Para cualquier estado coherente, se tiene: ⑥

$$e^{-\lambda\hat{N}}|\alpha\rangle = \sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}e^{-\lambda\hat{N}}|\varphi_n\rangle = \sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}e^{-\lambda n}|\varphi_n\rangle$$

$$= \sum_{n=0}^{\infty}\frac{(\alpha e^{-\lambda})^n}{\sqrt{n!}}|\varphi_n\rangle = |e^{-\lambda}\alpha\rangle$$

luego $e^{-i\Omega t\hat{N}}|\alpha\rangle = |e^{-i\Omega t}\alpha\rangle$

Por lo tanto:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}e^{-\frac{i\Omega t}{2}}\left(|e^{-i\Omega t}\alpha\rangle + i|e^{-i\Omega t}(-\alpha)\rangle\right)$$

(f) $\langle x(t)\rangle = \langle\psi(t)|\hat{x}|\psi(t)\rangle \quad ; \quad \beta \equiv \sqrt{\frac{m\Omega}{\hbar}}$

$$= \frac{1}{\sqrt{2}\beta}\langle\psi(t)|\hat{a}+\hat{a}^{\dagger}|\psi(t)\rangle$$

$$= \frac{1}{2\sqrt{2}\beta}\left(\langle\alpha e^{-i\Omega t}| - i\langle(-\alpha)e^{-i\Omega t}|\right)(\hat{a}+\hat{a}^{\dagger})\left(|\alpha e^{-i\Omega t}\rangle + i|-\alpha e^{-i\Omega t}\rangle\right)$$

$$= \frac{1}{2\sqrt{2}\beta}\left(\alpha e^{-i\Omega t} + \alpha^{*}e^{i\Omega t}\right)\langle\alpha e^{-i\Omega t}|\alpha e^{-i\Omega t}\rangle + (\text{sigue})$$