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SISTEMAS DE CONTROLE - 1ª PROVA

Questão ①: a) $f(t) = 3 + 7t + t^2 + \delta(t)$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3 + 7t + t^2 + \delta(t)\} = 3 \cdot \frac{1}{s} + 7 \cdot \frac{1}{s^2} + \frac{2}{s^3} + 1 =$$

$$= F(s) = \frac{3}{s} + \frac{7}{s^2} + \frac{2}{s^3} + 1$$

$$F(s) = \frac{s^3 + 3s^2 + 7s + 2}{s^3}$$

b) $f(t) = t.\cos 3t = -(-t)\cos 3t$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{-(-t)\cos 3t\} = -\frac{d}{ds}\left(\frac{s}{s^2+9}\right) = -\left(\frac{1.(s^2+9) - s.2s}{(s^2+3^2)^2}\right) =$$

$$= -\left(\frac{s^2+9-2s^2}{(s^2+9)^2}\right) \Rightarrow F(s) = \frac{s^2-9}{(s^2+9)^2}$$

c) $F(s) = \frac{1}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} = \frac{A(s+2)^2 + Bs(s+2) + Cs}{s(s+2)^2}$

$$1 = A(s+2)^2 + Bs(s+2) + Cs$$

$s \to 0: 1 = A.2^2 \to 1 = A.4 \to A = 1/4$

$s \to 2: 1 = C.-2 \to C = -1/2$

$s \to -1: 1 = A.1 - B - C \to 1 = 1/4 - B + 1/2 \to B = 3/4 - 1 \to B = -1/4$

$$F(s) = \frac{1}{s(s+2)^2} = \frac{1}{4}\cdot\frac{1}{s} - \frac{1}{4}\cdot\frac{1}{s+2} - \frac{1}{2}\cdot\frac{1}{(s+2)^2}$$

$\hookrightarrow$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{4}\cdot\frac{1}{s} - \frac{1}{4}\cdot\frac{1}{s+2} - \frac{1}{2}\cdot\frac{1}{(s+2)^2}\right\} = \frac{1}{4}\cdot 1 - \frac{1}{4}e^{-2t} - \frac{1}{2}e^{-2t}t$$

$$f(t) = \frac{e^{-2t}}{4}\left(-2t + e^{2t} - 1\right)$$

Questão ② d)

$$\frac{A}{s}+\frac{B}{s+1}+\frac{Cs+D}{s^2+4s+10}=\frac{1}{5}\cdot\frac{1}{s}+\frac{1}{7}\cdot\frac{1}{s+1}+\frac{\left(-\frac{12}{35}s-\frac{43}{35}\right)}{s^2+4s+10}$$

$$\frac{1}{5}\cdot\frac{1}{s}+\frac{1}{7}\cdot\frac{1}{s+1}+\left(-\frac{1}{35}\right)\cdot\frac{12s+43}{s^2+4s+10}=\frac{1}{35}\left(\frac{7}{s}+\frac{5}{s+1}-\frac{12s-43}{s^2+4s+10}\right)$$

$$\frac{1}{35}\left(\frac{7}{s}+\frac{5}{s+1}-12\left(\frac{s+2}{(s+2)^2+6}\right)-\frac{19}{\sqrt{6}}\cdot\frac{\sqrt{6}}{(s+2)^2\sqrt{6}^2}\right.$$

$$\mathcal{L}^{-1}$$

$$\frac{1}{35}\left(7.1+5e^{-t}-12e^{-2t}\cos(\sqrt{6}t)-\frac{19}{\sqrt{6}}\cdot e^{-2t}\operatorname{sen}(\sqrt{6}t)\right)$$

$$\mathcal{L}^{-1}\{F(s)\}=\frac{1}{35}e^{-2t}\left(7e^{2t}+5e^{t}-12\cos\sqrt{6}t-\frac{19}{\sqrt{6}}\operatorname{sen}(\sqrt{6}t)\right)$$

Questão 2: a)

$$-6x_1(t) - 4\ddot{x}_1(t) - 2\dot{x}_1(t) + 2\dot{x}_2(t) = 0$$

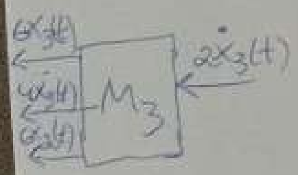
$$4s^2X_1(s) + 2sX_1(t) + 6X_1(s) = 2sX_2(s)$$

$$X_1(s)[4s^2+2s+6] = X_2(s)[2s] \quad \ldots(1)$$

$$-4\ddot{x}_2(t) - 4\dot{x}_2(t) - 6x_2(t) + 2\dot{x}_1(t) + 6x_3(t) + f(t) = 0$$

$$4s^2X_2(s) + 4sX_2(s) + 6X_2(s) - 2sX_1(s) - 6X_3(s) = F(s)$$

$$X_2(s)[4s^2+4s+6] - X_1(s)[2s] - X_3(s)[6] = F(s) \quad \ldots(2)$$



$$-6x_3(t) - 4\ddot{x}_3(t) - 2\dot{x}_3(t) + 6x_2(t) = 0$$

$$X_3[4s^2+2s+6] = X_2(s)[6] \quad \ldots(3)$$

> Substituindo (1) em (3):

$$X_3(t)[4s^2+2s+6] = 6\cdot\left[\frac{(4s^2+2s+6)}{2s}\right]X_1(s) \Rightarrow X_3(s) = \frac{3}{s}X_1(s) \quad \ldots(4)$$

→ Substituindo (1) em (2) e em (4):

$$X_1(s)\left[\frac{4s^2+2s+6}{2s}\right]\left[4s^2+4s+6\right]+X_1(s)\left[-2s\right]+X_1(s)\left[-\frac{3}{s}\right][6]=F(s)$$

$$X_1(s)\left[\frac{(2s^2+s+3)(4s^2+4s+6)-2s^2-18}{s}\right]=F(s)$$

$$\frac{X_1(s)}{F(s)}=\frac{s}{8s^4+12s^3+26s^2+18s}$$

→ Substituindo a equação acima em (4):

$$\frac{X_3(s)}{F(s)}=\frac{3}{8s^4+12s^3+26s^2+18s}$$

Questão (2): b) $\begin{bmatrix} 0 \\ T(s) \end{bmatrix} = \begin{bmatrix} J_1 s^2 + D_1 s & -(D_2 s + k_1) \\ -(D_2 s + k_1) & J_2 s^2 + k_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 5s^2+8s & -(s+9) \\ -(s+9) & 3s^2+3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(s)$ $-Ts$

$$\theta_1(s) = \begin{bmatrix} 0 & -(s+9) \\ 1 & 3s^2+3 \end{bmatrix} ; \quad \theta_2(s) = \begin{bmatrix} 5s^2+8s & 0 \\ -(s+9) & 1 \end{bmatrix}$$

$$\Rightarrow \frac{|\theta_1(s)|}{|T(s)|} = \frac{0 - [-(s+9)]}{(5s^2+8s)(3s^2+3) - [-(s-9)^2]} = \frac{s+9}{15s^4+15s^2+24s^3+24s-s^2-18s-81}$$

$$\frac{\theta_1(s)}{T(s)} = \frac{s+9}{15s^4+24s^3+16s^2+6s-81}$$

$$\frac{|\theta_2(s)|}{|T(s)|} = \frac{5s^2+8s-0}{15s^4+24s^3+16s^2+6s-81} \Rightarrow \frac{\theta_2(s)}{T(s)} = \frac{5s^2+8s}{15s^4+24s^3+16s^2+6s-81}$$

c) $T(s) = (J_1 s^2 + k_1)\theta_1(s) + (J_2 s^2 + k_2)\theta_2(s) + (J_3 s^2 + D s)\theta_3 \quad \dots (1)$

$\rightarrow \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} \Rightarrow \theta_1 = \frac{N_2}{N_1}\theta_2 = \frac{50}{5}\theta_2 \Rightarrow \theta_1(s) = 10\,\theta_2 \quad \dots (2)$

$\rightarrow \frac{N_1}{N_3} = \frac{\theta_3}{\theta_1} \Rightarrow \theta_3 = \frac{N_3}{N_1}\theta_1 = \frac{25}{5}(10\theta_2) \Rightarrow \theta_3(s) = 50\,\theta_2 \quad \dots (3)$

substituindo (2) e (3) em (1):

$T(s) = (3s^2+3)\,10\,\theta_2(s) + (150s^2+300)\theta_2(s) + (100s^2+500s)\,50\,\theta_2(s)$

$T(s) = [30s^2+30+150s^2+300+5000s^2+25000s]\,\theta_2(s)$

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{5180s^2+25000s+330}$$

Questão ③ a) $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{\omega_n^2}{s(s+2\zeta\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\zeta\omega_n)}} = \frac{\omega_n^2}{s^2+2\zeta\omega_n . s+\omega_n^2}$

$\Rightarrow$ Param. desempenho ($\xi = 0{,}8$ e $\omega_n = 25 \frac{rad}{s}$):

$\cdot \omega_d = \omega_n\sqrt{1-\xi^2} = 25\sqrt{1-0{,}8^2} = 25\sqrt{0{,}36} = 25 . 0{,}6 \Rightarrow \omega_d = 15 \frac{rad}{s}$

$\sigma_d = \xi\omega_n = 0{,}8 . 25 \Rightarrow \sigma_d = 20$

$T_r \cong \frac{1{,}8}{\omega_n} = \frac{1{,}8}{25} \Rightarrow T_r \cong 0{,}0725$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{20} \Rightarrow T_p = 0{,}157\, s$

$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\sigma_d} = \frac{4}{20} \Rightarrow T_s = 0{,}2s$

$M_p = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} . 100\% = e^{-\left(\frac{0{,}8\pi}{\sqrt{0{,}36}}\right)} . 100\% \Rightarrow M_p = 1{,}516\%$

$\theta = \cos^{-1}\xi = \cos^{-1} 0{,}8 \Rightarrow \theta \cong 36{,}87^\circ$

b) $F(s) - f_v . sX(s) - kX(s) = M . s^2X(s)$

$F(s) = m . s^2X(s) + f_v . sX(s) + kX(s) \Rightarrow F(s) . \frac{1}{M} = X(s)\left[s^2 + \frac{f_v}{M}s + \frac{k}{M}\right]$

$\frac{X(s)}{F(s)} = \frac{1/M}{s^2+\frac{f_v}{M}s+\frac{k}{M}} \Rightarrow \frac{X(s)}{F(s)} = \frac{0{,}2}{s^2+0{,}4s+4}$

$\Rightarrow$ Param. desempenho: $\omega_n^2 = 4 \Rightarrow \omega_n = 2\frac{rad}{s}$

$\cdot 2\xi\omega_n = 0{,}4 \Rightarrow \xi = \frac{0{,}4}{2.2} \Rightarrow \xi = 0{,}1$

$\cdot \sigma_d = \xi\omega_n = 0{,}1.2 \Rightarrow \sigma_d = 0{,}2$

$\cdot \omega_d = \omega_n\sqrt{1-\xi^2} = 2 . \sqrt{0{,}99} \Rightarrow \omega_d = 1{,}989 \frac{rad}{s}$

$\cdot \theta = \cos^{-1}\xi = \cos^{-1} 0{,}1 \Rightarrow \theta \cong 84{,}26^\circ$

$\cdot M_p = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} . 100\%$

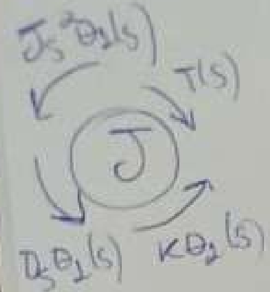
$\Rightarrow M_p = 72{,}92\%$

$T_r \cong \frac{1{,}8}{\omega_n} = \frac{1{,}8}{2} \Rightarrow T_r \cong 0{,}9s$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{1{,}989} \Rightarrow T_p = 1{,}579s$

$T_s = \frac{4}{\sigma_d} = \frac{4}{0{,}2} \Rightarrow T_s = 20s$

Questão (3): c) Diagrama de torque em J $(\Theta_1(s) = \Theta_2(s))$



$$T(s) - Js^2\Theta_1(s) - D_s\Theta_1(s) - K\Theta_1(s) = 0$$

$$\frac{1}{2} \cdot T(s) = \Theta_1(s)\left[s^2 + \frac{1}{2}s + \frac{1}{2}\right]$$

$$\frac{\Theta_1(s)}{T(s)} = \frac{\Theta_2(s)}{T(s)} = \frac{-1/2}{s^2+\frac{1}{2}s+\frac{1}{2}}$$

→ Param. desempenho:

- $\omega_n^2 = 1/2 \Rightarrow \omega_n = 0{,}707 \frac{rad}{s}$ //
- $2\xi\omega_n = \frac{1}{2} \Rightarrow \xi = \frac{1}{2} \cdot \frac{1}{2 \cdot 0{,}707} \Rightarrow \xi = 0{,}354$ //
- $\sigma d = \xi\omega_n = 0{,}354 \cdot 0{,}707 \Rightarrow \sigma d = 0{,}25$ //
- $\omega d = 0{,}707\sqrt{1-0{,}354^2} \Rightarrow \omega d = 0{,}661 \frac{rad}{s}$ //
- $\theta = \cos^{-1}\xi = \cos^{-1} 0{,}354 \Rightarrow \theta = 69{,}30°$ //
- $M_p = e^{-\left(\frac{0{,}354\pi}{\sqrt{0{,}935}}\right)} \cdot 100\% \Rightarrow M_p = 30{,}443\%$ //
- $T_r \cong \frac{1{,}8}{0{,}707} \Rightarrow T_r \cong 2{,}546\,s$ //
- $T_p = \frac{\pi}{0{,}661} \Rightarrow T_p = 4{,}753\,s$ //
- $T_s = \frac{4}{0{,}25} \Rightarrow T_s = 16s$ //

Questão ①. a) $H(s) = \dfrac{G(s)}{1+G(s)H(s)} = \dfrac{\dfrac{450}{s^4+3s^3+10s^2+30s+150}}{1+\dfrac{450}{s^5+3s^4+10s^3+30s^2+150s}} = \dfrac{450 \cdot s}{s^5+3s^4+10s^3+30s^2+150s+450}$

Polos

$\Downarrow$

$s^5+3s^4+10s^3+30s^2+150s+450=0 \Rightarrow (s+3)(s^4+10s^2+150)=0$

$$\begin{array}{l|l} s^5+3s^4+10s^3+30s^2+150s+450 & s+3 \\ \hline -s^5-3s^4 & s^4+10s^2+150 \\ 0+0 \quad +10s^3+30s^2 & \\ \quad -10s^3-30s^2 & \\ \quad 0+0 \quad +150s+450 & \\ \quad -150s-450 & \\ \quad 0+0 & \end{array}$$

$\Rightarrow s+3=0 \rightarrow s=-3$

$\Rightarrow s^4+10s^2+150=0 \rightarrow s^2=x \rightarrow \underbrace{x^2+10x+150=0}$

$\Delta = 10^2-4.1.150 = 100-600 = -500$
$= 500\, i^2$

$x = \dfrac{-10 \pm 10\sqrt{5}\, i}{2} = -5 \pm 5\sqrt{5}\, i$

$s^2 = x \rightarrow s = \pm\sqrt{x} \Rightarrow s = \pm\sqrt{5 \pm 5\sqrt{5}\, i}$

Sendo assim, teremos 03 polos em R_-, sendo dois deles pares conjugados, e 02 polos em R_+, sendo eles um par conjugado. Logo esse sistema é instável, pois temos polos com parte real positiva.

Questão (4):

b) $$H(s) = \frac{G(s)}{1 - G(s)H(s)} = \frac{\dfrac{18}{s^5+s^4-7s^3-7s^2-18s}}{1 - \dfrac{18}{s^5+s^4-7s^3-7s^2-18s}} = \frac{18}{s^5+s^4-7s^3-7s^2-18s-18}$$

Polos

$\Downarrow$ $$\underbrace{s^5+s^4-7s^3-7s^2-18s-18=0}$$

$s_0 = 3;\ s_1 = -1;\ s_2 = -3;\ s_3 = -\sqrt{2}i;\ s_4 = \sqrt{2}i$

Portanto, o sistema é instável pois o polo $(s=3)$ encontra-se no semi-eixo dos R_+.

Questão ⑤ a) $f_v = 1,5$; $T_s = 4$; $T_p = 1$

→ Fazendo a modelagem do sistema

$$\Sigma F(t) = m \cdot a(t)$$

$$f(t) - f_v \cdot v(t) - k \cdot x(t) = m \cdot a(t)$$

$$m \cdot a(t) + f_v \cdot v(t) + k \cdot x(t) = f(t)$$

$$m\ddot{x} + f_v \dot{x} + kx = f(t)$$

$$\ddot{x} + \frac{f_v}{m}\dot{x} + \frac{k}{m}x = f(t) \cdot \frac{1}{m}$$

$$s^2 X(s) + \frac{f_v}{m} s X(s) + \frac{k}{m} X(s) = F(s) \cdot \frac{1}{m}$$

$$X(s)\left[s^2 + \frac{f_v}{m}s + \frac{k}{m}\right] = F(s) \cdot \frac{1}{m}$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{\frac{1}{m}}{s^2 + \frac{f_v}{m}s + \frac{k}{m}}}$$

- $T_p = \frac{\pi}{\omega d} \Rightarrow \omega d = \frac{\pi}{T_p} = \frac{\pi}{1} \Rightarrow \boxed{\omega d = \pi}$

- $T_s \cong \frac{4}{\sigma d} = \sigma d \cong \frac{4}{T_s} = \frac{4}{4} \Rightarrow \boxed{\sigma d \cong 1}$

- $\sigma d = \frac{f_v}{m} \cdot \frac{1}{2} = \frac{1,5}{2m} = \frac{0,75}{m} \Rightarrow m = \frac{0,75}{\sigma d} = \frac{0,75}{1} \Rightarrow \boxed{m = 0,75 \text{ kg}}$

- $\omega d = \omega_n \sqrt{1 - \xi^2} \Rightarrow \pi = \omega_n \sqrt{1 - \frac{1}{\omega_n^2}} \Rightarrow \omega_n = \frac{\pi}{\sqrt{\frac{\omega_n^2 - 1}{\omega_n^2}}} = \frac{\pi \omega_n}{\sqrt{\omega_n^2 - 1}}$

$\omega_n = \sqrt{\pi^2 + 1} \Rightarrow \boxed{\omega_n \cong 3,3}$

↪

- $\frac{f_v}{m} = 2\xi \cdot \omega_n \Rightarrow \frac{1,5}{0,75} \cdot \frac{1}{2} \cdot \frac{1}{3,3} = \xi \Rightarrow \boxed{\xi \cong 0,3}$ //

- $\theta = \cos^{-1} \xi \Rightarrow \boxed{\theta \cong 72,50^\circ}$ //

- $T_r \cong \frac{1,8}{\omega_n} = \frac{1,8}{3,3} \Rightarrow \boxed{T_r \cong 0,545}$ //

- $\omega_n^2 = \frac{K}{m} \Rightarrow \omega_n^2 \cdot m = K \Rightarrow \boxed{K = 8,168}$ //

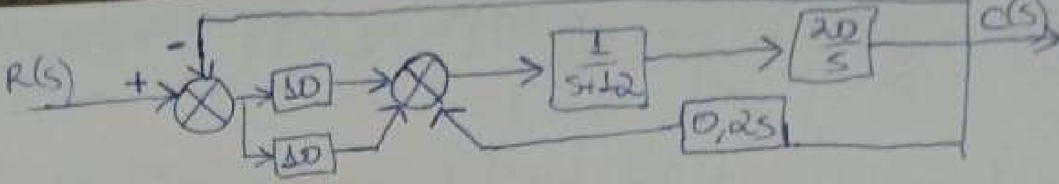
- $M_p = e^{-\left(\frac{3\pi}{\sqrt{1-\xi}}\right)} \cdot 100\% = e^{-\left(\frac{0,3\pi}{\sqrt{0,7}}\right)} \cdot 100\% \Rightarrow \boxed{M_p \cong 32,42\%}$ //

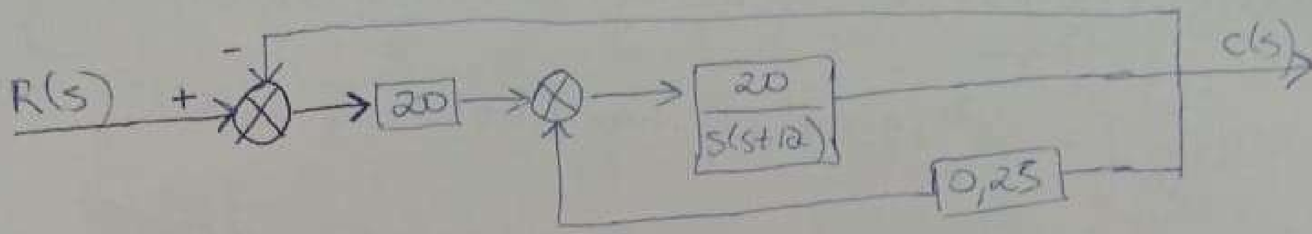
b) $\frac{X(s)}{F(s)} = \frac{1}{s^2+1,5s+10,433}$

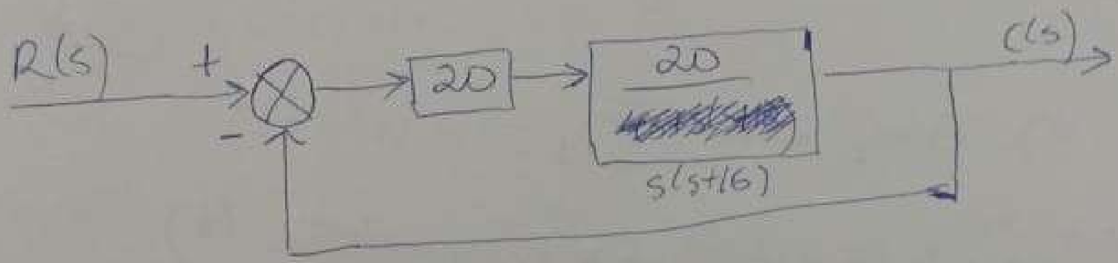
* Simulação feita no python:

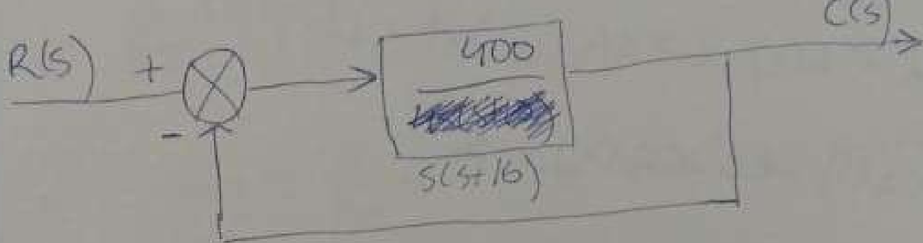
```
import matplotlib.pyplot as plt
import numpy as np
from scipy import signal

#Transfer function
 num = [1]
 denm = [1, 1.5, 10.433]
 system = signal.TransferFunction(num, denm)

#Degree enter function
 t, y = signal.step(system)

#Plot
 plt.plot(t,y)
 plt.title('System Degree')
 plt.xlable('Time(s)')
 plt.ylable('out')
 plt.grid(True)
 plt.show()
```

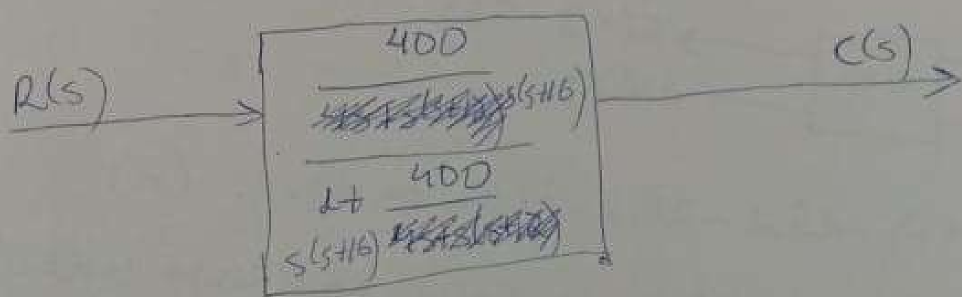
Questão ⑥: a)

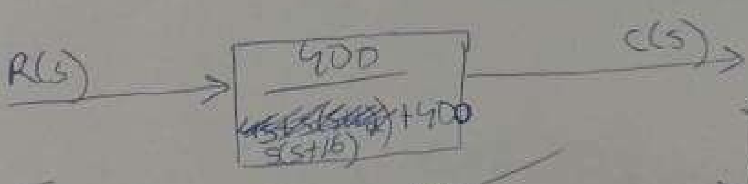








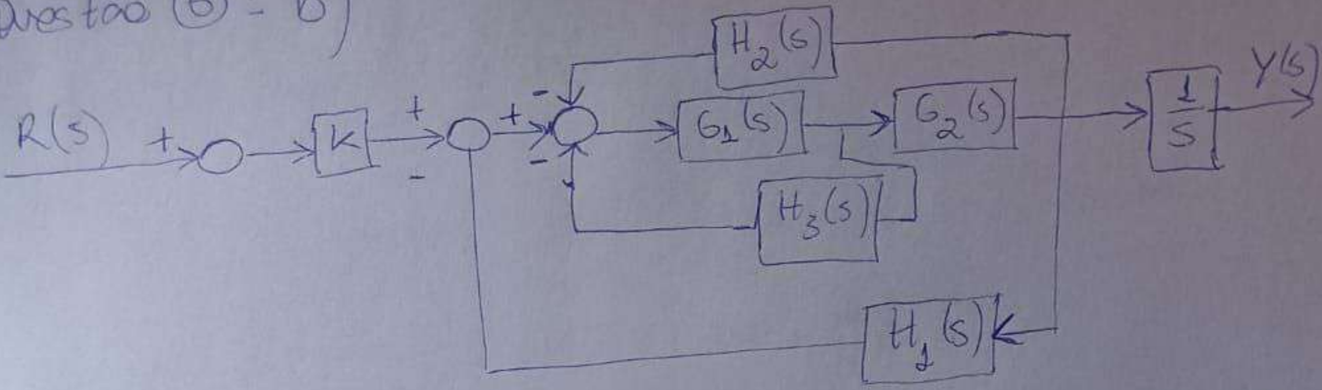


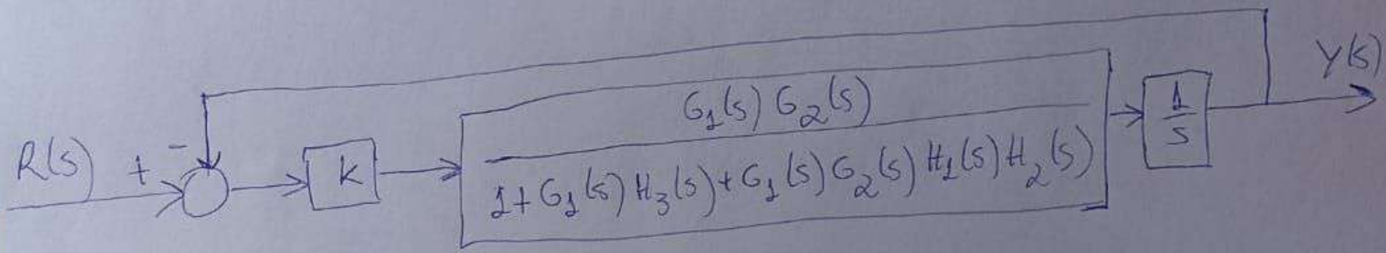


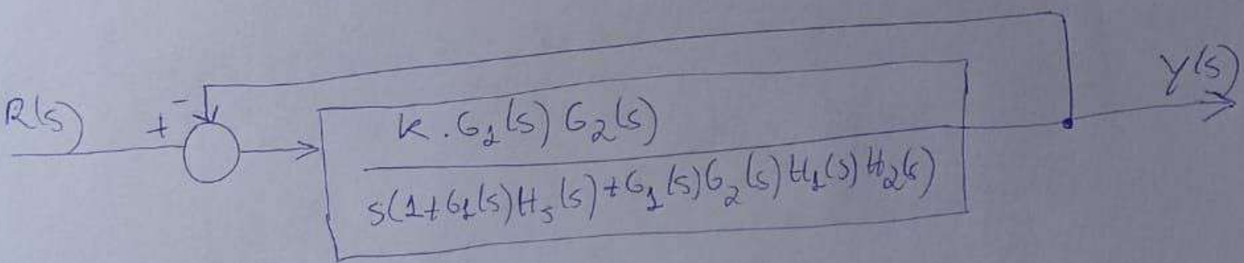
~~$\frac{C(s)}{R(s)} = H(s) = \frac{40}{s^2+16s+40}$~~

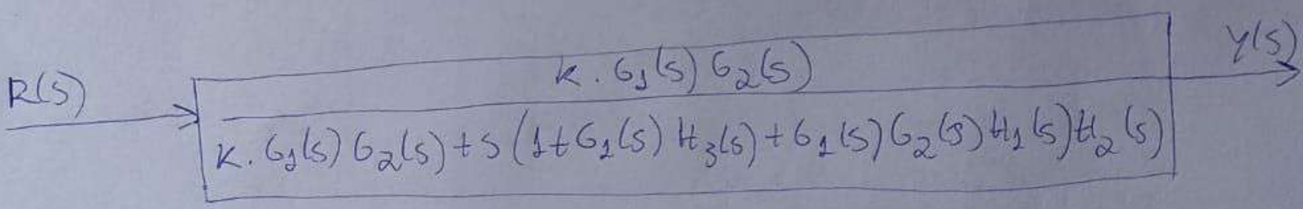
$$\frac{C(s)}{R(s)} = \frac{400}{s(s+16)+400} = H(s)$$

Questão ⑥ : b)









$$H(s) = \frac{Y(s)}{R(s)} = \frac{K . G_1 G_2}{K . G_1 G_2 + s(1 + G_1 H_3 + G_1 G_2 H_1 H_2)}$$