$$A^{i'}_{k}\tilde{A}^{j}_{i'} = \delta^{j}_{k}$$

$$\begin{pmatrix} \cos\phi & \operatorname{sen}\phi & 0 \\ -\operatorname{sen}\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & -\operatorname{sen}\phi & 0 \\ \operatorname{sen}\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$= \delta^{j}_{k}$

$A^{i'}_{k} \Rightarrow \cos(\theta_{i'k})$ es el angulo entre los ejes i' y k.

$A^{i'}_{k}\tilde{A}^{j}_{i'} \rightarrow k=j \Rightarrow A^{i'}_{j}A^{j}_{i'} = \cos(\phi_{i'j})\cos(\phi_{ji'}) \rightarrow$

$$\cos(\phi_{ij})^2 = \cos(\phi_1)^2 + \cos(\phi_2)^2 + \cos(\phi_3)^2 = 1$$

I) $(x\ y) \rightarrow (-y, x)$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix} \rightarrow A^{T}A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$\underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{A}$

II) $(x, y) \rightarrow (x, -y)$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

III) $(x, y) \rightarrow (x-y, x+y)$

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ x+y \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$$

IV) $(x, y) \rightarrow (x+y, x-y)$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$$

Solo las transformaciones I y II mantienen la escala, mientras III y IV cambian la escala

1.5.7)

2a) $\nabla(\phi\psi) = \partial_i(\phi\psi)^i \Rightarrow ((\partial_i\phi)\psi) + \phi(\partial_i\psi)$

2d) $\nabla\cdot(\nabla\times a) = \nabla\cdot(\varepsilon_{ijk}\,\partial_j a^k) \rightarrow$

$$\partial_i\,\varepsilon_{ijk}\,\partial_j a_k = \varepsilon_{ijk}\,\underbrace{\partial^i\partial^j}_{\partial^j\partial^i} a^k$$

$$\varepsilon_{ijk}\,\partial^i\partial^j a^k = -\varepsilon_{jik}\,\partial^j\partial^i a^k = 0$$

$\nabla\times(\nabla\cdot a) \rightarrow \nabla\cdot a \rightarrow$ escalar no está definido

2f) $\nabla\times(\nabla\times a) \rightarrow \varepsilon_{ijk}\,\partial^j(\nabla\times a)^k \rightarrow \varepsilon_{ijk}\,\varepsilon^{knm}\,\partial^j\partial_n a_m$

$$(\delta^i_n\delta^j_m - \delta^i_m\delta^j_n)\,\partial^j\partial_n a_m$$

$$\delta^i_n\,\partial_n(\delta^j_m\,\partial^j a_m) - \partial^i_m a_m(\delta^j_n\,\partial^j\partial_n)$$

$$\partial_n(\partial^j a_j) - a_m(\partial^j\partial_j)$$

$$\nabla(\nabla\cdot a) - a(\nabla\cdot\nabla)$$

1.65

2) $(e^{i3\phi}) = (e^{i\phi})^3$

$$\cos(3\phi) + i\,\mathrm{sen}(3\phi) = (\cos\phi + i\,\mathrm{sen}\,\phi)^3$$

$$= \cos(\phi)^3 + 3\cos^2\phi\,\mathrm{sen}\,\phi\, i + 3\cos\phi\,(i\,\mathrm{sen}\,\phi)^2 + (i\,\mathrm{sen}\,\phi)^3$$

$$\mathrm{Re} = \cos(3\phi) = \cos(\phi)^3 - 3\cos(\phi)\,\mathrm{sen}(\phi)^2$$

$$\mathrm{Im} = \mathrm{sen}(3\phi) = -\mathrm{sen}(\phi)^3 + 3\cos(\phi)^2\,\mathrm{sen}(\phi)$$

5)

a) $\sqrt{2i} \Rightarrow \quad 2e^{i\pi/2}$

raices $\rightarrow \sqrt{2}\, e^{i\left(\frac{\pi/2 + 2\pi k}{n}\right)} \quad k = (0,1)$

1) $\sqrt{2}\, e^{i\pi/4} \rightarrow \sqrt{2}\left(\cos(\pi/4) + i\,\text{sen}(\pi/4)\right) \rightarrow 1+i$

2) $\sqrt{2}\, e^{i(\pi/4 + \pi)} \rightarrow (-1-i)$

B) $\sqrt{1-\sqrt{3}i} \rightarrow \sqrt{4}\, e^{i\,\text{artan}(\sqrt{3})} = 2e^{i\pi/3}$

1) $\sqrt{2}\, e^{i\pi/6} \rightarrow \sqrt{2}(\cos(\pi/6) + i\,\text{sen}(\pi/6)) \rightarrow \sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

2 $\sqrt{2}\, e^{i(\pi/6 + \pi)} \rightarrow \sqrt{2}e^{i\pi/6}\, e^{i\pi} = -1 \rightarrow \sqrt{2}(-\sqrt{3}/2 + 1/2 i)$

c) $(-1)^{1/3} \rightarrow e^{i\pi}$

1) $e^{i\pi/3} = e^{i\pi/3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

2) $e^{i(\pi/3 + 2\pi/3)} = \pi i$
$= -1$

3) $e^{i(\pi/3 + 4\pi/3)} = e^{i5\pi/3} = \frac{1}{2} + -\frac{\sqrt{3}}{2}i$

d) $\sqrt[6]{8} = 2^{3/6} = \sqrt{2} = \sqrt{2}\, e^{i0}$

1) $\sqrt{2}e^{i0} \Rightarrow \sqrt{2}$

2) $\sqrt{2}\, e^{i\pi/3} \rightarrow (1/2 + \sqrt{3}/2 i)\sqrt{2}$

3) $\sqrt{2}\, e^{i2\pi/3} = (-1/2 + \sqrt{3}/2 i)\sqrt{2}$

4) $\sqrt{2}e^{i\pi} \rightarrow -\sqrt{2}$

5) $\sqrt{2}e^{i4\pi/3} = -(1/2 + \sqrt{3}/2 i)\sqrt{2}$

6 $\sqrt{2}e^{i5\pi/3} = (1/2 + (-\sqrt{3}/2)i)\sqrt{2}$

e) $\sqrt[4]{-8 + 8\sqrt{3}i} \rightarrow \sqrt{64}\sqrt{4}\, e^{i\,\text{artan}(\sqrt{3}/1)} = i(\pi/3) \quad 4\pi/3$

1) $2e^{\pi/3 i} = 1 + \sqrt{3}i$

2) $2e^{(\pi/3 + \pi/2)i} = -\sqrt{3} + 1i$

3) $2e^{(\pi/3 + \pi)i} = -1 - \sqrt{3}i$

4) $2e^{(\pi/3 + \frac{\pi \cdot 3}{2})i} = \sqrt{3} - 1i$

6)

$\ln(-ie) = \ln(e) + \ln(-i)$

$e^{w} = -i \quad ; \quad w = x + yi$

$e^{x} e^{yi} = -i \rightarrow e^{x}(\cos(y) + i\,\mathrm{sen}(y)) = -1$

$Re = e^{x}\cos(y) = 0 \quad \rightarrow (y = (\pi/2, 3\pi/2)$

$Im = e^{x}\,\mathrm{sen}(y) = -1 \qquad (y = (-\pi/2) \vee x = 0$

$e^{-\pi/2 i} = -i$

$\ln(-ie) = \ln(e) + \ln(e^{-\pi/2 i}) = 1 - \pi/2$

b) $\ln(1-i)$

$e^{w} = (1-i) \rightarrow e^{x}(\cos(x) + i\,\mathrm{sen}(y))$

$e^{x} e^{yi} = \sqrt{2}\, e^{-i\cdot\pi/4}$

$e^{x} = \sqrt{2} \rightarrow x = 1/2 \ln(2)$

$e^{y} = e^{\pi/4} \rightarrow y = \pi/4$

$\ln(1-i) = \ln(e^{\frac{1}{2}\ln(2) - \pi/4}) = \frac{1}{2}\ln(2) - \pi/4$

c) $\ln(e)$

$e^{w} = e$

$e^{x} e^{yi} = e$

$(\cos(\varphi)) e^{x} = e$

$e^{x}(i\,\mathrm{sen}(\varphi)) = 0 \rightarrow e^{x} \neq 0 \rightarrow \mathrm{sen}(\varphi) \Rightarrow 2\pi n = 0$

$\ln(e^{(1 + 2\pi n i)}) = 1 + 2\pi n i$

D)

$$\operatorname{Ln}(i)$$

$$e^{w} = i$$

$$e^{x} e^{yi} = 1 e^{(\pi/2 + 2\pi n)i}$$

$$x = 0$$

$$yi = (\pi/2 + 2\pi n)i$$

$$\operatorname{Ln}\left(e^{(\pi/2 + 2n\pi)i}\right) = \pi/2 + 2n\pi i$$