ALGA

Fichas

2016/2017

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Folha prática 4 – AGA

① ⓐ $x = -2y$ ⟶ $(-2y, y, 1)$
$z = 1$

$A = \{(x, y, z) \in \mathbb{R}^3 : x = -2y \wedge z = 1\}$, $S \subseteq \mathbb{R}^3$

Se $y = 0$, vem que $(0, 0, 1)$.

Para que A seja subespaço vetorial de $\mathbb{R}^3$, então $0_3 \in A$. Como $z = 1$, não existe nenhum $0_3 \in A$, logo A não é e.v. real.

ⓑ

$A = (K, 2K, 3K)$
$B = (K_m, 2K_m, 3K_m)$

- $S \subseteq \mathbb{R}^3$, pois existe um valor nulo $(0_{\mathbb{R}^3})$ ✓
- $S \neq \emptyset$ ✓
- adição

$A + B = (K + K_m, 2(K + K_m), 3(K + K_m)) \in S$ ✓

- multiplicação:

$\alpha A = (\alpha K, 2\alpha K, 3\alpha K) \in S$ ✓

Assim, estão confirmadas todas as condições para que S seja um espaço vetorial real, ou seja, um subespaço de $\mathbb{R}^n$

(2)

(a) Pelas condições definidas sabe-se que $\mathcal{V}$ é fechado em relação à adição $\mathbb{R}^2$ bem como em relação à multiplicação por escalar de $\mathbb{R}^2$

- $\mathcal{V} \neq \emptyset$ ✓

Falta verificar se $0_{\mathcal{V}} \in \mathcal{V}$:

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 - 1 \\ x_2 + y_2 + 1 \end{bmatrix}$$

para este ser 0_V de $X \Rightarrow$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 - 1 \\ x_2 + y_2 + 1 \end{bmatrix} \Rightarrow$

$$\Rightarrow \begin{cases} \cancel{x_1} = \cancel{x_1} + y_1 - 1 \\ \cancel{x_2} = \cancel{x_2} + y_2 + 1 \end{cases} \Leftrightarrow \begin{cases} y_1 = 1 \\ y_2 = -1 \end{cases}$$

$$0_V \text{ dad} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Assim, o conjunto $\mathcal{V}$ é um espaço vetorial real

Elemento simétrico

$S + X = 0_{\mathcal{V}}$, sendo S o elemento simétrico de X

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} x_1 + S_1 - 1 \\ x_2 + S_2 + 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} S_1 = 2 - x_1 \\ S_2 = -2 - x_2 \end{cases} \qquad -X = \begin{bmatrix} 2 - x_1 \\ -2 - x_2 \end{bmatrix}$$

b)

$$S = \left\{ \begin{bmatrix} 1-2t \\ t-1 \end{bmatrix} : t \in \mathbb{R} \right\}$$

Verificar se $S \leq \mathbb{R}^2$

- Soma e multiplicação ✓
- $S \neq \emptyset$ ✓

Se $0_V \in S$ então S é subespaço de V

$$\begin{cases} \begin{bmatrix} 1-2t \\ t-1 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1-2t \\ t-1 \end{bmatrix} \\ \begin{bmatrix} 1-2t \\ t-1 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1-2t+a-1 \\ t-1+b+1 \end{bmatrix} \end{cases}$$

Logo: $\begin{bmatrix} \cancel{1-2t} \\ \cancel{t} \end{bmatrix} = \begin{bmatrix} \cancel{1-2t} + a - 1 \\ \cancel{t-1} + b + 1 \end{bmatrix} \Leftrightarrow \begin{cases} a = 1 \\ b = -1 \end{cases}$

$0_V = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Longrightarrow$ Assim, como $0_V \in S$, S é subespaço de V

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b)

$$(1,1,0) = \alpha(2,1,-2) + \beta(1,0,0) + \theta(1,1,1)$$

$$\begin{cases} 1 = 2\alpha + \beta + \theta \\ 1 = \alpha + \theta \\ 0 = -2\alpha + \theta \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ \theta = 1-\alpha \\ 0 = -2\alpha + 1 - \alpha \end{cases} \Leftrightarrow \begin{cases} \beta = 1 - \theta - 2\alpha \\ \theta = \frac{2}{3} \\ \alpha = \frac{1}{3} \end{cases}$$

$$\Leftrightarrow \begin{cases} \beta = -\frac{1}{3} \\ \theta = 2/3 \\ \alpha = 1/3 \end{cases}$$

$$(1,1,0) = \frac{1}{3}(2,1,-2) - \frac{1}{3}(1,0,0) + \frac{2}{3}(1,1,1)$$

d)

$$\begin{cases} 1 = \alpha + \beta + \theta \\ 1 = -\beta + 2\theta \\ 0 = 2\alpha - \theta \\ 2 = \alpha + 3\beta + \theta \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ \beta = 4\alpha - 1 \\ \theta = 2\alpha \\ 2 = \alpha + 12\alpha - 3 + 2\alpha \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 5 = 15\alpha \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ \alpha = \frac{1}{3} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{---} \\ \beta = \frac{1}{3} \\ \theta = \frac{2}{3} \\ \alpha = \frac{1}{3} \end{cases} \Leftrightarrow \left\{ 1 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \right.$$ Impossível

7 a)

$$S = \{\alpha_1(0,1) + \alpha_2(2,1) + \alpha_3(2,2) : \alpha_1, \ldots, \alpha_3 \in \mathbb{R}\} =$$

$$= \{(2\alpha_2 + 2\alpha_3, \alpha_1 + \alpha_2 + 2\alpha_3) : \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}\} =$$

$$= \mathbb{R}^2$$

c)

$S = \{\alpha_1(2,2,3) + \alpha_2(-1,-2,1) + \alpha_3(0,1,0) : \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}\} =$

$= \{(2\alpha_1 - \alpha_2,\ 2\alpha_1 - 2\alpha_2 + \alpha_3,\ 3\alpha_1 + \alpha_2) : \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}\} =$

$= \mathbb{R}^3$

d) $\{a(t^2+1) + b(t^2+t) + c(t+1) : a, b, c \in \mathbb{R}\} =$

$= \{(a+b)t^2 + (b+c)t + (a+c) : a, b, c \in \mathbb{R}\} =$

$= P_2$

8) $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 2 & 0 \\ 2 & 1 & 3 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \end{array}\right] \rightarrow \left\{\begin{array}{l} a + c = 0 \\ a + 2b + 3c + 2d = 0 \\ 2a + b + 3c + d = 0 \\ a + b + 2c + d = 0 \end{array}\right. \quad \begin{array}{l} a = -c \\ b = -d - c \\ 0 = 0 \\ 0 = 0 \end{array}$

$(-c, -d-c, c, d) = c(-1,-1,1,0) + d(0,-1,0,1)$

$K = \{c(-1,-1,1,0) + d(0,-1,0,1) : c, d \in \mathbb{R}\}$

$K = \langle (-1,-1,1,0), (0,-1,0,1) \rangle$

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(11)

(a) $A = \langle (1,1,0), (0,2,3), (1,2,3), (1,-1,1) \rangle$

$$\begin{cases} a + c + d = 0 \\ a + 2b + 2c - d = 0 \\ 3b + 3c + d = 0 \end{cases} \Rightarrow \begin{cases} a = -c - d \\ -2d + c + 2b = 0 \\ — \end{cases} \Rightarrow \begin{cases} — \\ c = 2d - 2b \\ 3b + 6d - 6b + d = 0 \end{cases}$$

$$\Rightarrow \begin{cases} a = -3d + 2b \\ c = 2d - 2b \\ 7d - 3b = 0 \end{cases}$$

linearmente dependente

(b) $$\begin{cases} a + b + c = 0 \\ 2a + b = 0 \\ 3a + b + c = 0 \end{cases} \Rightarrow \begin{cases} a = c \\ b = -2a \\ 3a + a - 2a = 0 \end{cases} \Rightarrow \begin{cases} c = 0 \\ b = 0 \\ a = 0 \end{cases} \Rightarrow$$ linearmente independente

(15)

(a) $(2,4) = 2(1,2)$ → linearmente dependente → não é base

(b) $$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$\det(A) =$

$= (2 + 0 + 0) - (0) = 2 \longrightarrow \neq 0 \rightarrow$ vetores linearmente independente

(c) $$B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\det(B) = 0 \Rightarrow$ vetores linearmente dependentes → NÃO

c)

$$\begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} + \begin{bmatrix} 0 & d \\ d & d \end{bmatrix} \Rightarrow$$

$$\rightarrow \begin{cases} a + c = 0 \\ a + d = 0 \\ b + d = 0 \\ b + d + c = 0 \end{cases} \Leftrightarrow \begin{cases} a = 0 \\ d = 0 \\ \\ c = 0 \end{cases} \rightarrow \text{é uma base}$$

→ LI

16)

a) $a(1,3,0) + b(-1,1,0) = (x,y,z) = (x,y,0)$

$x(1,0,0) + y(0,1,0)$

b) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$A = \{(1,-1,1),(0,2,1)\}$ $\dim(A) = 2$

c) $\left[\begin{array}{cc|c} 1 & 1 & x \\ 0 & -1 & y \\ 1 & 1 & z \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & x \\ 0 & -1 & y \\ 0 & 0 & z-x \end{array}\right]$

$\{(x,y,z) \in \mathbb{R}^3 : z = x\}$, (x,y,x)

$\{(x,y,z) \in \mathbb{R}^3 : x(1,0,1) + y(0,1,0)\}$

$\langle (1,0,1),(0,1,0) \rangle = \langle t^2+1, t \rangle$ dimensão 2

(13)

Para que os 3 vetores sejam uma base de $\mathbb{R}^3$, estes têm de ser linearmente independentes

$$\left[\begin{array}{ccc|c} a^2 & 0 & 1 & x \\ 0 & a & 0 & y \\ 1 & 2 & 1 & z \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & z \\ 0 & a & 0 & y \\ a^2 & 0 & 1 & x \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & z \\ 0 & a & 0 & y \\ 0 & -2a^2 & 1-a^2 & z-xa^2 \end{array}\right] \underset{L_3+2a}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 1 & z \\ 0 & a & 0 & y \\ 0 & 0 & 1-a^2 & z-xa^2+2ya \end{array}\right]$$

$$a \neq 0 \ \wedge \ 1-a^2 \neq 0 \Rightarrow a \in \mathbb{R} \setminus \{-1, 0, 1\}$$

(14)

(a) $S = \{(x,y,z) \in \mathbb{R}^3 : x - y + 3z = 0\} =$

$= \{(x,y,z) \in \mathbb{R}^3 : x = y - 3z\} =$

$= (y-3z, \ y, \ z) = y(1,1,0) + z(-3,0,1)$

Se $x = y = z = 0$ então $0_{\mathbb{R}^3}$ está contido em $\mathbb{R}^3$

$A + B = (y_1 - 3z_1 - 3z_2 + y_2, \ y_1 + y_2, \ z_1 + z_2) \subseteq \mathbb{R}^3$

$\alpha S = y(\alpha, \alpha, 0) + z(-3\alpha, 0, \alpha) \subseteq \mathbb{R}^3$

Verifica-se que $S \subseteq \mathbb{R}^3$

(b)

$S = \langle (1,1,0), (-3,0,1) \rangle$

$$\begin{cases} a - 3b = 0 \\ a = 0 \\ b = 0 \end{cases} \Rightarrow \{ a = b = 0 \qquad \underline{\underline{LI}}$$

ⓒ dimensão $= 2$, uma vez que bastam 2 vetores l.i para formar S

(2)

ⓐ $A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

car $(A) = 2$

$$\begin{cases} a + b + 2c = 0 \\ c - d = 0 \end{cases} \Leftrightarrow \begin{cases} a = -b - 2c \\ d = c \end{cases} \Rightarrow X = \begin{bmatrix} -b - 2c \\ b \\ c \\ c \end{bmatrix} =$$

$$= b \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$\{(-1, 1, 0, 0), (-2, 0, 1, 1)\}$

nul $(A) = 2$

ⓑ $\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -2 & 1 \end{bmatrix}_{3\times 4} \times \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}_{4\times 1} = \begin{bmatrix} x + y + 2z \\ -z + w \\ -2z + w \end{bmatrix}$

Erro: Solução: $\{(a, b, c) \in \mathbb{R}^3 : c = a + 2b\}$

c) $(a,b,c) = \alpha_1(1,-1,-1) + \alpha_2(4,-3,-2)$

$$\left[\begin{array}{cc|c} 1 & 4 & a \\ -1 & -3 & b \\ -1 & -2 & c \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 4 & a \\ 0 & 1 & a+b \\ 0 & 2 & c+a \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 4 & a \\ 0 & 1 & a+b \\ 0 & 0 & c-a-2b \end{array}\right]$$

Para que seja um sistema possível: $c-a-2b=0 \Leftrightarrow$
$\Leftrightarrow c = a+2b$

Logo, o espaço gerado é $\{(a,b,c) \in \mathbb{R}^3 : c = a+2b\}$

23) Como queríamos provar

a)

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 2 & 6 \\ 0 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 12 \\ 0 & 0 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} a+c=0 \\ b-3c=0 \\ c=0 \end{cases} \quad \begin{cases} a=0 \\ b=0 \\ c=0 \end{cases} \longrightarrow (a,b,c) = (0,0,0) \rightarrow 0_{\mathbb{R}^3}$$

$\mathcal{B}_{\mathcal{L}(A)} = \{(1,0,0),(0,1,0),(0,0,1)\}$

$\text{car}(A) = 3 \longrightarrow$ Não!

$\text{nul}(A) = 0$

$\mathcal{N}(A) = \emptyset$??

c)
$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & 0 & -1 & 2 \\ 0 & 2 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & -5 & -9 & -7 & -1 \\ 0 & 2 & 1 & 1 & 1 \end{bmatrix} + \frac{2}{5}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & -5 & -9 & -7 & -1 \\ 0 & 0 & -\frac{13}{5} & -\frac{9}{5} & \frac{3}{5} \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 1 & 9/5 & 7/5 & 1/5 \\ 0 & 0 & 1 & 9/13 & -3/13 \end{bmatrix}$$

$$\begin{cases} a + 2b + 3c + 2d + e = 0 \\ b + \frac{9}{5}c + \frac{7}{5}d + \frac{1}{5}e = 0 \\ c + \frac{9}{13}d - \frac{3}{13}e = 0 \end{cases}$$

d)
$$\begin{bmatrix} 1 & 2 & -3 \\ -1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 0 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathcal{N}(A) = \emptyset$

$nul(A) = 0$

$car(A) = 3 \longrightarrow$ As linhas de A são todas l. independentes

$\mathcal{B}_{\mathcal{L}(A)} = \{(1,0,0),(0,1,0),(0,0,1)\}$

$\mathcal{B}_{\mathcal{C}(A)} = \{(1,-1,0),(2,2,1)(-3,3,1)\}$

b

$$\begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 4 & 6 & 1 \\ 0 & 0 & -3 & 1 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 6 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathcal{N}(A) = \emptyset$

$nul(A) = 0$

$ran(A) = 4$

$B\,d_{(A)} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

linhas $\rightarrow$ l.i

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$B = \{(1,1,0,0), (1,0,0,0), (1,1,1,0), (1,1,1,1)\}$

a)

$\vec{v} = (-1, 2, -6, 5)$

$$[(-1,2,-6,5)]_B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$-6 - 8 \rightarrow$

$$\begin{cases} a+b+c+d = -1 \\ a+c+d = 2 \\ c+d = -6 \\ d = 5 \end{cases} \Rightarrow \begin{cases} b = -1 - d - c - a = 3 \\ a = 8 \\ c = -11 \\ d = 5 \end{cases}$$

$$[(-1,2,-6,5)]_B = (8, 3, -11, 5)$$

c) $$[(1,2,3,4)]_B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{cases} a+b+c+d = 1 \\ a+c+d = 2 \\ c+d = 3 \\ d = 4 \end{cases} \Rightarrow \begin{cases} b = 1 - d - c - a = -1 \\ a = 2 - c - d = -1 \\ c = -1 \\ d = 4 \end{cases}$$

$$[(1,2,3,4)]_B = (-1, -1, -1, 4)$$

(22)

a) $[(1,5)]_{\mathcal{J}} = \begin{bmatrix} a \\ b \end{bmatrix}$ $\left\{ \begin{array}{l} a + 2b = 1 \\ a + 3b = 5 \end{array} \right. \sim \left| \begin{array}{l} a = 1 - 2b \\ 3b = 5 - 1 + 2b \end{array} \right. \sim$

$\sim \left| \begin{array}{l} a = -7 \\ b = 4 \end{array} \right.$

$[(1,5)]_{\mathcal{J}} = (-7, 4)$

b) $[(a,b)]_{\mathcal{J}} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ $\begin{array}{l} a = 1 - 6 = -5 \\ b = -8 \end{array}$

$z = (-5, -8)$

c) $M_{S \leftarrow \mathcal{J}} = ?$

$S = ((1,2), (0,1))$
$\mathcal{J} = ((1,1), (2,3))$

$[X]_{\mathcal{J}} = \begin{bmatrix} a \\ b \end{bmatrix} \longrightarrow [X]_S = a[(1,1)]_S + b[(2,3)]_S$

$[(1,1)]_S = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$[(2,3)]_S = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\left| \begin{array}{l} a = 1 \\ 2a + b = 1 \end{array} \right. \sim \left| \begin{array}{l} a = 1 \\ b = -1 \end{array} \right.$

$a = 2$
$b = -1$

$$[x]_S = a\begin{bmatrix}1\\-1\end{bmatrix} + b\begin{bmatrix}2\\-1\end{bmatrix} = \underbrace{\begin{bmatrix}1&2\\-1&-1\end{bmatrix}}_{M_{S\leftarrow T}}\begin{bmatrix}a\\b\end{bmatrix}$$

(d)

$[x]_S = ?$

$$[x]_S = M\underbrace{[x]_T}_{\begin{bmatrix}-7\\4\end{bmatrix}}$$

$$[x]_S = \begin{bmatrix}1&2\\-1&-1\end{bmatrix}\begin{bmatrix}-7\\4\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$$

(e)

$$[(1,5)]_S = \begin{bmatrix}a\\b\end{bmatrix} \qquad \begin{cases}a = 1\\2+b=5\end{cases} \Rightarrow \begin{cases}a=1\\b=3\end{cases}$$

$$[x]_S = \begin{bmatrix}1\\3\end{bmatrix}$$

(f)

$$M_{T\leftarrow S} \quad \left[\begin{array}{cc|cc}1&2&1&0\\-1&-1&0&1\end{array}\right] \sim \left[\begin{array}{cc|cc}1&2&1&0\\0&1&1&1\end{array}\right]_{-2\times L_2} \sim$$

$$\sim \left[\begin{array}{cc|cc}1&0&-1&-2\\0&1&1&1\end{array}\right] \qquad M_{T\leftarrow S} = \begin{bmatrix}-1&-2\\1&1\end{bmatrix}$$

$$[X]_{\mathcal{J}} = [X]_S N = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \times \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

(29)

Se $X_i * X_j = 0 \Rightarrow$ o conjunto $\{X_1, \ldots, X_k\}$ é ortogonal

(a) Não, pois $X_1 * X_3 = 5$

(b) $X_1 \cdot X_2 = 0$

$X_1 \cdot X_3 = 0$ É ortogonal

$X_2 \cdot X_3 = 0$

(31) (a)

Se o conjunto de vetores for ortonormado a base também é ortonormada!

$X_i \cdot X_j = 0 \wedge X_i \cdot X_i = 1$

$\mathcal{B} = \{(\frac{4}{5}, 0, \frac{3}{5}), (0,1,0), (-\frac{3}{5}, 0, \frac{4}{5})\}$

$X_1 * X_2 = 0$ $X_1 \cdot X_1 = \frac{16}{25} + \frac{9}{25} = 1$

$X_1 \cdot X_3 = 0$ $X_3 \cdot X_3 = +\frac{9}{25} + \frac{16}{25} = 1$

$X_2 \cdot X_3 = 0$ $X_2 \cdot X_2 = 1$

$\mathcal{B}$ é uma base ortonormada de $\mathbb{R}^3$

6

$$[x]_{\mathcal{B}} = [(1,1,1)]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{cases} \left(\frac{4}{5}\right)a - \left(\frac{3}{5}\right)c = 1 & \longrightarrow \frac{4}{5}a = 1 + \frac{3}{5}c \Rightarrow 4a = 5 + 3c \\ b = 1 \\ \frac{3}{5}a + \frac{4}{5}c = 1 & \longrightarrow 4c = 5 - 3a \end{cases}$$

$$\begin{cases} 4a = 5 + 3c \ (\times 4) \\ 4c = 5 - 3a \ (\times 3) \end{cases} \Rightarrow \begin{cases} 16a = 20 + 12c \\ 12c = 15 - 9a \end{cases} \Rightarrow \begin{cases} 16a = 20 + 15 - 9a \end{cases}$$

$$25a = 35 \Rightarrow a = \frac{35}{25} = \frac{7}{5}$$

$$c = \frac{5 - \frac{21}{5}}{4} = \frac{4}{20} = \frac{1}{5}$$

$$[x]_{\mathcal{B}} = \left(\frac{7}{5}, 1, \frac{1}{5}\right)$$

$$\mathcal{D} = \{(0,0,1), (0,1,1), (1,1,1)\} \qquad x = (1,1,1)$$

$$\mathcal{B} = \left\{\left(\frac{4}{5}, 0, \frac{3}{5}\right), (0,1,0), \left(-\frac{3}{5}, 0, \frac{4}{5}\right)\right\}$$

$$M_{\mathcal{B} \leftarrow \mathcal{D}} = ?$$

$$[x]_{\mathcal{D}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \Leftrightarrow \begin{cases} c = 1 \\ b = 0 \\ a = 0 \end{cases} \qquad [x]_{\mathcal{D}} = (1,0,0)$$

Dados:

$S_1 = M_{\cancel{\mathcal{B}} \leftarrow \cancel{\mathcal{B}}} = M_{S \leftarrow \mathcal{T}}$

$\mathcal{T} = \left\{ \left(\frac{4}{5}, 0, \frac{3}{5}\right), (0,1,0), \left(-\frac{3}{5}, 0, \frac{4}{5}\right) \right\}$

$S = \left\{ (0,0,1), (0,1,1), (1,1,1) \right\}$

$[x]_{\mathcal{T}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$[x]_S = a\left[\left(\frac{4}{5}, 0, \frac{3}{5}\right)\right]_S + b\left[(0,1,0)\right]_S + c\left[\left(-\frac{3}{5}, 0, \frac{4}{5}\right)\right]_S$

$[(0,0,1)]_S = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \longrightarrow \begin{cases} 0 = \frac{4}{5}a - \frac{3}{5}c & \Rightarrow a = \frac{3c}{4} \\ 0 = b & \\ 1 = \frac{3}{5}a + \frac{4}{5}c & \rightarrow 5 = 3a + 4c \end{cases}$

$4c = 5 - \frac{9c}{4} \Leftrightarrow 25c = 20 \Rightarrow c = \frac{4}{5}$

$a = \frac{3}{5}$

$\rightarrow = \left(\frac{3}{5}, 0, \frac{4}{5}\right)$

$[(0,1,1)]_S = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \rightarrow \begin{cases} 4a = 3c \\ b = 1 \\ 5 = 3a + 4c \end{cases}$

$\rightarrow \left(\frac{3}{5}, 1, \frac{4}{5}\right)$

$$[(1,1,1)]_S = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \to \begin{cases} 5 = 4a - 3c \\ b = 1 \\ 5 = 3a + 4c \end{cases} \to 4a - 3c = 3a + 4c \Rightarrow a = 7c$$

$$5 = 25c \Leftrightarrow c = \frac{1}{5}$$

$$a = \frac{7}{5}$$

$$\left(\frac{7}{5}, 1, \frac{1}{5}\right)$$

$$M_{\mathcal{B} \leftarrow \tilde{\mathcal{B}}} = \frac{1}{5}\begin{bmatrix} 3 & 3 & 7 \\ 0 & 5 & 5 \\ 4 & 4 & 1 \end{bmatrix}$$

(d)

$$[x]_{\mathcal{B}} = M_{\mathcal{B} \leftarrow \tilde{\mathcal{B}}} [x]_{\tilde{\mathcal{B}}}$$

$$[Y]_{\mathcal{B}} = \frac{1}{5}\underset{3\times 3}{\begin{bmatrix} 3 & 3 & 7 \\ 0 & 5 & 5 \\ 4 & 4 & 1 \end{bmatrix}} \times \underset{3\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} = \frac{1}{5}\begin{bmatrix} 30 \\ 25 \\ 15 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 3 \end{bmatrix}$$

$$[Y]_{\mathcal{B}} = (6, 5, 3)$$

(33) (a)

$\mathcal{S}: a(1,1,0) + b(0,0,1) \to$ já é ortogonal

Para ver:

$$(a, a, 0) \cdot (a, a, 0) = 1 \Rightarrow$$

$$\left\{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right), (0,0,1)\right\} \qquad a^2 + a^2 = 1 \Leftrightarrow a^2 = \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow a = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$

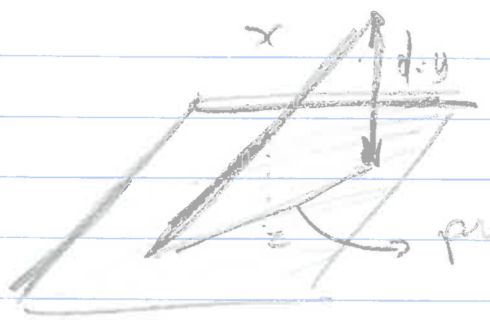
(b)

$\text{proj}_\beta\, x = (x \cdot x_1)\, x_1 + (x \cdot x_2)\, x_2$

$0 + (0,0,1) =$

$\text{proj}_\beta\, x = (0,0,1)$

(c)



$\vec{x} = \vec{y} + \vec{z}$

$\beta = \left\{ \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0 \right), (0,0,1) \right\}$

$\begin{cases} \frac{\sqrt{2}}{2} a + \frac{\sqrt{2}}{2} b = 0 \\ c = 0 \end{cases} \rightarrow \begin{cases} a = -b \\ c = 0 \end{cases} \rightarrow (-b, b, 0)$???

ALGA - ficha capítulo 4

3 a) $(-y, y, z)$

- [illegible] ✓
- 0_V incluído

É subespaço de $\mathbb{R}^2$

$A = a(-y, y)$

$B = b(-y, \ )$

$A+B = (-ay - by,\ ay + by) \in \mathbb{R}$

$\alpha A = (\alpha(-by), \alpha a y) \in \mathbb{R}^2$

b)
- $S \neq \emptyset$
- $0_V \notin S$

não é subespaço

c) i → $(a, b, 0)$ é subespaço vetorial

ii → não

iii → não

2)

[illegible], para que exista um 0_V podemos considerar que $\{(0,0)\}$, logo:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 + 1 \\ x_2 + y_2 + 1 \end{bmatrix}$$

→ Se este for elemento neutro ⇒ $\begin{cases} x_1 = x_1 + y_1 + 1 \\ y_2 + 1 = 0 \end{cases}$ ⇔ $\begin{cases} y_1 = 1 \\ y_2 = -1 \end{cases}$

Elemento neutro ⇒ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Assim, $0_V \in V$ e V é fechado em relação à adição e multiplicação

→ V é um subespaço vetorial real

1

Seja (y_1, y_2) o simétrico de X

$\underline{\underline{X + Y = 0_V}}$

$$\begin{cases} x_1 + y_1 - 1 = 1 \\ x_2 + y_2 + 1 = -1 \end{cases} \Leftrightarrow \begin{cases} y_1 = 2 - x_1 \\ y_2 = -2 - x_2 \end{cases}$$

$$-X = \begin{bmatrix} 2 - x_1 \\ -2 - x_2 \end{bmatrix}$$

b)

$S = (1 - 2t, t - 1)$

$S \neq \emptyset$ ✓

$\boxed{0_V \in S\ ??}$

Seja $K = (a, b)$ o elemento neutro de S:

Sabemos que $X + Y = \begin{bmatrix} x_1 + y_1 - 1 \\ x_2 + y_2 + 1 \end{bmatrix}$, então, se Y for elemento neutro, tem que

$$X = X + Y = \begin{bmatrix} x_1 + y_1 - 1 \\ x_2 + y_2 + 1 \end{bmatrix}$$

Substituindo por S, e como $Y = (y_1, y_2) = 0_V$, vem que

$$\begin{bmatrix} 1 - 2t \\ t - 1 \end{bmatrix} = \begin{bmatrix} \cancel{1} - \cancel{2t} + y_1 - \cancel{1} \\ \cancel{t} - \cancel{1} + y_2 + \cancel{1} \end{bmatrix} \longrightarrow \begin{cases} y_1 = 1 \\ y_2 = -1 \end{cases}$$

$$0_V = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \in V \quad ✓$$

Seja $A = a(1-2t, t-1)$ e $B = b(1-2t, t-1)$, $\forall a, b \in \mathbb{R} \setminus \{0\}$

$A+B = (a+b-2at-2bt, at-a+bt-b)$

$A+B \subseteq V$ ✓

$\alpha(1-2t, t-1) = (\alpha-2\alpha t, \alpha t-\alpha)$, $\forall \alpha \in \mathbb{R}$

$\alpha(1-2t, t-1) \subseteq V$ ✓

Assim S é subespaço de V

(6) (a)

$$\begin{cases} a+4b=2 & a=-2 \\ 2a+b=-3 ✓ \\ a+2b=4 ✓ \\ 3b=3 \longrightarrow b=1 \end{cases}$$

$(2,-3,-4,3) = (-2)(1,2,1,0) + (1)(4,1,-2,3)$

(c) $F = (-1,1,4)$

$A=(1,2,1)$
$B=(1,0,3)$
$C=(0,1,-1)$

$$\begin{cases} a+b=-1 \\ 2a+c=1 \\ a+3b-c=4 \end{cases} \quad \begin{cases} b=-a-1 \\ c=1-2a \\ \cancel{a}+(-3\cancel{a})-3-1+2a=4 \end{cases}$$

(e) Impossível

(2)

(b) $a(0,1) + b(0,2)\ ,\ a,b \in \mathbb{R} =$

$= (0,\ a+2b)\ ,\ a,b \in \mathbb{R}$

$\{(x,y) \in \mathbb{R}^2 : x = 0\}$

(d) $(2,2,2) = 2(1,1,1)$ l.d

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & x \\ 1 & 0 & 2 & y \\ 1 & 0 & 2 & z \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 1 & 0 & x-y \\ 1 & 0 & 2 & y \\ 0 & 0 & 0 & z-y \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & y \\ 0 & 1 & 0 & x-y \\ 0 & 0 & 0 & z-y \end{array}\right]$$

Para que seja possível

E. $K = \{(x,y,z) \in \mathbb{R}^3, z = y\}$, ou seja, é o plano $z - y = 0 \Rightarrow z = y$

$z = y$

(c)

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & x \\ 2 & -2 & 1 & y \\ 3 & 1 & 0 & z \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1/2 & 0 & x/2 \\ 0 & -1 & 1 & y-x \\ 0 & \frac{5}{2} & 0 & z-3x/2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1/2 & 0 & x/2 \\ 0 & 1 & -1 & x-y \\ 0 & 1 & 0 & \frac{2z}{5} - \frac{3x}{10} \end{array}\right] \ldots$$

S. Possível

$\Downarrow$

$\mathbb{R}^3$

$\{(x,y,z) \in \mathbb{R}^3\}$

(8)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 2 \\ 2 & 1 & 3 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x + z = 0 \\ y + z + w = 0 \end{cases} \Leftrightarrow \begin{cases} x = -z \\ y = -z - w \end{cases} \Longrightarrow \begin{bmatrix} -z \\ -z-w \\ z \\ w \end{bmatrix} = z \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + w \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathcal{N}(A): \langle (-1,-1,1,0), (0,-1,0,1) \rangle$$

(9) Seja $v = (a,b)$

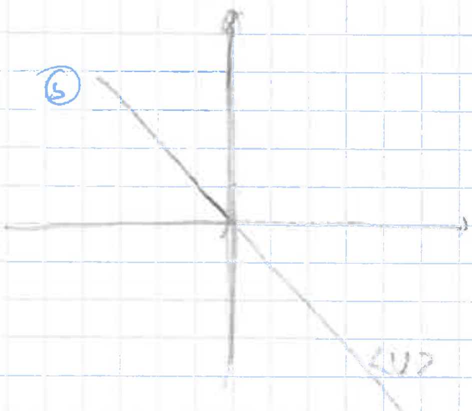
(a) $\langle v \rangle = \alpha_1 (a,b), \alpha \in \mathbb{R}$

$(x,y) = \alpha_1 (a,b)$

$$\begin{cases} x = \alpha_1 a \\ y = \alpha_1 b \end{cases} \Leftrightarrow \begin{cases} \alpha_1 = a/x \\ \alpha_1 = b/y \end{cases} \Leftrightarrow \frac{a}{x} = \frac{b}{y} \Leftrightarrow ya = bx \Leftrightarrow$$

$$\Leftrightarrow y = \frac{bx}{a}$$

(b)



$m = \frac{b}{a}$

ordenada na origem é 0, logo

⟨v⟩ passa pela origem e

tem direção $\vec{v}$

(11)

(c)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 3 \\ 1 & 3 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 2 \\ 0 & 2 & -1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} a - d = 0 \\ b = 0 \\ c + 2d = 0 \end{cases} \quad \begin{cases} d = a \\ b = 0 \\ c = -2a \end{cases} \rightarrow (a, 0, -2a, a)$$

→ não são linearmente independentes!

(d)

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \quad (0, 0, 0) \longrightarrow \text{linearmente independentes!}$$

(12)

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \text{ logo } \begin{bmatrix} x_1 + x_2 \\ x_1 + x_3 \\ x_2 + x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_3 + x_1 \\ x_2 + x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \\ x_2 \end{bmatrix} + \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix} =$$

$\in V$

$$= 2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \times 0 \\ 2 \times 0 \\ 2 \times 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow B_s = B_i$$

15. As bases são constituídas por vetores l.i

d)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow x = y = z = w = 0$$

l.I $\Rightarrow$ é base!

g)
$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & -1 & -2 \\ 1 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x = y = z = 0$

$\hookrightarrow$ l.I $\rightarrow$ é base!

a)
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x = -2y$$

$(-2y, y) \Rightarrow$ L.D $\Rightarrow$ não é base!

6

a) $\left[\begin{array}{cc|c} 1 & -1 & x \\ 3 & 1 & y \\ 0 & 0 & z \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & x \\ 0 & 4 & y-3x \\ 0 & 0 & z \end{array}\right]$ para que sea PD $z=0$

$\hookrightarrow (x, y, 0)$

$x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\mathcal{B} = \{(1,0,0), (0,1,0)\}$ $\quad \dim(\mathcal{B}) = 2$

b) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \sim \left[\begin{array}{ccc|c} -1 & 2 & 1 & y \\ 0 & 2 & 2 & x+y \\ 0 & 3 & 3 & z+y \end{array}\right] \sim \left[\begin{array}{ccc|c} -1 & 2 & 1 & y \\ 0 & 1 & 1 & \frac{x+y}{2} \\ 0 & 1 & 1 & \frac{z+y}{3} \end{array}\right] \sim$

$\sim \left[\begin{array}{ccc|c} -1 & 2 & 1 & y \\ 0 & 1 & 1 & \frac{x+y}{2} \\ 0 & 0 & 0 & \frac{z+y}{3} - \frac{x+y}{2} \end{array}\right]$

Para que sea PD:

$\frac{2z + 2y - 3x - 3y}{6} = 0 \iff$

$\iff -3x + 2z - y = 0 \iff$

$\iff y = -3x + 2z$

$\downarrow$

$(x, -3x + 2z, z)$

$x \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

$\mathcal{B} = \{(1,-3,0), (0,2,1)\}$ $\quad \dim(\mathcal{B}) = 2$

(e)

$$\left[\begin{array}{cc|c} 1 & 1 & x \\ 0 & -1 & y \\ 1 & 1 & z \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & x \\ 0 & -1 & y \\ 0 & 0 & z-x \end{array}\right]$$

para ser p.d.

$x = z \Rightarrow z = x$

(x, y, x)

$$x\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$S = \{(1,0,1) ; (0,1,0)\}$ $\quad \dim(S) = 2$

$(t^2 + 1, t)$

(17)

$$\left[\begin{array}{ccc|c} a^2 & 0 & 1 & x \\ 0 & a & 2 & y \\ 1 & 0 & 1 & z \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 0 & 1-a^2 & x - a^2 z \\ 0 & a & 2 & y \\ 1 & 0 & 1 & z \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & z \\ 0 & a & 2 & y \\ 0 & 0 & 1-a^2 & x - a^2 z \end{array}\right]$$

para ser possível determinado:

$a \neq 0 \wedge 1 - a^2 \neq 0 \Rightarrow$

$\Rightarrow a \neq 0 \wedge a \neq -1 \wedge a \neq 1$

$a \in \mathbb{R} \setminus \{-1, 0, 1\}$

(19) (a) $x = y - 3z$

$A = (y - 3z, y, z)$ $\qquad A + B = (y + y_2 - 3z - 3z_2, y + y_2, z + z_2) \subseteq \mathbb{R}^3$

$B = (y_2 - 3z_2, y_2, z_2)$ $\qquad \alpha A = (\alpha y - \alpha 3z, \alpha y, \alpha z) \subseteq \mathbb{R}^3$

$S \neq \emptyset$

se $y = z = 0$ então $S = (0,0,0)$ $(0_v) \subseteq \mathbb{R}^3$ ✓

(9)

(b)

$(y-3z, y, z) = y(1,1,0) + z(-3,0,1)$

$$\begin{bmatrix} 1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1/3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 1/3 \end{bmatrix}$$

$$\begin{cases} x - \frac{1}{3}z = 0 \\ y = -\frac{1}{3}z \end{cases} \sim \begin{cases} x - y = 0 \\ y = x + \frac{1}{3} z \end{cases}$$

$y = x + 3z$

$(x, x+3z, z) = x(1,1,0) + z(0,3,1)$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 1/3 \end{bmatrix}$$

$\alpha_1(1,1,0) + \alpha_2(0,3,1) = (0,0,0)$

$$\begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0 \end{cases}$$

(c) dim = 2 uma vez que bastam 2 vetores [illegible] para gerar o conjunto C em $\mathbb{R}^3$

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a)

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} a + b + 2c = 0 \\ d = c \end{cases} \Leftrightarrow \begin{cases} a = -b - 2c \\ d = c \end{cases} \to (-b-2c, b, c, c)$$

b) ~~Sem solução~~

$$N(A) = \{(-1, 1, 0, 0) ; (-2, 0, 1, 1)\}$$

$$\mathrm{nul}(A) = 2$$

$$\{(a,b,c) \in \mathbb{R}^3 : c = a + 2b\}$$

c) Seja K uma base de S. O espaço gerado por K será igual ao espaço gerado por S.

$$\left[\begin{array}{cc|c} 1 & 4 & x \\ -1 & -3 & y \\ -1 & -2 & z \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 4 & x \\ 0 & 1 & y+x \\ 0 & 2 & z+x \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 4 & x \\ 0 & 1 & y+x \\ 0 & 0 & z+x-2x-2y \end{array}\right]$$

para ser um S.P.D.:

$$z - x - 2y = 0 \Leftrightarrow z = 2y + x$$

$$(x, y, 2y + x)$$

$U = \{(a,b,c) \in \mathbb{R}^3 : c = 2b + a\} = S$, logo U é uma base de S, uma vez que o espaço gerado é o mesmo

11

23

a)

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 4 & -1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 4 & -7 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -7/4 & -5/4 \end{bmatrix}$$

$\mathcal{L}(A) = \{ (1,0,2,1) \, ; \, (0,1,-\frac{7}{4},-\frac{5}{4}) \}$

$\mathcal{C}(A) = \{ (1,3) \, ; \, (0,4) \}$

$car(A) = 2$

Diferente das soluções

a)

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -5 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -5 & 0 & 4 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\mathcal{L}(A) = \{ (1,0,0,0), (0,1,0,0), (0,0,0,1) \}$

$\mathcal{C}(A) = \{ (1,2,0,2), (2,1,0,-1), (0,4,0,1) \}$

$= \{ (1,0,0,0), (0,1,0,0), (0,0,0,1) \} = \mathcal{L}(A)$

$car(A) = 3$

$$\begin{cases} a + b = 0 \\ 5b = 4d \\ d = 0 \end{cases} \quad \begin{cases} a = -b = 0 \\ b = 0 \\ d = 0 \end{cases} \rightarrow (0,0,c,0) \rightarrow c \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$\mathcal{N}(A) = \{ (0,0,1,0) \}$

$nul(A) = 1$

Não são independentes

①

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathcal{C}(A) = \{ (1,0,0), (0,1,0), (0,0,1) \}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \\ 3 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathcal{F}(A) = \{ (1,0,0), (0,1,0), (0,0,1) \}$

$\operatorname{ran}(A) = 3 = m \Longrightarrow$ dil... 0

$$\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \longrightarrow (0,0,0) \Longrightarrow \mathcal{N}(A) = \emptyset$$

$\operatorname{nul}(A) = 0$

⑤

ⓑ

$\mathcal{B} = \{ (1,1,0,0), (1,0,0,0), (1,1,1,0), (1,1,1,1) \}$

$X = (2,1,0,0)$

$$[(2,1,0,0)]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} a + b + \cancel{c} + \cancel{d} = 2 \Rightarrow b = 1 \\ a + \cancel{c + d} = 1 \Rightarrow a = 1 \\ c + d = 0 \longrightarrow c = 0 \\ d = 0 \longrightarrow d = 0 \end{cases}$$

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$\mathcal{A} = \{ (1,2,1), (0,2,0), (0,0,-1) \}$

a

$\mathcal{B} = \{ (1,0,-1), (1,1,1), (2,3,-1) \}$

i $[(2,3,5)]_{\mathcal{A}} = \begin{bmatrix} 2 \\ -1/2 \\ -3 \end{bmatrix}$

$$\begin{cases} a = 2 \\ 2a + 2b = 3 \\ a - c = 5 \end{cases} \Leftrightarrow \begin{cases} a = 2 \\ b = -1/2 \\ c = -3 \end{cases}$$

$A = (A_1, A_2) \quad B(B_1, B_2,$

b

$M_{\mathcal{B} \leftarrow \mathcal{A}}$

$[X]_A = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$X = a(1,2,1) + b(0,2,0) + c(0,0,-1)$

$[X]_{\mathcal{B}} = a[(1,2,1)]_{\mathcal{B}} + b[(0,2,0)] + c[(0,0,-1)]_{\mathcal{B}}$

$M_{A \leftarrow B}$

$[X]_B = \begin{bmatrix} x \\ y \end{bmatrix}$

$X = a(B_1) + b(X_2)$

$[X]_S = \underbrace{a[B_1]_S}_{\text{Valor 1}} + \underbrace{b[B_2]_S}_{\text{Valor 2}}$

$[X]_S = a \begin{bmatrix} \text{Valor} \\ 1 \end{bmatrix} + b \begin{bmatrix} \text{Valor} \\ 2 \end{bmatrix}$

$M_{A \leftarrow B} = \begin{bmatrix} \text{Valor } 1 & \text{Valor } 2 \end{bmatrix}$

• $[(1,2,1)]_{\mathcal{B}} = \frac{1}{5} \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}$

$$\begin{cases} 1 = a + b + 2c \\ 2 = b + 3c \\ 1 = -a + b - c \end{cases}$$

• $[(0,2,0)]_{\mathcal{B}} \Longrightarrow = \frac{1}{5} \begin{bmatrix} -6 \\ -2 \\ 4 \end{bmatrix}$

$$\begin{cases} 0 = a + b + 2c \\ 2 = b + 3c \\ 0 = -a + b - c \end{cases}$$

• $[(0,0,-1)]_{\mathcal{B}} = \frac{1}{5} \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$

$[X]_{\mathcal{B}} = \frac{1}{5} \begin{bmatrix} -3 & -6 & 1 \\ 4 & -2 & -3 \\ 2 & 4 & 1 \end{bmatrix} [X]_{\mathcal{A}}$

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$$a\frac{\sqrt{2}}{2} - b\frac{\sqrt{2}}{2} = 0 \implies a = b$$

$$a^2 + b^2 + \frac{2}{4} = 1 \iff$$

$$2a^2 = \frac{4}{4} - \frac{2}{4} = \frac{2}{4} \implies a^2 = \frac{2}{8} = \frac{1}{4} \implies a = \pm\frac{1}{2} \wedge b = \pm\frac{1}{2}$$

$$a = b = \frac{1}{2} \vee a = b = -\frac{1}{2}$$

34 X

$$X = (4, 0, -9)$$

$$A_1 = (0, 1, 0)$$

$$A_2 = \left(\frac{1}{2}, 0, \frac{\sqrt{3}}{2}\right)$$

$$\operatorname{proj}_W X = (X \cdot A_1)A_1 + (X \cdot A_2)A_2 = \left(2 - \frac{9\sqrt{3}}{2}\right)\left(\frac{1}{2}, 0, \frac{\sqrt{3}}{2}\right) =$$

$$= \frac{4 - 9\sqrt{3}}{2}\left(\frac{1}{2}, 0, \frac{\sqrt{3}}{2}\right) = \left(\frac{4 - 9\sqrt{3}}{4}, 0, \frac{4\sqrt{3} - 27}{4}\right) =$$

$$= \left(1 - \frac{9}{4}\sqrt{3}, 0, \sqrt{3} - \frac{27}{4}\right)$$

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$$[x]_S = M_{S \leftarrow T} [x]_{\mathcal{T}}$$

$$\left[M_{C \leftarrow S} \;\middle|\; M_{C \leftarrow \mathcal{T}} \right] = \left[I_n \;\middle|\; M_{S \leftarrow T} \right] \qquad \cdot 1 \quad -1 \; -2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & \\ 1 & 0 & 0 & M_{C \leftarrow T} \\ 0 & 1 & 1 & \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 & 2 \end{array}\right]$$

$$\mathcal{T} : \left((1,1,1), (0,1,0), (-1,2,2) \right)$$

(27)

- $S = ((1,2),(0,1))$
- $J = ((1,1),(2,3))$
- $X \hookrightarrow (1,5)$

(a) $[(1,5)]_J = \begin{bmatrix} -7 \\ 4 \end{bmatrix} = (-7,4)$

$\begin{cases} a+2b=1 \\ a+3b=5 \end{cases} \quad \begin{cases} a=1-2b \\ 1+b=5 \end{cases} \quad \begin{cases} a=-7 \\ b=4 \end{cases}$

(b) $[z]_J = (1,-3)$

$z = (-5,-8)$

(c) $M_{S \leftarrow T}$

$X = a(1,1) + b(2,3)$

$[x]_S = a[(1,1)]_S + b[(2,3)]_S$

$\begin{cases} 1 = a \\ 1 = 2a+b \Rightarrow b=-1 \end{cases}$ $\quad \begin{cases} 2 = a \\ 3 = 2a+b \Rightarrow b=-1 \end{cases}$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$M_{S\leftarrow J} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

(d) $[X]_S = \underbrace{\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}}_{M_{S\leftarrow T}} [X]_J = \underbrace{\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}}_{2\times 2} \times \underbrace{\begin{bmatrix} -7 \\ 4 \end{bmatrix}}_{1\times 2} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\quad [X]_S = (1,3)$

(e) $[X]_S = (1,3)$ $\quad \begin{cases} 1 = a \\ 5 = 2a+b \Rightarrow \end{cases}$ $\quad a \Rightarrow 1$, $\quad b = 3$

b)

$$N = M_{T \leftarrow S} = \left(M_{S \leftarrow T} \right)^{-1}$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

$$N = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$[X]_T = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 4 \end{bmatrix} = (-7, 4)$$

Ficha - Capítulo 5

1)

a) $X \to (x, y, z)$

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} = \qquad \det(A - \lambda I_n) = 0$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} -\lambda & 2 & 1 \\ 0 & -\lambda & 4 \\ 0 & 0 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I_3) = (-\lambda^3 + 0) - 0 = -\lambda^3 = 0$$

$$\Downarrow$$

$$\lambda = 0$$

Para $\lambda = 0$ vem que:

$$\underset{3\times 3}{\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}} \times \underset{3\times 1}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \begin{bmatrix} 2y + z \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Longrightarrow \begin{cases} y = 0 \\ z = 0 \\ x = ? \end{cases}$$

$$X \to (x, 0, 0), \; x \in \mathbb{R}$$

$A_{3\times 3}$ não é diagonizável, pois $n = 3$ e esta matriz apenas possui 1 vetor próprio l.i.

b)

$$A = \begin{bmatrix} 1-\lambda & 0 & 0 \\ -1 & 3-\lambda & 0 \\ 3 & 2 & -2-\lambda \end{bmatrix} \qquad \det(A) = \big((1-\lambda)(3-\lambda)(-2-\lambda)\big) - (0) =$$

$$\det(A) = 0 \Longrightarrow \lambda = 1 \lor \lambda = 3 \lor \lambda = -2$$

Para $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & 2 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 0 \\ 4 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ x-2y \\ 4x-3z \end{bmatrix}$$

$$\begin{cases} x = 2y \\ x = \frac{3z}{4} \end{cases}$$

$$X_1 \to \left(2y,\ y,\ \frac{8y}{3}\right) = \alpha(6, 3, 8),\ \alpha \in \mathbb{R}\setminus\{0\}$$

$\lambda = 3$

$$\begin{bmatrix} -2 & 0 & 0 \\ -1 & 0 & 0 \\ 3 & 2 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} x \\ 0 \\ x+2y-5z \end{bmatrix}$$

$$\begin{cases} x = 0 \\ \\ 2y = 5z \end{cases} \sim \begin{cases} x = 0 \\ y = \frac{5}{2} z \end{cases}$$

$$X_3 \to \left(0,\ \frac{5}{2}z,\ z\right) = \beta(0, 5, 2),\ \beta \in \mathbb{R}\setminus\{0\}$$

$\lambda = -2$

$$\begin{bmatrix} 3 & 0 & 0 \\ -1 & 5 & 0 \\ 3 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ y \end{bmatrix} = 0$$

$$X_{-2} = (0, 0, z) \to \theta(0, 0, 1),\ \theta \in \mathbb{R}\setminus\{0\}$$

$$\begin{cases} 6x + 3y + 8z = 0 \\ 5y + 2z = 0 \\ z = 0 \end{cases} \sim \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \Longrightarrow \text{Os 3 vetores são l.i.}$$

• Como $n = 3$ e a matriz tem 3 vetores próprios linearmente independentes, podemos afirmar que a matriz é diagonalizável

$$P_1 = \begin{bmatrix} 0 & 6 & 0 \\ 0 & 3 & 5 \\ 1 & 8 & 2 \end{bmatrix} \qquad D_2 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

1)

$$A = \begin{bmatrix} 1-\lambda & 1 & -2 \\ 2 & -\lambda & -2 \\ 3 & 1 & -4-\lambda \end{bmatrix}$$

$$\det(A) = 0 = \left((1-\lambda)(-\lambda)(-4-\lambda) + \cancel{1(-6)} \right) - \left(6y + (\cancel{-2} + 2y) + (\cancel{-8} - \cancel{2y}) \right)$$

$$= (1-\lambda)(-\lambda)(-4-\lambda) \Leftrightarrow \underline{\lambda = -2 \lor \lambda = -1 \lor \lambda = 0}$$

$\underline{\lambda = -2}$

$$\begin{bmatrix} 3 & 1 & -2 \\ 2 & 2 & -2 \\ 3 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 3 & 1 & -2 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & -2 & 1 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} z = 2y \\ x + y = z \end{cases} \sim \begin{cases} 2y = x + y \Rightarrow y = x \\ z = 2x \end{cases}$$

$$X_{-2} = (x, x, 2x) = \alpha(1, 1, 2),\ \alpha \in \mathbb{R} \setminus \{0\}$$

$\lambda = -1$

$$\begin{bmatrix} 2 & 1 & -2 \\ 2 & 1 & -2 \\ 3 & 1 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0{,}5 & -1 \\ 3 & 1 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 1 & \frac{1}{2} & -1 \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}$$

$$\begin{cases} x + \cancel{\frac{1}{2}y} = z \\ y = 0 \end{cases}$$

$$X_{-1} = (z, 0, z) \to \beta(1, 0, 1), \quad \beta \in \mathbb{R} \setminus \{0\}$$

$\lambda = 0$

$$\begin{bmatrix} 1 & 1 & -2 \\ 2 & 0 & -2 \\ 3 & 1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & -2 & 2 \\ 0 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x + y = 2z \\ y = z \end{cases} \quad \begin{cases} x = z \\ y = z \end{cases}$$

$$X_0 = (z, z, z) \longrightarrow \theta(1, 1, 1), \quad \theta \in \mathbb{R} \setminus \{0\}$$

Verificar se os vetores são l.i.:

$$\begin{cases} x + y + 2z = 0 \\ x + y + z = 0 \\ x + z = 0 \end{cases} \to \begin{cases} z + 0 = 0 \\ y = 0 \\ x = -z \end{cases} \to \begin{cases} z = 0 \\ y = 0 \\ x = 0 \end{cases} \quad \text{São l.i}$$

Matriz diagonalizável:

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \qquad B_1 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2)

- $AX = \lambda X$
- $\det(A - \lambda I_n) = 0$

$$\det(A - I_n \times 1) = \begin{vmatrix} 0 & 0 & 0 \\ 2 & -2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = \det(A - I_n \times 1) = 0$$

$\hookrightarrow \lambda = 1$ é um valor próprio da matriz

a)

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 5 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{cases} 2x = -5z \\ y = -2z \end{cases} \begin{cases} x = -\frac{5}{2}z \\ y = -2z \end{cases}$$

$$X_1 = \left(-\frac{5}{2}z, -2z, z\right) \rightarrow \alpha(-5, -4, 2), \alpha \in \mathbb{R} \setminus \{0\}$$

$$U_1 = \langle (-5, -4, 2) \rangle$$

b)

$$B = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 2 & -1-\lambda & 1 \\ 0 & 1 & 3-\lambda \end{bmatrix} \quad \det(B) = (1-\lambda)(-1-\lambda)(3-\lambda) - (1-\lambda)$$

???

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a)

$$A\,\cancel{x} = \begin{bmatrix} 2-\lambda & -2 & 3 \\ 0 & 3-\lambda & -2 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

$$\det(A) = \left((2-\lambda)(3-\lambda)(2-\lambda) \right) - \left((4-2\lambda) \right) =$$

$$= (2-\lambda)(3-\lambda)(2-\lambda) - 2(2-\lambda) =$$

$$= (2-\lambda)(6-5\lambda+\lambda^2) - 2(2-\lambda)$$

$$= (2-\lambda)(6-5\lambda+\lambda^2-2) =$$

$$= (2-\lambda)(\lambda^2-5\lambda+4)$$

$$\lambda = \frac{5 \pm \sqrt{25-16}}{2} = \frac{8}{2} \vee \frac{2}{2}$$

$$= \underline{\lambda = 2 \vee \lambda = 4 \vee \lambda = 1}$$

Para $\lambda = 1$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -2 \\ 0 & -1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{cases} x = -2 \\ y = 2 \end{cases} \qquad U_1 = \langle (-1, 1, 1) \rangle$$

$\lambda = 2$

$$\begin{bmatrix} 0 & -2 & 3 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{cases} z = 0 \\ z = 0 \\ y = 0 \end{cases} \qquad U_2 = \langle (1,0,0) \rangle$$

$\lambda=4$

$$\begin{bmatrix} -2 & -2 & 3 \\ 0 & -1 & -2 \\ \cancel{0} & \cancel{-1} & -2 \end{bmatrix} \sim \begin{bmatrix} -2 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{cases} 7z = 2x \\ 2z = -y \end{cases} \begin{cases} x = \frac{7}{2} z \\ y = -2z \end{cases}$$

$$V_4 = \langle (7, -4, 2) \rangle$$

(b)

$$\begin{cases} -x + y + z = 0 \\ x = 0 \\ 7x - 4y + 2z = 0 \end{cases} \rightarrow \begin{cases} y = -z \\ x = 0 \\ 0 + 4z + 2z = 0 \end{cases} \begin{cases} y = 0 \\ x = 0 \\ z = 0 \end{cases}$$

Matriz 3x3 → 3 vetores próprios l.i. → matriz diagonalizável

$$P = \begin{bmatrix} -1 & 1 & 7 \\ 1 & 0 & -4 \\ 1 & 0 & 2 \end{bmatrix}$$ → matriz diagonalizante de A, que satisfaz a condição $D = P^{-1}AP$

(c)

$$A^5 = PD^5P^{-1}$$

$$P = \begin{bmatrix} -1 & 1 & 7 \\ 1 & 0 & -4 \\ 1 & 0 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & 1/3 & 2/3 \\ 1 & 3/2 & -1/2 \\ 0 & -1/6 & 1/6 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} -1 & 1 & 7 \\ 1 & 0 & -4 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 32 & 0 \\ 0 & 0 & 1024 \end{bmatrix} \times$$

$$\begin{bmatrix} -1 & 32 & 7168 \\ 1 & 0 & -4096 \\ 1 & 0 & 2048 \end{bmatrix} \begin{bmatrix} 0 & 1/3 & 2/3 \\ 1 & 3/2 & -1/2 \\ 0 & -1/6 & 1/6 \end{bmatrix} =$$

$$= \begin{bmatrix} 32 & -1147 & 1178 \\ 0 & 683 & -682 \\ 0 & -341 & 342 \end{bmatrix} = A^5$$

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$$A = \begin{bmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \delta & \theta & \mu \end{bmatrix} \qquad \begin{matrix} \vec{u} \to (1,1,1) \\ \vec{v} \to (1,0,-1) \\ \vec{w} \to (1,-1,0) \end{matrix}$$

$$A X_i = \lambda_i X_i \qquad \begin{bmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \delta & \theta & \mu \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_i \\ \lambda_i \\ \lambda_i \end{bmatrix} \Leftrightarrow \begin{bmatrix} 3 \\ \alpha+\beta+\gamma \\ \delta+\theta+\mu \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_1 \\ \lambda_1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} \lambda i = 3 \\ \alpha+\beta+\gamma = 3 \\ \delta+\theta+\mu = 3 \end{cases}$$

$$A X_i = \lambda_i x_i = \begin{bmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \delta & \theta & \mu \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \lambda_2 \\ 0 \\ -\lambda \end{bmatrix} \Leftrightarrow \begin{bmatrix} 2 \\ \alpha-\gamma \\ \delta-\mu \end{bmatrix} \to \begin{cases} \lambda_2 = 2 \\ 2 = \alpha-\gamma \\ 2 = \delta-\mu \end{cases}$$

$$\ldots \begin{bmatrix} 0 \\ \alpha-\beta \\ \delta-\theta \end{bmatrix} \to \begin{cases} \lambda_3 = 0 \\ 0 = \alpha-\beta \\ 0 = \delta-\theta \end{cases}$$

$$\begin{cases} \delta = 3-\theta-\mu \\ \alpha = 2+\gamma \\ \delta = 2+\mu \\ \alpha = 3-\beta-\gamma \\ \alpha = \beta \\ \theta = \delta \end{cases}$$

Resolvendo o sistema:

$\alpha = \beta = \gamma = \delta = \phi = \mu = 1$

$\det(B - \lambda I_n) = 0$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ a & b & c-\lambda \end{vmatrix} = \left(\lambda^2(c-\lambda) + a\right) - \left(-b\lambda\right) =$$

$$= -\lambda^3 + c\lambda^2 + b\lambda + a = 0$$

Como $\lambda = -1$ / $\lambda = 0$ / $\lambda = 1$

$$\begin{cases} 1 + c - b + \alpha = 0 \\ a = 0 \\ -1 + c + b + \alpha = 0 \end{cases} \Leftrightarrow \begin{cases} b = c + 1 \\ \\ -\cancel{1} + c + c + \cancel{1} = 0 \end{cases} \Leftrightarrow \begin{cases} b = 1 \\ a = 0 \\ c = 0 \end{cases}$$

15 a) $Ax = \lambda x$ / $\det(A - \lambda I) = 0$

$$\det(A - 9I) = \begin{vmatrix} -6 & -4 & -4 \\ -4 & -8 & 0 \\ -4 & 0 & -4 \end{vmatrix} = \left((-192) + (0) + 0\right) - \left((-128) + 0 + (-64)\right) = -192 + 192 = 0$$

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(b)

$$A - \lambda I = \begin{vmatrix} 3-\lambda & -4 & -4 \\ -4 & 1-\lambda & 0 \\ -4 & 0 & 5-\lambda \end{vmatrix} = \left((3-\lambda)(1-\lambda)(5-\lambda) + 0 \right) - \left(16(1-\lambda) + 16(5-\lambda) \right) =$$

$$= -16\left((1-\lambda) + (5-\lambda)\right) + \left((3-\lambda)(1-\lambda)(5-\lambda)\right) =$$

$$= \dots - 16(6-2\lambda) =$$

$$= \dots - 32(3-\lambda)$$

$$= (3-\lambda)(1-\lambda)(5-\lambda) - 32(3-\lambda) =$$

$$= (3-\lambda)(5-\lambda-5\lambda+\lambda^2-32) =$$

$$= (3-\lambda)(\lambda^2 - 6\lambda - 27) = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda = 3 \vee \lambda = \frac{6 \pm \sqrt{36 - 4 \times 1 \times (-27)}}{2}$$

$$\Leftrightarrow \lambda = 3 \vee \lambda = \frac{6 \pm 12}{2} \Leftrightarrow$$

$$\Leftrightarrow \lambda = 3 \vee \lambda = 9 \vee \lambda = -3$$

$$\lambda = -3 \quad \begin{bmatrix} 6 & -4 & -4 \\ -4 & 4 & 0 \\ -4 & 0 & 8 \end{bmatrix} \sim \begin{bmatrix} 6 & -4 & -4 \\ 0 & -\frac{8}{3} & \frac{16}{3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} 6x - 4y = 4z \\ 16z = 8y \end{cases} \Leftrightarrow \begin{cases} 6x - 8z = 4z \\ y = 2z \end{cases} \Leftrightarrow \begin{cases} x = 2z \\ y = 2z \end{cases}$$

$$U_{-3} = \langle (2, 2, 1) \rangle$$

$y=3$ $\begin{bmatrix} 0 & -4 & -4 \\ -4 & -2 & 0 \\ -4 & 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{cases} 2x = -y \\ z = -y \end{cases} \to \begin{cases} x = -\frac{y}{2} \\ z = -y \end{cases}$

$U_3 = \langle (-1, 2, -2) \rangle$

$y=9$ $\begin{bmatrix} -6 & -4 & -4 \\ -4 & -8 & 0 \\ -4 & 0 & -4 \end{bmatrix} \sim \begin{bmatrix} -6 & -4 & -4 \\ 0 & -\frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{cases} 3x + 2y + 2z = 0 \\ 16y = 8z \end{cases} \to \begin{cases} 3x = -2y - 4y \to x = -2y \\ z = 2y \end{cases}$

$U_9 = \langle (-2, 1, 2) \rangle$

$P = \begin{bmatrix} 2/3 & -1/3 & -2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix}$ $\| x_1 \| = \| x_3 \| = \| x_9 \| = 3$

$D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{bmatrix} = P^{-1} A P$

(16) (a)

· A simétrica $\longrightarrow P^{-1} = P^T$

Com $\lambda = 1 \Rightarrow \vec{a} = (1,0,0)$, $\vec{b} = (0,1,1)$

$$\left[\begin{array}{cc|c} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 1 & z \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & z-y \end{array}\right]$$

↳ Tem que ser = 0

$z = y$

$V = \{ (x,y,z) \in \mathbb{R}^3 : z = y \}$

(b) · A é uma matriz 3x3 com 3 valores próprios li, logo A é diagonalizável

$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, $A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$AX = \lambda X = A(1,0,0) = (1,0,0) =$

$= \begin{bmatrix} a & b & c \\ b & e & h \\ c & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e & h \\ 0 & h & i \end{bmatrix}$

$A(0,1,1) = (0,1,1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & e & h \\ 0 & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e+h \\ h+i \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{cases} e+h=1 \\ h+i=1 \end{cases} \Rightarrow \begin{cases} e = i \\ h = 1-i \end{cases}$$

$A(0,-1,1) = (0,3,-3)$

$$\begin{cases} 0 \\ -e+h = 3 \\ i-h = -3 \end{cases}$$

$$\begin{cases} e=i \\ h=1-i \\ i-h=-3 \end{cases} \quad \begin{cases} h=1-i \\ h=3+i \end{cases} \rightarrow 1-i = 3+i \Leftrightarrow$$

$\Leftrightarrow 2i = -2 \Leftrightarrow$

$\Leftrightarrow \underline{i = -1}$

$\underline{e = -1}$

$h = 2$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix}$$

GA – capítulo 5

ⓓ $\begin{vmatrix} 4-\lambda & 2 & 3 \\ 2 & 1-\lambda & 2 \\ -1 & -2 & -\lambda \end{vmatrix} = [(4-\lambda)(1-\lambda)(-\lambda) + (-12) + (-12)] - [\ldots] - 16$

$[(-3+3\lambda) + (-16+16\lambda) + (-4\lambda)]$ $(-19+15y)$

$= (4-\lambda)(1-\lambda)(-\lambda) + 3 - 15y \ldots \Rightarrow \lambda = 1 \vee \lambda = 3$

$\lambda = 1$

$\begin{bmatrix} 3 & 2 & 3 \\ 2 & 0 & 2 \\ -1 & -2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 0 \\ 0 & 4 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} x = -z \\ y = 0 \end{matrix}$

$(-z, 0, z) = z \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$v_1 = (-1, 0, 1)$

$\lambda = 3$

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 2 \\ -1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} 3y = -2z \\ x - y + z = 0 \end{cases}$

$\Rightarrow \begin{cases} y = -\frac{2}{3} z \\ x = -\frac{2}{3} z - z = -\frac{5}{3} z \end{cases}$ $\left(-\frac{5}{3} z, -\frac{2}{3} z, z\right) \to (-5z, -2z, 3z)$

$v_2 = (-5, -2, 3)$

[illegible]

(c)

$$\begin{vmatrix} 2-\lambda & -1 & -1 \\ 0 & 3-\lambda & 1 \\ 0 & 1 & 3-\lambda \end{vmatrix} = (2-\lambda)\left((3-\lambda)^2 - 1\right) =$$

$$= (2-\lambda)(9 - 6\lambda + \lambda^2 - 1) = (2-\lambda)(\lambda - 2)(\lambda - 4) = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda = 2 \lor \lambda = 2 \lor \lambda = 4$$

$\boxed{\lambda = 2}$

$$\begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow y + z = 0 \to y = -z$$

$$X_2 = (x, -z, z) = x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$X_2 = x(1,0,0) + z(0,-1,1), \quad \forall x, z \in \mathbb{R} \setminus \{0\}$$

$\boxed{\lambda = 4}$

$$\begin{bmatrix} -2 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{cases} x = -z \\ y = z \end{cases}$$

$$X_4 = z(-1, 1, 1)$$

[illegible] 3x3 (n = 3) [illegible] LI [illegible]

$$P = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

[illegible]

(2)

a) · Se $\det(A - 1I_n)$ for zero, então 1 é valor próprio de A

$$\det(A - 1I_n) = \begin{vmatrix} 0 & 0 & 0 \\ 2 & -2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 0 \text{, pois tem uma linha de zeros}$$

→ 1 é valor próprio de A

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 5 \\ & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{cases} x = -\frac{5}{2}z \\ y = -2z \end{cases}$$

$$v_1 = z\left(-\frac{5}{2}, -2, 1\right) = z(-5, -4, 2) = z(5, 4, -2)$$

$$U_1 = \langle (5, 4, -2) \rangle$$

b)

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & -1-\lambda & 1 \\ 0 & 1 & 3-\lambda \end{vmatrix} = \left[(1-\lambda)(-1-\lambda)(3-\lambda)\right] - (1-\lambda) =$$

$$= (1-\lambda)\left(-3 + \lambda^2 + \lambda - 3\lambda - 1\right) =$$

$$= (1-\lambda)(\lambda^2 - 2\lambda - 4) = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda = 1 \lor \lambda = 1-\sqrt{5} \lor \lambda = 1+\sqrt{5}$$

- Matriz 3x3
- 3 vetores próprios l.i.

} é diagonalizável → $D = \begin{bmatrix} 1+\sqrt{5} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\sqrt{5} \end{bmatrix}$

(2)

$$\begin{vmatrix} 2-\lambda & -2 & 3 \\ 0 & 3-\lambda & -2 \\ 0 & -1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2(3-\lambda) + 0 - \left(2(2-\lambda)\right) =$$

$$= (2-\lambda)\left((2-\lambda)(3-\lambda) - 2\right) =$$

$$= (2-\lambda)(6 - 5\lambda - 2 + \lambda^2) =$$

$$= (2-\lambda)(\lambda^2 - 5\lambda + 4) = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda = 1 \vee \lambda = 2 \vee \lambda = 4$$

$\lambda = 1$

$$\begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & -2 \\ 0 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{cases} x = -z \\ y = z \end{cases}$$

$$X_1 = z(-1, 1, 1) \qquad U_1 = \langle (-1, 1, 1) \rangle$$

$\lambda = 2$

$$\begin{pmatrix} 0 & -2 & 3 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -2 \\ 0 & -1 & 0 \end{pmatrix} \Longrightarrow \begin{cases} z = 0 \\ y = 0 \\ x = ? \end{cases}$$

$$X_2 = x(1, 0, 0) \qquad U_2 = \langle (1, 0, 0) \rangle$$

$\lambda = 4$

$$\begin{pmatrix} -2 & -2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} -2 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{cases} x = \frac{7}{2} z \\ y = -2z \end{cases}$$

$$X_4 = z\left(\frac{7}{2}, -4, 2\right) \qquad U_4 = \langle (7, -4, 2) \rangle$$

b) A é do tipo 3x3
A tem 3 valores próprios (...) } A é diagonalizável

$$P = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & -4 \\ 1 & 0 & 2 \end{bmatrix} \quad D = P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

c)

$$A^5 = PD^5P^{-1}$$

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & -4 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 9 & 1 & 0 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 6 & 0 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1/6 & 1/6 \end{array}\right]$$

$$\therefore \quad P^{-1} = \begin{bmatrix} 0 & 1/3 & 2/3 \\ 1 & 3/2 & -1/2 \\ 0 & -1/6 & 1/6 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 32 & -1147 & 1178 \\ 0 & 683 & -682 \\ 0 & -341 & 342 \end{bmatrix}$$ via Symbolab.

(a)

$$\det \begin{vmatrix} 1-\lambda & 1 \\ a & b-\lambda \end{vmatrix} = (1-\lambda)(b-\lambda) - a =$$

$$= b - \lambda - b\lambda + \lambda^2 - a =$$

$$= \lambda^2 - \lambda(1+b) + (b-a) = 0$$

$$\text{Se } \lambda = 0 \Rightarrow b - a = 0 \Rightarrow \underline{b = a}$$

$$\begin{bmatrix} 1 & 1 \\ a & b \end{bmatrix} = \begin{cases} x = -y \\ ax = -by \end{cases} \qquad V_0 = y(-1, 1)$$

$$x = -\frac{by}{a}$$

$$\begin{bmatrix} 1-\lambda & 1 \\ b & a-\lambda \end{bmatrix} = (1-\lambda)(a-\lambda) - a =$$

$$= \cancel{a} - \lambda - a\lambda + \lambda^2 - \cancel{a} =$$

$$= \lambda(\lambda - (1+a)) \Rightarrow$$

$$\Rightarrow \lambda = 0 \ \vee \ \lambda = 1 + a$$

$\underline{\lambda = 1 + a}$

$$\begin{bmatrix} -a & 1 \\ a & -1 \end{bmatrix} \Rightarrow \begin{cases} x = \frac{1}{a} \\ y = \frac{1}{a} \end{cases} \qquad a = b = 1$$

(b) (a)

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0 \Rightarrow \lambda = -1 \ \vee \ \lambda = 1$$

$\lambda = -1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \sim \quad x = -y \qquad V_{-1} = y(-1, 1) \qquad U_1 = \langle (-1, 1) \rangle$$

$\lambda = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \sim \quad x = y \qquad V_1 = y(1, 1) \qquad P_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$P_2 = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

diagonalizante

$$P = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(6)

(h)

$$\begin{vmatrix} -\lambda & -1 & -1 \\ -1 & -\lambda & -1 \\ -1 & -1 & -\lambda \end{vmatrix} = (-\lambda^3 + 1 - 1) - (-\lambda - \lambda - \lambda) =$$

$$= -\lambda^3 - 2 - 3\lambda =$$

$$= 0 \Rightarrow \lambda(\lambda^2 + 3) - 2$$

$$\Leftrightarrow \underline{\lambda = -2 \vee \lambda = 1}$$

$\lambda = -2$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & -3 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 0 & 3 & 3 \\ 0 & 3 & -3 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} x = z \\ y = z \end{cases}$$

$X_2 = z(1, 1, 1)$

$W_2 = \langle (1, 1, 1) \rangle$

$$P_2 = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$$

$\boxed{\lambda = 1}$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \Rightarrow \begin{cases} x + y + z = 0 \\ x = -y - z \\ y = -y + z - z \end{cases} \longrightarrow (-y + z, y, z)$$

$$X_1 = y(-1, 1, 0) \quad z(-1, 0, 1)$$

$$P_1 = \left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$$

$$P_1' = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right) \longrightarrow$$ Dá certo, mas esta matriz não é ortogonal!

$(1, 1, 1)(-1, 1, 0)$ ✓

$(1, 1, 1)(-1, 0, 1)$ ✓

$(-1, 1, 0)(-1, \ldots)$

15

a) Se $\det(A - 9I_3) = 0$, então 9 é vp

$$\det(A - 9I_3) = \begin{vmatrix} -6 & -4 & -4 \\ -4 & -8 & 0 \\ -4 & 0 & -4 \end{vmatrix} = (24(-8) + 0 - 0) - (16(-8) + 16(-4)) =$$

$$= -192 - (-192) = 0$$

b)

$$\begin{vmatrix} 3-\lambda & -4 & -4 \\ -4 & 1-\lambda & 0 \\ -4 & 0 & 5-\lambda \end{vmatrix} = ((3-\lambda)(1-\lambda)(5-\lambda) - 0) - (16(1-\lambda) + 16(5-\lambda))$$

$$= (3-\lambda)(1-\lambda)(5-\lambda) - 16(1-\lambda + 5-\lambda) =$$

$$= (3-\lambda)(1-\lambda)(5-\lambda) - 16(6-2\lambda) =$$

$$= (3-\lambda)(1-\lambda)(5-\lambda) - 32(3-\lambda) =$$

$$= (3-\lambda)(5 - 6\lambda + \lambda^2 - 32) =$$

$$= (3-\lambda)(\lambda^2 - 6\lambda - 27) = 0 \Rightarrow$$

$$\Rightarrow \lambda = -3 \vee \lambda = 3 \vee \lambda = 9$$

$\boxed{\lambda = -3}$

$$\begin{bmatrix} 6 & -4 & -4 \\ -4 & 4 & 0 \\ -4 & 0 & 8 \end{bmatrix} \sim \begin{bmatrix} 3 & -2 & -2 \\ 0 & -\frac{2}{3} & \frac{16}{3} \\ & & \end{bmatrix} \Rightarrow \begin{cases} 3x = 2y + 2z \\ \frac{2}{3}y = \frac{16}{3}z \end{cases} \Rightarrow \begin{cases} 3x = 6z \rightarrow x = 2z \\ y = 2z \end{cases} \quad y = 2z$$

$$V_{-3} = z(2, 2, 1)$$

$$P_{-3} = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$$

Capítulo 6 - ALGA

1

a) $x^2 + y^2 - 2xy + 2x + 4y + 5 = 0$

$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \end{bmatrix}$

$\begin{vmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 1 = \lambda^2 - 2\lambda + 0 = 0 \Leftrightarrow$

$\Leftrightarrow \lambda(\lambda - 2) = 0 \Leftrightarrow$

$\lambda = 0 \lor \lambda = 2$

$\boxed{\lambda = 0}$

$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \rightarrow \{ x = y \qquad v_1 = y(1,1) \qquad P_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\lambda = 2$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \rightarrow x = -y \qquad v_2 = y(1,-1) \qquad P_2 = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$, $P = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

$\hat{X}^T D \hat{X} + BP\hat{X} + 5 = 0 \Leftrightarrow \begin{bmatrix} \hat{x} & \hat{y} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \cdot \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} + 5 = 0$

$= \begin{bmatrix} 2\hat{x} & 0 \end{bmatrix} + \begin{bmatrix} \sqrt{2} - 2\sqrt{2} & \sqrt{2} + 2\sqrt{2} \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} + 5 = 0 \Leftrightarrow$

$\Leftrightarrow 2\hat{x}^2 - \sqrt{2}\hat{x} + 3\sqrt{2}\hat{y} = -5$

$\Leftrightarrow 2\left(\hat{x}^2 - \frac{\sqrt{2}}{2}\hat{x} + \left(\frac{\sqrt{2}}{4}\right)^2\right) + 3\sqrt{2}\hat{y} = -5 + 2\left(\frac{2}{16}\right) \Leftrightarrow$

$\Leftrightarrow 2\left(\hat{x} - \frac{\sqrt{2}}{4}\right)^2 + 3\sqrt{2}\hat{y} = -\frac{19}{4} \Leftrightarrow 2\left(\hat{x} - \frac{\sqrt{2}}{4}\right)^2 + 3\sqrt{2}\hat{y} - \frac{19}{4} = 0 \Leftrightarrow$

$\Leftrightarrow 2\tilde{x}^2 + \tilde{y} = 0 \Leftrightarrow \underline{\tilde{y} = -2\tilde{x}^2 \Rightarrow \text{parábola}}$

$\begin{cases} \tilde{x} = \hat{x} - \frac{\sqrt{2}}{4} \\ \tilde{y} = 3\sqrt{2}\hat{y} - \frac{19}{4} \end{cases}$

①

c)

$$4xy - 2x + 6y + 3 = 0$$

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \quad e \quad B = \begin{bmatrix} -2 & 6 \end{bmatrix} \quad , \quad \mu = 3$$

$$\begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = \lambda^2 - 4 \Rightarrow \lambda = -2 \vee \lambda = 2$$

$\lambda = -2$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \sim \left\{ x = -y \right. \qquad X_2 = y(1, -1) \Rightarrow P_2 = \left(\frac{\sqrt{2}}{2} , -\frac{\sqrt{2}}{2} \right)$$

$$\lambda = 2 \quad \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \Longrightarrow P_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \qquad P = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\hat{X}^T D \hat{X} = 2\hat{x}^2 - 2\hat{y}^2$$

$$BP\hat{X} = 2\sqrt{2}\,\hat{x} - 4\sqrt{2}\,\hat{y}$$

$$\Rightarrow 2\hat{x}^2 - 2\hat{y}^2 + 2\sqrt{2}\hat{x} - 4\sqrt{2}\hat{y} = -3 \Leftrightarrow$$

$$\Leftrightarrow 2\left(\hat{x}^2 + \sqrt{2}\hat{x} + \left(\frac{\sqrt{2}}{2}\right)^2\right) - 2\left(\hat{y}^2 + 2\sqrt{2}\hat{y} + (\sqrt{2})^2\right) = -3 + \frac{2 \times 2}{4} - 4 \Leftrightarrow$$

$$\Leftrightarrow 2\left(\hat{x} + \frac{\sqrt{2}}{2}\right)^2 - 2\left(\hat{y} + \sqrt{2}\right) = -6 \Leftrightarrow$$

$$\Leftrightarrow 2\tilde{x}^2 - 2\tilde{y}^2 + 6 = 0 \longrightarrow \text{hipérbole} \qquad \begin{cases} \tilde{x} = \hat{x} + \sqrt{2}/2 \\ \tilde{y} = \hat{y} + \sqrt{2} \end{cases}$$

2

b) $x^2 + 2y^2 + z^2 - 2x + 4y = 0$

$(x^2 - 2x + (-1)^2) - 1 + 2(y^2 + 4y + 1) - 1 + z^2 = 0 \Leftrightarrow$

$\Leftrightarrow (x-1)^2 + 2(y+1)^2 + z^2 = 2 \Leftrightarrow$

$\Leftrightarrow \hat{x}^2 + 2\hat{y}^2 + \hat{z}^2 = 2 \Leftrightarrow$ $\quad \hat{x} = x - 1$

$\Leftrightarrow \frac{\hat{x}^2}{2} + \hat{y}^2 + \frac{\hat{z}^2}{2} = 1 \rightarrow$ elipsoid $\quad \hat{y} = y + 1, \ \hat{z} = z$

d) $x^2 + 4y^2 + 4xy - 2x - 4y + 2z + 1 = 0$

$$A = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} x \\ y \\ z \end{array} \qquad B = [-2 \quad -4 \quad 2] \qquad \mu = 1$$

Diagonalització: $[-2, -4, 2]$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad P = \begin{bmatrix} \sqrt{5}/5 & 0 & -2\sqrt{5}/5 \\ 2\sqrt{5}/5 & 0 & \sqrt{5}/5 \\ 0 & 1 & 0 \end{bmatrix}$$

$\hat{x}^T D \hat{x} = 5\hat{x}^2$

$BP = [-2, -4, 2] \cdot P = [-2\sqrt{5} \quad 2 \quad 0]$

$5x^2 - 2\sqrt{5}\,x + 2y + 1 = 0 \Leftrightarrow$

$5\left(x^2 - \frac{2\sqrt{5}}{5}\left(-\frac{\sqrt{5}}{5}\right)^2\right) + 2y + \cancel{1} - 5\frac{\cancel{5}}{25} = 0 \Leftrightarrow$

$\Leftrightarrow 5\left(x - \frac{\sqrt{5}}{5}\right)^2 + 2y = 0 \Leftrightarrow$

$5\tilde{x}^2 - 2\tilde{y} = 0 \Leftrightarrow \tilde{y} = \frac{5}{2}\tilde{x}^2 \Longrightarrow$ cilindre parabòlic

3

3

$5x^2 + 5y^2 + 2xy + 2x - 2y + \alpha = 0$

$A = \begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad B = [2 \;\; -2 \;\; 0] \quad \gamma = \alpha$

$\det(A - \lambda I_n) = \begin{vmatrix} 5-\lambda & 1 & 0 \\ 1 & 5-\lambda & 0 \\ 0 & 0 & 0 \end{vmatrix} = (5-\lambda)^2 - 1$

$\lambda = 4 \;\vee\; \lambda = 6$

$\lambda = 4$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \longrightarrow x = -y \qquad v_4 = (1, -1) \qquad P_4 = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$P_6 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$D = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix} \quad P = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

$\hat{X}^T D \hat{X} = 6\hat{x}^2 + 4\hat{y}^2$

$BP = [2 \;\; -2] \; P = [0 \quad 2\sqrt{2}]$

$6\hat{x}^2 + 4\hat{y}^2 + 2\sqrt{2}\,\hat{y} + \alpha = 0 \Rightarrow 6\hat{x}^2 + 4\left(\hat{y}^2 + \frac{\sqrt{2}}{2}\hat{y} + \left(\frac{\sqrt{2}}{4}\right)^2\right) - 4\left(\frac{\sqrt{2}}{4}\right)^2 + \alpha$

$\Rightarrow 6\hat{x}^2 + 4\left(\hat{y} + \frac{\sqrt{2}}{4}\right)^2 - \frac{1}{2} + \alpha = 0 \Rightarrow 6\hat{x}^2 + 4\hat{y}^2 + \left(-\frac{1}{2} + \alpha\right)$

Elipse $\longrightarrow -\frac{1}{2} + \alpha < 0 \Rightarrow \alpha > \frac{1}{2}$

Ficha 6 - ALGA

1) a) $x^2 + y^2 - 2xy + 2x + 4y + 5 = 0$

$Y = 1$

$y = \begin{bmatrix} x \\ y \end{bmatrix} \qquad X^T = \begin{bmatrix} x & y \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 4 \end{bmatrix}$

$\mu = 5$

$\det(A - \lambda I_n) = 0 \quad \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 1 = 0$

$\lambda = 2 \quad \vee \quad \lambda = 0$

$\lambda = 0$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{cases} x + y = 0 \\ \end{cases} \ldots \begin{cases} x = -y \end{cases}$

$X_0 = \alpha(-1, 1);\ \alpha \in \mathbb{R} \setminus \{0\}$

$P_0 = \frac{X_0}{\|X_0\|} = \frac{(1,-1)}{\sqrt{2}} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\lambda = 2$

$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{cases} x = y \end{cases} \qquad X_2 = \beta(1\ 1),\ \beta \in \mathbb{R} \setminus \{0\}$

$P_2 = \frac{X_2}{\|X_2\|} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$P = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$P^{T}AP = D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

Como $X = P\hat{x}$: $\longrightarrow$ $\hat{x} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$

$$\hat{x}^T P^T A P \hat{x} + BP\hat{x} = -5$$

$$BP = \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}_{2\times 2} = \begin{bmatrix} -\sqrt{2} & 3\sqrt{2} \end{bmatrix}$$

$$\hat{x}^T \times D =$$

$$= \begin{bmatrix} \hat{x} & \hat{y} \end{bmatrix}_{1\times 2} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 2\hat{x} & 0 \end{bmatrix} \times \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 2\hat{x}^2 & 0 \end{bmatrix}$$

$$\underbrace{2\hat{x}^2 - \sqrt{2}\hat{x}}\ + 3\sqrt{2}\hat{y} + 5 = 0$$

$$2\left(\hat{x}^2 - \frac{\sqrt{2}}{2}\hat{x}\right) + 3\sqrt{2}\hat{y} + 5 = 0$$

$$2\left(\hat{x}^2 - \frac{\sqrt{2}}{2}\hat{x} + \left(\frac{\sqrt{2}}{4}\right)^2\right) + \left(\frac{\sqrt{2}}{4}\right)^2 + 3\sqrt{2}\hat{y} + 5 = 0$$

$$\Rightarrow \underbrace{2\left(\hat{x} - \frac{\sqrt{2}}{4}\right)^2}_{\tilde{x}} + \underbrace{\frac{1}{8} + 5 + 3\sqrt{2}\hat{y}}_{\tilde{y}} = 0$$

$$2\tilde{x} + \tilde{y} = 0 \Rightarrow$$ parábola

???

b)

$$4xy - 2x + 6y + 3 = 0$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \qquad \mu = 3$$

$$B = \begin{bmatrix} -2 & 6 \end{bmatrix}$$

$$\det(A - \lambda I_2) = \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = \lambda^2 - 4 = 0 \Rightarrow$$

$$\Rightarrow \lambda = -2 \lor \lambda = 2$$

$\boxed{\lambda = -2}$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Longrightarrow \{ x = -y \qquad X_2 = \alpha(-1, 1), \alpha \in \mathbb{R} \setminus \{0\}$$

$$P_{-2} = \frac{(-1, 1)}{\sqrt{2}} = \left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} \right)$$

$\boxed{\lambda = 2}$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \Longrightarrow \{ x = y \qquad X_2 = \beta(1, 1), \beta \in \mathbb{R} \setminus \{0\}$$

$$P_2 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$$

$$P = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \quad \hat{X} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \hat{x} & \hat{y} \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}} + \begin{bmatrix} -2 & 6 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix} \cdot \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} + 3 = 0 \Rightarrow$$

$$2\hat{x}^2 - 2\hat{y}^2 + \begin{bmatrix} (-\sqrt{2} + 3\sqrt{2})\hat{x} & (-\sqrt{2} - 3\sqrt{2})\hat{y} \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} + 3 = 0 \Leftrightarrow$$

$$\Rightarrow 2\hat{x}^2 - 2\hat{y}^2 + 2\sqrt{2}\,\hat{x} - 4\sqrt{2}\,\hat{y} + 3 = 0 \Rightarrow$$

$$\Leftrightarrow 2\left(\hat{x}^2 + \sqrt{2}\,\hat{x}\right) - 2\left(\hat{y}^2 - 2\sqrt{2}\,\hat{y}\right) = -3 \Rightarrow$$

$$\Rightarrow 2\left(\hat{x}^2 + \sqrt{2}\,\hat{x} + \left(\frac{\sqrt{2}}{2}\right)^2\right) - 2\left(\hat{y}^2 - 2\sqrt{2}\,\hat{y} + (\sqrt{2})^2\right) = -3 + 1 - 4 \Rightarrow$$

$$\Rightarrow 2\left(\hat{x}^2 + \frac{\sqrt{2}}{2}\right)^2 - 2\left(\hat{y}^2 - \sqrt{2}\right)^2 = -6 \Rightarrow$$

$$\Leftrightarrow -\frac{\left(\hat{x}^2 + \sqrt{2}/2\right)^2}{3} + \frac{\left(\hat{y}^2 - \sqrt{2}\right)^2}{3} = 1$$

$$\underbrace{-\frac{\tilde{x}^2}{3} + \frac{\tilde{y}}{3} = 1}_{\text{Hipérbole de equação reduzida}} \quad \text{onde} \quad \begin{cases} \tilde{x} = \hat{x}^2 + \sqrt{2}/2 \\ \tilde{y} = \hat{y}^2 - \sqrt{2} \end{cases}$$

2) a)

$$x^2 - y^2 - 6z^2 + 4x - 6y - 9 = 0 \Leftrightarrow$$

$$\Leftrightarrow (x^2+4x) - (y^2+6y) - 6z^2 = 9$$

$$\Leftrightarrow (x^2+4x+\underline{\underline{2^2}}) - (y^2+6y+\underline{\underline{3^2}}) - 6z^2 = 9+4-9 \Leftrightarrow$$

$$\Leftrightarrow \underbrace{(x+2)}_{\tilde{x}}^2 - \underbrace{(y+3)}_{\tilde{y}}^2 - 6(\underset{\tilde{z}}{z})^2 = 4$$

$$\Leftrightarrow \tilde{x}^2 - \tilde{y}^2 - 6\tilde{z} = 4 \Leftrightarrow$$

$$\Leftrightarrow \frac{\tilde{x}^2}{4} - \frac{\tilde{y}^2}{4} - \frac{3\tilde{z}}{2} = 1, \text{ sendo } \begin{cases} \tilde{x} = x+2 \\ \tilde{y} = y+3 \\ \tilde{z} = z \end{cases}$$

b)

$$x^2 + y^2 + 4x - 6y - z = 0$$

$$(x^2+4x+\underline{\underline{2^2}}) + (y^2-6y+\underline{\underline{3^2}}) - z = 4+9$$

$$\Rightarrow \underset{\tilde{x}}{(x+2)}^2 + \underset{\tilde{y}}{(y-3)}^2 = 13+z$$

$$\Rightarrow \tilde{x}^2 + \tilde{y}^2 - z - 13 = 0$$

- 2 Valores próprios não nulos:

$\eta \neq 0 \Longrightarrow$ parabolóide elíptico.

~~(c)~~ $\cancel{3y^2 + 4xz}$

(d) $x^2 + 4y^2 + 4xy - 2x - 4y + 2z + 1 = 0$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad A = \begin{array}{c|c} & \begin{matrix} x & y & z \end{matrix} \\ \hline \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad B = \begin{array}{c} \begin{matrix} x & y & z \end{matrix} \\ \begin{bmatrix} -2 & -4 & 2 \end{bmatrix} \end{array}$$

$\mu = 1$

Equação geral de uma quadrática: $X^T A X + BX + \mu = 0$

$X = P\hat{X}$

$$\hat{X}^T \underbrace{P^T A P}_{=D} \hat{X} + BP\hat{X} + \mu = 0$$

Vetores próprios de A:

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & 4-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0 = ((-\lambda)(4-\lambda)(1-\lambda) + 0) - (-4\lambda)$$

$= (-\lambda)(4-\lambda)(1-\lambda) + (-4)(\lambda) =$

$= (-\lambda)(\cancel{4} - 5\lambda + \lambda^2 \cancel{-4}) =$

$= (-\lambda)(\lambda)(-5+\lambda) = 0 =$

$- \lambda = 0 \wedge \lambda = 5 \wedge \lambda = 0$

$\boxed{\lambda = 0}$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Longrightarrow \left\{ x = -2y \right.$$

$$X_0 = (-2y, y, z) = \{(-2, 1, 0), (0, 0, 1)\}$$

$$P_{01} = \left(-\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, 0 \right)$$

$$P_{02} \ (0, 0, 1)$$

$\boxed{\lambda = 5}$

$$\begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Longrightarrow \left\{ \begin{array}{l} 2x = y \\ z = 0 \end{array} \right.$$

$$X_5 = \left(\frac{1}{2}y, y, 0 \right) = (1, 2, 0)$$

$$P_5 = \left(\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}, 0 \right)$$

$$P = \begin{bmatrix} \sqrt{5}/5 & 0 & -2\sqrt{5}/5 \\ 2\sqrt{5}/5 & 0 & \frac{\sqrt{5}}{5} \\ 0 & 1 & 0 \end{bmatrix} \quad ; \quad D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{B} = BP = \begin{bmatrix} -2 & -4 & 2 \end{bmatrix} \begin{bmatrix} \sqrt{5}/5 & 0 & -2\sqrt{5}/5 \\ 2\sqrt{5}/5 & 0 & \sqrt{5}/5 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -2\sqrt{5} & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} 5\hat{x} & 0\hat{y} & 0\hat{y} \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = 5\hat{x}^2$$

$$\underbrace{5\hat{x}^2 - 2\sqrt{5}\,\hat{x}} + 2\hat{y} + 1 = 0$$

$$5\left(\hat{x}^2 - \frac{2\sqrt{5}}{5}\hat{x} + \left(\frac{\sqrt{5}}{5}\right)^2\right) + 2y + \cancel{1} = \cancel{1} - \ldots$$

$$\Rightarrow 5\left(x - \frac{\sqrt{5}}{5}\right)^2 + 2y = 0 \Rightarrow$$

$$\Rightarrow \underline{5\tilde{x}^2} + 2\tilde{y} = 0 \quad \text{em que:} \begin{cases} \tilde{x} = x - \frac{\sqrt{5}}{5} \\ \tilde{y} = y \end{cases}$$

$\downarrow$ 1 valor próprio não nulo

$\hookrightarrow \eta \neq 0 \longrightarrow$ cilindro parabólico!

(c)

$$-x^2 + y^2 - 2x - 4y + 2 = 0$$

$$-(x^2 + 2x + 1^2) + (y^2 - 4y + (-2)^2) = -2 - 1^2 + 4 \Rightarrow$$

$$\Rightarrow -(x+1)^2 + (y-2)^2 = 1 \Rightarrow$$

$$\Rightarrow \underline{\tilde{y}^2 - \tilde{x}^2 = 1} \quad \text{sendo que} \begin{cases} \tilde{y} = y - 2 \\ \tilde{x} = x + 1 \end{cases}$$

$\hookrightarrow$ 2 valores próprios $\rightarrow \tilde{y}$ e $\tilde{x}$ com sinais opostos e $\mu \neq 0$

Cilindro hiperbólico

4) a) Se os vetores de P forem ortogonais, a matriz P também o vai ser:

$\vec{u} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$

$\vec{v} = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$

$\vec{u} \cdot \vec{v} = -\frac{2}{4} + \frac{2}{4} = 0 \Rightarrow$ Vetores ortogonais

$\downarrow$

logo a matriz P é ortogonal

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \sqrt{3}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \ldots = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

b)

$$4xy + x + y = 0$$

$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 1 \end{bmatrix}$ $\quad X = \begin{bmatrix} x \\ y \end{bmatrix}$

$\det(A - \lambda I_2) = \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - 4 = 0 \Rightarrow$

$\Leftrightarrow \lambda = -2 \vee \lambda = 2$

$\lambda = -2$

$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \{ x = -y$ $\qquad X_{-2} = \alpha(-1, 1), \alpha \in \mathbb{R} \setminus \{0\}$

$P_{-2} = \frac{X_{-2}}{\|X_{-2}\|} = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$

$\lambda = 2$

$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \Rightarrow \{ x = y$ $\qquad P_2 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$

$P = \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$ $\quad$ e $D = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$

A

$$\hat{x}^T D \hat{x} + \hat{b}^T x = -\mu$$

$$\hookrightarrow = BP = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} & \hat{y} \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} =$$

$$= \begin{bmatrix} -2\hat{x} & 2\hat{y} \end{bmatrix}$$

$$-2\hat{x}^2 + 2\hat{y}^2 + \sqrt{2}\,\hat{y} = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\left(\hat{y}^2 + \frac{\sqrt{2}}{2}\hat{y}\right) - 2\hat{x}^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\left(\hat{y}^2 + \frac{\sqrt{2}}{2}\hat{y} + \left(\frac{\sqrt{2}}{4}\right)^2\right) - 2\hat{x}^2 = 2\left(\frac{\sqrt{2}}{4}\right)^2 \Leftrightarrow$$

$$\Leftrightarrow 2\left(y + \frac{\sqrt{2}}{4}\right)^2 - 2\hat{x}^2 = \frac{1}{4} \Leftrightarrow$$

$$\Leftrightarrow 2\tilde{x}^2 - 2\tilde{y}^2 = \frac{1}{4}$$, sendo que $\begin{cases} \tilde{x} = \left(y + \frac{\sqrt{2}}{4}\right) \\ \tilde{y} = (\hat{x}) \end{cases}$

$$\Leftrightarrow \frac{\tilde{x}^2}{8} - \frac{\tilde{y}^2}{8} = 1$$

$\hookrightarrow$ hipérbole.

6) $P \to (x, y, z)$

$A \to (0, 0, -2)$

$\alpha = z + 18 \longrightarrow a = b = 0, \; c = 1$

$d(P, A) = \frac{1}{3} \, d(\alpha, A)$

$dp = \frac{|0 \times 0 + 0 \times 0 + 1 \times z + 18|}{1} = z + 18$

$\vec{AP} = P - A = (x, y, z + 2)$

$d(\vec{AP}) = \sqrt{x^2 + y^2 + (z+2)^2} = \frac{|z + 18|}{3} \Leftrightarrow$

$\Leftrightarrow x^2 + y^2 + z^2 + 4z + 4 = \frac{z^2 + 36z + 324}{9} \Leftrightarrow$

$\Leftrightarrow 9x^2 + 9y^2 + 9z^2 + \cancel{36z} + 36 = z^2 + \cancel{36z} + 324 \Leftrightarrow$

$\Leftrightarrow 9x^2 + 9y^2 + 8z^2 = 288 \Leftrightarrow$

$\Leftrightarrow \frac{x^2}{32} + \frac{y^2}{32} + \frac{z^2}{36} = 1 \longrightarrow$ elipsoide

Ficha 7 – ALGA

1

b) $\phi(x,y,z) = (x+y,\, y,\, x-z)$

- $\phi(0,0,0) = (0,0,0)$ ✓

Seja $\vec{u} = (x_1, y_1, z_1)$

Seja $\vec{v} = (x_2, y_2, z_2)$

$\phi(\vec{u}) + \phi(\vec{v}) = (x_1 + x_2 + y_1 + y_2,\; y_1 + y_2,\; x_1 + x_2 - z_1 - z_2)$ ✓

$\phi(\vec{u} + \vec{v}) = \phi(x_1 + x_2,\, y_1 + y_2,\, z_1 + z_2) =$ $\qquad$ $\phi(\vec{u}+\vec{v}) = \phi(\vec{u}) + \phi(\vec{v})$

$= (x_1 + x_2 + y_1 + y_2,\; y_1 + y_2,\; x_1 + x_2 - z_1 - z_2)$

- $\phi(\alpha \vec{u}) = \phi(\alpha x_1,\, \alpha y_1,\, \alpha z_1) = (\alpha x_1 + \alpha y_1,\; \alpha y_1,\; \alpha x_1 - \alpha z_1)$

- $\alpha\, \phi(\vec{u}) = \alpha(x_1 + y_1,\; y_1,\; x_1 - z_1) = (\alpha x_1 + \alpha y_1,\, \alpha y_1,\; \alpha x_1 - \alpha z_1)$

$\phi(\alpha \vec{u}) = \alpha\, \phi(\vec{u})$ ✓

É aplicação linear

3) $\phi(at^2 + bt + c) = a + (t+1)(bt + c) =$

$= a + bt^2 + tc + bt + c =$

$= bt^2 + (c+b)t + a + c$

$\phi(a,b,c) = (b,\; c+b,\; a+c)$

$\phi(0,0,0) = (0,0,0)$ ✓

$\vec{u} = (x_1, y_1, z_1)$ e $\vec{v} = (x_2, y_2, z_2)$

$\phi(\vec{u}+\vec{v}) = \phi(x_1 + x_2,\, y_1 + y_2,\, z_1 + z_2) = (y_1 + y_2,\; y_1 + y_2 + z_1 + z_2,\; x_1 + x_2 + z_1 + z_2)$

$\phi(\vec{u}) + \phi(\vec{v}) = (y_1,\; y_1 + z_1,\; x_1 + z_1) + (y_2,\; y_2 + z_2,\; x_2 + z_2) =$

$= (y_1 + y_2,\; y_1 + z_1 + y_2 + z_2,\; x_1 + z_1 + x_2 + z_2)$

$\phi(\vec{u}+\vec{v}) = \phi(\vec{u}) + \phi(\vec{v})$ ✓

1

• $\phi(\alpha U) = \phi(\alpha x_1, \alpha y_1, \alpha z_1) = (\alpha y_1, \alpha y_1 + \alpha z_1, \alpha x_1 + \alpha z_1)$

• $\alpha(\phi U) = \alpha(y_1, y_1 + z_1, x_1 + z_1) = (\alpha y_1, \alpha y_1 + \alpha z_1, \alpha x_1 + \alpha z_1)$

• $\phi(\alpha U) = \alpha(\phi(U))$ ✓

Logo, $\phi(ax^2 + bx + c)$ é uma AL

(2)

Sejam A, B 2 matrizes não singulares

$\phi(A+B) = (A+B)^{-1} = A^{-1} + B^{-1}$

$\phi(A) + \phi(B) = A^{-1} + B^{-1}$

$\phi(A+B) = \phi(A) + \phi(B)$ ✓

$\phi(\alpha A) = (\alpha A)^{-1} = \alpha^{-1} A^{-1} = \frac{A^{-1}}{\alpha}$

$\alpha(\phi A) = \alpha A^{-1}$

Como $\alpha(\phi A) \neq \phi(\alpha A)$

$\phi(A)$ não é aplicação linear

(1)

$\phi(1,1) = (2,-3)$

$\phi : V \to W$

$\phi(0,1) = (1,2)$

$B_V = ((1,1),(0,1))$

$\phi(a(1,1) + b(0,1)) = a(\phi(1,1)) + b(\phi(0,1)) \Leftrightarrow$

$(x_1, x_2) = a(1,1) + b(0,1) \Leftrightarrow$

$\Rightarrow \begin{cases} x_1 = a \\ x_2 = a + b \end{cases} \Rightarrow \begin{cases} a = x_1 \\ b = x_2 - a \end{cases} \Rightarrow \begin{cases} a = x_1 \\ b = x_2 - x_1 \end{cases}$

• $\phi(x_1, x_2) = x_1\,\phi(1,1) + (x_2 - x_1)\,\phi(0,1) =$

$= x_1(2,-3) + (x_2 - x_1)(1,2) =$

$= (2x_1 + x_2 - x_1, -3x_1 + 2x_2 - 2x_1) =$

$= (x_1 + x_2, 2x_2 - 5x_1)$

ⓐ $\phi(x,y) = (x-y,\ 2y-5x)$

$\phi(3,-4) = (1,\ -19)$

ⓑ $\phi(a,b) = (a+b,\ -5a+2b)$

⑥

ⓐ $C: \{(1,0,0), (0,1,0), (0,0,1)\}$

$S: \{(1,0,1), (0,1,1), (2,0,1)\}$

$\phi(x,y,z) = (x+2y+z,\ 2x-y,\ 2y+z)$

ⓐ

$$AX = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad [\phi]_{C \leftarrow C} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

ⓑ $[\phi]_{B \leftarrow C} = [\phi(C_1)]_B + [\phi(C_2)]_B + [\phi(C_3)]_B$

$\phi(1,0,0) = (1,2,0)$

$\phi(0,1,0) = (2,-1,2)$

$\phi(0,0,1) = (1,0,1)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 1 & 1 & 1 & 0 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3 & 1 & 0 \end{array}\right]$$

$$[\phi]_{B \leftarrow C} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 0 \\ -3 & 1 & 0 \end{bmatrix}$$

c) $[\phi]_{S \leftarrow B} = ?$

$S = \big((1,0,0), (0,1,0), (0,0,1) \big)$

$\phi(1,1,1) = (2,2,1) \qquad \phi(0,1,1) = (3,-1,3) \qquad \phi(0,0,1) = (1,0,1)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array}\right] \longrightarrow [\phi]_{S \leftarrow B} = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

d) $[\phi]_{B \leftarrow S}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 1 & 1 & 1 & 1 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3 & 1 & 0 \end{array}\right]$$

$$\downarrow$$

$[\phi]_{B \leftarrow S}$

b)

$[\phi(1,1,-2)]_S = [\phi]_{S \leftarrow B} \, [(1,1,-2)]_B$

$$[\phi(1,1,-2)]_S = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix} = [1 \;\; 1 \;\; 0]$$

$$[(1,1,-2)]_B = \begin{cases} a = 1 \\ b = 1 \\ a+b+c = -2 \end{cases} \longrightarrow \begin{cases} a = 1 \\ b = 1 \\ c = -4 \end{cases}$$

2

$\phi(x,y) = (x+y,\ x-y,\ x+2y)$

$S = ((1,-1),(0,1))$

$T = ((1,1,0),(0,1,1),(1,-1,1))$

a) $[\phi]_{\mathcal{B}_3 \leftarrow \mathcal{B}_2}$

$\mathcal{B}_2 = ((1,0),(0,1))$

$$\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$$

$\phi(1,0) = (1,1,1)$

$\phi(0,1) = (1,-1,2)$

$$[\phi]_{\mathcal{B}_3 \leftarrow \mathcal{B}_2} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix}$$

b) $[\phi]_{T \leftarrow S}$

$\phi(1,-1) = (0,2,-1)$

$\phi(0,1) = (1,-1,2)$

$$\left[\begin{array}{ccc|cc} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 2 & -1 \\ 0 & 1 & 1 & -1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 2 & -2 \\ 0 & 0 & 3 & -3 & 4 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 2 & -2 \\ 0 & 0 & 1 & -1 & 4/3 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & -1/3 \\ 0 & 1 & 0 & 0 & 2/3 \\ 0 & 0 & 1 & -1 & 4/3 \end{array}\right]$$

$$[\phi]_{T \leftarrow S} = \frac{1}{3}\begin{bmatrix} 3 & -1 \\ 0 & 2 \\ -3 & 4 \end{bmatrix}$$

c) $\phi(2,-3) = (-1, 5, -4)$

$[\phi(2,-3)]_T = [\phi]_{TeS}\, [(2,-3)]_S$

$\begin{cases} 2 = a \\ -3 = -a + b \end{cases} \quad \begin{cases} a = 2 \\ b = -1 \end{cases}$

$= \frac{1}{3} \begin{pmatrix} 3 & -1 \\ 0 & 2 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \frac{1}{3}(7, -2, -10) = \left(\frac{7}{3}, -\frac{2}{3}, -\frac{10}{3}\right)$

$[\phi(2,-3)]_T = \begin{pmatrix} 7/3 \\ -2/3 \\ -10/3 \end{pmatrix} \Rightarrow \phi(2,-3) = \frac{7}{3}(1,1,0) - \frac{2}{3}(0,1,1) - \frac{10}{3}(1,-1,1) =$

$= \left(\frac{7}{3} - \frac{10}{3}, \frac{7}{3} - \frac{2}{3} + \frac{10}{3}, -\frac{2}{3} - \frac{10}{3}\right) =$

$= \left(-\frac{3}{3}, \frac{15}{3}, -\frac{12}{3}\right) = (-1, 5, -4) =$

$= \phi(2,-3)$

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$\emptyset : P_2 \to P_2$

$\emptyset(at^2 + bt + c) = (a+c)t^2 + (b+c)t$

a

- Se os elementos pertencerem a $Ker(\emptyset)$, então vão ser geradores [illegible] geradores de $Ker(\emptyset)$

$$\begin{cases} a+c=0 \\ b+c=0 \end{cases} \Leftrightarrow \begin{cases} a=-c \\ c=b \end{cases} \Leftrightarrow \begin{cases} a=b \\ c=-b \end{cases} \to (1,1,-1)$$

$Ker(\emptyset) = \langle (1,1,-1) \rangle$

1) $t^2 - t - 1 \to (1,-1,-1)$
$\begin{cases} 1 = \alpha 1 \\ -1 = \alpha 1 \\ -1 = \alpha \cdot 1 \end{cases}$ Impossível, logo $t^2 - t - 1 \notin Ker(\emptyset)$

2) $t^2 + t - 1 = (1,1,-1)$
$\begin{cases} 1 = \alpha 1 \\ 1 = \alpha 1 \\ -1 = \alpha \cdot 1 \end{cases}$ SPD $\to$ $t^2 + t - 1 \in Ker(\emptyset)$

b

$im(\emptyset) = \{ \emptyset(X) \in P_2 : X \in P_2 \}$

$\to \langle ((a+c), (b+c), 0) \rangle =$

$= \langle (1,0,0), (0,1,0), (1,1,0) \rangle =$

$\to$ Não é l.i

$= \langle (1,0,0), (0,1,0) \rangle =$

$= \langle (t^2, t) \rangle$ $\to$ O primeiro elemento pertence e o segundo não!

c $Ker(\emptyset) = \langle (t^2 + t - 1) \rangle$

$im(\emptyset) = \langle t^2, t \rangle$

d $\dim(Ker(\emptyset)) \neq 0 \Rightarrow$ Não é injetiva!

$\dim(Im(\emptyset)) = 2 \longrightarrow$ Não é sobrejetiva!

$\dim V = \dim W = 3$

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(a)

$\phi(x) = AX = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = (x+2z+w,\ 3x+y-w)$

$\phi(x,y,z,w) = (x+2z+w,\ 2x+y-w)$

$S_2 = \langle (1,0), (0,1) \rangle$

$S_4 = \langle (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \rangle$

$[\phi]_{\mathbb{R}^2 \leftarrow \mathbb{R}^4}$

$\phi(1,0,0,0) = (1,3)$

$\phi(0,1,0,0) = (0,1)$

$\phi(0,0,1,0) = (2,0)$

$\phi(0,0,0,1) = (1,-1)$

$$\left[\begin{array}{cc|cccc} 1 & 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 & 0 & -1 \end{array}\right]$$

$\underbrace{}_{I_n}$

$[\phi]_{\mathbb{R}_2 \leftarrow \mathbb{R}_2}$

(b)

$\dim V = 4$
$\dim W = 2$

$\dim V - \dim W = 2$

$\dim V = \dim \ker(\phi) + \dim \operatorname{im}(\phi) = 4$

$\operatorname{car}([\phi]_{\mathbb{R}_2 \leftarrow \mathbb{R}^4}) = 2 = \dim(\operatorname{im}(\phi)) \longrightarrow \dim \ker(\phi) = 2$!

Verdadeira !

(c)

i) $[\phi]_{G \leftarrow S}$

$\phi(1,1,1,0) = (3,4)$

$\phi(1,1,1,1) = (4,3)$

$\quad \dots = (4,2)$

$\quad \dots (3,0)$

$[\phi]_{G \leftarrow S} = \begin{bmatrix} 3 & 4 & 4 & 3 \\ 4 & 3 & 2 & 0 \end{bmatrix}$

ii)

$[\phi]_{J \leftarrow S}$

$$\left[\begin{array}{cc|cccc} 1 & 1 & 3 & 4 & 4 & 3 \\ 1 & -1 & 4 & 3 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|cccc} 1 & 1 & 3 & 4 & 4 & 3 \\ 0 & -2 & 1 & -1 & -2 & -3 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cc|cccc} 1 & 1 & 3 & 4 & 4 & 3 \\ 0 & 1 & -1/2 & 1/2 & 1 & 3/2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cccc} 1 & 0 & 7/2 & 7/2 & 3 & 5/2 \\ 0 & 1 & -1/2 & 1/2 & 1 & 3/2 \end{array}\right]$$

$\downarrow$

$[\phi]_{J \leftarrow S}$

$$[\phi]_{J \leftarrow S} = \frac{1}{2} \begin{bmatrix} 7 & 7 & 6 & 5 \\ -1 & 1 & 2 & 3 \end{bmatrix}$$

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a

$Ax = (x + y + 2z,\ 2x + y + z,\ 3x + 2y + 3z)$

$\phi(x,y,z) = (x + y + 2z,\ 2x + y + z,\ 3x + 2y + 3z)$

$\phi(1,2,3) = (9, 7, 16)$

b

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$car(A) = 2$

$\hookrightarrow = \dim im(A) = 2$

$\dim \mathbb{R}^3 = 3 = \dim im(\phi) + \dim Ker(\phi) \Rightarrow$

$\Rightarrow \dim Ker(\phi) = 1$

$\hookrightarrow \neq 0 \Rightarrow$ não é injetiva, logo não é isomorfismo

c)

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 3 & 0 \end{array}\right] \sim \quad X_1 = (z, -3z, z)$$

$$\mathrm{Ker}(\phi) = \langle (1,-3,1) \rangle$$

$$\phi(x,y,z) = (x+y+2z,\ 2x+y-z,\ 3x+2y+3z)$$

$$\mathrm{Im}(\phi) = \langle (1,2,3), (1,1,2), (2,1,3) \rangle$$

$$\begin{cases} x + 2y + 3z = 0 \\ x + y + 2z = 0 \\ 2x + y + 3z = 0 \end{cases}$$

Não são linearmente independentes!

↳ não é base!

???

(pôr numa matriz e reduzir!)

d)

$[\phi]_{\mathcal{B} \leftarrow \mathcal{B}}$

$\mathcal{B} = \langle (1,1,0), (1,1,1), (1,0,0) \rangle$

$\phi(1,1,0) = (2,3,5)$

$\phi(1,1,0) = (4,4,8)$

$\phi(1,0,0) = (1,2,3)$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 2 & 4 & 1 \\ 1 & 1 & 0 & 3 & 4 & 2 \\ 0 & 1 & 0 & 5 & 8 & 3 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -4 & -1 \\ 0 & 1 & 0 & 5 & 8 & 3 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$\underbrace{\begin{matrix} -2 & -4 & -1 \\ 5 & 8 & 3 \\ -1 & 0 & -1 \end{matrix}}_{[\phi]_{\mathcal{B} \leftarrow \mathcal{B}}}$$

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a) $\phi: \mathbb{R}^2 \to \mathbb{R}^3$

$\phi(1,1) = (3,0,2)$

$\phi(1,-1) = (1,0,2)$

$(x,y) = a(1,1) + b(1,-1) = (a+b, a-b)$

$\begin{cases} x = a+b \\ y = a-b \end{cases} \Rightarrow \begin{cases} a = x-b \\ b = -y+a \end{cases} \Leftrightarrow \begin{cases} a = x+y-a \\ - \end{cases} \Leftrightarrow \begin{cases} a = \frac{x+y}{2} \\ b = \frac{-2y+x+y}{2} = \frac{x-y}{2} \end{cases}$

$(x,y) = \left(\frac{x+y}{2}\right)(1,1) + \left(\frac{x-y}{2}\right)(1,-1)$

$\phi(x,y) = \frac{x+y}{2}(3,0,2) + \frac{x-y}{2}(1,0,2) =$

$= \left(\frac{3x+3y+x-y}{2}, 0, \frac{(2x+\cancel{2y}+2x-\cancel{2y})}{2}\right) =$

$= (2x+y, 0, 2x)$

b) $\phi(x,y) = (2x+y, 0, 2x) = x(2,0,2) + y(0,1,0)$

$\operatorname{im}(\phi) = \langle (1,0,1) + (0,1,0) \rangle$

c) $\dim(\mathbb{R}^2) = \dim \operatorname{Ker}(\phi) + \dim \operatorname{im}(\phi) \Leftrightarrow$

$\Leftrightarrow 2 = 2 + \dim \operatorname{Ker}(\phi)$

$\hookrightarrow = 0 \Rightarrow$ é injetiva

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(d)

$[\phi]_{J \leftarrow S}$

$\phi(1,1) = (3, 0, 2)$

$\phi(1,-1) = (1, 0, 2)$

$$\left[\begin{array}{ccc|cc} 0 & 1 & 0 & 3 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 2 \end{array}\right] \to \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right]$$

$$[\phi]_{J \leftarrow S} = \begin{bmatrix} 0 & 0 \\ 3 & 1 \\ -1 & 1 \end{bmatrix}$$

(e)

$$[\phi(x)]_J = [\phi]_{J \leftarrow S} [x] = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$\left\{\begin{array}{l} \cancel{0 = a} \\ \cancel{1 = b} \\ \cancel{1 = b + c} \end{array}\right. \quad \left\{\begin{array}{l} a = 0 \\ b = 1 \\ c = -2 \end{array}\right.$

- $\cancel{x} = (1, 0, -1) = (x, y, z)$

- $(1,0,0) = (2x + y, 0, 2x) \Rightarrow \left| \begin{array}{l} y = 1 \\ x = 0 \end{array}\right. \quad x = (0, 1)$

$(0,1) = \alpha(1,1) + \beta(1,-1) = (\alpha + \beta, \alpha - \beta) \Rightarrow \left\{\begin{array}{l} \alpha = 1/2 \\ \beta = -1/2 \end{array}\right.$

$[x]_S = \left(\frac{1}{2}, -\frac{1}{2}\right)$

(22)

$\phi : \mathbb{R}^4 \to \mathbb{R}^6$

(a) $V \to W$

$\dim(V) = 4 = \longrightarrow \dim \operatorname{im}(\phi) = 2$

(b) $\dim \ker(\phi) = 1$

(23)

(a) $\dim \ker(\phi) = 2$

Sobrejetiva $\longrightarrow \dim \operatorname{im}(\phi) = \dim(W) = 6$

~~dim~~ $\dim \operatorname{im}(\phi) = 4$

(b)

$\dim \ker(\phi) = 0$
$\dim \operatorname{im}(\phi) = 5$

$\dim V = \dim \ker(\phi) + \dim \operatorname{im}(\phi) \Leftrightarrow$

$\Rightarrow x = 0 + 5 \Leftrightarrow x = 5$

$\dim(V) = 5$