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Semana 08 - Redução de Sistemas e Projeto

Questão (2): a) $\frac{K}{s+a}$ (transfer function)

$$(x = \frac{550}{s}) : Y = \frac{550 K}{s(s+a)}$$

$$= \frac{550 K}{as} - \frac{550 K}{a(s+a)}$$

$$y(t) = \frac{550 K}{a} (1 - e^{-at})$$

$$y(1.1) = 20$$

$$\frac{550 K}{a} = 40$$

$$1 - e^{-at} = 0.5 \Rightarrow a = 9.9$$

$$\therefore K = \frac{40}{550 \times 9.9} = 0.72$$

$$T = \frac{0.72}{s+9.9} ; y = 40(1 - e^{-9.9t})$$

$$y(0.17) = 32.57$$

b)

$$\begin{aligned}
 y(t) &= \int_0^t C \Phi(t-\tau) B u(\tau) d\tau \\
 &= \int_0^t e^{-s(t-\tau)} (4s e_a - s t_c) d\tau \\
 &= \int_0^t 4s e_a e^{-s(t-\tau)} d\tau - \int_0^t s t_c e^{-s(t-\tau)} d\tau \\
 &= \int_0^t 4s e_a e^{-s(t-\tau)} d\tau - \int_4^t s t_c e^{-s(t-\tau)} d\tau \\
 &= 900(1-e^{-st}) - 10(1-e^{-s(t-4)}) \quad (\text{para } t \geq 4)
 \end{aligned}$$

$$y(t) = \begin{cases} 900(1-e^{-st}) & t \leq 4 \\ 890 - 900e^{-st} + 10e^{-s(t-4)} & t \geq 4 \end{cases}$$

$$c) f(t) = 2x + f_v \frac{dx}{dt} \Rightarrow F(s) = 2X(s) + f_v s X(s)$$

$$\Rightarrow \frac{X(s)}{F(s)} = \frac{1}{f_v \frac{s+2}{f_v}}$$

$$T_s = \frac{4}{a}, a = \frac{2}{f_v} \Rightarrow T_s = 2t_v / f_v = 0.5 T_s$$

$$f_v = 0.5 T_s$$

$$d) f(t) - f_v \frac{dx}{dt} - k_v = M \frac{d^2x}{dt^2}$$

$$\Rightarrow f(t) = f_v \frac{dx}{dt} + x + M \frac{d^2x}{dt^2}$$

$$\Rightarrow F(s) = f_v s X(s) + X(s) + s^2 M X(s)$$

$$\Rightarrow \frac{X(s)}{F(s)} = \frac{1}{M} \frac{1}{s^2 + \frac{f_v}{M} s + \frac{1}{M}}$$

$$e^{\frac{-\pi \delta}{\sqrt{1-\delta^2}}} \times 100 = 17 \rightarrow \delta = 0.49$$

$$T_s = \frac{4}{\delta \omega_n} \Rightarrow T_s = 4s \Rightarrow \delta \omega_n = 0.4$$

$$\omega_n = 0.815$$

$$\omega_n^2 = \frac{1}{M} \text{ ou } M = \frac{1}{0.815^2} = 1.51$$

$$2 \delta \omega_n = \frac{f_v}{M} \Rightarrow 2 \times 0.4 = \frac{f_v}{1.51}$$

$$f_v = 1.21$$

$$\therefore f_v = 1.21 \quad M = 1.51$$

$$e) T(t) = J \frac{d\theta}{dt} + k\theta = J \frac{d^2\theta}{dt^2}$$

$$\Rightarrow T(t) = J \frac{d\theta}{dt} + k\theta + J \frac{d^2\theta}{dt^2}$$

$$T(s) = s\Theta(s) + k\Theta(s) + s^2 J\Theta(s)$$

$$\Rightarrow \frac{\Theta(s)}{F(s)} = \frac{\frac{1}{J}}{s^2 + \frac{1}{J}s + \frac{k}{J}}$$

$$e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \times 100 = 30$$

$$\delta = 0.358$$

$$2\delta\omega_n = \frac{1}{J} \Rightarrow \delta\omega_n = \frac{1}{2J}$$

$$T_s = \frac{4}{\delta\omega_n}$$

$$T_s = 3s \Rightarrow \frac{4}{\delta\omega_n} = \frac{4}{\frac{1}{2J}} = 3$$

$$J = \frac{3}{8}$$

$$\delta\omega_n = \frac{4}{3}, \text{ como } \delta = 0.358, \text{ temos } \omega_n = 3.72$$

$$\omega_n^2 = \frac{k}{J} \text{ ou } k = 3.72^2 \times \frac{3}{8} = 5.20$$

$$J = \frac{3}{8}, k = 5.20$$

Questão ④ : a) $\frac{50}{s+1}$ e $\frac{2}{s} \Rightarrow \frac{50s}{s^2+s+100}$

x2 blocos paralelos (s e 2) $\Rightarrow s-2$

$\frac{1}{s^2}, \frac{50s}{s^2+s+100}, s-2 \Rightarrow \frac{50s(s-2)}{s^2(s^2+s+100)}$

$\frac{50s(s-2)}{s^2(s^2+s+100)} = \frac{50(s-2)}{s^3+s^2+100s-100}$

$s = tf('s');$

$num\ 1 = [50];$

$den1 = [1\ 1];$

$num\ 2 = [2];$

$den\ 2 = [1\ 0];$

$R1 = feedback(tf(num1, den1), tf(num2, den2), -1);$

$R2 = parallel(s, -2);$

$R3 = 1/s^2;$

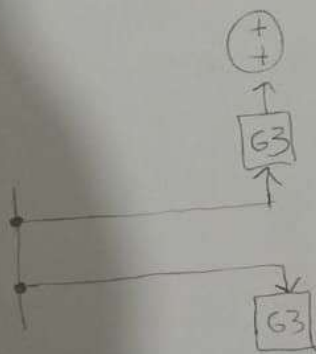
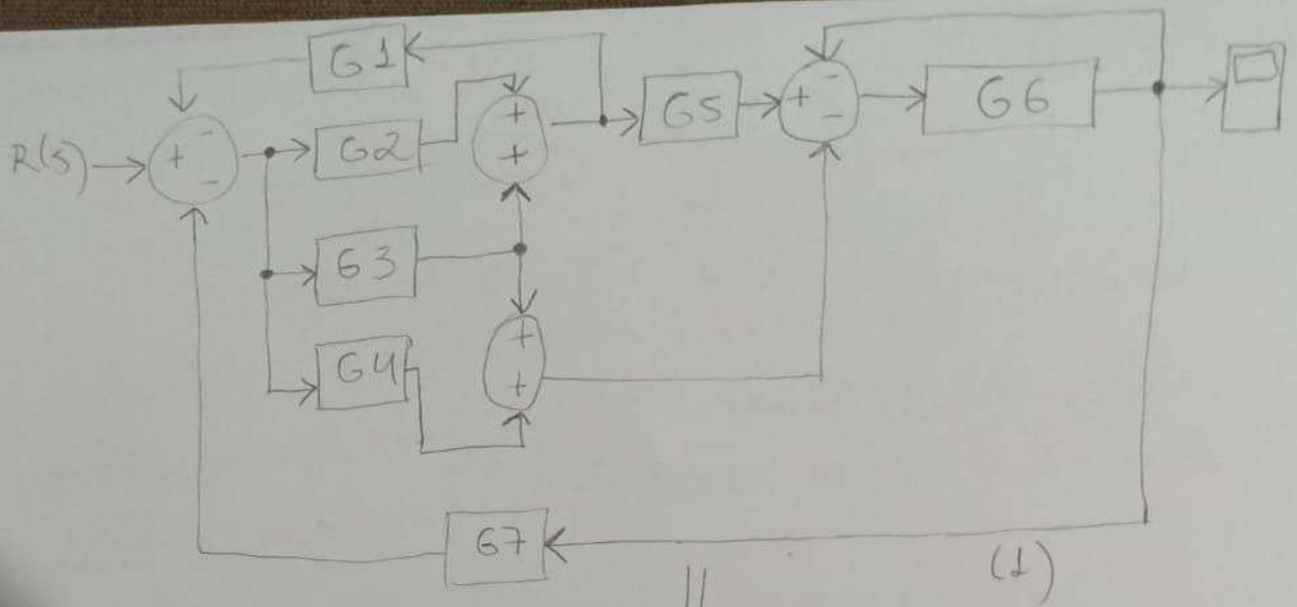
$R4 = series(R1, R2);$

$R4 = series(R4, R3);$

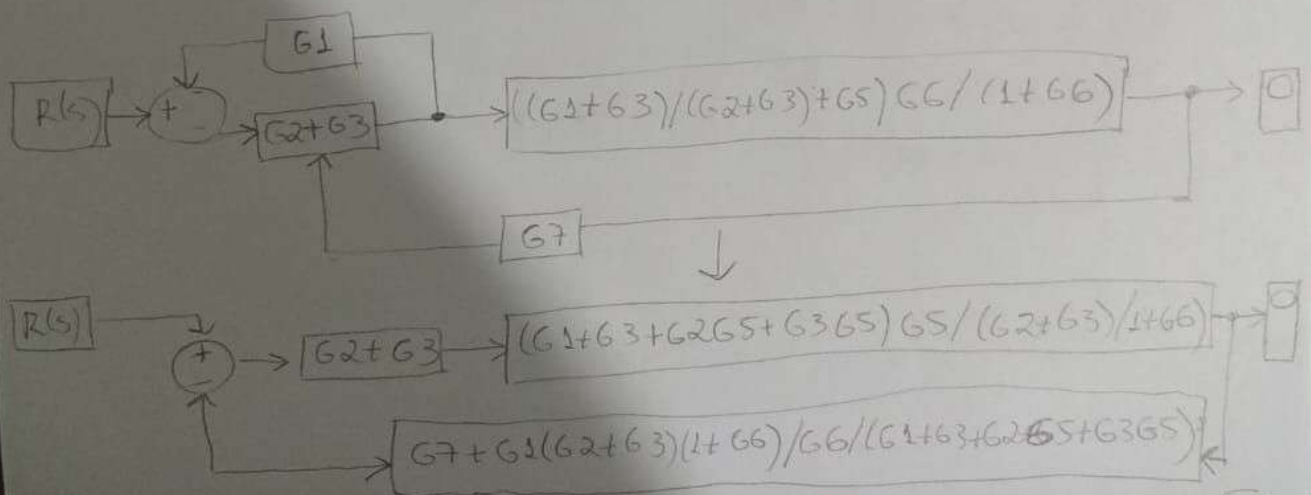
$Final = minreal(feedback(R4, 1, -1))$

$\Gamma\ Final = \frac{50s - 100}{s^3 + s^2 + 100s - 100}$

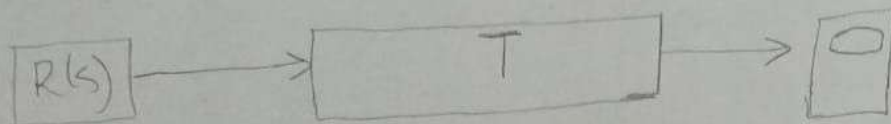
b)



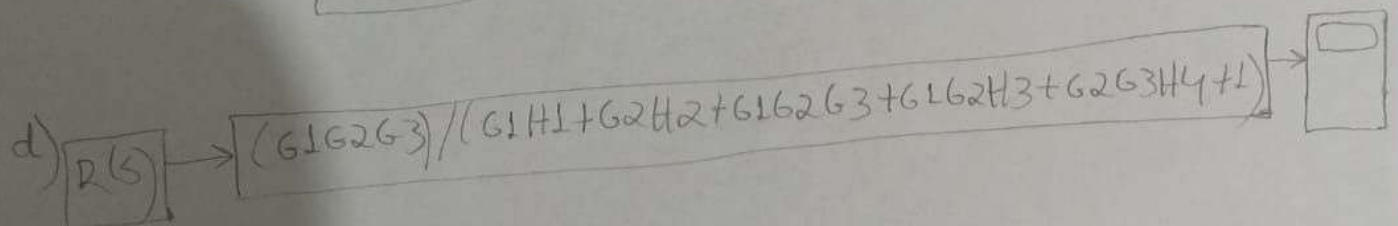
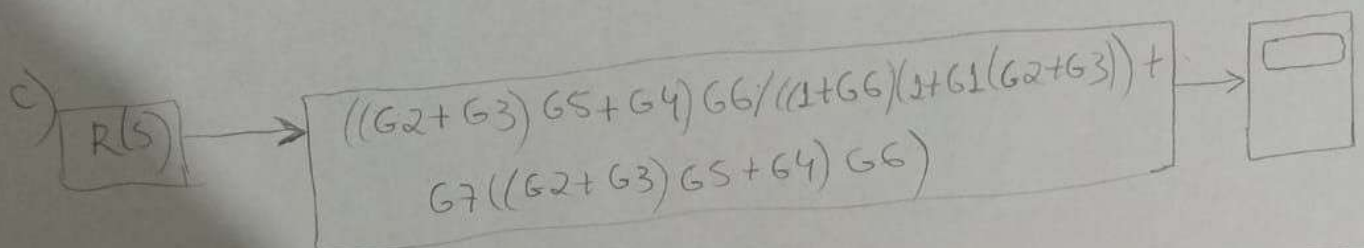
\Downarrow
 \vdots
 \Downarrow



Continuação questão (21) b):



$$T = \frac{(G_2 + G_3 + (G_6(G_1 + G_3 + G_2 G_5 + G_3 G_5)) / ((G_6 + 1)(G_2 + G_3))) / ((G_7 + (G_1(G_6 + 1)(G_2 + G_3)) / (G_6(G_1 + G_3 + G_2 G_5 + G_3 G_5))) (G_2 + G_3 + (G_6(G_1 + G_3 + G_2 G_5 + G_3 G_5)) / ((G_6 + 1)(G_2 + G_3))) + 1)}{1}$$



e) função de trans.

$$\Rightarrow \frac{4s+1}{s}$$

$$\rightarrow \frac{s(20s+5)}{25s^2+18s+3}$$

$$\therefore \frac{-9 \pm \sqrt{6}}{25}$$