(c) ②

$$\langle x(t)\rangle = \langle \psi(t)|\hat{x}|\psi(t)\rangle$$

$$= \left(\frac{e^{\frac{i}{\hbar}\varepsilon_0 t}}{\sqrt{3}}\langle\varphi_0| - i\sqrt{\frac{2}{3}}\,e^{\frac{i}{\hbar}\varepsilon_1 t}\langle\varphi_1|\right)\hat{x}\left(\frac{e^{-\frac{i}{\hbar}\varepsilon_0 t}}{\sqrt{3}}|\varphi_0\rangle + i\sqrt{\frac{2}{3}}\,e^{-\frac{i}{\hbar}\varepsilon_1 t}|\varphi_1\rangle\right)$$

$$= \frac{1}{3}\langle\varphi_0|\hat{x}|\varphi_0\rangle + \frac{2}{3}\langle\varphi_1|\hat{x}|\varphi_1\rangle$$

$$+ 2\,\frac{\sqrt{2}}{3}\,\mathrm{Im}\left(e^{\frac{i}{\hbar}(\varepsilon_1-\varepsilon_0)t}\langle\varphi_1|\hat{x}|\varphi_0\rangle\right)$$

Usando: $\langle\varphi_{n'}|\hat{x}|\varphi_n\rangle = \sqrt{\frac{\hbar}{2m\Omega}}\left(\sqrt{n}\,\delta_{n',n-1} + \sqrt{n+1}\,\delta_{n',n+1}\right)$

$$\Rightarrow \langle\varphi_0|\hat{x}|\varphi_0\rangle = \langle\varphi_1|\hat{x}|\varphi_1\rangle = 0$$

$$\langle\varphi_1|\hat{x}|\varphi_0\rangle = \sqrt{\frac{\hbar}{2m\Omega}}\cdot\sqrt{2} = \sqrt{\frac{\hbar}{m\Omega}}$$

de donde:

$$\langle x(t)\rangle = 2\,\frac{\sqrt{2}}{3}\sqrt{\frac{\hbar}{m\Omega}}\,\sin\left(\frac{\varepsilon_1-\varepsilon_0}{\hbar}t\right)$$

$$= \frac{2\sqrt{2}}{3}\sqrt{\frac{\hbar}{m\Omega}}\,\sin(\Omega t)$$