5)

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$|b\rangle \begin{pmatrix} a+ & x+yi \\ +x+yi & a- \end{pmatrix} = \begin{pmatrix} a & v \\ v & b \end{pmatrix}$

$\alpha_0 I + \alpha_1 \sigma_1 + \alpha_2 \sigma_2 + \alpha_3 \sigma_3 = |b\rangle$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & a \\ 0 & 1 & -i & 0 & x+yi \\ 0 & 1 & +i & 0 & +x-yi \\ 1 & 0 & 0 & -1 & b \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & a \\ 0 & 1 & -i & & x+yi \\ 0 & 1 & i & & +x+yi \\ 0 & & & -2 & \frac{b+a}{2} \end{array}\right) \Rightarrow \begin{array}{l} \alpha_1 = 1 \\ \alpha_2 = -1 \\ \alpha_3 = \frac{a-b}{2} \end{array} \quad \alpha_4 = \frac{a+b}{2}$$

b)

$\langle A | B \rangle = traz(A^{\dagger} B)$ // $\sigma_i^{\dagger} = \sigma_i$

$\sigma_i \sigma_j \Rightarrow i = j = \begin{bmatrix} i_j & 0 \\ 0 & -i_j \end{bmatrix}$

$\sigma_i \sigma_j \Rightarrow i \neq j = \begin{bmatrix} 0 & i_j \\ i_j & 0 \end{bmatrix}$

$traz(\sigma_i^{\dagger} \sigma_j) = tr(\sigma_i \sigma_j) \Rightarrow 0 \rightarrow = 0$

$\hookrightarrow i_j - i_j = 0$

C) si es solo compleja

$\alpha_0 \sigma_0 + \alpha_1 \sigma_1 + \alpha_2 \sigma_2 + \alpha_3 \sigma_3 = iM \quad \backslash M \in M_{2\times 2}$

$\alpha_0 = \alpha_1 = \alpha_3 = 0$

$i \alpha_2 \sigma_2 = iM$

$\alpha_2 \sigma_2 = M$

$\begin{pmatrix} 0 & -\alpha_2 \\ \alpha_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -A \\ A & 0 \end{pmatrix} \Rightarrow$ sub espacio es $\begin{pmatrix} 0 & -A \\ A & 0 \end{pmatrix}$

Si es Todo Mod I $a_2 = 0$

$a_0\sigma_0 + a_1\sigma_1 + a_3\sigma_3 = M \Rightarrow M \in M_{2\times2}$

$$\begin{pmatrix} a_0 & 0 \\ 0 & a_0 \end{pmatrix} + \begin{pmatrix} 0 & a_1 \\ a_1 & 0 \end{pmatrix} + \begin{pmatrix} a_3 & 0 \\ 0 & -a_3 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$a_0 + a_3 = A$

$a_1 = B = C$

$a_0 - a_3 = d$

$a_0 = \frac{1}{2}(d+a)$ $\Rightarrow$ $a_0\sigma + a_1\sigma_1 + a_3\sigma_3 = \begin{bmatrix} A+d & C \\ C & A-d \end{bmatrix}$

$a_3 = 1/2(A-d)$