(9)

(c) El estado $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}|\alpha\rangle + iA|-\alpha\rangle$ debe estar normalizado:

$$\langle\psi(0)|\psi(0)\rangle = \left(\tfrac{1}{\sqrt{2}}\langle\alpha| - iA\langle-\alpha|\right)\left(\tfrac{1}{\sqrt{2}}|\alpha\rangle + iA|-\alpha\rangle\right)$$

$$= \frac{1}{2}\underbrace{\langle\alpha|\alpha\rangle}_{=1} + A^2\underbrace{\langle-\alpha|-\alpha\rangle}_{=1} - i\frac{A}{\sqrt{2}}\underbrace{\left(\langle-\alpha|\alpha\rangle - \langle\alpha|-\alpha\rangle\right)}_{2i\,\mathrm{Im}(\langle-\alpha|\alpha\rangle)}$$

$$1 = \frac{1}{2} + A^2 + 2A\,\underbrace{\mathrm{Im}(\langle-\alpha|\alpha\rangle)}$$

$$\mathrm{Im}\left(\exp\left(-2|\alpha|^2\right)\right) = 0.$$

real.

Luego, $A^2 = 1/2 \Rightarrow \boxed{A = \frac{1}{\sqrt{2}}}$

En $t > 0$,

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t}|\psi(0)\rangle = e^{-i\Omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})t}|\psi(0)\rangle$$

$$= \frac{1}{\sqrt{2}}e^{-i\Omega t(\hat{a}^\dagger\hat{a} + 1/2)}|\alpha\rangle + \frac{i}{\sqrt{2}}e^{-i\Omega t(\hat{a}^\dagger\hat{a} + \frac{1}{2})}|-\alpha\rangle$$

$$= \frac{1}{\sqrt{2}}e^{-i\frac{\Omega t}{2}}\left(e^{-i\Omega t\hat{N}}|\alpha\rangle + i\,e^{-i\Omega t\hat{N}}|-\alpha\rangle\right)$$

con $\hat{N} = \hat{a}^\dagger\hat{a}$