Atividade Semana 03 - 03 B

Nise - Capítulo 2

Exercício 1 -

a) $u(t)$: $F(s) = \int_0^{\infty} e^{-st} \cdot f(t)\, dt = \int_0^{\infty} e^{-st} \cdot 1\, dt$

$$F(s) = \left.\frac{e^{-st}}{-s}\right|_0^{\infty} = \left(\frac{e^{-s.\infty}}{-s}\right) - \left(\frac{e^{-s.0}}{-s}\right) = \frac{1}{s}$$

$$\boxed{\mathcal{L}\{u(t)\} = F(s) = \frac{1}{s}}$$

b) $t.u(t)$: $F(s) = \int_0^{\infty} e^{-st} \cdot f(t)\, dt = \int_0^{\infty} e^{-st} \cdot t\, dt$

Por partes: $\int f g' = f.g - \int f' g$ com $f = x \mid g = -\frac{e^{-st}}{s}$

$f' = 1 \mid g' = + e^{-st}$

Assim:

$$= -\frac{t \cdot e^{-st}}{s} + \int_0^{\infty} \frac{e^{-st}}{s} dt$$

substituindo $u = -st \rightarrow dt = -\frac{1}{s} du$

$$= -\frac{t \cdot e^{-st}}{s} + \left(-\frac{1}{s^2}\int e^u du\right) = -\frac{t \cdot e^{st}}{s} + \left(-\frac{e^{-st}}{s^2}\right)$$

$$= \left.\frac{t \cdot e^{-st}}{s} - \frac{e^{-st}}{s^2}\right|_0^{\infty} = \left[(0 - 0) - \left(0 - \frac{1}{s^2}\right)\right]$$

$$\boxed{\mathcal{L}\{t\, u(t)\} = \frac{1}{s^2}}$$

Exercício ① - Continuação:

c) $\sin \omega t \, u(t)$

$$F(s) = \int_0^{\infty} \sin \omega t \, e^{-st} \, dt \rightarrow \int f g' = fg - \int f' g$$

$$f = \sin(\omega t) \,\Big|\, g = \frac{e^{-st}}{s}$$

$$f' = \omega \cos(\omega t) \,\Big|\, g' = e^{-st}$$

$$1^{\circ} = \frac{e^{-st} \sin(\omega t)}{s} - \int - \frac{\omega e^{-st} \cos(\omega t)}{s} \, dt$$

$$2^{\circ} = \frac{e^{-st} \sin(\omega t)}{s} \left(\frac{\omega e^{-st} \cos(\omega t)}{s^2} - \int - \frac{\omega^2 e^{-st} \sin(\omega t)}{s^2} \right.$$

$$= \frac{e^{-st} \sin(\omega t)}{s} - \left(\frac{\omega e^{-st} \cos(\omega t)}{s^2} + \frac{\omega^2}{s^2} \int e^{-st} \sin(\omega t) \, dt \right.$$

$$= \left. \frac{-s e^{-st} \sin(\omega t) - \omega e^{-st} \cos(\omega t)}{\omega^2 + s^2} \right|_0^{\infty}$$

$$F(s) = \frac{\omega}{\omega^2 + s^2}$$

d) $\cos \omega t \, u(t)$:

De maneira similar:

$$F(s) = \int_0^{\infty} \cos \omega t \, e^{-st} \, dt = \frac{s}{s^2 + \omega^2}$$

Exercicio 2 -

a) Usando teorema da frequencia e Transformada de Laplace do sen wt:

$$F(s) = \frac{w}{(s+a)^2 + w^2}$$

b) Semelhantemente:

$$\frac{(s+a)}{(s+a)^2 + w^2}$$

c) Usando integração sucessiva

$$\int dt = t \rightarrow \int t\,dt = \frac{t^2}{2} \rightarrow \int \frac{t^2}{2}dt = \frac{t^3}{6}$$

$$\mathcal{L}\{t^3 u(t)\} = \frac{6}{s^4}$$

3 - Exercicio Referente ao c, e, p d?

4 - :

8 - Um sistema descrito por:

$$\frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + \frac{5dy}{dt} + y = \frac{d^3x}{dt^3} + \frac{4d^2x}{dt^2} + \frac{6dx}{dt} + 8x$$

Encontre a função de transferência $\frac{Y(s)}{X(s)}$:

Questão (8) Resolução:

Laplace:

$$\mathcal{L}\left\{ \frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + \frac{5dy}{dt} + y = \frac{d^3x}{dt^3} + \frac{4d^2x}{dt^2} + \frac{6dx}{dt} + 8x \right\}$$

$$s^3y(s) + 3s^2y(s) + 5sy(s) + y(s) = s^3x(s) + 4s^2x(s) + 6\ldots$$

Evidência:

$$y(s)(s^3 + 3s^2 + 5s + 1) = X(s)(s^3 + 4s^2 + 6s + 8)$$

$$\boxed{\frac{y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}}$$

Questão (9):

a) $$\frac{X(s)}{F(s)} = \frac{7}{s^2 + 5s + 10} \rightarrow 7F(s) = X(s)(s^2 + 5s + 10)$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 10x = 7f}$$

b) $$\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)} = X(s)[s^2 + 21s + 110] = 15F(s)$$

$$\boxed{\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 110x = 15f}$$

Continuação (8).

Letra c) $\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18}$

$X(s)(s^3+11s^2+12s+18) = F(s)(s+3)$

$$\frac{d^3x}{dt^3} + 11\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 18x = \frac{df}{dt} + 3f$$

Questão (10):

$$R(s) \rightarrow \frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+5} \rightarrow C(s)$$

$\frac{C(s)}{R(s)} =$

Assim:

$$\boxed{\frac{d^6c}{dt^6} + 7\frac{d^5c}{dt^5} + 3\frac{d^4c}{dt^4} + 2\frac{d^3c}{dt^3} + \frac{d^2c}{dt^2} + 5c = \frac{d^5R}{dt^5} + 2\frac{d^4R}{dt^4} + 4\frac{d^3R}{dt^3} + \frac{d^2R}{dt} + 4R}$$

Questão (11): $R(s) \rightarrow \underbrace{\frac{s^4+3s^3+2s^2+s+1}{s^5+4s^4+3s^3+2s^2+3s+2}}_{G(t)} \rightarrow C(s)$

$\frac{C(s)}{R(s)} = G(t)$

Assim: $\frac{d^5c}{dt^5} + 3\frac{d^4c}{dt^4} + 2\frac{d^3c}{dt^3} + 4\frac{d^2c}{dt^2} + 5\frac{dc}{dt} + 2c = \frac{d^4r}{dt^4} + 2\frac{d^3r}{dt^3} + 5\frac{dr^2}{dt^2} + \frac{dr}{dt} + r$

Continua →

Continuação questão (11):

Substituindo $r(t) = t^3$:

$$\frac{d^5c}{dt^5} + \frac{3d^4c}{dt^4} + \frac{2d^3c}{dt^3} + \frac{4d^2c}{dt^2} + \frac{5dt}{dt} + 2c = \underbrace{18K(t)}_{\text{substituição}} + 4t(36+90t+9t^2+3t^3)$$

Questão (12):

$$F = \frac{d^2x}{dt^2} + \frac{4dx}{dt} + 5x = 1$$

$x(0) = 1$ e, $\dot{x}(0) = -1$.

$$\mathcal{L}\{F\} = X(s)\,(s^2 + 4s + 5) = g(s)$$

$$\frac{d^2x}{dt^2} + 4.1 + 5.(-1) = 1$$

$$\ddot{x} = 2$$

Exercícios Franklin ~~K~~ Capítulo 3.

3.2 : b) $f(t) = 3 + 7t + t^2 + \delta(t)$

$\mathcal{L}\{f(t)\} = \frac{3}{s} + 7\frac{1}{s^2} + \frac{2}{s^3} + \frac{1}{s}$

d) $f(t) = (t+1)^2 = t^2 + 2t + 1$

$\mathcal{L}\{f(t)\} = \frac{2}{s^3} + 2\frac{1}{s^2} + \frac{1}{s}$

e) $f(t) = \text{senh}\, t \rightarrow \text{senh}(at)$

$\mathcal{L}\{f(t)\} = \frac{a}{s^2 - a^2} \quad \{ a = 1$

$\mathcal{L}\{f(t)\} = \frac{1}{s^2 - 1}$ //

$\text{Sinh}\, at = \left\{ \frac{e^{at} - e^{-at}}{2} \right\}$

$\frac{1}{2} \mathcal{L}\{e^{at}\} - \mathcal{L}\{e^{-at}\}$

$\frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right) = \frac{a}{s^2 - a}$ //

Com $a = 1$

$\mathcal{L}\{f(t)\} = \frac{1}{s^2 - 1}$ //

3.3 :

a) $f(t) = 3\cos 6t$

$\mathcal{L}\{f(t)\} = 3\,\mathcal{L}\{\cos 6t\}$

$\mathcal{L}\{f(t)\} = 3\left(\frac{s}{s^2+6^2}\right) = \frac{3s}{s^2+36}$

b) $f(t) = \operatorname{sen}(2t) + 2\cos(2t) + e^{-t}\operatorname{sen} 2t$

$\mathcal{L}\{f(t)\} = \frac{2}{s^2+4} + \frac{2s}{s^2+4} + \frac{2}{(s+1)^2+4}$

c) $f(t) = t^2 + e^{-2t}\operatorname{sen} 3t$

$\mathcal{L}\{f(t)\} = \frac{2}{s^3} + \frac{3}{(s+2)^2+9}$

3.5 -

a) $\operatorname{sen} t \cdot \operatorname{sen} 3t$

$\sin(s)\sin(t) = \frac{1}{2}\left(-\cos(s+t) + \cos(s-t)\right)$

$\mathcal{L}\{f(t)\} = -\frac{1}{2}\mathcal{L}\{\cos(4t)\} + \frac{1}{2}\mathcal{L}\{\cos(-2t)\}$

$\mathcal{L}\{f(t)\} = -\frac{1}{2}\cdot\frac{s}{s^2+16} + \frac{1}{2}\,\frac{s}{s^2+4}$

$= \frac{6s}{(s^2+4)(s^2+16)}$

3.5

b) $f(t) = sen^2 t + 3\cos^2 t$

$\mathcal{L}\{f(t)\} \Rightarrow sen^2(x) = \frac{1}{2} - \cos(2x)\frac{1}{2}$

$= \frac{1}{2} - \cos(2t)\frac{1}{2} + 3\cos^2(t)$

$= \mathcal{L}\{\frac{1}{2}\} - \mathcal{L}\{\cos(2t)\frac{1}{2}\} + 3\mathcal{L}\{\cos^2(t)\}$

$\mathcal{L}\{f(t)\} = \frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+4} + 3\left(\frac{1}{2s} + \frac{s}{2(s^2+4)}\right)$

c) $f(t) = \frac{sen\, t}{t}$

se $\mathcal{L}\{f(t)\} = F(s) \rightarrow \mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(u)\,du$

$= \int_s^\infty \mathcal{L}\{\sin(t)\}\,du\,du$

$= \int_s^\infty \frac{1}{u^2+1}\,du = \frac{\pi}{2} - \arctan(s)$

3.7

a) $F(s) = \dfrac{2}{s(s+2)}$

$$\mathcal{L}^{-1}\left\{\frac{2}{s(s+2)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+2}\right\}$$

$$= u(t) - e^{-2t}$$

d) $F(s) = \dfrac{3s^2 + 9s + 12}{(s+2)(s^2+5s+11)}$

$$\mathcal{L}^{-1}\{F(s)\} =$$

l) $\dfrac{1}{s^2+4} = \dfrac{1}{s^2+2^2} = \dfrac{1}{2}\dfrac{1}{s^2+2^2}$

$$\mathcal{L}^{-1}\{f(s)\} = \frac{1}{2}\operatorname{sen}(2t)$$

i) $F(s) = \dfrac{4}{s^4+4} =$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{s+2}{2(s^2+2s+2)} + \frac{-s+2}{2(s^2-2s+2)}$$

$$\Rightarrow \frac{s+2}{2(s^2+2s+2)} = \frac{1}{2}\cdot\frac{s+1}{(s+1)^2+1} + \frac{1}{2}\,\frac{1}{(s+1)^2+1}$$

$$\rightarrow \frac{-s+2}{2(s^2-2s+2)} = -\frac{1}{2}\cdot\frac{s-1}{(s-1)^2+1} + \frac{1}{2}\,\frac{1}{(s-1)^2+1}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1}\right\}$$

$$= \frac{1}{2}e^{-t}\cos(t) + \frac{1}{2}e^{-t}\,\mathrm{sen}(t) - \frac{1}{2}e^{t}\cos(t) + \frac{1}{2}e^{t}\,\mathrm{sen}(t)$$

j) $F(s) = \dfrac{e^{-s}}{s^2}$ se $\mathcal{L}^{-1}\{F(s)\} = f(t)$

então $\mathcal{L}^{-1}\{e^{-as}F(s)\} = \mu(t-a).\,f(t-a)$

$$\mu(t-1).\,\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}(t-1)$$

$$= \mu(t-1)(t-1)$$