

# Formulário

**Fórmula fundamental dos erros**  $\left[ \delta_f \leq M_{x_1} \delta_{x_1} + M_{x_2} \delta_{x_2} + \dots + M_{x_n} \delta_{x_n} \right]$

em que  $\left| \frac{\partial f}{\partial x_i}(\xi) \right| \leq M_{x_i}$  com  $\xi \in [x_1 - \delta_{x_1}, x_1 + \delta_{x_1}] \times \dots \times [x_n - \delta_{x_n}, x_n + \delta_{x_n}]$ .

## Uma equação não linear

Método da Secante	Método de Newton
$x_{k+1} = x_k - \frac{(x_k - x_{k-1})f(x_k)}{f(x_k) - f(x_{k-1})}, k = 2, 3, \dots$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, k = 1, 2, \dots$

## Critério de Paragem:

$$\frac{|x_{k+1} - x_k|}{|x_{k+1}|} \leq \epsilon_1 \text{ e } |f(x_{k+1})| \leq \epsilon_2$$

## Método de Newton para sistemas de equações não lineares

Equação iterativa	Jacobiano	Critério de Paragem
$J(x^{(k)})\Delta_x = -f(x^{(k)})$ $x^{(k+1)} = x^{(k)} + \Delta_x$	$J = \begin{pmatrix} \frac{\partial f_1(x_1, x_2, \dots, x_n)}{\partial x_1} & \dots & \frac{\partial f_1(x_1, x_2, \dots, x_n)}{\partial x_n} \\ \dots & \dots & \dots \\ \frac{\partial f_n(x_1, x_2, \dots, x_n)}{\partial x_1} & \dots & \frac{\partial f_n(x_1, x_2, \dots, x_n)}{\partial x_n} \end{pmatrix}$	$\frac{\ \Delta_x\ }{\ x^{(k+1)}\ } \leq \epsilon_1$ e $\ f(x^{(k+1)})\  \leq \epsilon_2$

## Tabela das diferenças divididas

$$[x_j, x_{j+1}] = \frac{f_j - f_{j+1}}{x_j - x_{j+1}}, \quad j = 0, \dots, n-1 \quad (\text{diferença dividida de ordem 1}) \quad (dd1)$$

$$[x_j, x_{j+1}, x_{j+2}] = \frac{[x_j, x_{j+1}] - [x_{j+1}, x_{j+2}]}{x_j - x_{j+2}}, \quad j = 0, \dots, n-2 \quad (dd2)$$

$$[x_0, x_1, \dots, x_{n-1}, x_n] = \frac{[x_0, x_1, \dots, x_{n-2}, x_{n-1}] - [x_1, x_2, \dots, x_{n-1}, x_n]}{x_0 - x_n} \quad (ddn)$$

$$[x_0, x_1, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$

## Polinómio interpolador de Newton

$$p_n(x) = f_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + \dots + (x - x_0) \dots (x - x_{n-1})[x_0, \dots, x_n]$$

$$\text{Erro de truncatura } R_n(x) \leq |(x - x_0)(x - x_1) \dots (x - x_n) dd_{n+1}|$$

## Expressão do segmento $i$ da *spline* cúbica

$$s_3^i(x) = \frac{M_{i-1}}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{M_i}{6(x_i - x_{i-1})}(x - x_{i-1})^3 + \left[ \frac{f(x_{i-1})}{(x_i - x_{i-1})} - \frac{M_{i-1}(x_i - x_{i-1})}{6} \right] (x_i - x) + \left[ \frac{f(x_i)}{(x_i - x_{i-1})} - \frac{M_i(x_i - x_{i-1})}{6} \right] (x - x_{i-1}) \text{ para } i = 1, 2, \dots, n$$

## Expressão para os pontos interiores (nó $i$ )

$$\frac{(x_i - x_{i-1})M_{i-1} + 2(x_{i+1} - x_{i-1})M_i + (x_{i+1} - x_i)M_{i+1}}{6} = \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)} - \frac{f(x_i) - f(x_{i-1})}{(x_i - x_{i-1})}$$

<i>Spline</i> Natural	<i>Spline</i> Completa
$M_0 = 0$	$2(x_1 - x_0)M_0 + (x_1 - x_0)M_1 = \frac{6}{(x_1 - x_0)}(f(x_1) - f(x_0)) - 6f'(x_0)$
$M_n = 0$	$2(x_n - x_{n-1})M_n + (x_n - x_{n-1})M_{n-1} = 6f'(x_n) - \frac{6}{(x_n - x_{n-1})}(f(x_n) - f(x_{n-1}))$

$$\text{Erro de truncatura } spline \text{ cúbica } \left[ |f(x) - s_3(x)| \leq \frac{5}{384} h^4 M_4 \right] \left[ |f'(x) - s'_3(x)| \leq \frac{1}{24} h^3 M_4 \right] \text{ com}$$

$$\max_{\xi \in [x_0, x_n]} |f^{(iv)}(\xi)| \leq M_4 \quad h = \max_{0 \leq i \leq n-1} (x_{i+1} - x_i)$$

Fórmulas simples Newton-Cotes		
<b>Trapézio</b>	$\int_a^b f(x)dx \approx \frac{(b-a)}{2} [f(a) + f(b)]$	$ET =   - \frac{(b-a)^3}{12} f''(\xi)  , \xi \in [a, b]$
<b>Simpson</b>	$\int_a^b f(x)dx \approx \frac{(b-a)}{6} [f(a) + 4f(\frac{a+b}{2}) + f(b)]$	$ET =   - \frac{(b-a)^5}{2880} f^{(iv)}(\xi)  , \xi \in [a, b]$
$\frac{3}{8}$	$\int_a^b f(x)dx \approx \frac{(b-a)}{8} [f(a) + 3f(\frac{2a+b}{3}) + 3f(\frac{a+2b}{3}) + f(b)]$	$ET =   - \frac{(b-a)^5}{6480} f^{(iv)}(\xi)  , \xi \in [a, b]$
Fórmulas compostas Newton-Cotes		
<b>Trapézio</b>	$\int_a^b f(x)dx \approx \frac{h}{2} [f_0 + 2f_1 + 2f_2 + \dots + 2f_{n-2} + 2f_{n-1} + f_n]$ $ET =   - \frac{h^2}{12} (b-a) f''(\eta)  , \eta \in [a, b]$	
<b>Simpson</b>	$\int_a^b f(x)dx \approx \frac{h}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + \dots + 4f_{n-3} + 2f_{n-2} + 4f_{n-1} + f_n]$ $ET =   - \frac{h^4}{180} (b-a) f^{(iv)}(\eta)  , \eta \in [a, b]$	
$\frac{3}{8}$	$\int_a^b f(x)dx \approx \frac{3h}{8} [f_0 + 3f_1 + 3f_2 + 2f_3 + \dots + 2f_{n-3} + 3f_{n-2} + 3f_{n-1} + f_n]$ $ET =   - \frac{h^4}{80} (b-a) f^{(iv)}(\eta)  , \eta \in [a, b]$	

### Mínimos Quadrados (amostra $m$ )

#### Polinómios ortogonais

$$p_n(x) = c_0 P_0(x) + c_1 P_1(x) + \dots + c_n P_n(x)$$

$$P_{i+1} = A_i(x - B_i)P_i(x) - C_i P_{i-1}(x), \quad P_0(x) = 1, \quad P_{-1}(x) = 0$$

$$A_i = 1 \quad B_i = \frac{\sum_{j=1}^m x_j P_i(x_j) P_i(x_j)}{\sum_{j=1}^m P_i(x_j) P_i(x_j)} \quad C_0 = 0 \text{ e } C_i = \frac{\sum_{j=1}^m P_i(x_j) P_i(x_j)}{\sum_{j=1}^m P_{i-1}(x_j) P_{i-1}(x_j)}$$

#### Coefficientes do modelo polinomial

$$c_i = \frac{\sum_{j=1}^m f_j P_i(x_j)}{\sum_{j=1}^m P_i(x_j)^2}, \quad i = 0, \dots, n$$

#### Modelo não polinomial linear

$$M(x; c_1, c_2, \dots, c_n) = c_1 \phi_1(x) + c_2 \phi_2(x) + \dots + c_n \phi_n(x)$$

$$\begin{pmatrix} \sum_{j=1}^m \phi_1^2(x_j) & \dots & \sum_{j=1}^m \phi_1(x_j) \phi_n(x_j) \\ \vdots & \ddots & \vdots \\ \sum_{j=1}^m \phi_n(x_j) \phi_1(x_j) & \dots & \sum_{j=1}^m \phi_n^2(x_j) \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^m f_j \phi_1(x_j) \\ \vdots \\ \sum_{j=1}^m f_j \phi_n(x_j) \end{pmatrix}$$

$$\text{Resíduo} \quad \sum_{j=1}^m (f_j - M(x_j))^2$$