Empirical Conjecture on Prime Gaps

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Abstract

The study of prime gaps has been a central topic in number theory for centuries. In this paper, we present a conjecture, strongly supported by extensive computational evidence, that provides insights into the distribution of new prime gaps.

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1 Introduction

In pursuit of patterns and potential algorithms describing prime numbers, I have uncovered several significant regularities through empirical analysis. This article introduces a new conjecture concerning prime gaps, supported by extensive computational data. We examine the structure of prime gaps, their discrete derivatives, and recurrence patterns in the distribution of primes. The goal is to provide evidence that the gaps between prime numbers are not random, but rather exhibit an underlying, discernible structure.

2 Definitions and Notation

2.1 Prime Numbers

Prime numbers are natural numbers greater than 1 that have no positive divisors other than 1 and themselves. We denote the n-th prime number by p_n.

2.2 Prime Gaps

The prime gap g_n is defined as the difference between the n-th prime number p_n and the $(n-1)$-th prime number p_{n-1}:

$$g_n = p_n - p_{n-1}.$$

For example, the first few prime gaps are $g_2 = 1$ (between 2 and 3), $g_3 = 2$ (between 3 and 5), $g_4 = 2$ (between 5 and 7), $g_5 = 4$ (between 7 and 11), etc.

2.3 New Gap

Let the set of prime gaps be

$$G = \{g_2, g_3, g_4, \dots\}.$$

A *new gap* is defined as a gap g_n that appears for the first time in the sequence, i.e., it does not belong to the set of previous gaps

$$G_{n-1} = \{g_2, g_3, \dots, g_{n-1}\},$$

so that

$$g_n \notin G_{n-1}.$$

2.4 Discrete Derivatives

The discrete derivative of a sequence $\{a_n\}$ is defined as

$$\Delta a_n = a_{n+1} - a_n.$$

In the context of prime gaps, the first discrete derivative is

$$\Delta g_n = g_{n+1} - g_n,$$

and higher-order derivatives are defined similarly:

$$\Delta^2 g_n = \Delta g_{n+1} - \Delta g_n.$$

3 Main Result

Serer Conjecture (Inflection or Stability, Empirical Evidence).
All new prime gaps appear only at points of stability (where the first discrete derivative is zero) or at points of discrete inflection (where the second discrete derivative is negative), according to computational tests up to the 41 146 179*-th prime (*799 999 999*).*

$$
\begin{aligned}
\text{Stability points:} \quad & S = \{\, n \mid \Delta g_n = 0 \,\}, \\
\text{Inflection points:} \quad & I = \{\, n \mid \Delta^2 g_{n-1} < 0 \,\}, \\
\text{Inconsistency points:} \quad & C = \{\, n \mid \Delta^2 g_{n-1} \geq 0 \,\}.
\end{aligned}
$$

The second discrete derivative is indexed as $\Delta^2 g_{n-1} = g_n - 2g_{n-1} + g_{n-2}$ so that it corresponds to the same prime gap g_n in the original sequence.

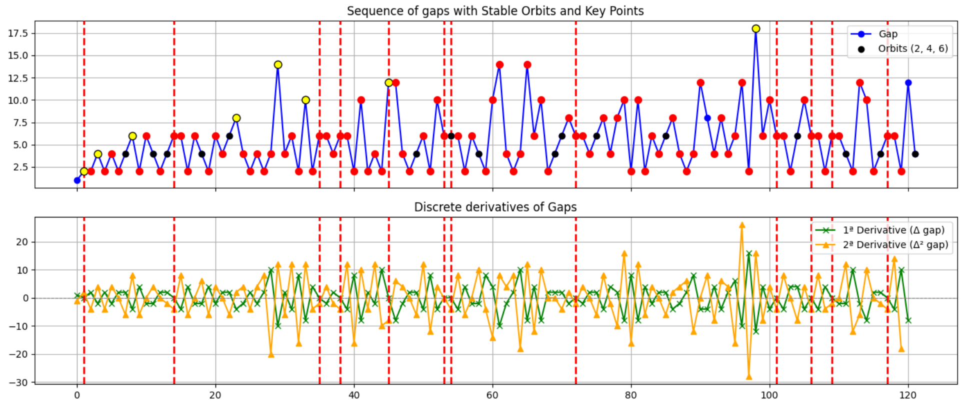

Figure 1: Top: Sequence of prime gaps g_n versus prime index n (natural numbers). New gaps are highlighted in yellow, local maxima and minima in red and black. Bottom: First (green) and second (orange) discrete derivatives of the prime gaps sequence. Vertical dashed red lines indicate positions of $\{\Delta g_n = 0\}$.

4 Empirical Evidence

We tested the conjecture on the first 41,146,179 prime numbers. Within this dataset, a total of 127 new prime gaps were identified. None of these new gaps appeared outside of the predicted stability or discrete inflection points. Therefore, there were **zero inconsistencies** found in the tested range, providing strong computational support for the conjecture.

Quantity	Value
Total numbers generated	800 000 000
Total primes generated	41 146 179
New prime gaps verified	127
Inconsistencies found	0

Table 1: Summary of computational results supporting Serer Conjecture (Inflection or Stability).

Remark: Despite the apparent randomness of prime gaps, all new gaps strictly conform to the predicted stability or inflection points, highlighting an underlying structure in the distribution.

5 Discussion

The computational results provide strong empirical evidence supporting Serer Conjecture. Although a formal mathematical proof is still lacking, the absence of inconsistencies across more than 41 million primes suggests that the conjecture captures a genuine pattern in the distribution of prime gaps.

Future work may include extending computational tests to larger primes, exploring statistical properties of new gaps, and introducing the topological model of prime numbers for partial mapping, which may reveal additional structural insights.

6 Conclusion

We presented Serer Conjecture, supported by extensive empirical verification, demonstrating that new prime gaps consistently occur at points of stability or discrete inflection. This evidence suggests that prime gaps are not entirely random and that underlying structural patterns can be identified, opening avenues for further mathematical exploration.

7 References

References

[1] G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, 6th Edition, Oxford University Press, 2008.

[2] P. Ribenboim, *The Little Book of Bigger Primes*, 2nd Edition, Springer, 2004.

[3] Pedro Serer, *Prime Gap Verification Code*, GitHub repository, `https://github.com/Pedro-Serer/teoria-dos-numeros`, 2025.