2.1.6

10)

1) $a\ P_{n_1} + P_{n_2} = (a_n x^{n-1} + a_{n-1} x^{n-2} \dots a_{01}) + (a_{n_2} x^{n-1} + a_{n-1_2} x^{n-2} + \dots + a_{02})$

$= x^{n-1}(a_{n_1} + a_{n_2}) + x^{n-2}(a_{n-1} a_{n-2}) + \dots + a_{01} + a_{02}$

$= P_{n_2} + P_{n_1} \Rightarrow \in V \ \forall \ P_{n_1}, P_{n_2} \in V$

2) $P_{n_1} + P_{n_2} = P_{n_2} + P_{n_1}$

3) $P_{n_1} + P_{n_2} + P_{n_3} = x^{n-1}(a_{n-1} + a'_{n-1} + a''_{n-1}) + x^{n-2}(a_{n-2} + a'_{n-2} + a''_{n-3})$

$+ \dots + x^{0}(a_{0} + a'_{0} + a''_{0}) \to a, a', a'' \in R$

4) $P_{n_1} + P_0 = (a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0 x^0) + (0x^{n-1} + 0x^{n-2} + \dots + 0x^0)$

$= a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0 = P_{n_1}$

5) $P_{n_1} + \tilde{P}_{n_1} = (a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0) + (-a_{n-1} x^{n-1} - a_{n-2} x^{n-2} + \dots + -a_0)$

$= \{0\} = P_0$

6) 1

4') $P_n + P_0 = P_n \quad \to \quad 2P_n = P_0 \quad \to \quad P_0 = P_0'$

$P_n + P_0' = P_n \quad \quad 2P_n = P_0'$

6) $\beta(\alpha P_n) = \beta(\alpha a_{n-1} x^{n-1} + \alpha a_{n-2} x^{n-2} + \dots + \alpha a_0)$

$= \beta(\alpha(a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0)$

$= (\beta\alpha) P_n$

7) $(\alpha + \beta) P_n = (\alpha + \beta)(a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0)$

$(\alpha a_{n-1} x^{n-1} + \alpha a_{n-2} x^{n-2} + \dots + \alpha a_0) + (\beta a_{n-1} x^{n-1} + \beta a_{n-2} x^{n-2} + \dots \beta a_0)$

$\alpha P_n + \beta P_n$

9) $(\alpha P_n + \alpha P'_{n}) = (\alpha a_{n-1} x^{n-1} + \alpha a_{n-2} x^{n-2} + \dots + \alpha a_0) + (\alpha a'_{n-1} x^{n-1} + \alpha a'_{n-2} x^{n-2} + \dots \alpha a'_0)$

$\alpha(a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0) + \alpha(a'_{n-1} x^{n-1} + a'_{n-2} x^{n-2} + \dots a'_0)$

$\alpha(P_n + P'_n)$

10 $\alpha P_n \to \in V \ \forall \alpha \in R$ y cualquier P_n

b) $n\theta$

$\alpha P_n \Rightarrow (\alpha a_{n-1}x^{n-1} + \alpha a_{n-2}x^{n-2} + \dots \alpha a_0) \rightarrow \alpha \in I \rightarrow$

$\{\alpha a_{n-1} + \alpha a_{n-2}, \dots, \alpha a_0\} \in I$

c)

III) $P_n \rightarrow \{a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2x^2 + \dots\}$

$P_n + P'_n = P''_n \rightarrow P''_n \in S_{n>1}$

$\alpha P_n = \alpha(a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2x^2)$

$(\alpha a_{n-1})x^{n-1} + (\alpha a_{n-2})x^{n-2} + \dots + (\alpha a_2)x^2) \in S_{n>1}$

$P_0 \in S_{n>1} = \{0x^{n-1} + 0x^{n-2} + \dots + 0x^2\}$

I) $P_{n-1} \rightarrow$ es un espacio $\rightarrow$ entonce es un sub espacio

II) $P_{2nk} \rightarrow \sum_{i=0}^{n-1} a_i x^{2ki} \in N$

1) $P_{2nk} + P'_{2nk} = \sum_{i=0}^{n-1} a_i x^{2ki} + \sum_{i=}^{n-1} b_i x^{2ki}$

$= \sum_{i=0}^{n-1} (a_i + b_i) x^{2ki} \Rightarrow \in S_{2kn}$

2) $\alpha P_{2kn} = \alpha \sum_{i=0}^{n-1} a_i x^{2ni} \rightarrow \sum_{i=0}^{n-1} \alpha a_i x^{2ni} \Rightarrow \in S_{2kn}$

3) $P_0 = \sum_{i}^{n-1} 0x^{2ni} = \{0\}$

IV $P_{(x-1)} = (x-1)P_n \rightarrow (x-1)\sum_{i=0}^{n-1} a_i x^i$

$P_{(x-1)} + P'_{(x-1)} \Rightarrow (x-1)\sum_{i=0}^{n-1} a_i x^i + (x-1)\sum_{i=0}^{n-1} b_i x^i$

$= (x-1)\sum_{i}^{n-1}(a_i + b_i)x^i$

$= (x-1) P_{(a_i+b_i)} \in S_{(x-1)}$

$\alpha P_{(x-1)} \Rightarrow \alpha(x-1)\sum_{i}^{n-1} a_i x_i$

$= (x-1)\sum \alpha a_i x_i$

$= (x-1)\sum(\alpha a_i)x_i =$

$\Rightarrow \in S$

$P(x-1)_0 \Rightarrow (x-1)\sum 0x^i = (x-1)(0) = 0$

2.24

b) $H = \{a + b\hat{i} + c\hat{j} + d\hat{k}\}$ $a \in R$ $b, c, d \in C$ y $c_1 + c_2 \in R$

1) $|c\rangle = |a\rangle + |b\rangle = (a^{\alpha} + b^{\alpha})|q_{\alpha}\rangle = b^{\alpha}|q_{\alpha}\rangle + |a^{\alpha}|q_{\alpha}\rangle$

2) $|a\rangle + |b\rangle = a^{\alpha}|q_{\alpha}\rangle + b^{\alpha}|q_{\alpha}\rangle = \sum a^{\alpha}|q_{\alpha}\rangle + \sum b^{\alpha}|q_{\alpha}\rangle = \sum_{\alpha=1}^{4}(a^{\alpha} + b^{\alpha})|q_{\alpha}\rangle$

$= \sum (b^{\alpha} + a^{\alpha})|q_{\alpha}\rangle = |b\rangle + |a\rangle$

3) $(|a\rangle + |b\rangle) + |c\rangle = (a^{\alpha}|q_{\alpha}\rangle + b^{\alpha}|q_{\alpha}\rangle) + c^{\alpha}|q_{\alpha}\rangle$

$= \sum a^{\alpha}|q_{\alpha}\rangle + b^{\alpha}|q_{\alpha}\rangle + c^{\alpha}|q_{\alpha}\rangle = \sum (a^{\alpha} + c^{\alpha})|q_{\alpha}\rangle + b^{\alpha}|q_{\alpha}\rangle$

$= (a^{\alpha}|q_{\alpha}\rangle + c^{\alpha}|q_{\alpha}\rangle) + b^{\alpha}|q_{\alpha}\rangle = (|a\rangle + |c\rangle) + |b\rangle$

4) $|a\rangle + |0\rangle = |a\rangle$; $|a\rangle + |0_2\rangle = |a\rangle$

$|a\rangle + |0_1\rangle = |a\rangle + |0_2\rangle \rightarrow |0_1\rangle = |0_2\rangle \rightarrow |0_1\rangle = 0_{\alpha}|q^{\alpha}\rangle = 0$

$[a + 0, 0, 0, 0] = a + 0i + 0j + 0k$

5) $|a\rangle + |-a\rangle = |a\rangle \Rightarrow |a\rangle - |a\rangle = 0 \Rightarrow \sum \underbrace{(a^{\alpha} - a^{\alpha})}_{\neq 0}|q_{\alpha}\rangle = |0\rangle$

6) $c_1|a\rangle$ $c_1 \in R$ $\Rightarrow$ $c_1 a^{\alpha}|q_{\alpha}\rangle \in H$

7) $(c_1 + c_2)|a\rangle = (c_1 + c_2)\sum a^{\alpha}|q_{\alpha}\rangle = c_1\sum a^{\alpha}|q_{\alpha}\rangle + c_2\sum a^{\alpha}|q_{\alpha}\rangle$

$= c_1 a^{\alpha}|q_{\alpha}\rangle + c_2 a^{\alpha}|q_{\alpha}\rangle \Rightarrow c_1|a\rangle + c_2|a\rangle$

8) $c_1|c_2 a\rangle = c_1 \sum c_2 a^{\alpha}|q_{\alpha}\rangle \Rightarrow c_1 c_2 \sum a^{\alpha}|q_{\alpha}\rangle = c_1 c_2 |a\rangle$

9) $c_1(|a\rangle + |b\rangle) = c_1\sum(a^{\alpha} + b^{\alpha})|q_{\alpha}\rangle = \sum(c_1 a^{\alpha} + c_1 b^{\alpha})|q_{\alpha}\rangle = c_1|a\rangle + c_1|b\rangle$

10) $1|a\rangle = 1\sum a^{\alpha}|q_{\alpha}\rangle \Rightarrow \sum a^{\alpha} 1|q_{\alpha}\rangle = \sum a^{\alpha}|q_{\alpha}\rangle = |a\rangle$

c) $|b\rangle = (b^0, b)$ y $|r\rangle = (r^0, r)$

$|d\rangle = |b\rangle \odot |r\rangle = ((b^0 r^0 - b \cdot r), rb + b^0 r + b \times r)$

demostrar

$i^2 = j^2 = k^2 = -1$; $ij = k$; $jk = i$; $ki = j$; $a^0 a^0 = 1$

$ji = -k$; $kj = -i$; $ik = -j$

$(b^0, 000)$; $(r^0, 000)$

$(b^0 r^0 - \overset{0}{b^i \cdot r_i}, \overset{0}{r^i b} - \overset{0}{b^0 r} + \overset{-0}{\varepsilon_{ijk} b^j r^k}) = (b^0 r^0, 0\,0\,0)$

$(b^0 r^0 - (b_1 r_1 \hat{i} + b_2 r_2 \hat{i} + b_3 r_3 \hat{k}), r_0(b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}) + b^0(r_1 \hat{i} + r_2 \hat{j} + r_3 \hat{k}) + ((b_2 r_3 - b_3 r_2)\hat{i} + (b_1 r_3 - b_3 r_1)\hat{j}$

$(b_1 r_2 - b_2 r_1)\hat{k}$

$(b^0 r^0 - (br)_1 \hat{i} + (br)_2 \hat{j} + (br)_3 \hat{k}), ((r_0 b_1 + b_0 r_1 + (b_2 r_3 - b_3 r_2))\hat{i}$

$+ (r^0 b_2 + b^0 r_2 + (b_1 r_3 - b_3 r_1))\hat{j}$

$+ (r^0 b_3 + b^0 r_3 + (b_1 r_2 - b_2 r_1))\hat{k}$

$\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = -1$

$(0, b_1, 0, 0) \odot (0, r_1, 0, 0) = (0, 0, b_2, 0) \odot (0, 0, r_2, 0) = (0, 0, 0, b_3) \odot (0, 0, 0, r_3)$

$= (0 - (b^i r_i)), 0 b_1 + 0 r_1 + b_i \times r_i) = (-b^i r_i, 0, 0, 0)$ $i = 1, 2, 3$

$= br(-1, 0, 0, 0)$

$ij = k$; $ki = j$; $jk = i$

$(0, b_1, 0, 0) \odot (0, 0, r_2, 0) = (0, 0 + 0 \cdot b \times r) = (0, 0\hat{i}, 0\hat{j}, Q)$

$\epsilon_{ijk} b^j c^k = (0, 0, 0, 1)$; $j = 1$ y $k = 2$

Anti simetrico

$ji = -k$; $ik = -j$; $kj = -i$

$|b\rangle = b_2$; $|r\rangle = r_1$

$(0, 0 + 0 + \epsilon_{ijk} b^j r^k)$; $j = 2$; $k = 1$

$(0, 0 + \epsilon_{ikj} r^k b^j) \Rightarrow (b_2 r_3 - b_3 r_2)(0, 1, 0, 0)$

c)

$$|d\rangle = b^0 c^0 - \partial_j b^i r_j \, |Q_0\rangle + (b^0 r^j + r^0 b^j)\,|Q_j\rangle + \varepsilon_{ijk}\, b^j r^k \,|Q_i\rangle$$

$$b^0 c^0 - b^i r_i = a \qquad j\cdot b^0 r^j + r^0 b^j = b^0 b^j + b^0 r^j ;$$

$$\rightarrow S^{(0j)} \delta^0_\alpha = S^{j0}$$

$$\rightarrow S^{0j} = b^0 r^j + r^0 b^j$$

$$|d\rangle = a\,|Q_0\rangle + S^{0j}\,|Q_i\rangle + \varepsilon_{ijk}\, b^j r^k \,|Q_i\rangle$$

$$\varepsilon_{ijk} = A^{[jk]i}\, b_j r_k \,|Q_i\rangle$$

D)

$$|d\rangle = (\underbrace{a^0 b^0 - b\cdot a}_{\text{Escalar}},\ \underbrace{a^0 b}_{\text{vector}} + \underbrace{b^0 a}_{} + \underbrace{a \times b}_{\text{pseudovector}})$$

no es ni vector ni seudovector, es una baina loca

F)

$$Q_1 \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix};\ Q_2 \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix};\ Q_3 \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix};\ I_4 \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$Q_1 Q_1 = \begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = -I_4$$

$$Q_2 Q_2 = \begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}\begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = -I_4$$

$$Q_3 Q_3 = \begin{pmatrix} & & -1 & 0 \\ 0 & & 0 & 1 \\ 1 & 0 & & \\ 0 & -1 & & \end{pmatrix}\begin{pmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{pmatrix} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = -I_4$$

$$Q_1 Q_2 = \begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}\begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix} = \begin{pmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{pmatrix} = Q_3 = -Q_2 Q_1 = -\begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} & & 1 & \\ & & & 1 \\ -1 & & & \\ & 1 & & \end{pmatrix} = Q_3$$

$$Q_1 Q_3 = \begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}\begin{pmatrix} & & -1 & 0 \\ & & 0 & 1 \\ 1 & 0 & & \\ 0 & -1 & & \end{pmatrix} = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix} = -Q_2$$

$$Q_3 Q_1 = \begin{pmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{pmatrix}\begin{pmatrix} & 1 & & \\ -1 & & & \\ & & & 1 \\ & & -1 & \end{pmatrix} = \begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix} = Q_2$$

$$Q_2 Q_3 = \begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}\begin{pmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{pmatrix} = \begin{pmatrix} & 1 & & \\ -1 & & & \\ & & & 1 \\ & & -1 & \end{pmatrix} = Q_1 = -Q_3 Q_2 = \begin{pmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{pmatrix}\begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}$$

G)

$\widetilde{\langle a|b\rangle} = a^{+} \circ b = (a_0, -a) \circ (b_0, b) \Rightarrow d_0 |a_0\rangle a_0 b - b_0 a - a \times b$

$d_0 = a_0 b_0 + a^i b_i$
$d_1 = a_0 b_1 - b_0 a_1 + b_2 a_3 + a_2 b_3$
$d_2 = a_0 b_2 - b_0 a_2 - a_3 b_1 + a_1 b_3$
$d_3 = a_0 b_3 - b_0 a_3 - a_1 b_2 + a_2 b_1$

$\widetilde{\langle a|B\rangle} = \langle d_0 | d_i\rangle$

1) $\widetilde{\langle a|b\rangle} = \overline{\widetilde{\langle b|A\rangle}}$

$\langle b|A\rangle = b^{+} \circ A = (b_0, -b) \circ (a_0, a) \Rightarrow d_0 |a_0\rangle + b_0 a - a_0 b + -b \times a$

$d_0 = a_0 b_0 + a^i b_i$

$d_1 = -a_0 b_1 + a_1 b_0 + b_2 a_3 + b_3 a_2$

$\vdots$

$\langle A|b\rangle$ $\langle b|A\rangle$

$d_0 = (a_0 b_0 + a b)$ $d_0' = a_0 b_0 + a b$

$d = a_0 b - b_0 a - a \times b$ $d' = b_0 a - A_0 b - - a \times b$

$\langle A|b\rangle = \overline{\langle b|A\rangle}$

$(d_0, d) = (d_0, -d')$

2) $\widetilde{\langle a, a\rangle} = (a_0^2 + a \cdot a, a_0 a - a_0 a - a \times a) = a_0^2 + a^2 \geq 0$

si $a \neq 0$

3) $\langle c_1 a + c_2 b, w\rangle = (c_1 a + c_2 b)^{+} \circ w$

$= c_1 a^{+} \circ w + c_2 b^{+} \circ w$

$(c_1 a_0, -c_1 a) \circ (w_0, w) + (c_2 b_0, -c_2 b_2) \circ (w_0, w)$

$= c_1 \langle a \circ w\rangle + c_2 \langle b \circ w\rangle$

4) $\langle a|0\rangle = 0 = (a^{+}, 0\,0) = (a_0 0 + a 0, 0 a\; a_0 0 + a \times 0)$

H) $\langle a|B\rangle = \frac{1}{2}[\langle a,b\rangle - |Q_1\rangle \circ \langle \widetilde{a|b}\rangle \circ |Q\rangle]$

$= \frac{1}{2}, (d_0, d) - (0, Q_1) \circ (d_0 d) \circ (0, Q_1)$

$(0\,1\,0\,0) \circ (d_0, d_1 d_2 d_3) = (-d_1, d_0, -d_3, d_2) = d' = (-d_1, d')$

$d' \circ Q_1 = (-d_0, -d_1, d_2, +d_3)$

$\frac{1}{2}((d_0, d) - (-d_0, d_1, +d_2, d_3))$

$= \frac{1}{2}\, 2(d_0 + d_1|Q_1\rangle)$

$\langle a, B\rangle = d_0 + d_1|Q_1\rangle \Rightarrow \mathbb{C}$

$\langle B, a\rangle = \frac{1}{2}\cancel{(d_0, d) - |Q\rangle \circ (d_0, d)} = (d_0 + d_1|Q_1\rangle)$

$(B^+ \circ a) =$

$d_0 = a_0 b_0 + ab$

$d_1 = -(a_0 b_1 + b_0 a_1 + a_2 b_3 - a_3 b_2)$

$\langle a|b\rangle = d_0 - d_1|Q_1\rangle$

$A^+ \circ b =$

$d_0 = a_0 b_0 + ab$

$d_1 = -b_0 a_1 + a_0 b_1 + b_3 a_2 - b_2 a_3$

1) $\langle B|a\rangle = \overline{\langle a|B\rangle}$

2) $\langle a|a\rangle = d_0 = a^2 + |a|^2 + 0|Q_1\rangle$

$d_0 = a_0^2 + |a|^2$

3) $\langle a|0\rangle = 0 = d_0 + d_1|Q_1\rangle \Rightarrow d_0 = 0,\ d_1 = 0$

$d_0 = a_0 \cdot 0 + a0 = 0$

$d_1 = a_0 0 + a0 + [a \times 0]_{23-32} = 0$

4) $\langle c_1 a + c_2 b, w\rangle = \frac{1}{2}[\langle c_1 a + c_2 b|w\rangle - |Q_1\rangle \circ \langle \widetilde{c_1 a + c_2 b|w}\rangle \circ |Q_1\rangle$

$\langle c_1 a|w\rangle + \langle c_2 b|w\rangle - Q_1(\langle c_1 a|w\rangle + \langle c_2 b|w\rangle) Q_1\rangle$

$\frac{1}{2}[c_1 \langle a|w\rangle + c_2 \langle b|w\rangle - c_1 |Q_1\rangle \circ \langle \widetilde{a|w}\rangle \circ |Q_1\rangle - c_2 |Q_1\rangle \langle \widetilde{b|w}\rangle |Q_1\rangle]$

$\frac{1}{2}[c_1(\langle a|w\rangle - |Q_1\rangle \langle \widetilde{a|w}\rangle |Q_1\rangle] + \frac{1}{2}[c_2(\langle b|w\rangle - |Q_1\rangle \langle \widetilde{b|w}\rangle |Q_1\rangle]$

$c_1 \langle a|w\rangle + c_2 \langle b|w\rangle$

I $n(v) = \sqrt{a^+ \circ a} = \sqrt{\langle a | a \rangle}$

1 $n(a) \geq 0$; $a = 0 \Rightarrow n(a) = 0$

$\langle a | a \rangle = (a_0^2 + |a|^2) = n(a)^2 \Rightarrow \sqrt{a_0^2 + |a|^2} \geq 0$

2 $n(c_1 a) = \sqrt{(c_1 a)^+ \circ c_1 a}$

$c_1 (a_0, -a) \circ c_1 (a_0, +a)$

$= \sqrt{c_1^2 (a_0, -a) \circ (a_0, a)}$

$= |c_1| \sqrt{(a_0, -a) \circ (a_0, a)} = |c_1| n(a)$

3 $n(a+b) \leq n(a) + n(b)$

$n(a+b)^2 = \langle a | a \rangle + \langle b | b \rangle + \langle a | b \rangle + \overline{\langle a | b \rangle}$

$|a|^2 + |b|^2 + 2 a_0 (\langle a | b \rangle)$

$2 a_0 [\langle a | b \rangle] \leq |2 \langle a | b \rangle| \leq 2 |a| |b|$

$|a|^2 + |b|^2 \; 2 a_0 \langle a | b \rangle \leq |a|^2 + |b|^2 + 2 |a| |b|$

$n(a+b)^2 = (n(a) + n(b))^2$

$\hat{J}$ 1

$\langle a | \bar{a} \rangle =$ $= 1$

$a \circ a^+ = a^+ \circ a = \|a^2\| = a_0^2 + |a|^2$

$\frac{\|a\|^2}{\|a\|^2} = 1$

R

1 $|v'\rangle = \frac{1}{|a|^2} \underbrace{\langle a^+ \rangle \circ |v\rangle \circ |a\rangle}_{v}$

$(\delta^{0J} |aJ\rangle + \varepsilon_{iJk} a^J v^k) \circ a$

$(a_0 v, \delta^{0J} |aJ\rangle + \varepsilon_{ikJ} v^k a^J) \circ a$

$\langle v' | v' \rangle = \frac{1}{a^2} \langle |a^+\rangle \circ |v\rangle \circ a\rangle \, |a^+\rangle |v\rangle \circ |a\rangle\rangle$

$(|a^+\rangle |v\rangle \circ a\rangle)^+ \circ |a^+\rangle \circ |v\rangle \circ |a\rangle$

$|a^+\rangle |v\rangle \circ |a^+\rangle \circ |a^+\rangle \circ |v\rangle \; |a\rangle$

4)

$|a\rangle \circ |V\rangle$

$0 + a^i v_i \,,\; a_0 v + \epsilon_{ijk} a^j v_k$

$\frac{1}{|a|^2} |a_i v_i \,,\; d) \;\; \frac{\circ 1}{|a|^2} (|a_0^2 - \|a\|^2, -2a_0 a^i)$

$(a_i v_i (a_0^2 - |a|^2) - (-2a_0 a_i)^j (a_i v^i)_j \,) |Q_0\rangle$

$((a_i v_i (a_0^2 - |a|^2) + (-2a_0 a_i)(a_0 v + \epsilon_{ijk} a^j v_k)) |Q_0\rangle$

$+$

$(d (a_0^2 - \|a\|^2) + (a_i v^i)(-2a_0 a_i) + d \times a)$

$d \times a = 0 + a_0 v \times a = -\epsilon_{ikj} a^k v^j$

$\frac{1}{|a|^2} |a^+\rangle \circ |V\rangle = \frac{1}{\|a^2\|} \langle a^i v_i, +a_0 v - \epsilon_{ijk} a^j v^k\rangle = \langle a^i v_i, d\rangle$

$\frac{1}{|a|^2} |d\rangle \circ |a\rangle = a_0 a^i v_i - [(a_0 v_i - \epsilon_{ijk} a^j v^k)^I a_I] |Q_0\rangle$

$(a_i v^i) a_k + a_0 [a_0 v_i - \epsilon_{ijk} a^j v^k]$

$+ \underbrace{[a_0 v - a \times v] \times a}\; |Q_s\rangle$

$v \times a \;\; - a \times v \times a$

$- (|a|^2 v - a (v \cdot a))$

$\overbrace{(a_i v^i) a_k}^{\epsilon_1} + \underbrace{|a_0|^2 v} - (a \times b) a_0 + a_0 v \times a + [-|a|^2 v + \overbrace{a_k (v_i a^i)}^{\epsilon_1}$

$v(a_0^2 - |a^2|) \;\; 2[(a_i v^i) a_k + [v \times a] a_0]$

$\frac{1}{|a|^2} |a^+\rangle \circ |V\rangle \circ |a\rangle = (0, v(a^+ \circ a) + 2[(av)^i a + [v \times a] a_0])$

$\frac{1}{|a|^2} |a\rangle \circ |V\rangle^+ \circ |a\rangle^+ = (0, -v(a^+ \circ a) - 2[(av)^i a + [v \times a] a_0])$

$a_0 = 0$

$= |v|^2 |a|^4 \frac{1}{|a|^4} = |V|^2 = |v^i|$