Composição

$$\begin{tikzcd} A & B \arrow[l, "f"'] & C \arrow[l, "g"'] \end{tikzcd} \Rightarrow y = f(g\,x)$$

$$\begin{tikzcd} A & C \arrow[l, "f \cdot g"'] \end{tikzcd} \Rightarrow y = (f \cdot g)\,x$$

Produto-X

Cartesian product

$$\begin{tikzcd} B & & C \\ & A \arrow[ul, "f"] \arrow[ur, "g"'] & \end{tikzcd} \quad (f\,a, g\,a)$$

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Split

$$\begin{tikzcd} B & B \times C & C \\ & A \arrow[ul, "f"] \arrow[u, "\langle f,g \rangle"'] \arrow[ur, "g"'] & \end{tikzcd}$$

$$\langle f,g \rangle\, a = (f\,a, g\,a)$$

Product

$$\begin{tikzcd} B & B \times C \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & C \\ & A \arrow[ul, "f"] \arrow[u, "\langle f,g \rangle"'] \arrow[ur, "g"'] & \end{tikzcd}$$

$$\begin{tikzcd} D & D \times E \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & E \\ B \arrow[u, "f"] & B \times C \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] \arrow[u, "f \times g"] \arrow[ul] \arrow[ur] & C \arrow[u, "g"'] \end{tikzcd}$$

$$\pi_1 \cdot \langle f,g \rangle = f \qquad \pi_2 \cdot \langle f,g \rangle = g$$

$$f \times g = \langle f \cdot \pi_1, g \cdot \pi_2 \rangle$$

X - fusion

$$\begin{tikzcd} B & B \times C \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & C \\ & A \arrow[ul, "f"] \arrow[u, "\langle f,g \rangle"'] \arrow[ur, "g"'] & \\ & D \arrow[u, "h"] \arrow[uul, "f \cdot h"] \arrow[uur, "g \cdot h"'] & \end{tikzcd}$$

$$\langle f,g \rangle \cdot h = \langle f \cdot h, g \cdot h \rangle$$

× - Absorption

$$\begin{tikzcd}
D & D \times E \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & E \\
B \arrow[u, "h"] & B \times C \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] \arrow[u, "h \times k"] & C \arrow[u, "k"'] \\
& A \arrow[ul, "f"] \arrow[u, "\langle f, g \rangle"] \arrow[ur, "g"'] &
\end{tikzcd}$$

$$\begin{tikzcd}
D & D \times E \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & E \\
B \arrow[u, "h"] & & C \arrow[u, "k"'] \\
& A \arrow[ul, "f"] \arrow[ur, "g"'] \arrow[uul, "h \cdot f"] \arrow[uu, "\langle h \cdot f, k \cdot g \rangle"', "(h \times k) \cdot \langle f, g \rangle"] \arrow[uur, "k \cdot g"'] &
\end{tikzcd}$$

$$(h \times k) \cdot \langle f, g \rangle = \langle h \cdot f, k \cdot g \rangle$$

Natural-π_1 , Natural-π_2

$$\begin{tikzcd}
D & D \times E \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & E \\
B \arrow[u, "h"] & B \times C \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] \arrow[u, "h \times k"] & C \arrow[u, "k"']
\end{tikzcd}$$

$$\pi_1 \cdot (h \times k) = h \cdot \pi_1$$
$$\pi_2 \cdot (k \times k) = k \cdot \pi_2$$

Functor - id - ×

$$id \times id = id$$

× - Functor

$$(f \times h) \cdot (g \times k) = (f \cdot g) \times (h \cdot k)$$

× - Reflexion

$$\langle \pi_1, \pi_2 \rangle = id$$

× - Eq

$$\langle i, j \rangle = \langle f, g \rangle \iff \begin{cases} i = f \\ j = g \end{cases}$$

$$\begin{tikzcd}
A & A \times B \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & B \\
& A \times B \arrow[ul, "\pi_1"] \arrow[u, "id", "\langle \pi_1, \pi_2 \rangle"'] \arrow[ur, "\pi_2"'] &
\end{tikzcd}$$

× - cancellation

$$\begin{tikzcd}
B & B \times C \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & C \\
& A \arrow[ul, "f"] \arrow[u, "\langle f, g \rangle"'] \arrow[ur, "g"'] &
\end{tikzcd}$$

$$\pi_1 \cdot \langle f, g \rangle = f \qquad \pi_2 \cdot \langle f, g \rangle = g$$

× - Universal

tal que

$$k = \langle f, g \rangle \iff \begin{cases} \pi_1 \cdot k = f \\ \pi_2 \cdot k = g \end{cases}$$

existe

solução única

Swap

$$\begin{tikzcd} A \times B \arrow[rr, bend left, "\langle \pi_2, \pi_1 \rangle"] \arrow[rr, phantom, "\cong"] & & B \times A \arrow[ll, bend left, "\langle \pi_2, \pi_1 \rangle"] \end{tikzcd}$$

$swap = \langle \pi_2, \pi_1 \rangle$

$swap \cdot swap = id$

$$\begin{tikzcd} (A \times B) \times C \arrow[rr, bend left, "\langle \pi_1 \cdot \pi_1, \pi_2 \times id \rangle"] \arrow[rr, phantom, "\cong"] & & A \times (B \times C) \arrow[ll, bend left, "\langle id \times \pi_1, \pi_2 \cdot \pi_2 \rangle"] \\ & D \arrow[ul, "\langle \langle f, g \rangle, h \rangle"] \arrow[ur, "\langle f, \langle g, h \rangle \rangle"'] & \end{tikzcd}$$

assocr

assocl

$assocr \cdot assocl = id$

$\boxed{\text{Coproduto} - +}$

$A + B = \{ i_1 a \mid a \in A \} \cup \{ i_2 b \mid b \in B \}$

$i_1 a = (1, a)$
$i_2 b = (2, b)$

$$\begin{tikzcd} A \arrow[r, "i_1"] & A + B \arrow[d, "{[f, g]}"] & B \arrow[l, "i_2"'] \\ & C \arrow[ul, "f"] \arrow[ur, "g"'] & \end{tikzcd}$$

+ - Universal

$$k = [f, g] \iff \begin{cases} k \cdot i_1 = f \\ k \cdot i_2 = g \end{cases}$$

Sum

$$\begin{tikzcd} D \arrow[r, "i_1"] \arrow[dr] & D + E \arrow[d, "f + g"] & E \arrow[l, "i_2"'] \arrow[dl] \\ B \arrow[u, "f"] \arrow[r, "i_1"'] & B + C & C \arrow[l, "i_2"] \arrow[u, "g"'] \end{tikzcd}$$

$f + g = [i_1 \cdot f, i_2 \cdot g]$

+ - Absorption

$[h, k] \cdot (f + g) = [h \cdot f, k \cdot g]$

+ - Fusion

$f \cdot [h, k] = [f \cdot h, f \cdot k]$

+ - Reflexion

$[i_1, i_2] = id$

+ - Equality

$$[h, k] = [f, g] \Leftrightarrow \begin{cases} h = f \\ k = g \end{cases}$$

× - Functor

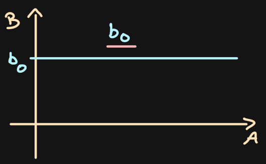
$$(h + k) \cdot (f + g) = h \cdot f + k \cdot g$$

× - Functor - id

$$id + id = id$$

$\boxed{\text{Constant functions}}$

$$\begin{cases} \underline{b_0} : A \longrightarrow B \\ \underline{b_0}\ a = b_0 \end{cases}$$



Propriedades

$$\underline{b} \cdot f = \underline{b}$$

$$g \cdot \underline{b} = \underline{g\,b}$$

$$\begin{tikzcd}
 & C \arrow[dl, "\underline{b}"'] \arrow[d, "f"] \\
B \arrow[d, "g"'] & A \arrow[l, "\underline{b}"'] \arrow[dl, "\underline{gb}"] \\
D &
\end{tikzcd}$$

Data duplication

$$dup = \langle id, id \rangle$$

Data joins

$$join = [id, id]$$

if - then - else

"Point - free"

$$\begin{tikzcd}
2 \times A \arrow[rr, bend left, "\alpha^\circ"] \arrow[rr, phantom, "\cong"] & & A + A \arrow[ll, bend left, "\alpha"]
\end{tikzcd}$$

$$\alpha \cdot \alpha^\circ = id$$
$$\alpha^\circ \cdot \alpha = id$$

$\rightarrow \alpha = [\langle T, id \rangle, \langle F, id \rangle]$

$$\begin{tikzcd}
 & A \arrow[d, "\langle p, id \rangle"] & \\
 & 2 \times A \arrow[d, "\alpha^\circ"] & \\
A \arrow[r, "i_1"] \arrow[d, "f"'] & A + A \arrow[d, "f + g"] & A \arrow[l, "i_2"'] \arrow[d, "g"] \\
B \arrow[r, "i_1"'] & B + B \arrow[d, "join"] & B \arrow[l, "i_2"] \\
 & B &
\end{tikzcd}$$

McCarthy Conditional

$$p \rightarrow f, g = [f, g] \cdot \underbrace{\alpha^\circ \cdot \langle p, id \rangle}_{p?}$$

$$\begin{tikzcd}
A \arrow[r, "\langle p, id \rangle"] \arrow[rr, bend right, "p?"'] & 2 \times A \arrow[r, "\alpha^\circ"] & A + A
\end{tikzcd}$$

$$\underbrace{(p \rightarrow f, g)\ a}_{p\,!\,a} = \begin{cases} p\,a & \Rightarrow f\,a \\ \neg(p\,a) & \Rightarrow g\,a \end{cases}$$

Conditional → 1st fusion law

$$h \cdot (p \rightarrow f, g) = p \rightarrow (h \cdot f), (h \cdot g)$$

Conditional → 2nd fusion law

$$(p \rightarrow f, g) \cdot h = (p \cdot h) \rightarrow (f \cdot h), (g \cdot h)$$

Isomorphisms

$$A\times(B\times C) \cong (A\times B)\times C$$
$$A\times B \cong B\times A$$

$$A+(B+C) \cong (A+B)+C$$
$$A+B \cong B+A$$

$$A\times A \cong 2\times A$$

$$\begin{tikzcd} A\times(B+C) \arrow[r, bend left, "\text{distributivity}"] & A\times B + A\times C \arrow[l, bend left, "\text{indistributivity}"] \end{tikzcd}$$

$$\begin{tikzcd} A+(B\times C) \arrow[r, bend left, "\text{distributivity}"] & A\times B + A\times C \arrow[l, bend left, "\text{indistributivity}"] \end{tikzcd}$$

$$= [\langle f, g\rangle, \langle h, k\rangle] = [\langle \pi_1, i_1\cdot\pi_2\rangle, \langle \pi_1, i_2\cdot\pi_2\rangle] = [id\times i_1, id\times i_2]$$

Exchange law

$$\langle [f,g],[h,k]\rangle = [\langle f,h\rangle,\langle g,k\rangle]$$

$A \xleftarrow{f} A\times B$ — $A \xleftarrow{h} A\times C$

$B+C \xleftarrow{g} A\times B$ — $B+C \xleftarrow{k} A\times C$

$A \xleftarrow{\pi_1} A\times B$ — $A \xleftarrow{\pi_1} A\times C$

$B+C \xleftarrow{i_1\cdot\pi_2} A\times B$ — $B+C \xleftarrow{i_1\cdot\pi_2} A\times C$

Type 1

Funções deste tipo são sempre constantes.

$A \xrightarrow{!} 1$ — $1 \xrightarrow{\underline{a}} A$

$! = \underline{()}$ — $\underline{a}\,() = a$

Type 0

$0 = \{\}$; $\forall a \to \neg(a\in 0)$

$\Rightarrow$ Impossível $f: A \to 0$ para qualquer $A \neq 0$

$\hookrightarrow A\times 0 = \{(a,b) \mid a\in A \wedge b\in 0\}$
$A\times 0 = \{(a,b) \mid a\in A \wedge F\}$
$A\times 0 = \{\}$

Type 1+A

$$\begin{tikzcd} 1 \arrow[r, "i_1"] \arrow[dr, "\underline{b_0}"'] & 1+A \arrow[d, "{[\underline{b_0}, f]}"] & A \arrow[l, "i_2"'] \arrow[dl, "f"] \\ & B & \end{tikzcd}$$

Maybe

$$\begin{tikzcd} 1 \arrow[r, "i_1"] \arrow[dr, "Nothing"'] & 1+A \arrow[d, "{[Nothing, Just]}"] & A \arrow[l, "i_2"'] \arrow[dl, "Just"] \\ & Maybe\ A & \end{tikzcd}$$

$$\begin{tikzcd} Maybe\ A \arrow[rr, bend left, "out = in^{\circ}"] \arrow[rr, phantom, "\cong"] & & 1+A \arrow[ll, bend left, "in = {[Nothing, Just]}"] \end{tikzcd}$$

$\Rightarrow out \cdot in = id$

data Maybe a = Nothing | Just a

Either

$$\begin{tikzcd} A \arrow[r, "i_1"] \arrow[dr, "Left"'] & A+B \arrow[d, "{[Left, Right]}"] & B \arrow[l, "i_2"'] \arrow[dl, "Right"] \\ & Either\ A\ B & \end{tikzcd}$$

$$\begin{tikzcd} Either\ A\ B \arrow[rr, bend left, "out = in^{\circ}"] \arrow[rr, phantom, "\cong"] & & A+B \arrow[ll, bend left, "in = {[Left, Right]}"] \end{tikzcd}$$

Natural-id

$f \cdot id = id \cdot f$

Natural-π_1

$(f) \cdot \pi_1 = \pi_1 \cdot (f \times g)$

Natural-a°

$(f+f) \cdot a^{\circ} \doteq a^{\circ} \cdot (id \times f)$

Curry | uncurry

curry :: ((a,b) -> c) -> a -> b -> c

curry f a b = f(a,b)

curry f = $\bar{f}$

$f = g \Leftrightarrow \bar{f} = \bar{g}$

$k = h \Leftrightarrow \hat{k} = \hat{h}$

uncurry :: (a -> b -> c) -> (a,b) -> c

uncurry g(a,b) = g a b

uncurry f = $\hat{f}$

$$\begin{tikzcd} A \times B \to C \arrow[rr, bend left, "curry"] \arrow[rr, phantom, "\cong"] & & A \to (B \to C) \arrow[ll, bend left, "uncurry"] \end{tikzcd}$$

$C^{A \times B}$

$A \to C^B$

$(C^B)^A$

type $B^A = \{ f \mid f : A \to B \}$

↳ função que faz o map do A no B

isomorphisms

$a^1 = a$

$$\begin{tikzcd} A^1 \arrow[r, bend left, "g"] \arrow[r, phantom, "\cong"] & A \arrow[l, bend left, "f"] \end{tikzcd}$$

$$\begin{cases} f\ a = a \\ g\ a = a \end{cases}$$

$1^a = 1$

$$\begin{tikzcd} 1^A \arrow[r, bend left, "g"] \arrow[r, phantom, "\cong"] & 1 \arrow[l, bend left, "f"] \end{tikzcd}$$

$$\begin{cases} f\ () = ! \\ g = ! \end{cases}$$

$a^0 = 1$

$$\begin{tikzcd} A^0 \arrow[r, bend left, "g"] \arrow[r, phantom, "\cong"] & 1 \arrow[l, bend left, "f"] \end{tikzcd}$$

$$\begin{cases} f\ () = \perp \\ g = ! \end{cases}$$

Universal property

$$k = \overline{f} \iff ap \cdot (k \times id)\, k = f$$

Reflexion

$$id = \overline{f} \iff ap = f$$

Fusion

$$\overline{f} \cdot g = \overline{f \cdot (g \times id)}$$

Absorption

$$g^B = \overline{g \cdot ap}$$

$$\exp g = \overline{g \cdot ap}$$

$$\overline{g \cdot f} = \exp g \cdot \overline{f}$$

$f \cdot g$			sequential	$A \times B$	records ("structs")
$\langle f, g \rangle$	$f \times g$		parallel	$A + B$	variant records ("unions")
$[f, g]$	$f + g$	$p \rightarrow f, g$	alternation	$1 + A$	"pointers"
$\overline{f}$	$\exp f$		higher order	B^A	"arrays", "streams"

$1 + A \times X^2$ polynomial structures