De la relación de Ehrenfest para el momentum y la posición:

$$\frac{d}{dt}\langle \vec{p} \rangle = \frac{1}{i\hbar}\langle [\hat{\vec{p}}, \hat{H}] \rangle \qquad (1)$$

$$\frac{d}{dt}\langle \vec{x} \rangle = \frac{1}{i\hbar}\langle [\hat{\vec{x}}, \hat{H}] \rangle = \frac{\langle \vec{p} \rangle}{m} \qquad (2)$$

notando que las componentes $\hat{p}_y$ y $\hat{p}_z$ conmutan con $\hat{H}$:

$$[\hat{p}_z, \hat{H}] = [\hat{p}_y, \hat{H}] = 0$$

$$[\hat{p}_x, \hat{H}] = \left[\hat{p}_x, \frac{(\hat{p}_y + eB_0 x)^2}{2m} + eE_0 x\right]$$

$$= \left[\hat{p}_x, \frac{\hat{p}_y^2}{2m} + \frac{eB_0}{m}\hat{p}_y x + \frac{(eB_0)^2}{2m}x^2 + eE_0 x\right]$$

$$= \frac{eB_0}{m}\hat{p}_y \underbrace{[\hat{p}_x, x]}_{=-i\hbar} + \frac{(eB_0)^2}{2m}\underbrace{[\hat{p}_x, x^2]}_{=2x(-i\hbar)} + eE_0 \underbrace{[\hat{p}_x, x]}_{=-i\hbar}$$

$$= -i\hbar\left(eB_0 \frac{\hat{p}_y}{m} + \frac{(eB_0)^2}{m}x + eE_0\right)$$