

$$A_K^i A_{ii} = \delta_K^i$$

$$\begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \delta_K^i \quad \text{dove } i \in \{1, 2, 3\} \quad \text{e } K \in \{1, 2, 3\}$$

$A_K^i \Rightarrow \cos(\theta_K^i)$ è l'angolo entre los tres eje's x k.

$$A_K^i \rightarrow K=3 \rightarrow A_3 A_{ii} = \cos(\theta_3) \cos(\theta_2) \rightarrow$$

$$\cos(\theta_3) = \cos(\theta_1)^2 + \cos(\theta_2)^2 + \cos(\theta_3)^2 = 1$$

$$m \cdot n \cdot 6 \cdot 6 \quad 3 \times 2;3 \leftarrow (0 \times V) \cdot 6 \times 2;3 \leftarrow (0 \times V) \times V, (75) \cdot 6 \cdot 6 \cdot 6 \quad (n_6 m_6 - m_6 n_6)$$

$$I)(x, y) \rightarrow (x+y, x-y) \quad m \cdot n \cdot 6 - (m \cdot 6 \cdot m \cdot 6) \cdot n \cdot n \cdot 6$$

$$\underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_A \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_V = \begin{pmatrix} -y \\ x \end{pmatrix} \rightarrow A^T \cdot A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \stackrel{II}{\rightarrow} (x, y) \rightarrow (x, -y) \quad \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$III)(x, y) \rightarrow (x+y, x-y) \quad (83) \cdot m \cdot 6 + (08) \cdot n \cdot 6$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$$

$$IV)(x, y) \rightarrow (x+y, x+y) \quad (145) \cdot m \cdot 6 + (045) \cdot n \cdot 6 = 2I$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x+y \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$$

solo los transformaciones I y II son lineales
la ecuación, mientras III y IV combinan la ecuación

$$1.5.7) \quad \nabla \cdot (\phi \mathbf{a}) = \partial_i (\phi a^i) = ((\partial_i \phi) a^i) + \phi (\partial_i a^i)$$

$$2a) \quad \nabla \cdot (\nabla \times \mathbf{a}) = \partial_i (\epsilon_{ijk} \partial_j a^k) \rightarrow$$

$$\partial_i \epsilon_{ijk} \partial_j a^k = \epsilon_{ijk} \partial^i \partial^j a^k$$

$$\epsilon_{ijk} \partial^i \partial^j a^k = -\epsilon_{ijk} \partial^j \partial^i a^k = 0$$

$$\nabla \times (\nabla \cdot \mathbf{a}) \rightarrow \nabla \cdot \mathbf{a} \rightarrow \text{es decir no tiene sentido}$$

$$= (1, 1, 1) \cdot (1, 1, 1) + (1, 1, 1) \cdot (1, 1, 1) = (1, 1, 1) \cdot (1, 1, 1)$$

$$2f) \quad \nabla \times (\nabla \times \mathbf{a}) \rightarrow \epsilon_{ijk} \partial^j (\nabla \times \mathbf{a})^k \rightarrow \epsilon_{ijk} \epsilon^{kmn} \partial^j \partial_m a_n$$

$$(\delta^i_m \delta^j_n - \delta^i_n \delta^j_m) \partial^j \partial_m a_n$$

$$\delta^i_m \delta^j_n (\delta^m_n \delta^p a_m) - \delta^i_m a_m (\delta^m_n \delta^p a_n)$$

$$\delta^i_m (\delta^p a_p) - a_m (\delta^p \delta^i_p)$$

$$I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \nabla (\nabla \cdot \mathbf{a}) = \mathbf{a} (\nabla \cdot \nabla) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

1.6.5

$$z = (e^{i3\theta}) = (e^{i\phi})^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}$$

$$\cos(3\theta) + i \sin(3\theta) = (\cos(\theta) + i \sin(\theta))^3$$

$$I_S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \cos^3(\theta) + 3 \cos^2(\theta) \sin(\theta)i + 3 \cos(\theta) (\sin(\theta))^2$$

$$Re = \cos(3\theta) = \cos(\theta)^3 - 3 \cos(\theta) \sin(\theta)^2$$

$$Im = \sin(3\theta) = -\sin(\theta)^3 + 3 \cos(\theta)^2 \sin(\theta)^2$$

$$I_S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}$$

5)

$$\text{a) } \sqrt{2}i \Rightarrow 2e^{i\frac{\pi}{12}(1-1)\pi j + (3)nj} = (9i)nj$$

$$\text{raices} \rightarrow \sqrt{2} e^{i\left(\frac{\pi/2+2\pi k}{n}\right)} \quad k=0,1,2, \dots, n-1$$

$$\begin{aligned} 1) \sqrt{2} e^{i\pi/4} &\rightarrow \sqrt{2} (\cos(\pi/4) + i \sin(\pi/4)) \rightarrow 1+i \\ 2) \sqrt{2} e^{i(\pi/4+\pi)} &\rightarrow (-1-i) \end{aligned}$$

$$\text{b) } \sqrt{1-\sqrt{3}i} \rightarrow \sqrt{q} e^{i\arctan(\sqrt{3})} = 2e^{i\pi/3}$$

$$d = \sqrt{r} \quad (\sqrt{\pi/6}) = \sqrt{2} \quad \theta = \arctan(\sqrt{3}) = \pi/3 = 37^\circ$$

$$1) \sqrt{2} e^{i\pi/6} \rightarrow \sqrt{2} (\cos(\pi/6) + i \sin(\pi/6)) \rightarrow \sqrt{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right)$$

$$2) \sqrt{2} e^{i(\pi/6+\pi)} \rightarrow \sqrt{2} e^{i\pi/6} e^{i\pi-1} \rightarrow \sqrt{2} \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right)$$

$$\text{c) } (-1)^{11/3} \rightarrow e^{i\pi/3} + (0)nj = (9i)nj$$

$$1) e^{i\pi/3} = e^{i\pi/3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i \quad (9i)nj$$

$$2) e^{i(\pi/3+2\pi/3)} = e^{i\pi} = -1 \quad (9i)nj$$

$$3) e^{i(\pi/3+4\pi/3)} = e^{i5\pi/3} = \frac{1}{2} - \frac{\sqrt{3}}{2}i \quad (9i)nj$$

$$\text{d) } \sqrt[6]{3} = 2^{3/6} = (\sqrt{2})^3 = 3\sqrt{2} e^{i0} \quad \rightarrow \quad \text{sr} = \frac{\pi}{6}$$

$$1) \sqrt{2} e^{i0} \rightarrow \sqrt{2} (\cos(0) + i \sin(0)) \rightarrow 1$$

$$2) \sqrt{2} e^{i\pi/3} \rightarrow (2/2 + \sqrt{3}/2i) \sqrt{2} \quad \rightarrow (1 + \sqrt{3}i) \sqrt{2} \quad (9i)nj$$

$$3) \sqrt{2} e^{i2\pi/3} = (-1/2 + \sqrt{3}/2i) \sqrt{2}$$

$$4) \sqrt{2} e^{i\pi} \rightarrow -\sqrt{2} \quad (9i)nj$$

$$5) \sqrt{2} e^{i4\pi/3} = -(\sqrt{2} + \sqrt{3}/2i) \sqrt{2} \quad (9i)nj$$

$$6) \sqrt{2} e^{i8\pi/3} = (1/2 + (\sqrt{3}/2)i) \sqrt{2} \quad (9i)nj$$

$$\text{e) } \sqrt[4]{-8+8\sqrt{3}i} \rightarrow \sqrt{64} \sqrt{2} e^{i\arctan(\sqrt{3}+1) + i(\pi/3+1)} = 4\sqrt{2} e^{i\pi/3}$$

$$1) 2e^{i\pi/3} = 1 + \sqrt{3}i \quad 4) 2e^{i(\pi/3+\pi/2)} = \sqrt{3} - 1i$$

$$2) 2e^{i(\pi/3+\pi/2)i} = -\sqrt{3} + 1i$$

$$3) 2e^{i(\pi/3+\pi)i} = -1 - \sqrt{3}i$$

6)

$$\ln(-i) = \ln(e) + \ln(-i)^{1/2} \quad \text{S}$$

$$e^w = -i \quad ; \quad w = x + iy \quad \left(\frac{-\pi}{2} + \frac{\pi}{2}n \right)$$

$$e^x e^{iy} = -i \rightarrow e^x (\cos(y) + i \sin(y)) = -1$$

$$RG \in \mathbb{R}^x; \cos(y) = 0 \rightarrow y = \pi/2, 3\pi/2$$

$$Im = e^x \sin(y) = -1 \quad (y = \pi/2, 3\pi/2) \quad x = 0$$

$$(1 + \frac{e^x}{2})^2 = ((-1)^2) \cos^2(y) + ((-1)^2) \sin^2(y) = 1$$

$$(\frac{e^x}{2} + i(e^x))^2 = 1 \quad \left(\frac{\pi}{2}, \frac{3\pi}{2} \right)$$

$$\ln(i) = \ln(e) + \ln(e^{-\pi/2i}) = 1 - \pi/2$$

$$b) \ln(1-i) \quad \left(\frac{\pi}{4} + \frac{\pi}{2}n \right) = \frac{e^{i\pi}}{2} \quad \left(\frac{3\pi}{4} + \frac{\pi}{2}n \right)$$

$$e^w = (1-i) \rightarrow e^x (\cos(x) + i \sin(y))$$

$$e^x e^{iy} = \sqrt{2} e^{-\pi/4}$$

$$e^x = \sqrt{2} \rightarrow x = \frac{1}{2} \ln(2) = \frac{\pi}{4} \quad (1)$$

$$e^y = e^{\pi/4} \rightarrow y = \pi/4 \quad (2)$$

$$\ln(1-i) = \ln \left(e^{\frac{i}{2}\ln(2)} - \frac{\pi}{4} \right) = \frac{1}{2} \ln(2) - \frac{\pi}{4}$$

c) $\ln(e)$

$$e^w = e \quad \left(\frac{\pi}{2} + \frac{\pi}{2}n \right)$$

$$e^x e^{iy} = e \quad \left(\frac{\pi}{2} + \frac{\pi}{2}n \right)$$

$$(\cos(\phi)) e^x = e \quad \left(\frac{\pi}{2} + \frac{\pi}{2}n \right)$$

$$e^x (\cos(\phi)) = 1 \rightarrow e^x \neq 0 \rightarrow \cos(\phi) \exists 2\pi n = 0$$

$$\ln(e^{(1+2\pi ni)}) = 1 + 2\pi ni$$

$$1 + 2\pi ni = \frac{1}{2} + \frac{\pi}{2}i \quad \left(\frac{\pi}{2} + \frac{\pi}{2}n \right)$$

D)

$$\ln(i)$$

$$e^w = i$$

$$e^x e^{yi} = 1 e^{(\pi/2 + 2\pi n)i}$$

$$x = 0$$

$$yi = (\pi/2 + 2\pi n)i$$

$$\ln(e^{(\pi/2 + 2n\pi)i}) = \pi/2 + 2n\pi i$$