

$$\begin{aligned}
 \langle \hat{p}_y \rangle_{\psi_{k_y k_z}} &= \underbrace{\int_{-a}^{+a} dx |\psi(x)|^2}_{=1} \cdot \underbrace{\frac{1}{L_z} \int_{-L_z/2}^{L_z/2} dz}_{=1} \cdot \frac{1}{L_y} \int_{-L_y/2}^{L_y/2} e^{-ik_y y} \left(-i\hbar \frac{\partial}{\partial y} \right) e^{ik_y y} dy \\
 &= \frac{1}{L_y} \int_{-L_y/2}^{+L_y/2} dy e^{-ik_y y} (\hbar k_y) e^{ik_y y} = \hbar k_y
 \end{aligned}$$

- El valor de expectación de la componente y de la velocidad:

$$\langle v_y \rangle \equiv \frac{\langle \hat{p}_y \rangle}{m} = \frac{\hbar k_y}{m}$$

Análogamente, se demuestra

$$\langle \hat{p}_z \rangle_{\psi_{k_y k_z}} = \hbar k_z, \quad \langle v_z \rangle = \frac{\hbar k_z}{m}$$