Capítulo 4 - NISE

Questão (2) -

a) $G(s) = \frac{5}{s(s+5)}$

$$\frac{5}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} \rightarrow \frac{5}{s(s+5)} = \frac{A(s+5) + Bs}{s(s+5)}$$

$$5 = As + 5A + Bs$$

$$5 = As + Bs + 5A$$

$$5 = (A+B)s + 5A$$

$$0 = (A+B)s \rightarrow A + B = 0$$

$$\therefore \boxed{A = 1} \rightarrow \boxed{5A = 5}$$

$$\boxed{B = -1}$$

$$\therefore \frac{5}{s(s+5)} = \frac{1}{s} + \frac{(-1)}{s+5} \rightarrow \frac{1}{s} - \frac{1}{s+5}$$

$$c(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+5}\right) \rightarrow c(t) = 1 - e^{-5t}$$

$$c(t) = (1 - e^{-at})$$

$$-5 = -a \rightarrow T = \frac{1}{a} \rightarrow T = \frac{1}{5}$$

$$\boxed{a = 5}$$

$$\boxed{T = 0{,}2}$$

$$T_r = \frac{2,2}{a} = \frac{2,2}{5} = 0,44\,s$$

$$T_s = \frac{4}{a} = \frac{4}{5} = 0,8\,s$$

b) $\frac{1}{s} \cdot \frac{20}{s+20} = C(s)$

$$\frac{20}{s(s+20)} = \frac{A}{s} + \frac{B}{s+20} \rightarrow 20 = As + 20A + Bs$$

$$20 = (A+B)s + 20A$$

$$0 = (A+B)s \rightarrow A+B=0 \rightarrow 20A = 20 \rightarrow \boxed{A=1}$$

$$\boxed{B=-1}$$

$$c(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+20}\right) \rightarrow \boxed{1 - e^{-20t}}$$

$$C(t) = 1 - e^{-at} = 1 - e^{-20t} \rightarrow a = 20$$

$$\tau = \frac{1}{a} = \frac{1}{20} = 0,05\,s$$

$$T_r = \frac{2,2}{a} = \frac{2,2}{20} = 0,11\,s$$

$$T_s = \frac{4}{a} = \frac{4}{20} = 0,2\,s$$

Questão (4).

$$5 - 2\frac{dq}{dt} - \frac{q}{0,5} = 0$$

$$2q = v \rightarrow 2\frac{dq}{dt} + 2q = 5 \rightarrow \frac{dv}{dt} + v = 5$$

$$V = \frac{5}{s+1} \rightarrow T_c = \frac{1}{1} = 1s, \quad T_v = \frac{2,2}{1} = 2,2s$$

$$T_s = \frac{4}{1} = 4s$$

Questão (6).

$$M\ddot{x} + 6\dot{x} = f$$

$$M\dot{v} + 6v = f$$

$$T(s) = \frac{1}{Ms+6} \rightarrow s = \frac{-6}{M}$$

$$T_v = \frac{2,2M}{6} = 0,366M$$

$$T_s = \frac{4M}{6} = \frac{2M}{3}$$

Questão (08).

a) Resposta = $a + Ke^{-2t}$

b) Resposta = $a + K_1 e^{-3t} + K_2 e^{-6t}$

c) Resposta = $a + K_1 e^{-10t} + K_2 e^{-20t}$

d) Resposta = $a + e^{-3t}(K_1 \cos(1,6t) + K_2 \sin(1,6t))$

e) Resposta = $a + K_1 \cos(3t) + K_2 \sin(3t)$

f) Resposta = $a + K_1 e^{-10t} + K_2 te^{-10t}$

Questão (10)

$T(s) = C(sI - A)^{-1}B + D$, onde B é uma coluna e C é um vetor linha, e $D = 0$.

$$A = \begin{bmatrix} 3 & -4 & 2 \\ -2 & 0 & 1 \\ 4 & 7 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}, \quad C = [1 \ 7 \ 1], \quad D = 0$$

Raízes = [1 0 -7,8 1,5]

Resposta = -7,5 4,81 0,69

Questão (12) -

$$\frac{V - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1}{sL} + \frac{sV_1}{1/C}$$

$$\frac{V_1}{V} = \frac{1}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{sL} + sC\right)}$$

$$\frac{V_1}{V} = \frac{10}{s^2 + 20s + 500}$$

Polos complexos:

$(-10 \pm 20i)$

Forma geral = $e^{-10t}(k_1 \cos(20t) + k_2 \operatorname{sen}(20t))$

(14) - $F = (Ms^2 + fvs + ks)X$

$$\frac{X}{F} = \frac{1}{Ms^2 + fvs + ks}$$

$$\frac{X}{F} = \frac{1}{2s^2 + 6s + 2} \rightarrow X = \frac{1}{2s(s^2 + 3s + 1)} \rightarrow \boxed{X = \frac{1}{2s(s^2 + 3s + 1)}}$$

Questão (19) - → $\frac{a}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\omega_n = 20$
$\zeta = 0,15$ → $\frac{a}{s^2 + 6s + 400}$

$s = 0 \rightarrow a = 400$

$T(s) = \frac{400}{s^2 + 6s + 400}$

$C(s) = \frac{400}{s(s^2 + 6s + 400)} \rightarrow \frac{1}{s} - \frac{s+3}{(s+3)^2 + 391} - \frac{3}{(s+3)^2 + 391}$

$$c(t) = 1 - e^{-3t}\left[\cos\left(\sqrt{391}\,t\right) + \frac{3}{\sqrt{391}}\sin\left(\sqrt{391}\,t\right)\right]$$

Questão (20) -

$T_s = \frac{4}{\zeta\omega_n}$, $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ e $\%OS = e^{-3\pi/\sqrt{1-\zeta^2}} \cdot 100$

a) $\omega_n = 4$ rad/s | $\zeta = 0,375$

$T_s = 2,66$ s, $T_p = 0,847$ s, $\%OS = 28\%$

b) $\omega_n = 0,2$ rad/s, $\zeta = 0,05$

$T_s = 400$ s, $T_p = 15,72$ e $\%OS = 85,4\%$

c) $\omega_n = 3240,37$ rad/s, $\zeta = 0,247$

$T_s = 5$ ms, $T_p = 1$ ms, $\%OS = 44,9\%$

Questão (23) - $\frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$

$Re = w_n \zeta$ e $Im = w\sqrt{1-\zeta^2}$

$Tan(\theta) = \frac{\zeta}{\sqrt{1-\zeta^2}}$

$T_s = \frac{4}{\zeta w_n}$, $T_p = \frac{\pi}{w_n\sqrt{1-\zeta^2}}$, $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

a) $tan(\theta) = 0,675$ e $Re = -6,67$

Polos $= -6,67 \pm 9,88i$

b) $tan(\theta) = 0,733$ e $Im = 0,628$

Polos $= -0,46 \pm 0,628i$

c) $tan(\theta) = -0,571$ e $Im = 1,045$

Polos $= -0,571 \pm 1,045i$

Questão (25) - $F - 2sX - 20X = 5s^2X$

$T(s) = \frac{X}{F} = \frac{1}{5s^2+2s+20}$

b) $w_n = \sqrt{\frac{20}{5}} = 2$ | $\zeta = \frac{2}{5 \cdot 2 \cdot 2} = 0,1$

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \cdot 100 \simeq 72,92\%$

$T_s = \frac{4}{0,2} = 20\,s \;\Big|\; T_p = \frac{\pi}{2\sqrt{1-0,2^2}} = 1,58\,s$

$T_r = \frac{1,104}{2} = 0,552\,s$

$C_{final} = \frac{1}{20} = 0,05 \quad (s \to 0)$

Questão (26) - Valores conhecidos: $\omega_n^2 = 0,5 \;|\; \omega_n = 0,707\,rad/s$

$2\xi\omega_n = 1 \rightarrow \xi = 0,707$

Equações: $T(t) = \frac{2d^2\theta_1(t)}{dt^2} + \frac{d[\theta_1(t) - \theta_2(t)]}{dt}$

e $0 = (1)\frac{d[\theta_2(t) - \theta_1(t)]}{dt} + (1)\theta_2(t)$

a) Laplace 1: $s\theta_1(s) = (s+1)\theta_2(s)$

$\theta_1(s) = \left(\frac{s+1}{s}\right)\theta_2(s)$

Laplace 2: $T(s) = (2s^2+1)\theta_1(s) - s\theta_2(s)$

Então: $\frac{\theta_2(s)}{T(s)} = \frac{0,5}{s^2+s+0,5}$

b) $G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \;\Big|\; T_s = \frac{4}{\xi\omega_n} \;\Big|\; T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\% = e^{-(\xi\pi/\sqrt{1-\xi^2})} \cdot 100\%$

$$T_s = \frac{4}{0{,}707 \cdot 0{,}707} = \frac{4}{0{,}49} = 8{,}16\,s$$

$$T_p = \frac{\pi}{0{,}505} = 6{,}22\,s$$

$$\% = e^{-(0{,}707\pi/\sqrt{1-(0{,}707)^2})} \cdot 100\% = 4{,}32\%$$

$$T_s = 8{,}16\,s \mid T_p = 6{,}22\,s \mid \% = 4{,}32\%$$