lista 03 (Distribuição Amostral da Média, Proporção e Aplicações)

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Questão 01

a)

$$P(90 < X < 110) = P\left(\frac{90-100}{10} < Z < \frac{110-100}{10}\right) = P(-1 < Z < 1)$$

$$= 2 \cdot P(0 \leq z \leq 1) = 2 \cdot 0,3413 = 0,6826$$

b)

$$P(90 < \bar{X} < 110) = P\left(\frac{90-100}{\sqrt{\frac{100}{16}}} < Z < \frac{110-100}{\sqrt{\frac{100}{16}}}\right) = P(-4 < Z < 4)$$

$$= 2 \cdot P(0 \leq z \leq 4) = 2 \cdot 0,5 = 1$$

c)

```
mean = 100
sdX = 10
sd_X = 100/sqrt(16)

curve(dnorm(x, mean, sdX), ylab = "dnorm(x)", from=60, to=140, col="blue")
curve(dnorm(x, mean, sd_X), from=60, to=140, add=TRUE, col="red")
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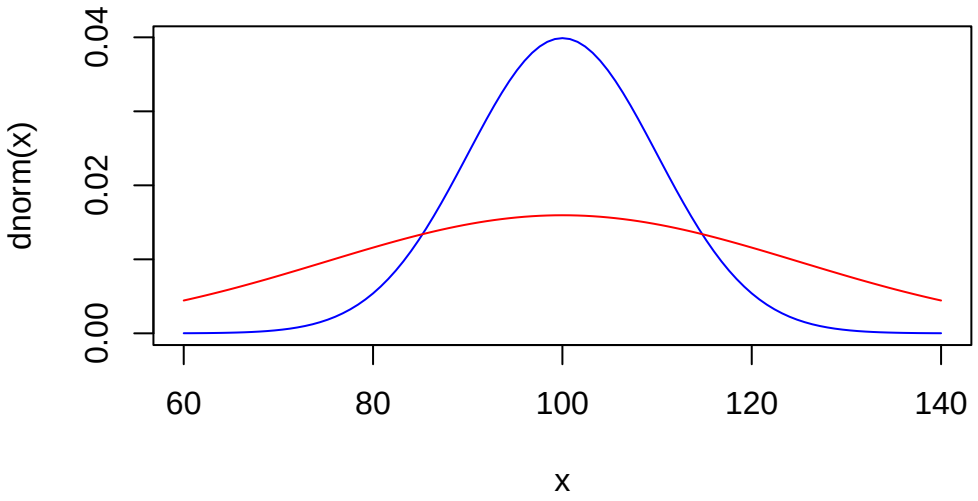


d)

$$P(90 < \bar{x} < 110) = 0{,}95$$

$$\hookrightarrow P\left(\frac{90-100}{\sqrt{\frac{100}{n}}} < Z < \frac{110-100}{\sqrt{\frac{100}{n}}}\right) = P(-\sqrt{n} < Z < \sqrt{n})$$

$$= 2 \cdot P(0 \leq Z \leq \sqrt{n}) = 0{,}95 = P(0 \leq Z \leq \sqrt{n}) = 0{,}475$$

$$= \sqrt{n} = 1{,}96$$

$$n = 1{,}96^2$$

$$n = 3{,}84 \simeq 4$$

Questão 02

a)

$$P(X < 500) = 0{,}10$$

$$\hookrightarrow P\left(Z < \underbrace{\frac{500-\mu}{10}}_{a}\right) = P(Z < a) = 0{,}10$$

* $P(0 \leq Z \leq a) = 0{,}5 - 0{,}10 = 0{,}40$

$-a = 1{,}28$

$a = 1{,}28$

* $\frac{500-\mu}{10} = -1{,}28$

$500 - \mu = -12{,}8$

$-\mu = -512{,}8$

$\mu = 512{,}8$

b)

$$X_1+X_2+X_3+X_4 < 2000 \leftrightarrow \bar{X} < 500$$

$$P(\bar{X} < 500) = P\left(Z < \frac{500 - 512,8}{\frac{10}{\sqrt{4}}}\right) = P(Z < -2,56) = P(Z > 2,56)$$

$$= 0,5 - P(0 \leq Z \leq 2,56) = 0,5 - 0,49948$$

$$= 0,00052$$

Questão 03

a)

$$P(\bar{X} < 495) + P(\bar{X} > 520) = P\left(Z < \frac{495 - 512,8}{\sqrt{25}}\right) + P\left(Z > \frac{520 - 512,8}{\sqrt{25}}\right)$$

$$= P(Z < -3,56) + P(Z > 1,44) = P(Z > 3,56) + P(Z > 1,44)$$

$$= [0,5 - P(0 \leq Z \leq 3,56)] + [0,5 - P(0 \leq Z \leq 1,44)]$$

$$= [0,5 - 0,49981] + [0,5 - 0,42511]$$

$$= 0,00021 + 0,07489$$

$$= 0,07511$$

b)

$$P(495 \leq \bar{x} \leq 520) = P\left(\frac{495-500}{\sqrt{25}} \leq Z \leq \frac{520-500}{\sqrt{25}}\right) = P(-1 \leq Z \leq 4)$$

$$= P(0 \leq Z \leq 1) + P(0 \leq Z \leq 4)$$

$$= 0{,}3413 + 0{,}5$$

$$= \underline{\underline{0{,}8413}}$$

Questão 04

$p = 0{,}10$ $\quad \bar{P} \sim N(0{,}10 ; 0{,}0045)$

$n = 20$ $\quad \hookrightarrow \dfrac{0{,}10 \cdot 0{,}90}{20}$

$$P(\bar{P} > 0{,}15) = P\left(Z > \frac{0{,}15 - 0{,}10}{\sqrt{0{,}0045}}\right) = P(Z > 0{,}74) = 0{,}5 - P(0 \leq Z \leq 0{,}74)$$

$$= 0{,}5 - 0{,}2734$$

$$= \underline{\underline{0{,}2266}}$$

Questão 05

a)

$$\bar{P} \sim N\left(0,1 ; \frac{0,1 \cdot 0,9}{100}\right)$$

$$\hookrightarrow 0,0009$$

$$P(\bar{P} \geq 0,1) = P\left(Z \geq \frac{0,1 - 0,1}{\sqrt{0,0009}}\right) = P(Z \geq 0) = 0,5$$

b)

$$P(X = K) = \binom{100}{K} \cdot 0,1^{K} \cdot 0,9^{100-K}$$

$$P(X = 0) = \binom{100}{0} \cdot 0,1^{0} \cdot 0,9^{100}$$

$$= 1 \cdot 1 \cdot 0,000002$$

$$= 0,000002$$

Questão 06

$\mu = 120$

$\sigma = 50$

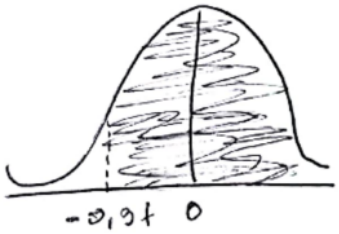
$\sigma^2 = 2500$

$n = 30$

$* \frac{3500}{30} = 116,67$ $\quad \bar{X} \sim N(120; 83,3)$

$* \frac{2500}{30} = 83,3$

$$P(\bar{X} \leq 116,67) = P\left(Z \leq \frac{116,67 - 120}{\sqrt{83,3}}\right) = P(Z \leq -0,365)$$



$$= 0,5 + P(0 \leq z \leq 0,365)$$

$$= 0,5 + 0,1443$$

$$= 0,6443$$