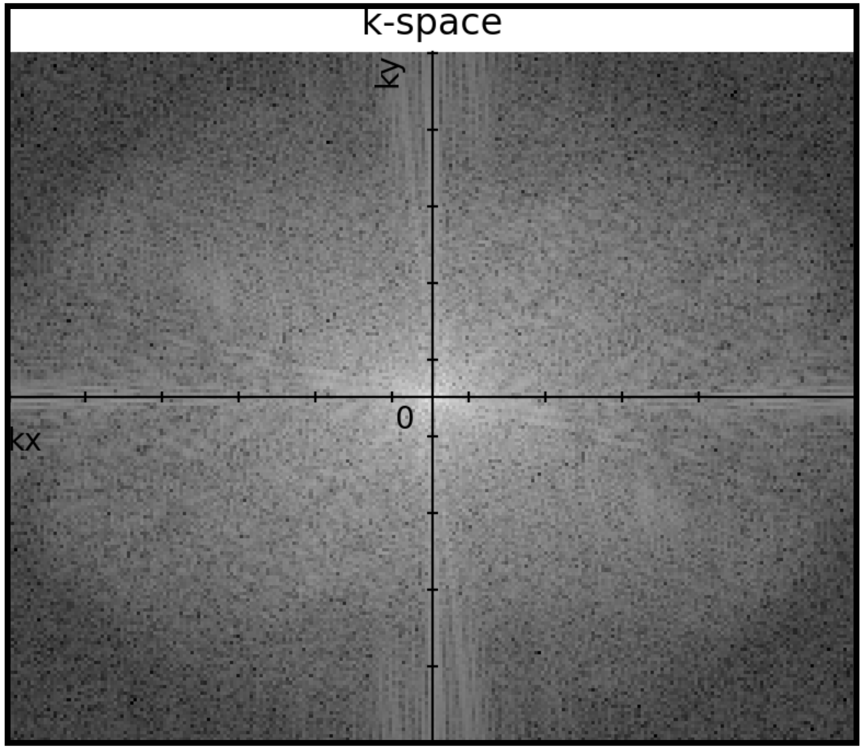
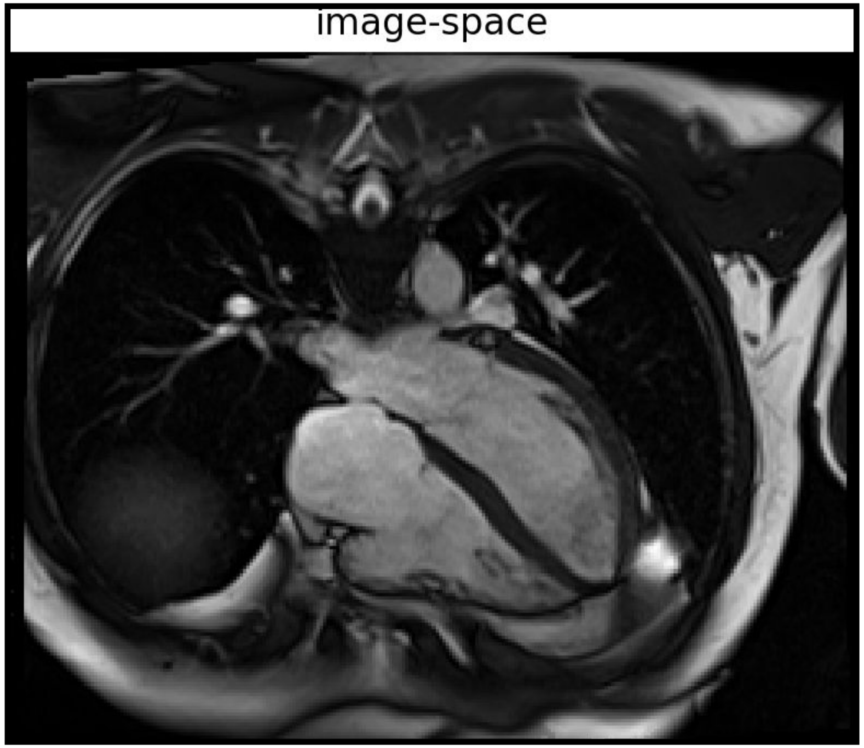
K-SPACE



2D inverse Fourier Transform



$$f(x, y) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_x, k_y) e^{i2\pi(k_x x + k_y y)} \, dk_x \, dk_y$$

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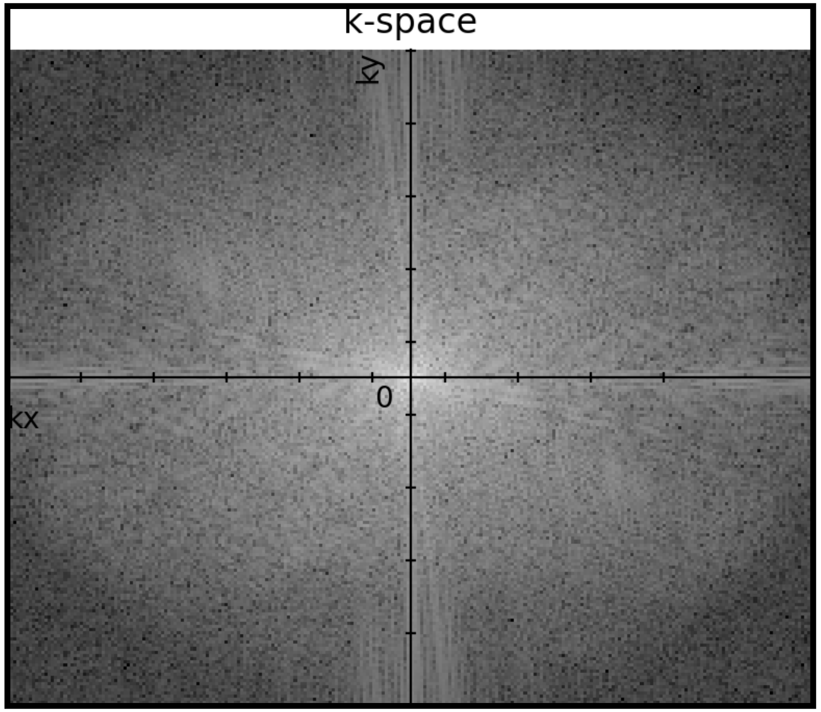
K-SPACE

k-space:

- **represents a large collection of many sinusoidal oscillations with weights given by the magnitude.**



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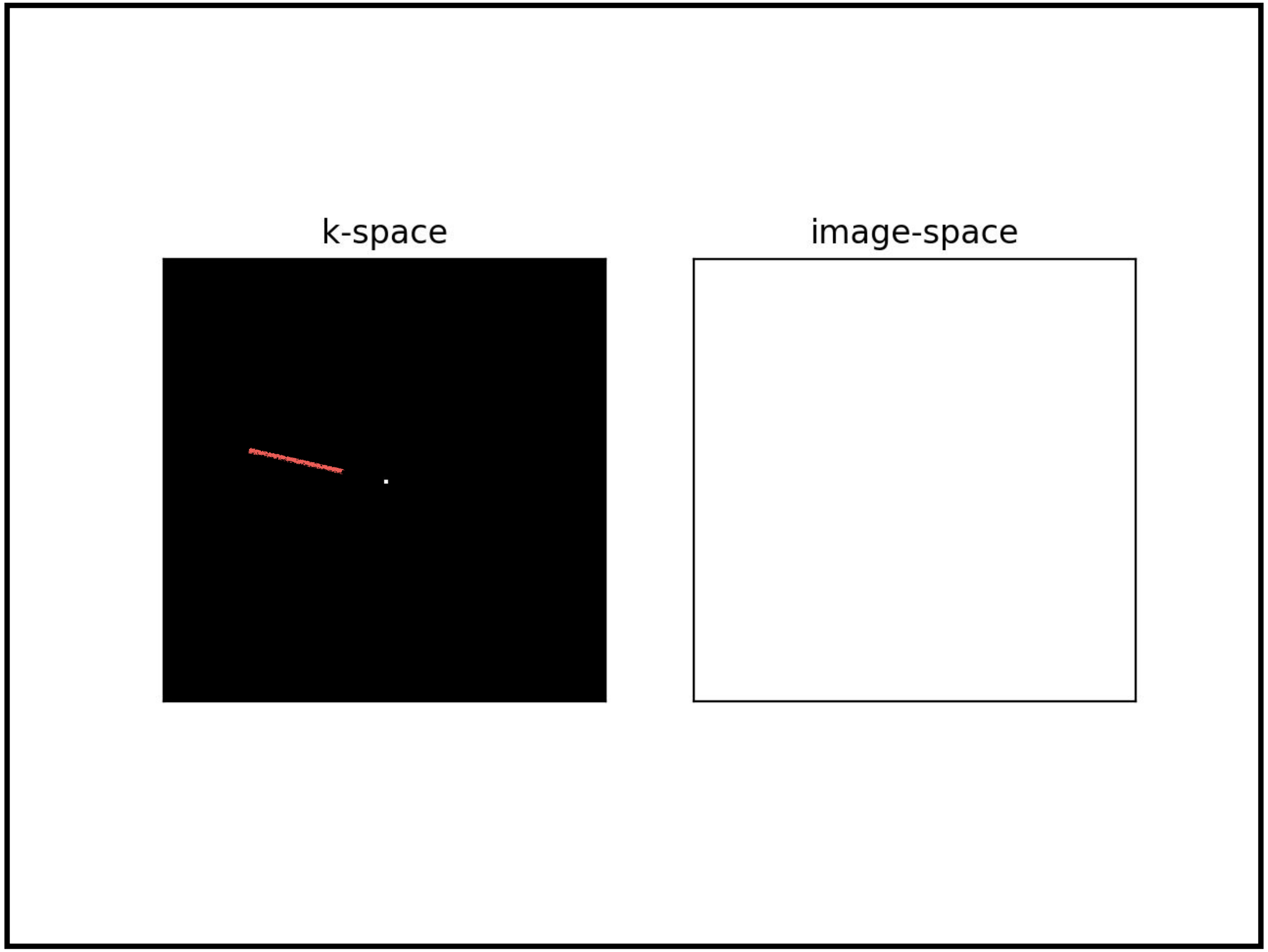
K-SPACE



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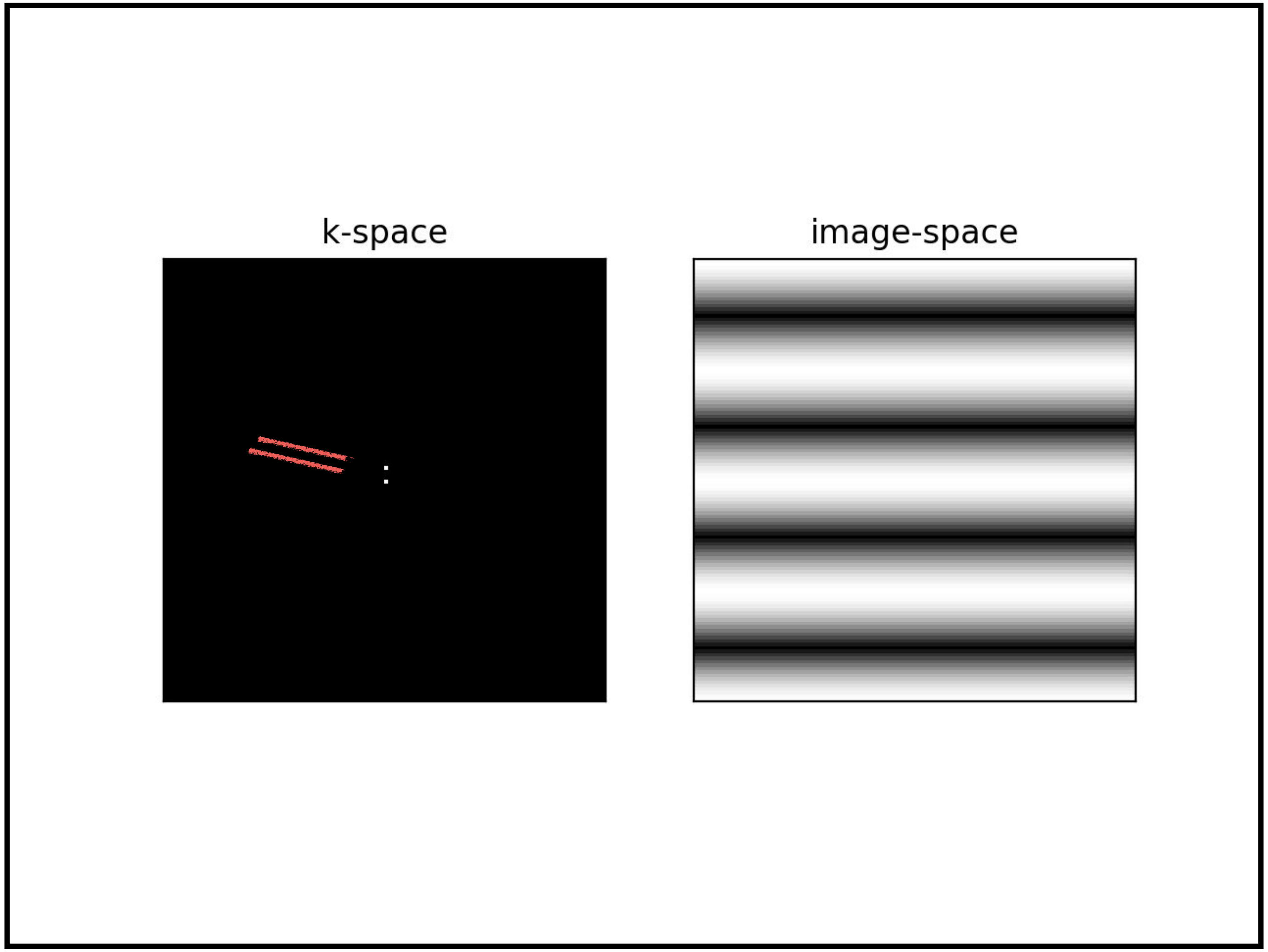

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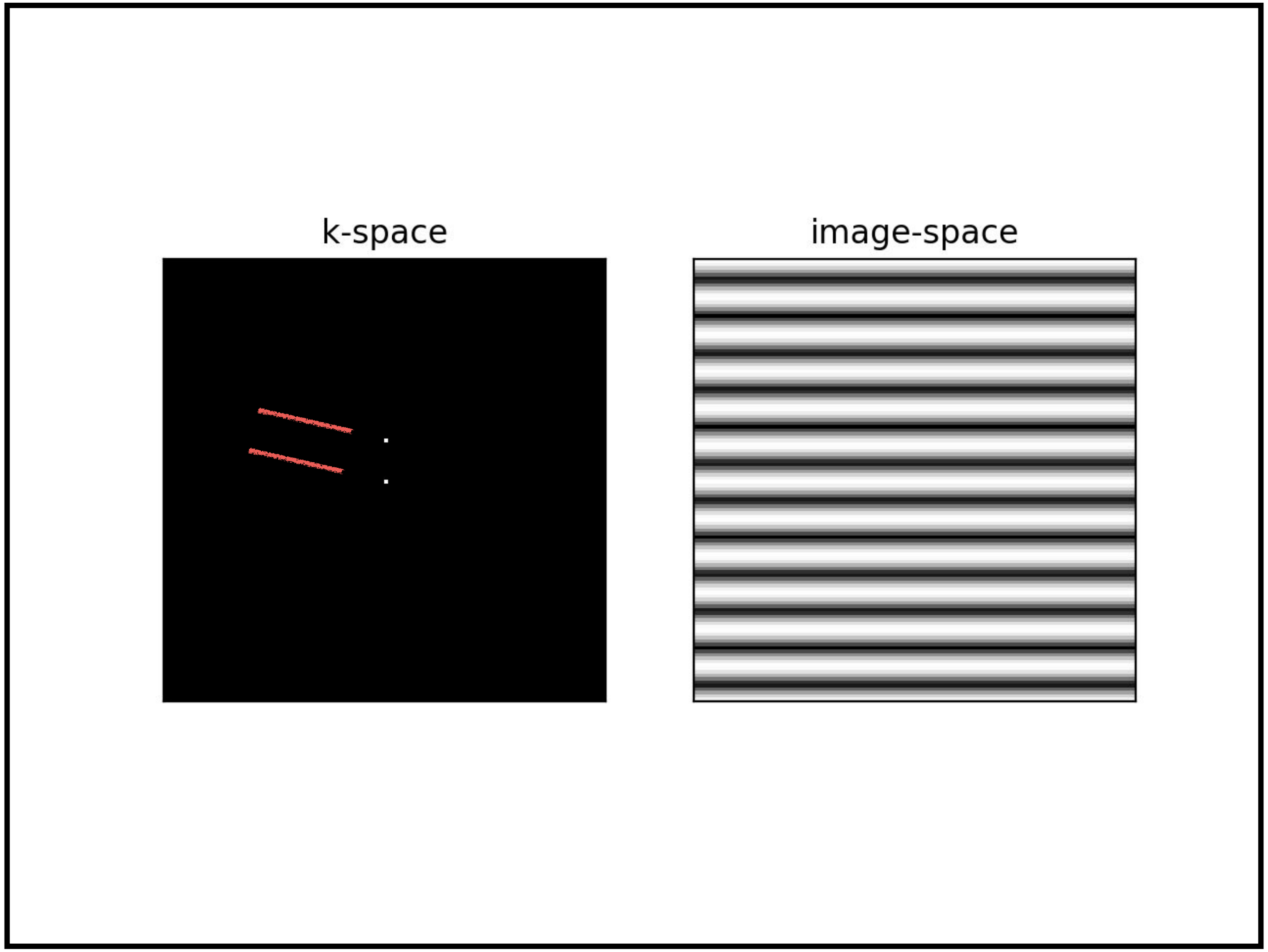

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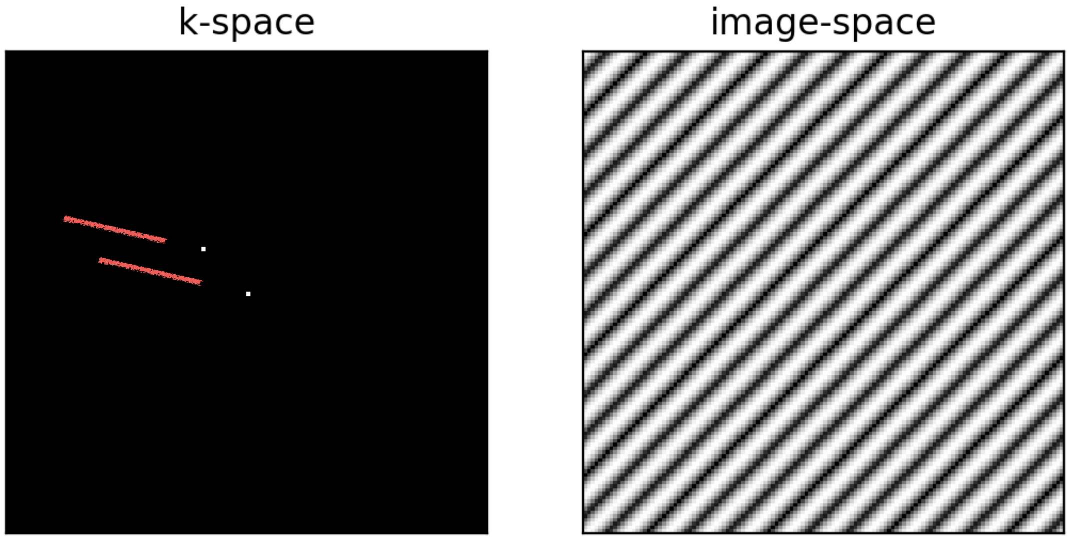

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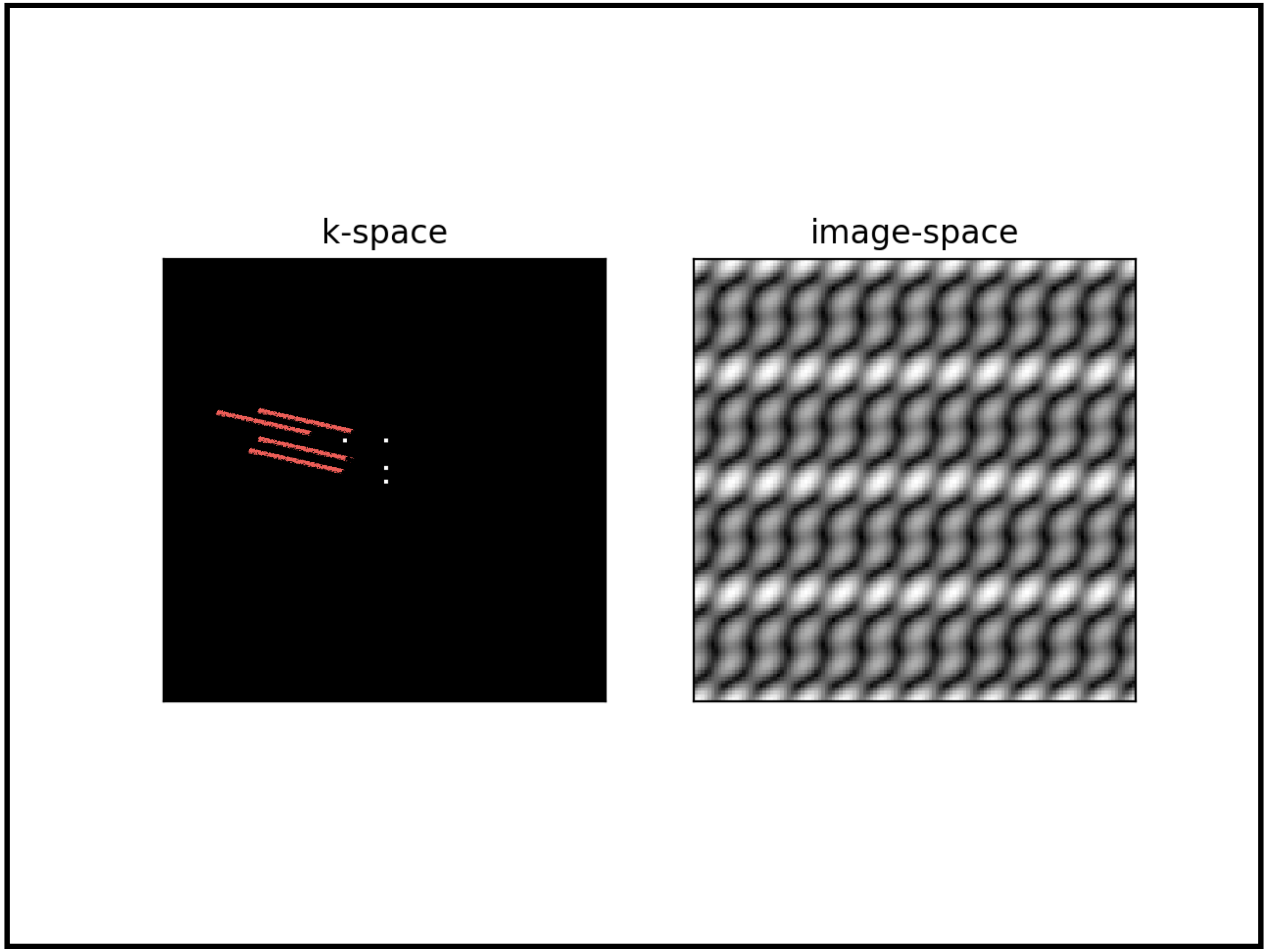

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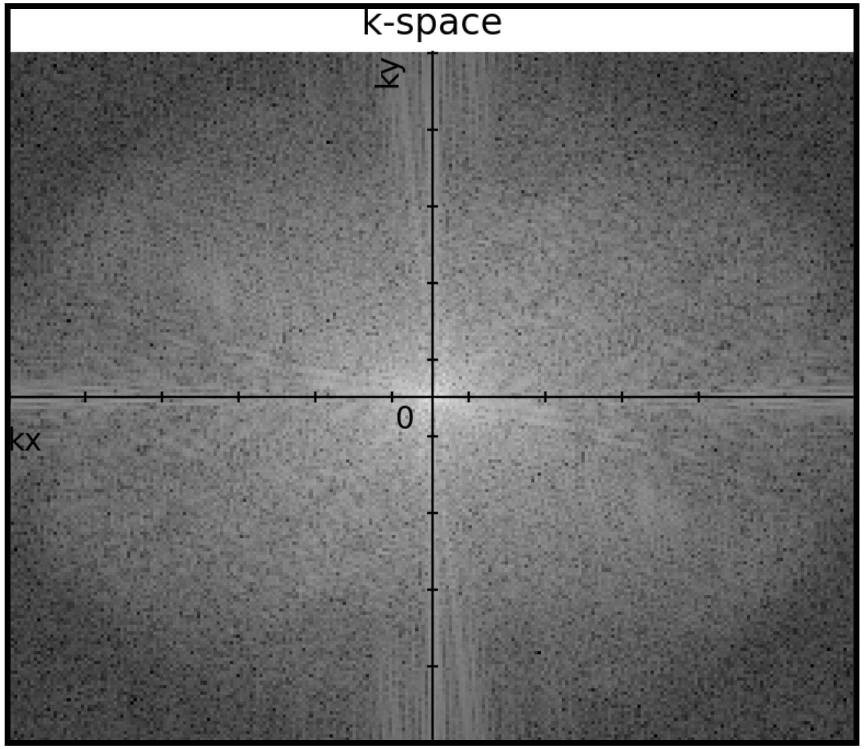
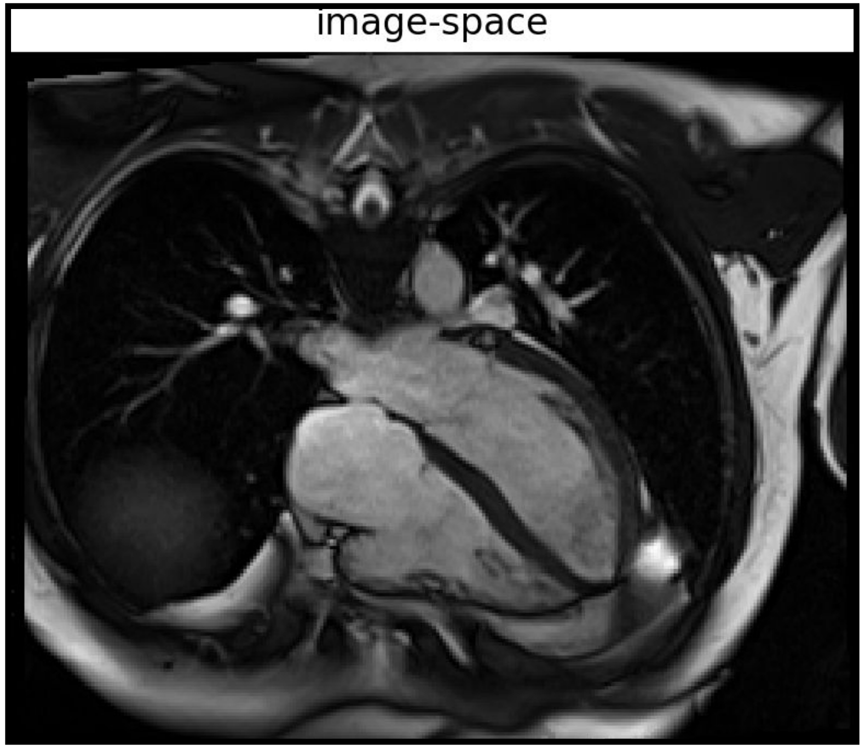


2D inverse Fourier Transform



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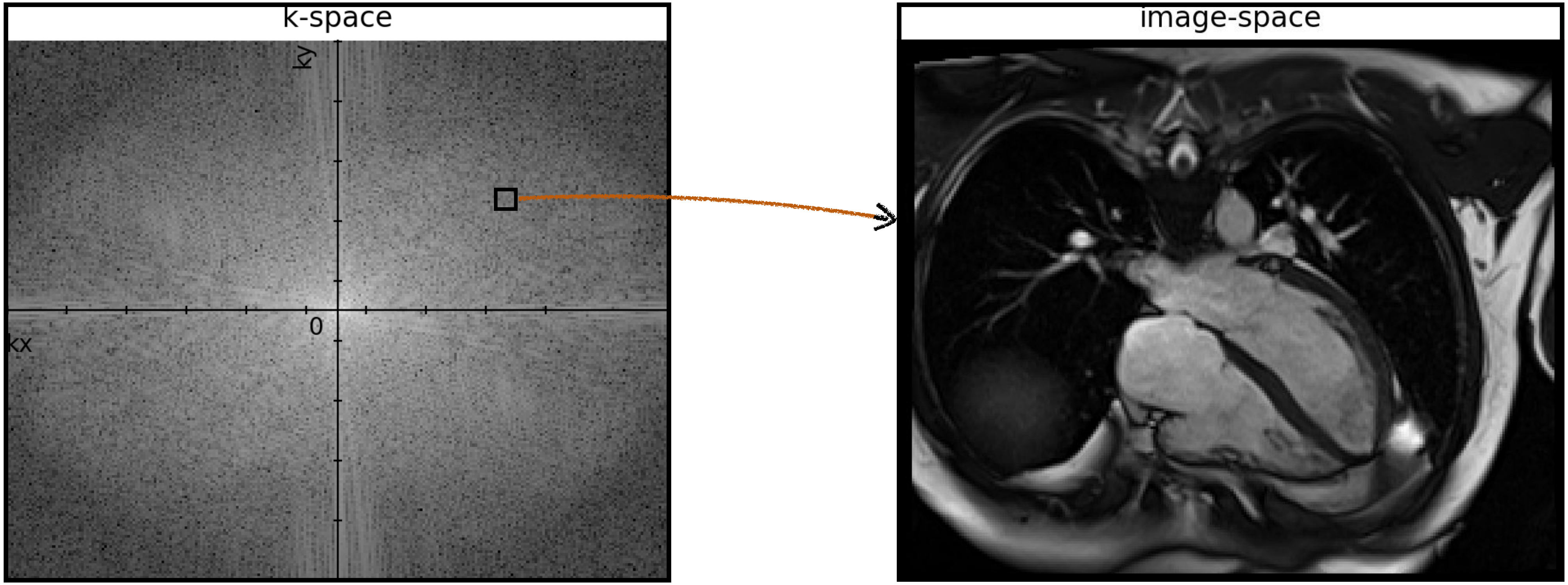

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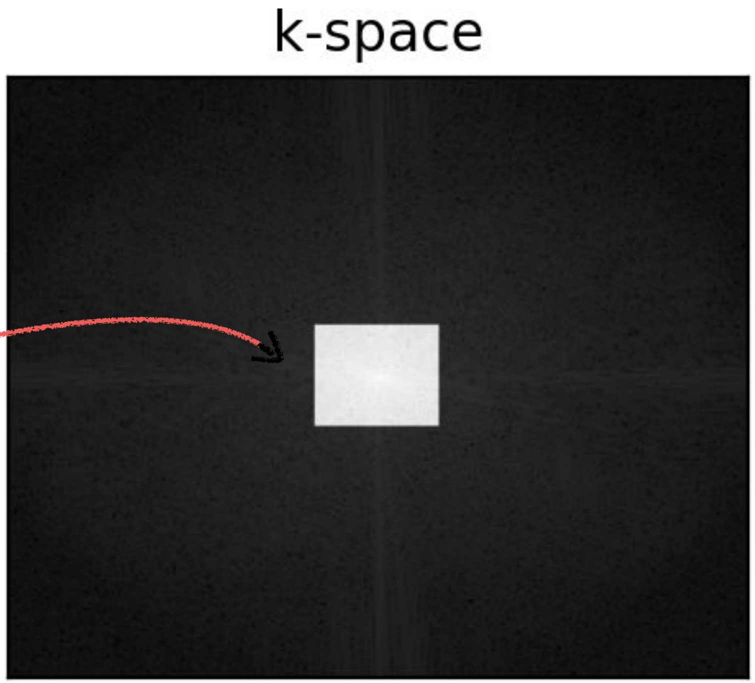
K-SPACE

central region of
k-space
(low frequencies)



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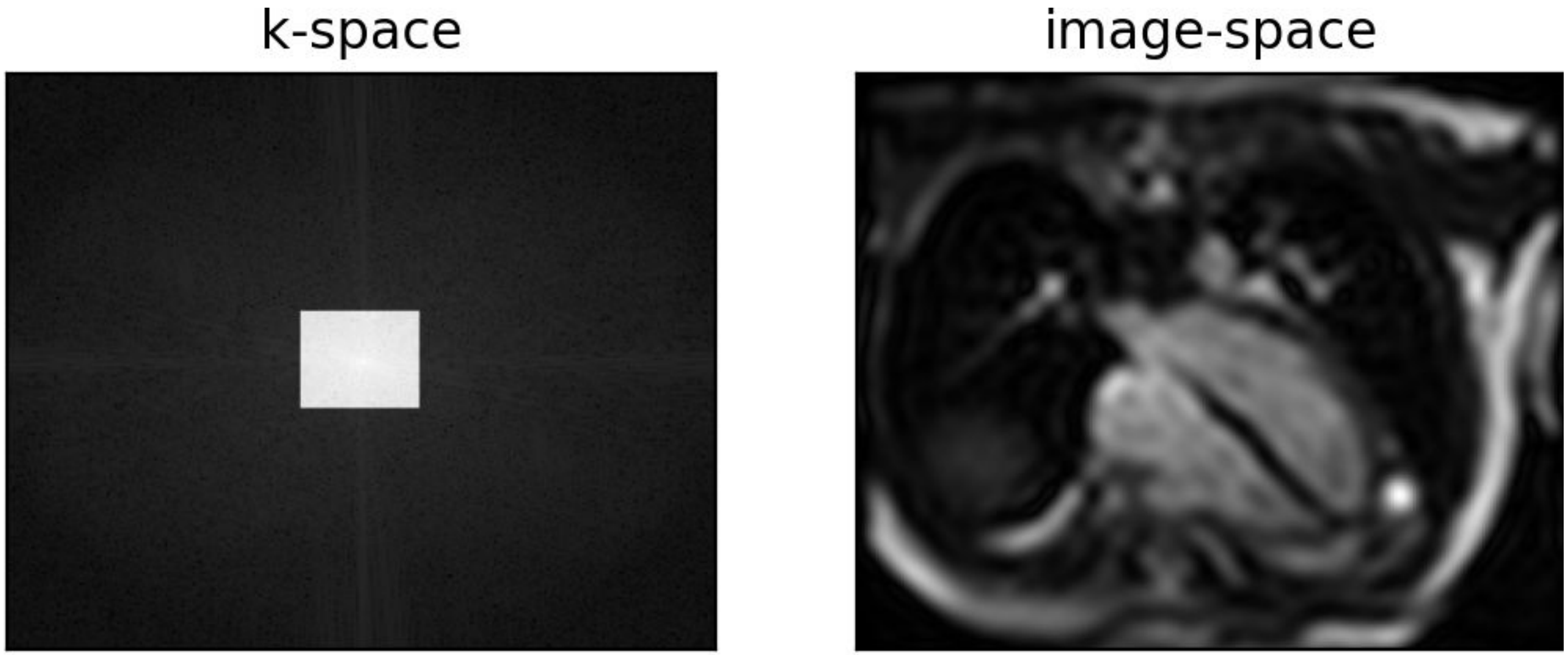
K-SPACE



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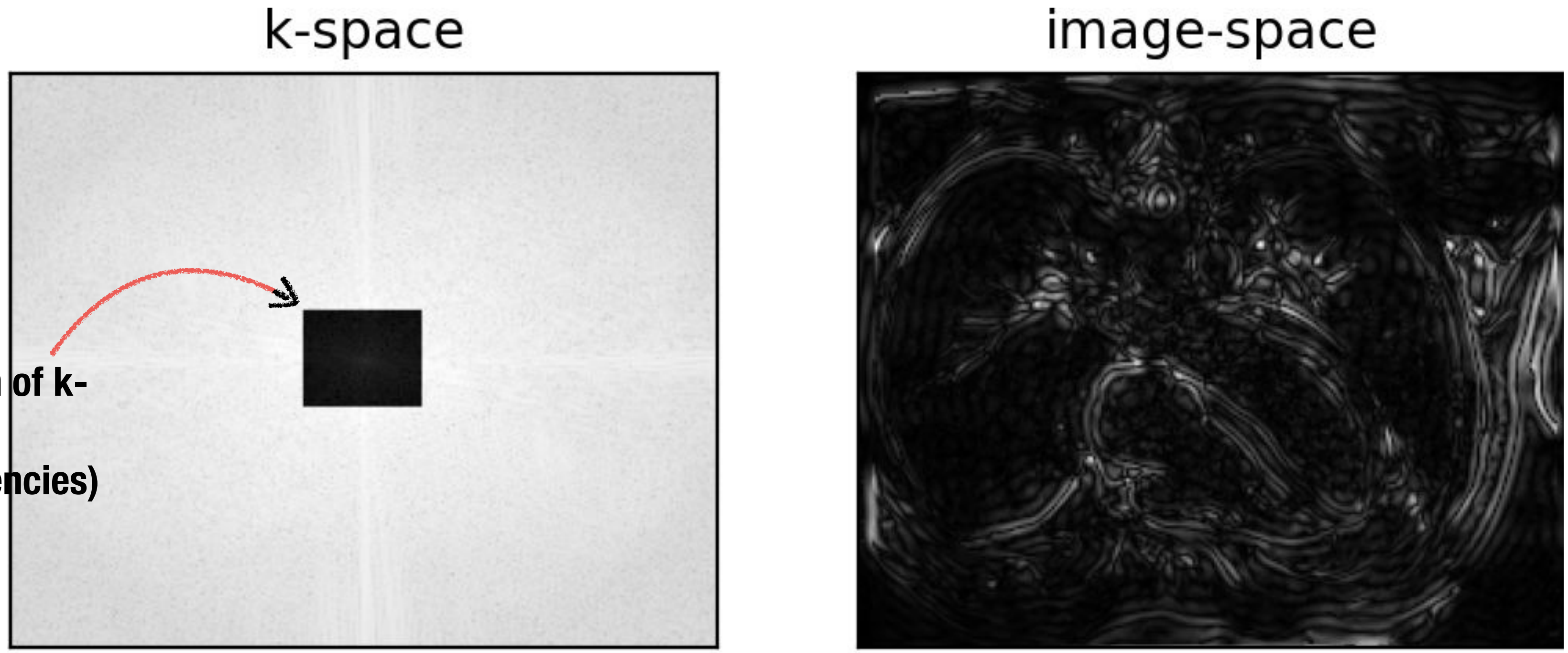
K-SPACE



outer region of k-space
(high frequencies)

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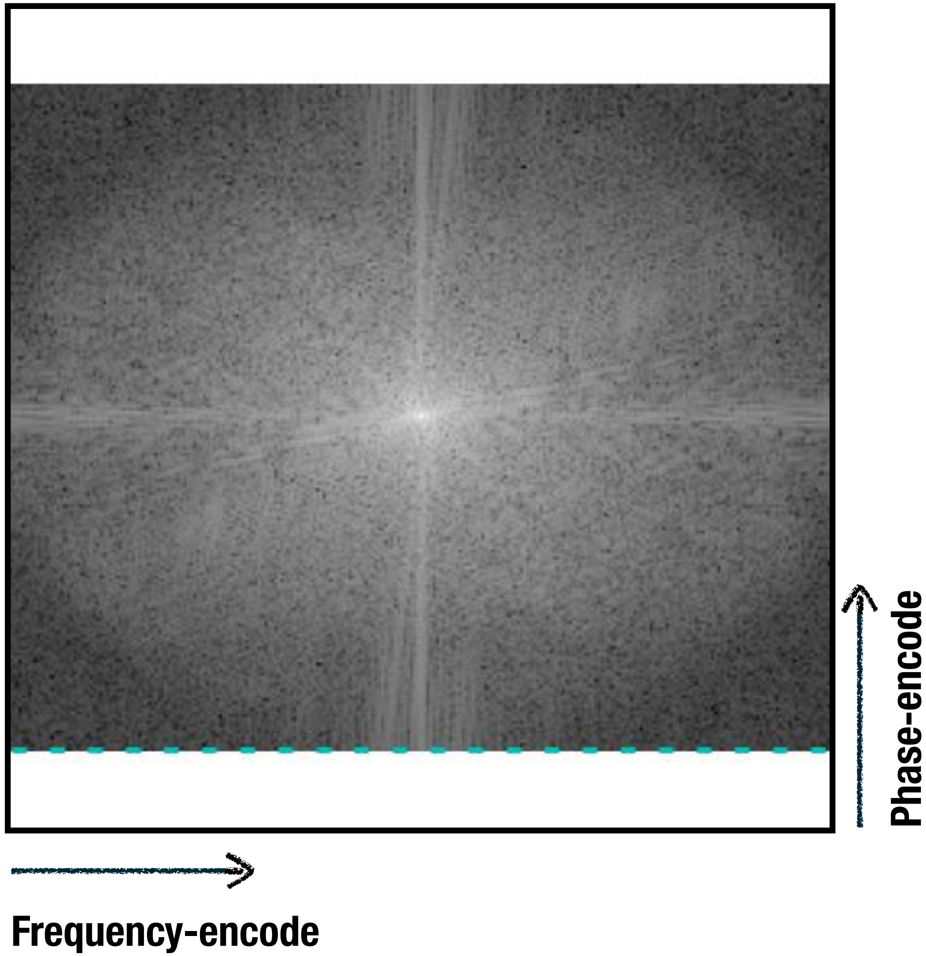
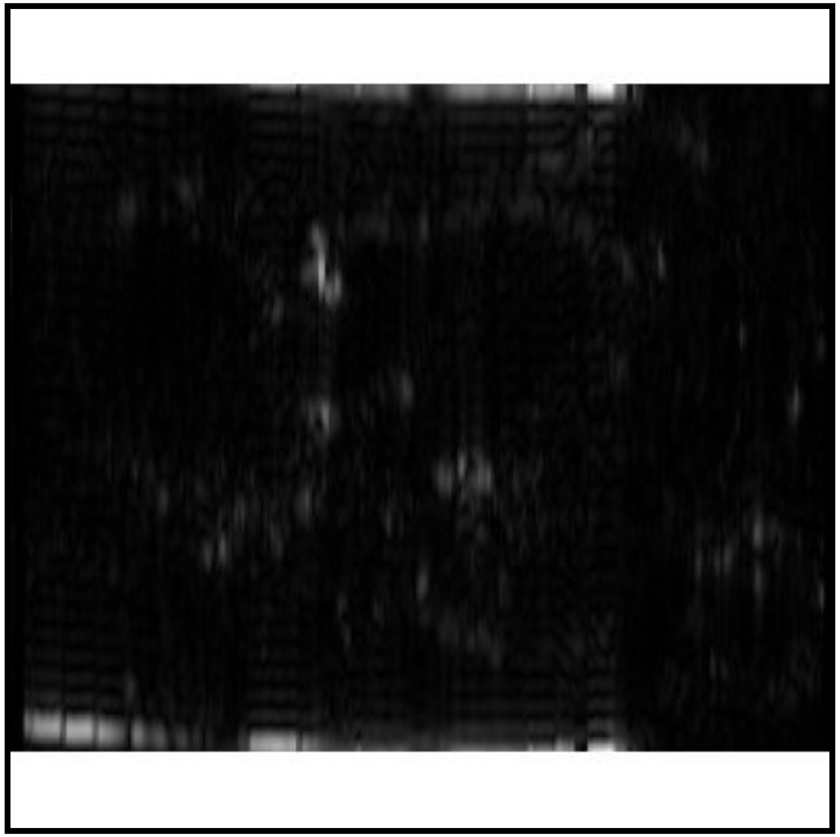
K-SPACE :: CARTESIAN SAMPLING





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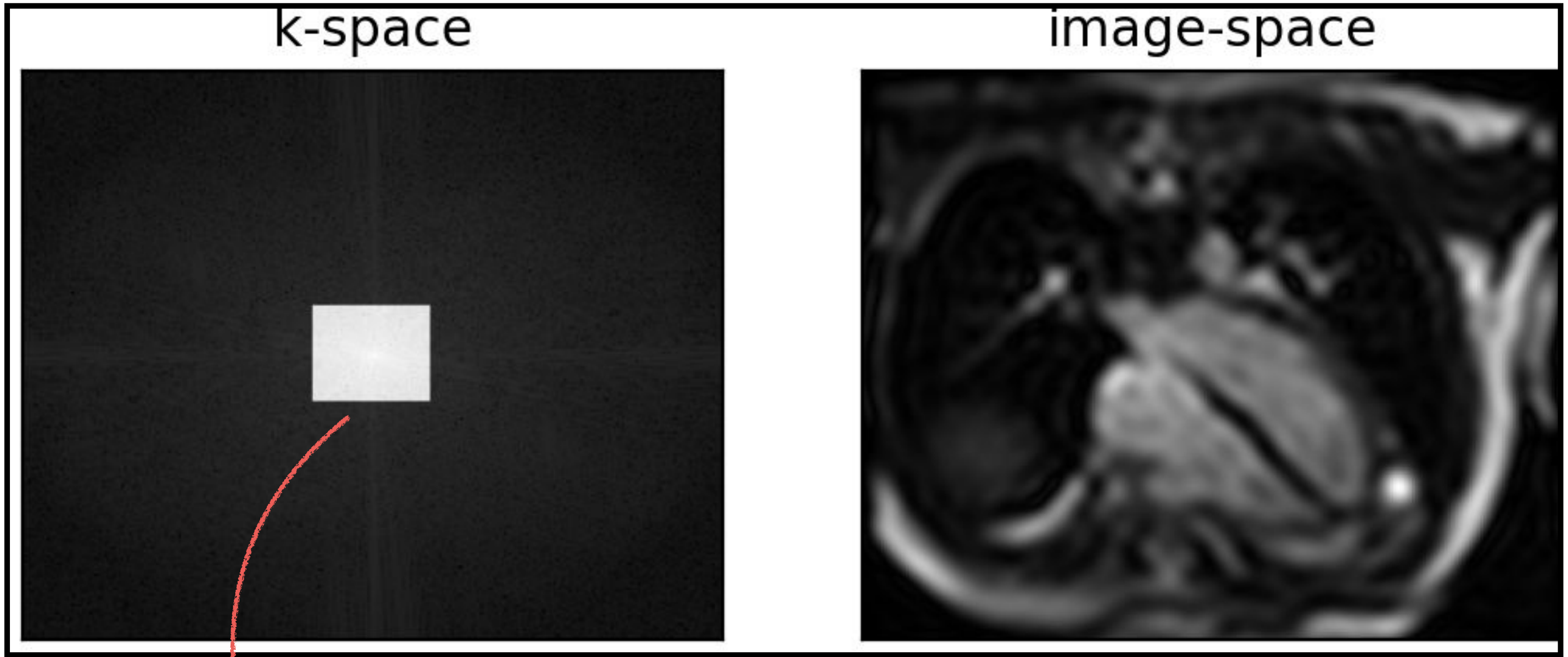
K-SPACE



k-space region covered by mask: 2.8%
k-space signal covered by mask: 38.0%

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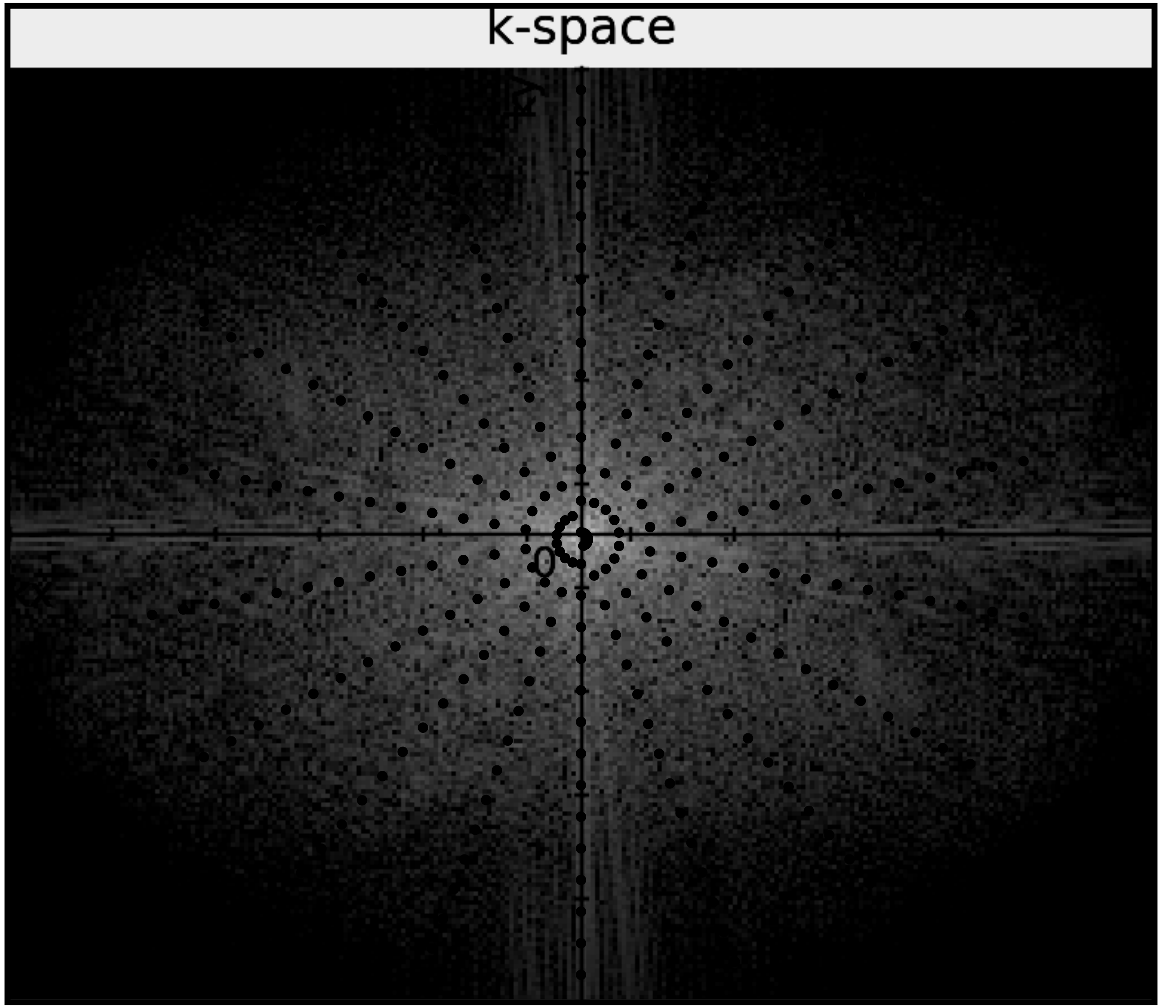
K-SPACE :: RADIAL ACQUISITION



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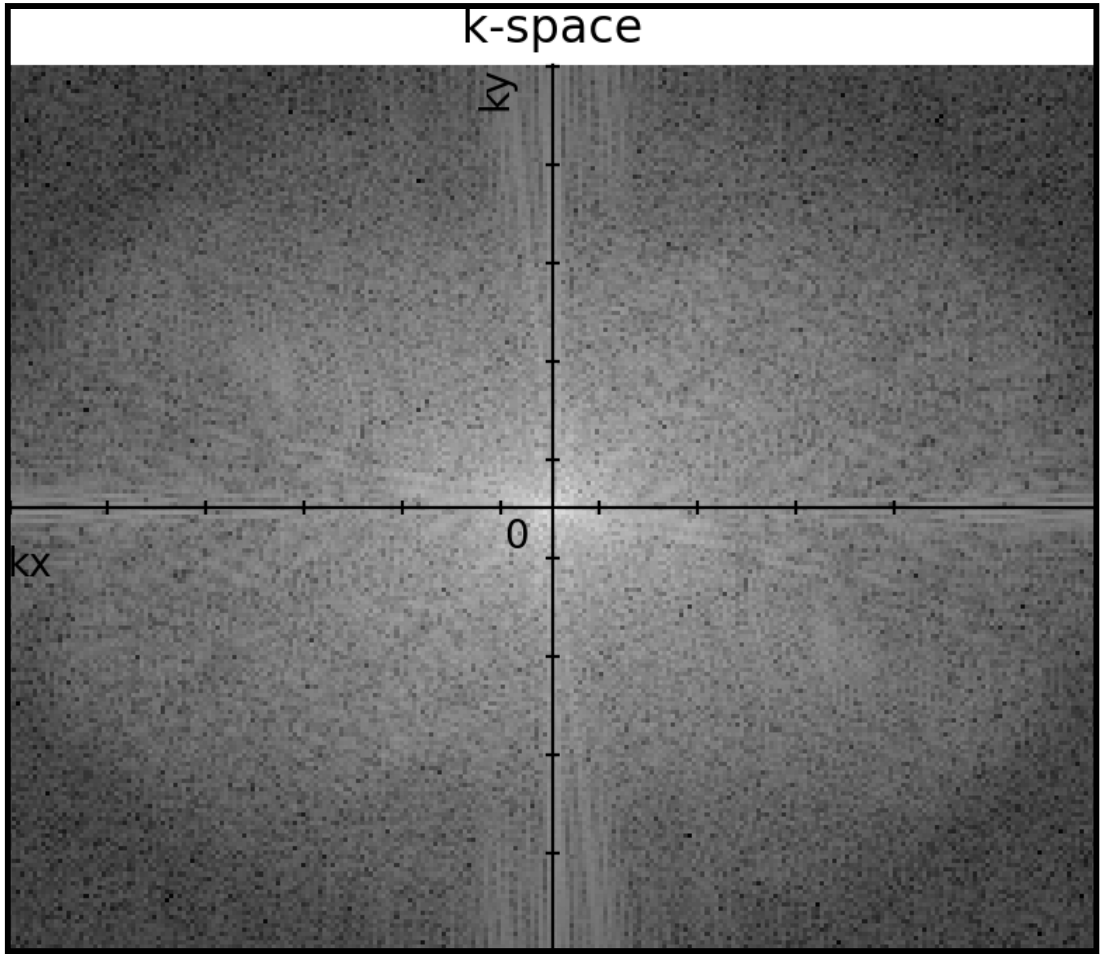
K-SPACE :: SAMPLING ARTEFACTS

k-space sampling artefacts:

- **finite sampling**
- **discrete sampling**



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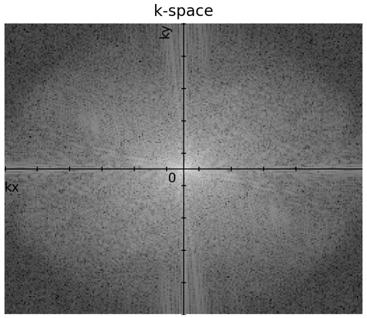
K-SPACE :: SAMPLING ARTEFACTS



...

ideal infinite continuous k-space

$\times$

$$H_{ws}(k) \equiv \mathrm{rect}\left(\frac{k + \frac{1}{2}\Delta k}{W}\right) \Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)$$

k-space filter

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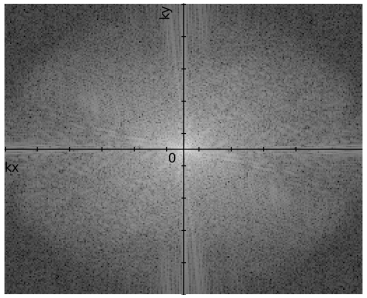
K-SPACE :: SAMPLING ARTEFACTS



$\times$

$$H_{ws}(k) \equiv \underbrace{\mathrm{rect}\left(\frac{k+\frac{1}{2}\Delta k}{W}\right)}_{\text{finite k-space}} \underbrace{\Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)}_{\text{discrete sampling}}$$

k-space filter

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FOURIER TRANSFORM MATHS

$$\mathscr{F}^{-1}(H(k) \times G(k)) = h(x) * g(x)$$

FT of the product of two functions is the convolution of the FT of each function

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FOURIER TRANSFORM MATHS

$$\mathscr{F}^{-1}(H(k) \times G(k)) = h(x) * g(x)$$

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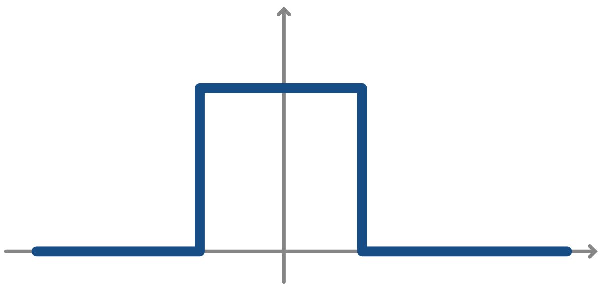
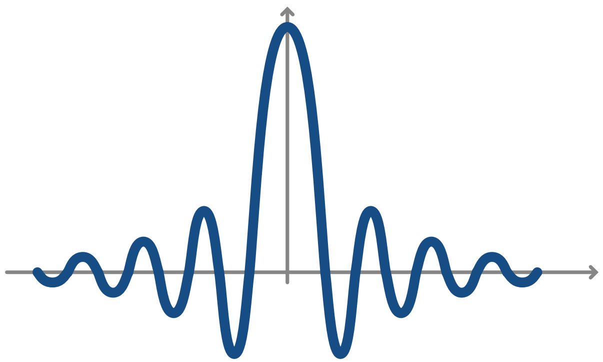
$$\text{rect}\left(\frac{x}{W}\right) <= \mathscr{F} => W\frac{\sin(\pi W k)}{\pi W k}$$

Fourier transform pair: Rectangular function & sinc function





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FOURIER TRANSFORM MATHS

$$\mathscr{F}^{-1}(H(k) \times G(k)) = h(x) * g(x)$$

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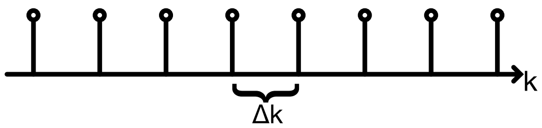
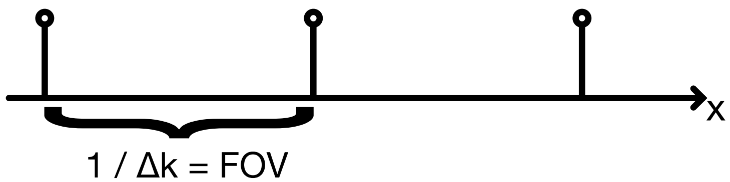
$$\mathrm{rect}\left(\frac{x}{W}\right) <= \mathscr{F} => W\frac{\sin(\pi W k)}{\pi W k}$$

Fourier transform pair: Rectangular function & sinc function

$$\mathrm{comb}\left(\Delta k\right) <= \mathscr{F} => \mathrm{comb}\left(\frac{1}{\Delta k}\right)$$

Fourier transform pair: comb function





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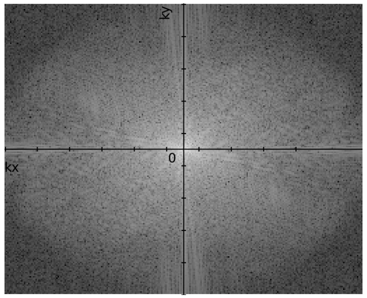
K-SPACE SAMPLING ARTEFACTS

k-space



$\times$

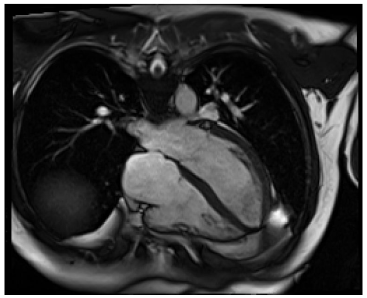
$$H_{ws}(k) \equiv \underbrace{\mathrm{rect}\left(\frac{k+\frac{1}{2}\Delta k}{W}\right)}_{\textbf{finite k-space}} \underbrace{\Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)}_{\textbf{discrete sampling}}$$

k-space filter

$\mathcal{F}^{-1}$

$\mathcal{F}^{-1}$

image-space



$*$

$h_{ws}(x)$

point spread function

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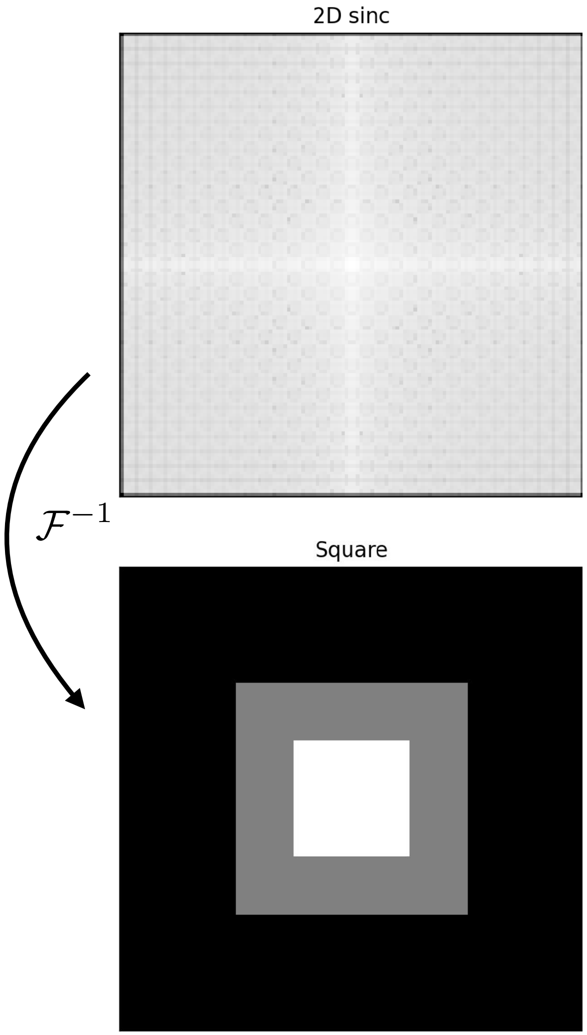
K-SPACE :: DATA TRUNCATION



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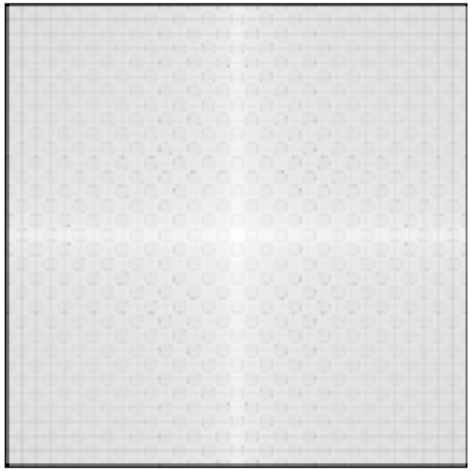
K-SPACE :: DATA TRUNCATION

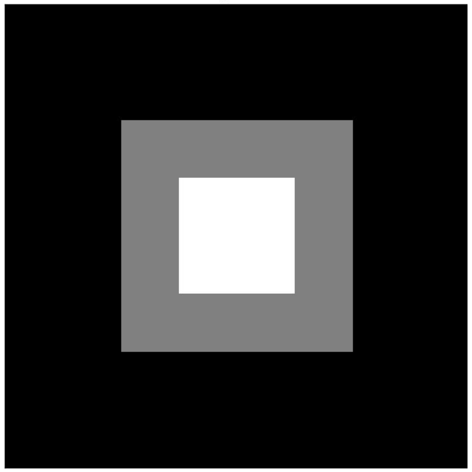
2D sinc



$$\times \, \mathrm{rect}\left(\frac{k + \frac{1}{2}\Delta k}{W}\right)$$

$\mathcal{F}^{-1}$

Square



$$* \, W \mathrm{sinc}(\pi W x) \; =$$

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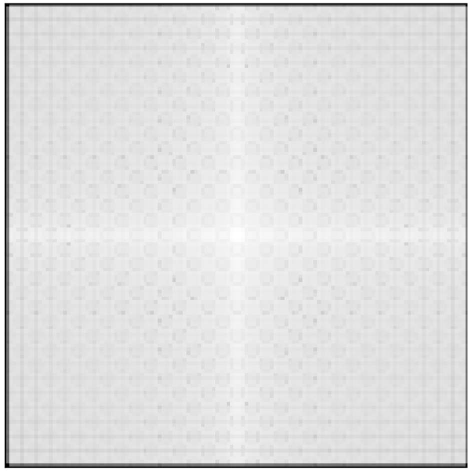
K-SPACE :: DATA TRUNCATION

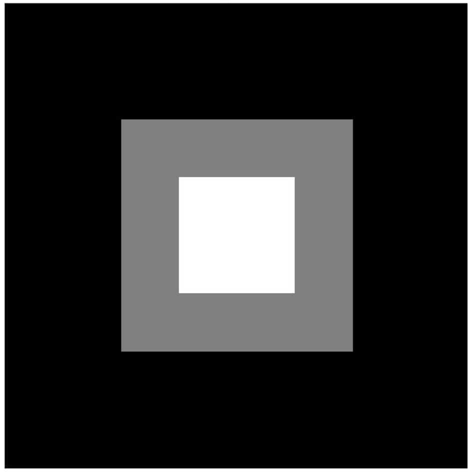
2D sinc



$$\times \mathrm{rect}\left(\frac{k+\frac{1}{2}\Delta k}{W}\right)$$

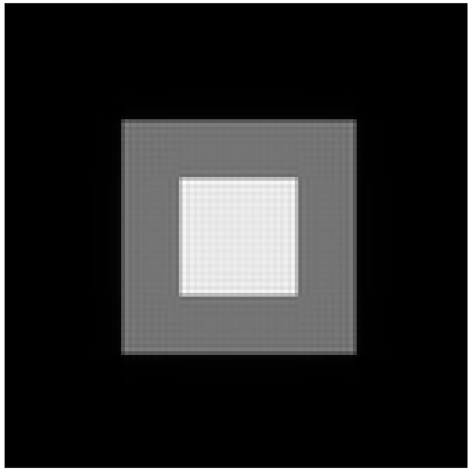
$\mathcal{F}^{-1}$

Square



$$* \, W \mathrm{sinc}(\pi W x) \; =$$

Square with Gibbs artefacts



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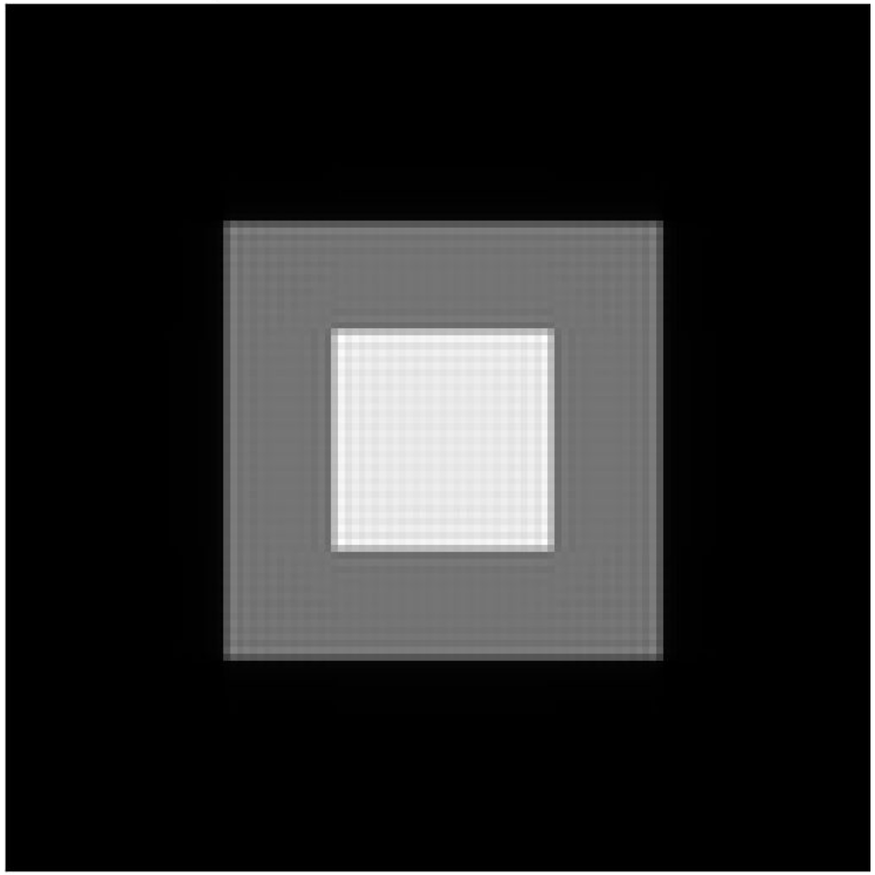
K-SPACE :: DATA TRUNCATION

Square with Gibbs artefacts



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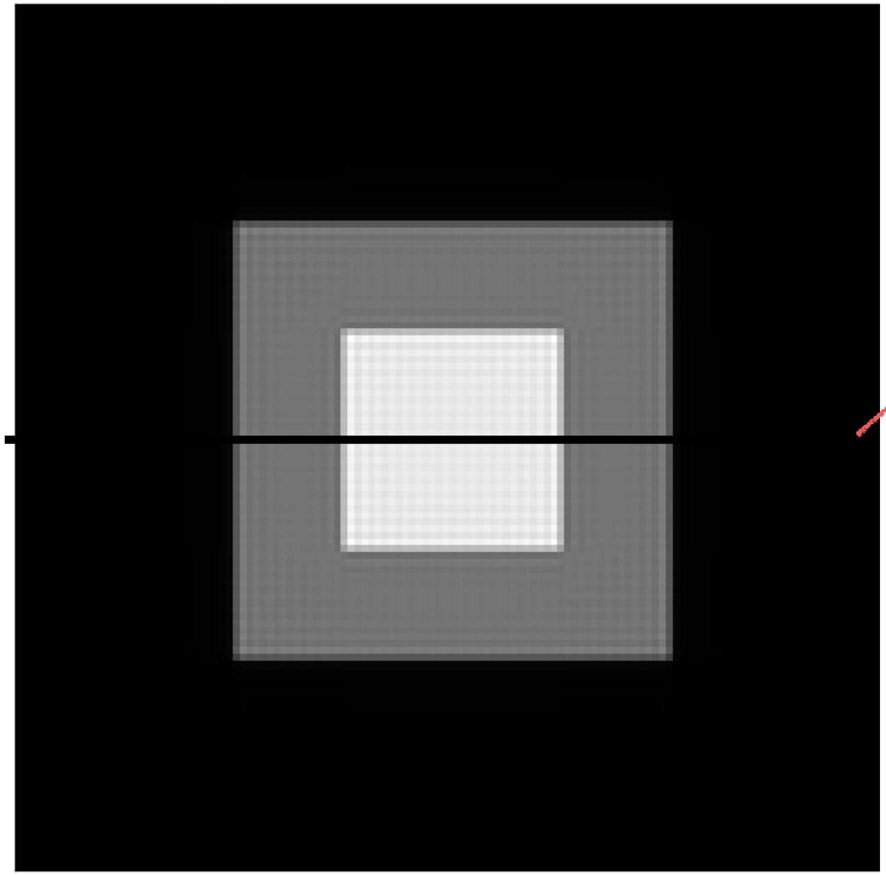

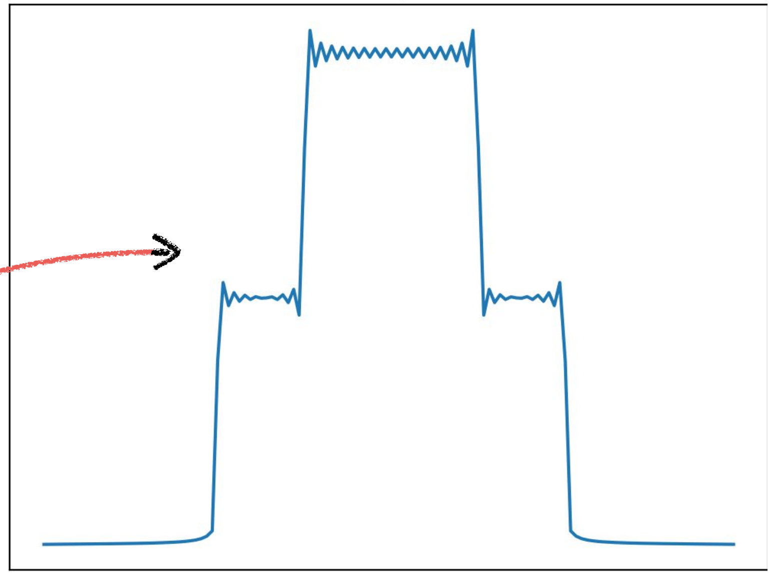

- **First overshoot and undershoot approx 9% of the signal jump**

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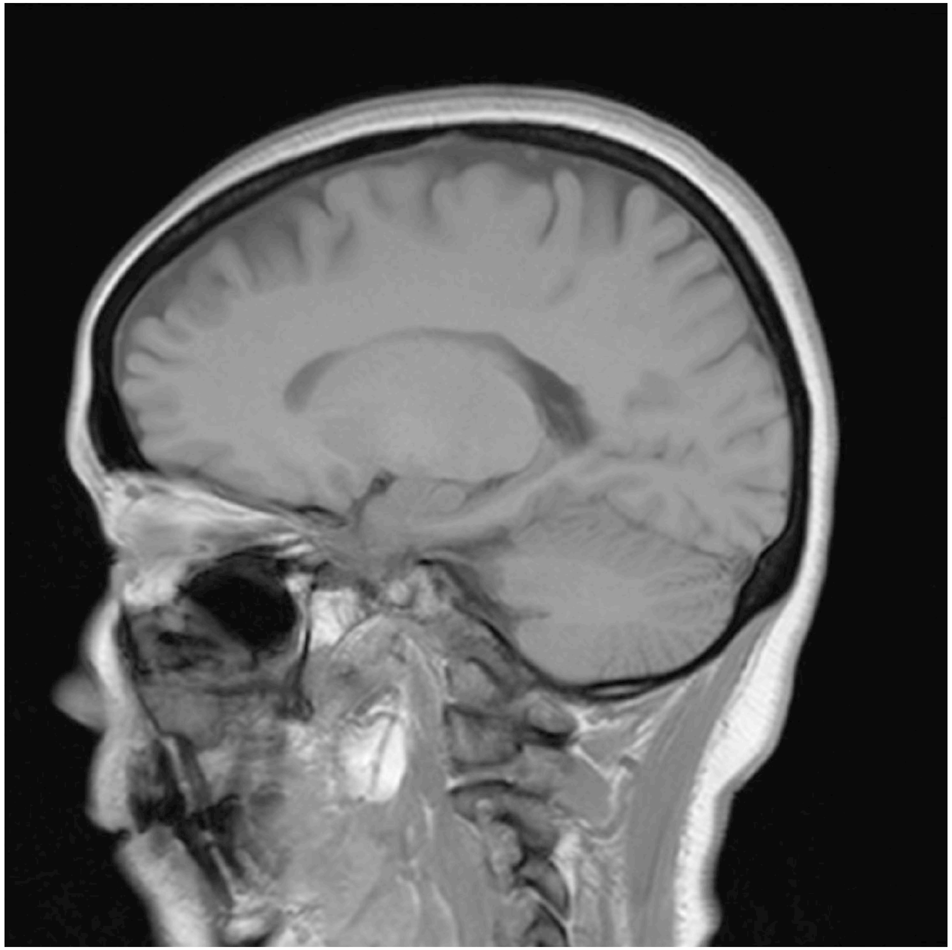
K-SPACE SAMPLING ARTEFACTS :: GIBBS



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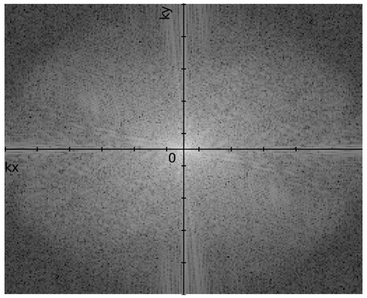
K-SPACE SAMPLING ARTEFACTS

k-space



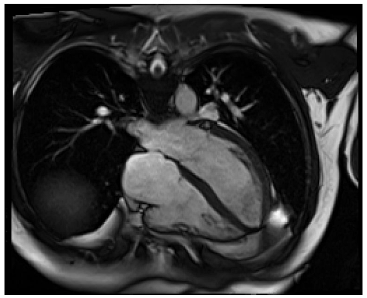
$\times$ $$H_{ws}(k) \equiv \underbrace{\mathrm{rect}\left(\frac{k+\frac{1}{2}\Delta k}{W}\right)}_{\text{finite k-space}} \underbrace{\Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)}_{\text{discrete sampling}}$$

k-space filter

Gibbs ringing

$\mathcal{F}^{-1}$ $\mathcal{F}^{-1}$

image-space



$*$ $h_{ws}(x)$

point spread function

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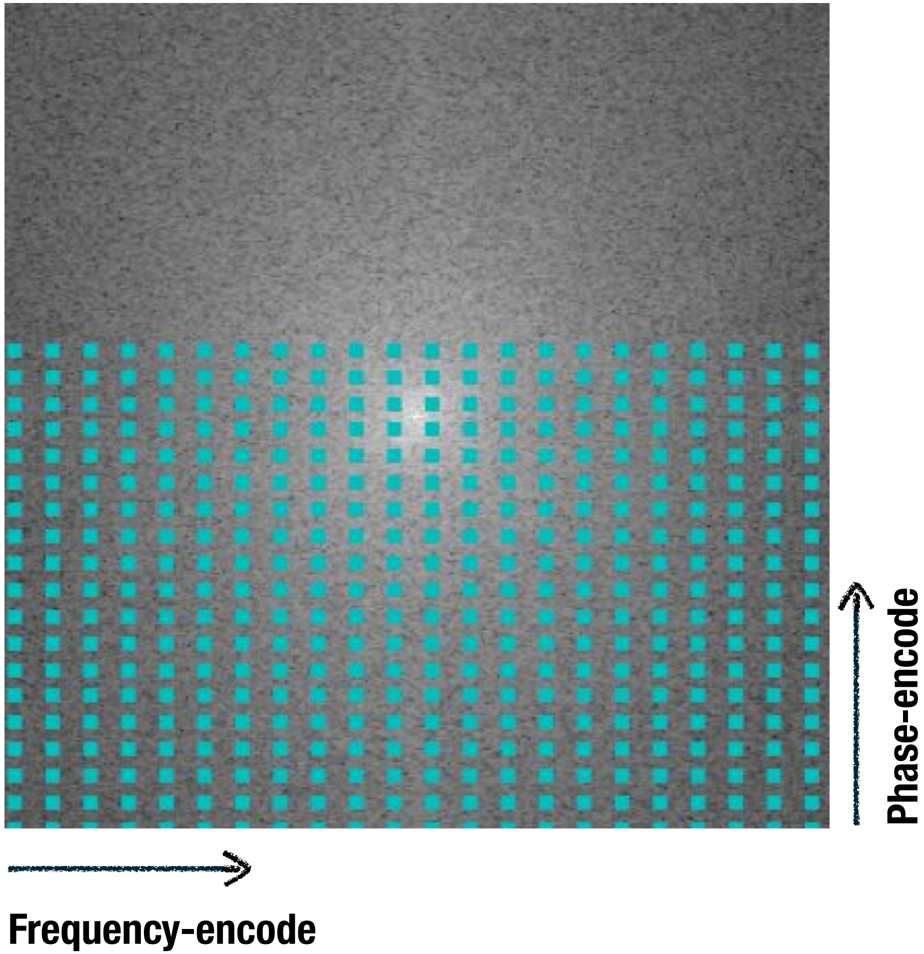
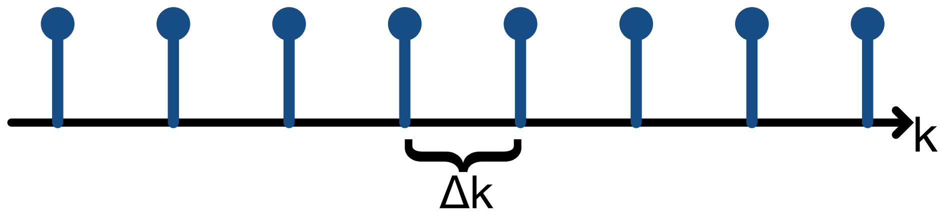
K-SPACE :: DISCRETE SAMPLING





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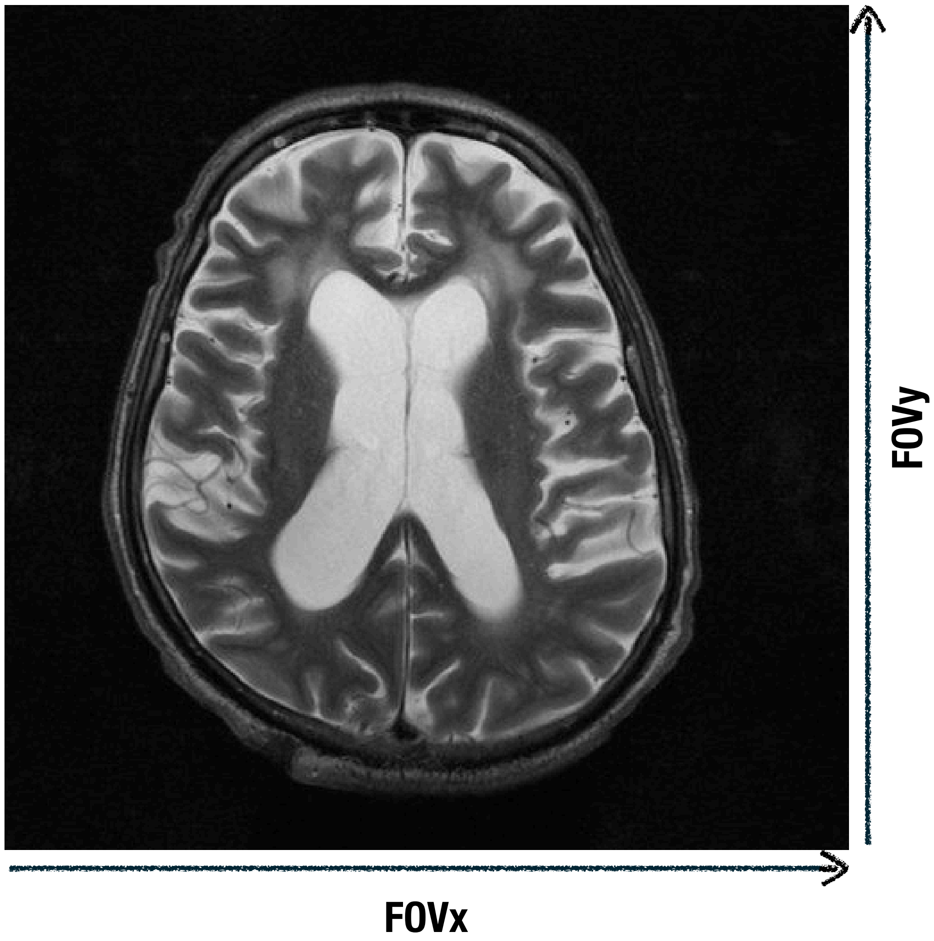

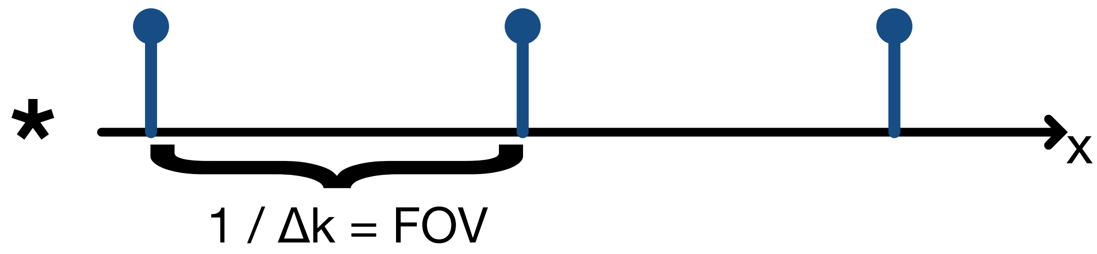

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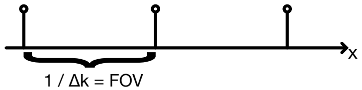
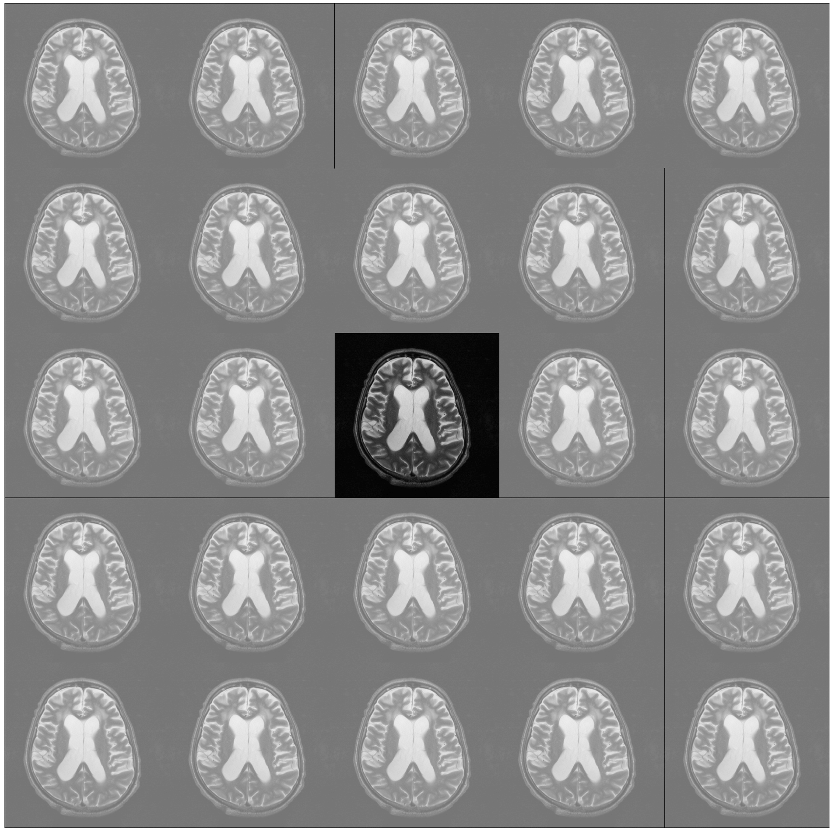
K-SPACE :: DISCRETE SAMPLING

*





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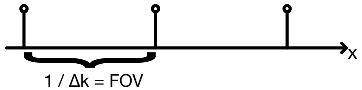
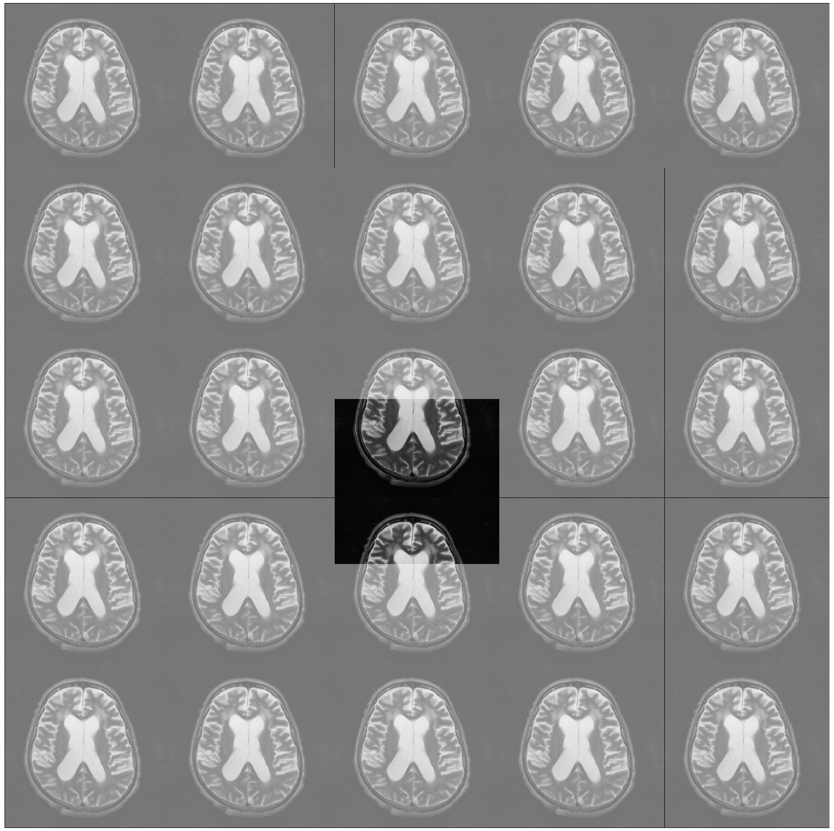
K-SPACE :: DISCRETE SAMPLING

$*$





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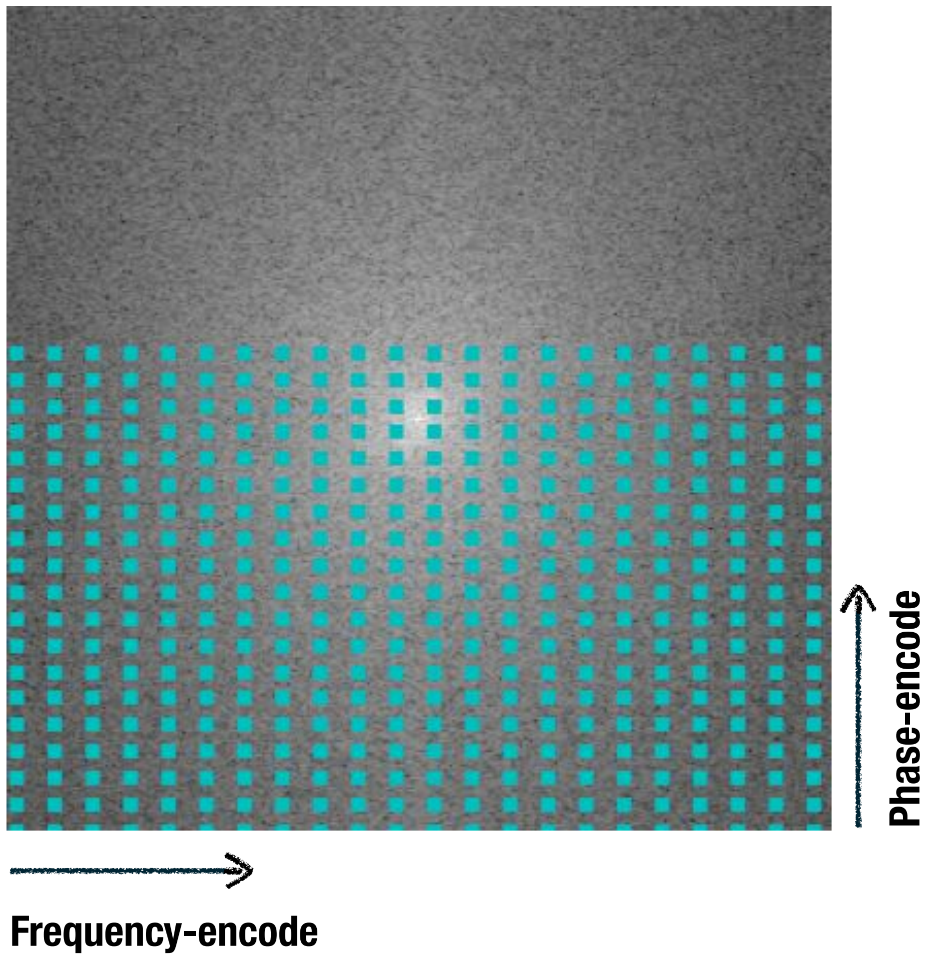

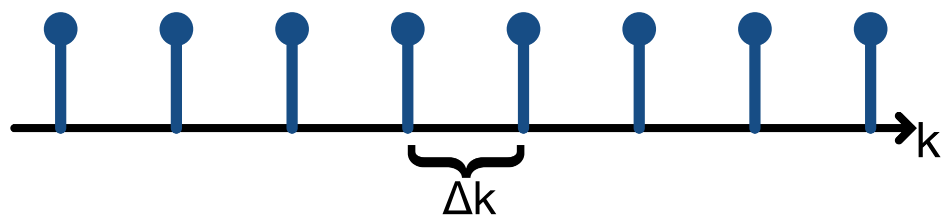

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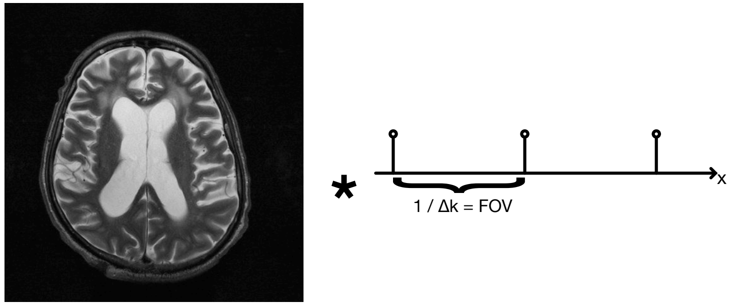

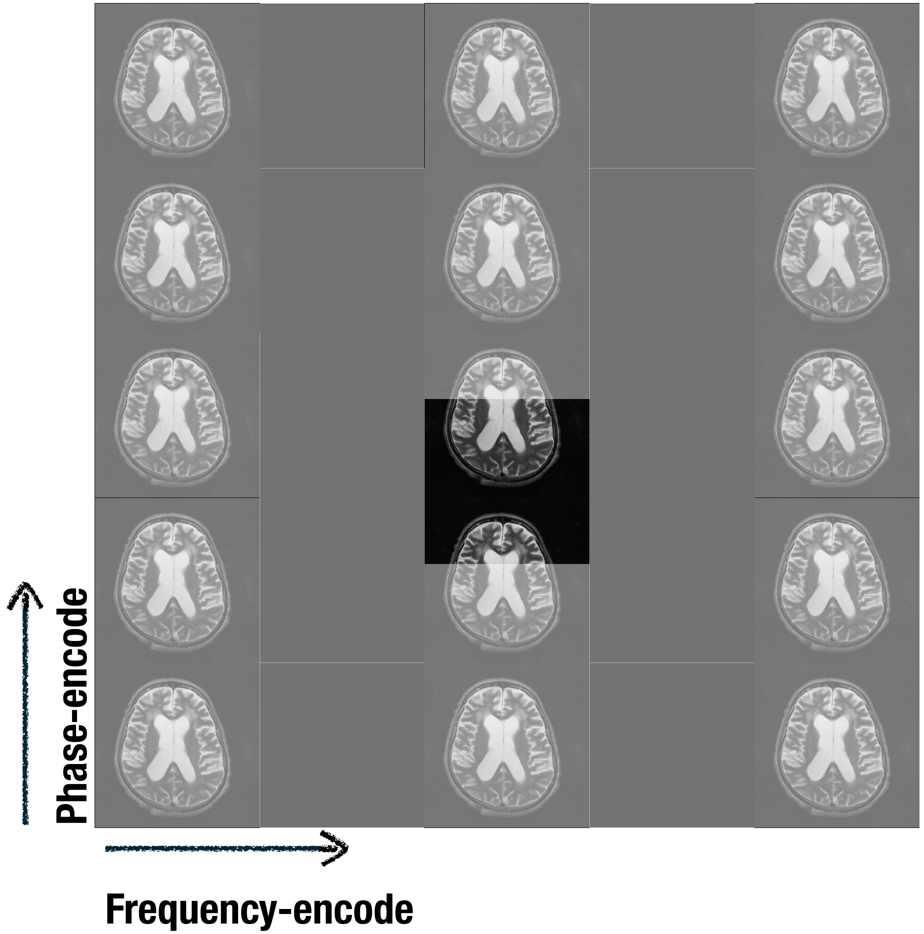

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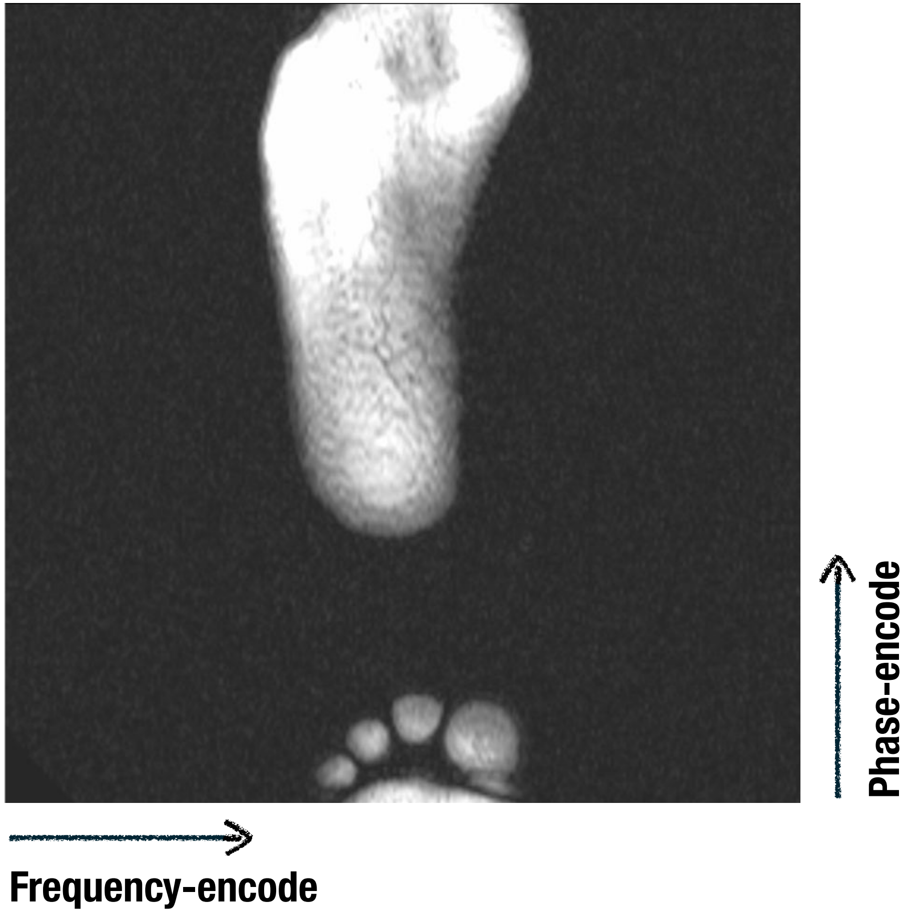
Wrap-around artefact



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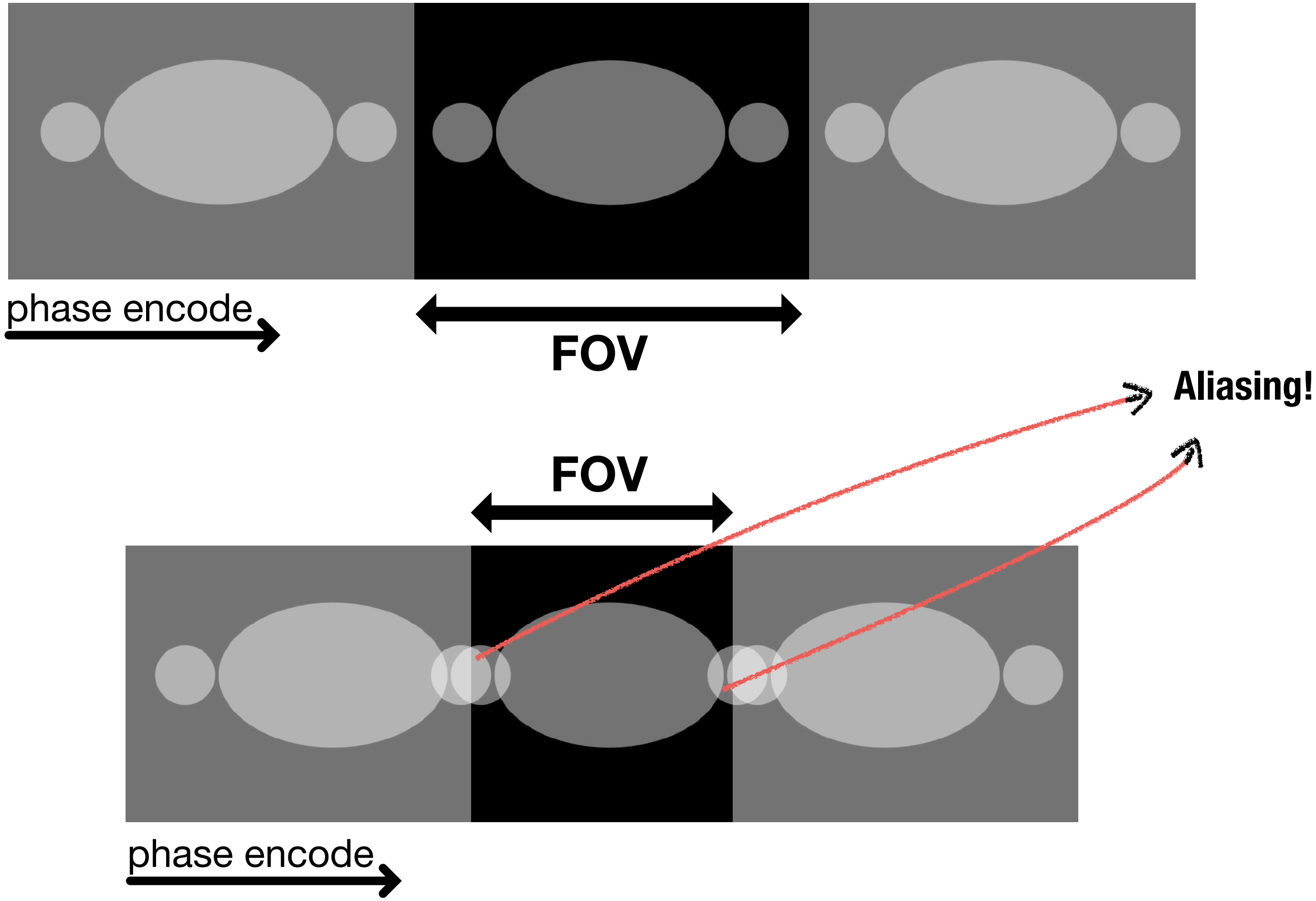
Aliasing artefact



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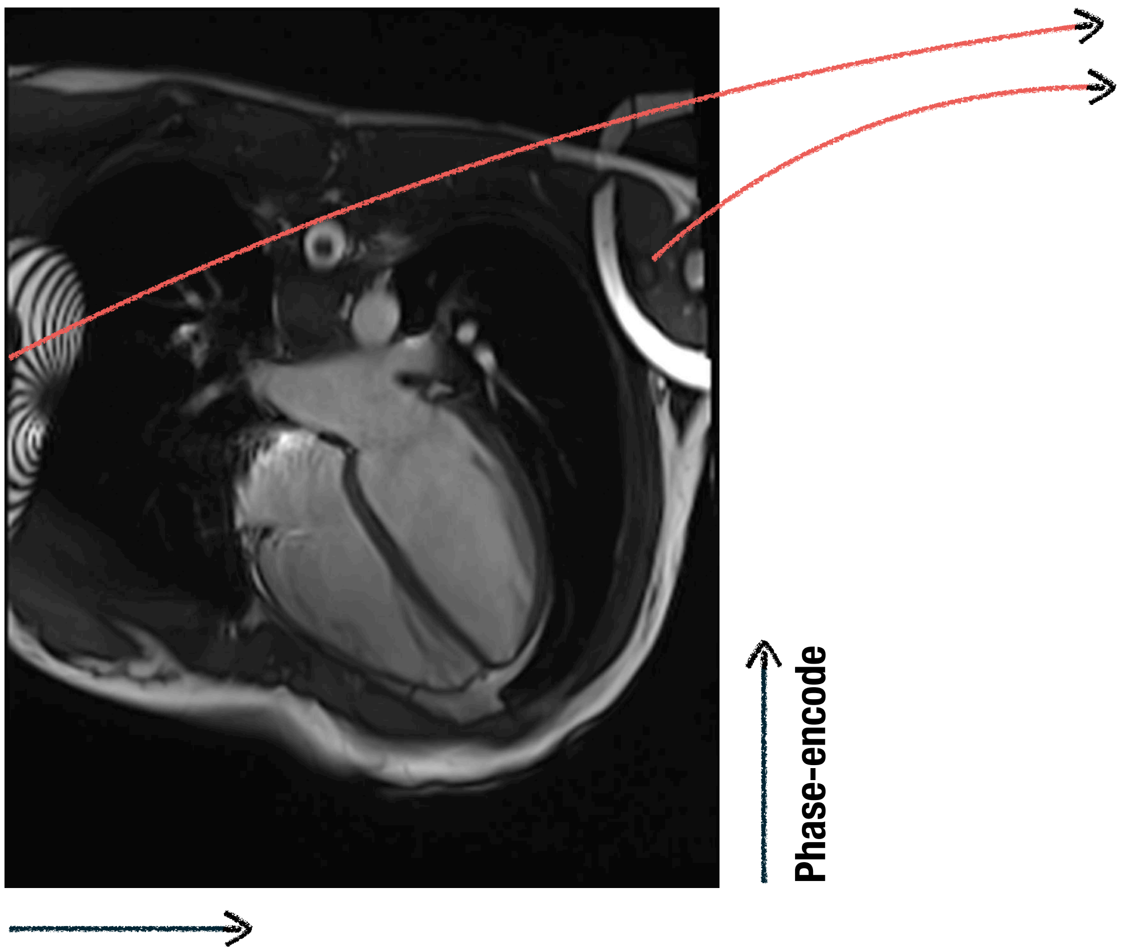
Aliasing artefact



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SUMMARY

- MRI raw-signal is known as k-space
  - It is in the frequency domain.
  - It allows for clever k-space under sampling tricks.
  - But its sampling creates image artefacts.

THANK YOU

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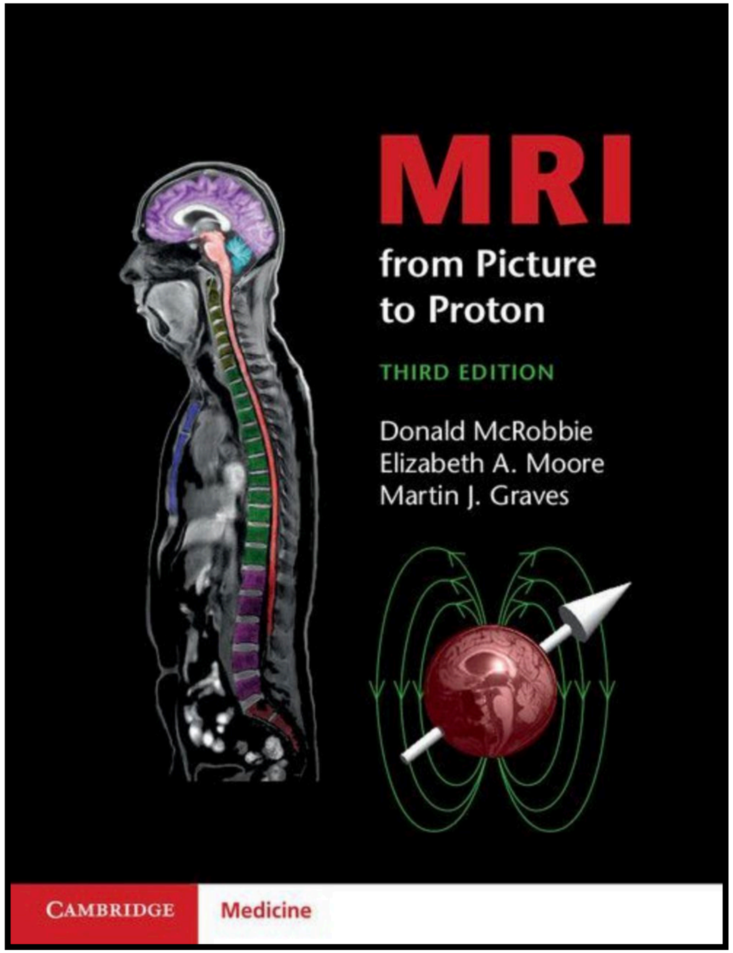
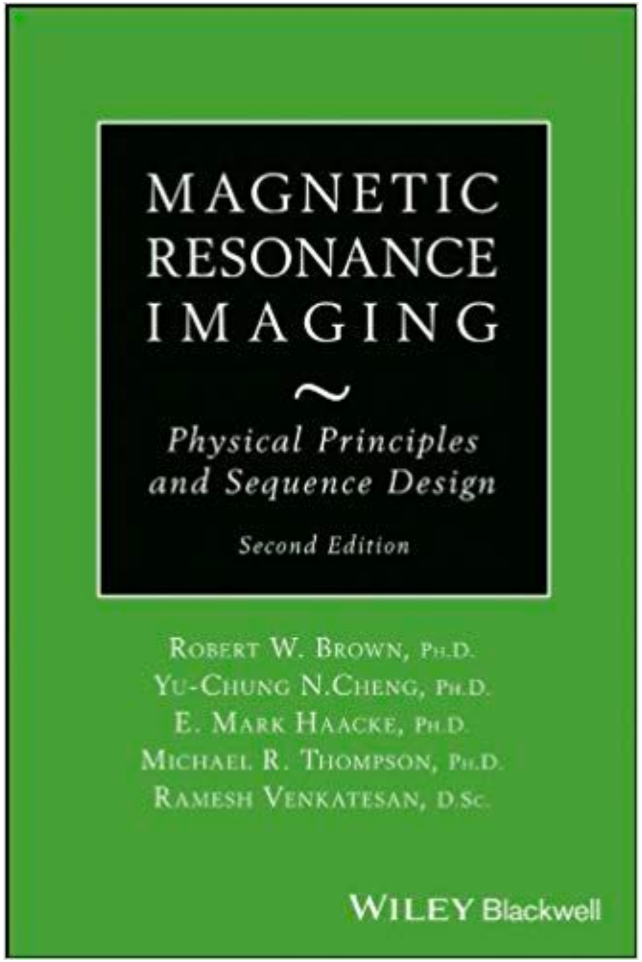
LITERATURE





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MATERIAL

- **Github:**
  - **Jupyter notebook** `https://github.com/Pedro-Filipe/k-space_simulations`
  - **Slides**

THANK YOU

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