Para la desviación cuadrática media, obtenemos: (4)

$$\langle x^2(t)\rangle = \langle \Psi(t)|\hat{x}^2|\Psi(t)\rangle$$

$$= \frac{1}{3}\langle \varphi_0|\hat{x}^2|\varphi_0\rangle + \frac{2}{3}\langle \varphi_1|\hat{x}^2|\varphi_1\rangle$$

$$+ 2\sqrt{\frac{2}{3}}\, \mathrm{Im}\left(e^{\frac{i}{\hbar}(\varepsilon_1-\varepsilon_0)t} \langle \varphi_1|\hat{x}^2|\varphi_0\rangle \right)$$

Usando:

$$\langle \varphi_{n'}|\hat{x}^2|\varphi_n\rangle = \frac{\hbar}{2m\Omega}\left(\sqrt{n(n-1)}\,\delta_{n',n-2} + \sqrt{(n+1)(n+2)}\,\delta_{n',n+2} + (2n+1)\delta_{n'n} \right)$$

se tiene:

$$\langle \varphi_0|\hat{x}^2|\varphi_0\rangle = \frac{\hbar}{2m\Omega}, \quad \langle \varphi_1|\hat{x}^2|\varphi_1\rangle = \frac{3\hbar}{2m\Omega},$$

$$\langle \varphi_1|\hat{x}^2|\varphi_0\rangle = 0$$

luego,

$$\langle x^2(t)\rangle = \frac{1}{3}\frac{\hbar}{2m\Omega} + \frac{2}{3}\cdot\frac{3\hbar}{2m\Omega} = \frac{7}{6}\frac{\hbar}{m\Omega}.$$

$$\Delta x(t) = \sqrt{\langle x^2(t)\rangle - \langle x(t)\rangle^2} = \sqrt{\frac{7}{6}\frac{\hbar}{m\Omega} - \frac{8}{9}\frac{\hbar}{m\Omega}\sin^2(\Omega t)}$$