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THE ORIGIN AND DEVELOPMENT OF FACTOR ANALYSIS

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Factor analysis was developed largely by psychologists for use in psychology, but it is a statistical method that has been applied to other fields. Mr Vincent's historical exposition should therefore have wide interest.

The Aim of Factor Analysis

The term 'factor analysis' is employed to describe a variety of analytical techniques. The question as to its origin depends upon what techniques are considered to be covered by this term. In my opinion a good definition of factor analysis is: Techniques, developed mainly by psychologists for dealing with their problems, which, by analysing the inter-correlations between sets of measurements, attempt to identify the causes that are operating to produce the variance within each set, and to evaluate the contribution due to each cause.

If a number of children of the same age were given, say, a test of English composition, part of the variance of the scores would be due to general intelligence, part to the differences in verbal ability of the children, part to the imperfections of the test (that is to chance), and part to other causes. Suppose g^2 were the proportion of the total variance due to intelligence, v^2 the proportion due to differences in verbal ability, e^2 the proportion due to chance, and s^2 the proportion due to all other causes. Then if these variances are uncorrelated,

$$g^2 + v^2 + s^2 + e^2 = 1$$

The influences operating to produce the variances g^2, v^2, and s^2 are called 'factors,' and the quantities g, v, and s are known as 'factor loadings.' Any operation for calculating them is a form of factor analysis. Such a calculation would, of course, be quite impossible if the scores of one test only were available, but it can be made from the correlations between the scores of a number of tests given to the same group of children. Actually, no method of factor analysis will determine the s and e loadings. The e loadings can be calculated from the 'reliabilities' of the tests, since e^2 is the difference between the reliability and unity. The reliability of a test is calculated by correlating the scores of parallel versions of it given to the same group of persons at the same time, or from two sets of scores obtained from the same test given to the same group on two occasions with a suitable time interval. The s loadings when they are required are found by difference.

As most methods of factor analysis do not require any advance knowledge of what factors are present, factor analysis is very frequently employed when it is required to find out how many factors actually are present in a group of tests or other measurements, and if possible to identify them, rather than to determine the numerical values of the factor loadings.

If the inter-correlations between the scores of a large group of tests for school subjects were subjected to a factor analysis, it is more than likely that more factors than g and v would be found. This would mean that in each test the proportion of the total variance s^2, ascribed above to other causes, has been broken up into component parts. Not all of it; in practice s^2 never disappears entirely. A psychologist would endeavour to identify the causes of the other factors; he might ascribe them to influences such as 'the child's reaction towards the teacher and the school' and 'home influence and tradition.' It is a maxim in interpreting the results of a factor analysis that there must be a reason for each factor that appears, and unless a satisfactory explanation can be found for all or nearly all the factors, the factorisation is considered to have failed.

According to the definition given above, all methods of factor analysis are an outgrowth of Galton's concept of the correlation coefficient. Galton showed that if the variances of two sets of measurements can each be regarded as made up of two parts, a part A^2 due to a cause operating in the case of both sets of measurements, and a part X^2 due to one cause in one set of measurements and a different cause in the other set, the correlation coefficient between the two sets of measurements is

$$\sqrt{\left(\frac{A_1^2}{A_1^2 + X_1^2}\right)} \times \sqrt{\left(\frac{A_2^2}{A_2^2 + X_2^2}\right)}$$

that is to say, the geometric mean of two variances. (The term 'variance' came into use long after Galton's time, but it is convenient to use it here as it is the term that is most familiar today.) If a_1^2 and a_2^2 represent the *proportions* of the variances due to the common cause, then

$$a_1^2 = A_1^2/(A_1^2 + X_1^2) = A_1^2/\sigma_1^2$$

and

$$a_2^2 = A_2^2/(A_2^2 + X_2^2) = A_2^2/\sigma_2^2$$

where σ_1^2 and σ_2^2 are the variances of the two sets of measurements. Thus the correlation between the two sets of measurements is

$$\sqrt{(A_1^2A_2^2/\sigma_1^2\sigma_2^2)} = a_1a_2$$

That is to say, the correlation coefficient is the product of what are today called the factor loadings. This principle can be extended to the case when there is more than one common cause. If a_1^2 and a_2^2 are the proportions of the total variance of the two sets of measurements due to one common cause, b_1^2 and b_2^2 are the proportions due to another common cause, and c_1^2 and c_2^2 are the proportions due to a third common cause; then, if the three causes operate independently, the correlation coefficient will be $a_1a_2 + b_1b_2 + c_1c_2$. For two tests similar to that used as an example above the correlation coefficient would be $g_1g_2 + v_1v_2$.

The simplest case of factor analysis is when there is only one common cause and, therefore, only one factor is present. If the inter-correlations of four sets of measurements have been calculated they can be arranged in the form of Table I. Since there is only one common cause, each

TABLE I

—	r_{12}	r_{13}	r_{14}
r_{21}	—	r_{23}	r_{24}
r_{31}	r_{32}	—	r_{34}
r_{41}	r_{42}	r_{43}	—

TABLE II

(m_1^2)	m_1m_2	m_1m_3	m_1m_4
m_2m_1	(m_2^2)	m_2m_3	m_2m_4
m_3m_1	m_3m_2	(m_3^2)	m_3m_4
m_4m_1	m_4m_2	m_4m_3	(m_4^2)

of the correlation coefficients can be expressed as the product of two factor loadings, as in Table II. The quantities in brackets, m_1^2, m_2^2, m_3^2, and m_4^2, which are called the 'communalities,' are not known. If the correlation coefficients were free from errors three sets of measurements would be enough to determine their factor loadings, since

$$m_1 = m_1m_2 \, . \, m_1m_3/m_2m_3 = r_{12} \, . \, r_{13}/r_{23}, \text{ etc.} \qquad \ldots .(1)$$

Correlation coefficients do, however, contain errors, and the error of a factor loading obtained in this way would be rather large. The problem is how to minimise the errors by averaging all the possible estimates of each factor. It would not involve much computation to work out each estimate separately when only four tests are involved, but when there are a dozen or more tests the amount of work would be very great indeed. A number of devices have been used to obtain an average value for each factor loading with a reasonable amount of work.

If c_1, c_2, c_3, and c_4 are the sums of the columns of Table II and S is the sum of the whole table,

$$c_1 = m_1^2 + m_1m_2 + m_1m_3 + m_1m_4$$
$$= m_1(m_1 + m_2 + m_3 + m_4)$$

and
$$c_2 = m_2(m_1 + m_2 + m_3 + m_4) \text{ etc.}$$

From this it follows that

$$\frac{c_1}{\sqrt{S}} = \frac{c_1}{\sqrt{(c_1 + c_2 + c_3 + c_4)}} = \frac{m_1(m_1 + m_2 + m_3 + m_4)}{\sqrt{(m_1 + m_2 + m_3 + m_4)^2}} = m_1$$

If the communalities were known they could be entered in the blank spaces in a table such as Table I and the averaged estimates of the factor loadings obtained from this table by the formula

$$m_1 = c_1/\sqrt{S} \qquad \ldots .(2)$$

The device most commonly used today is to make a guess at the

H*

communalities and to use this formula. It is not the most accurate method but it is accurate enough in most cases.

The Beginning of Factor Analysis

The first factor analyses ever to be made were all of this simple one-factor type. The reason is historical: they were a direct outgrowth of Galton's conception, which envisaged only the case when only one common cause was influencing the variances of two sets of measurements. Also, Galton's idea fitted in very well with the ideas of the man who began what has since developed into factor analysis.

In his autobiography[1] Spearman tells how, about 1901, he became interested in Galton's work and saw how he could use it to test his own ideas about the nature of intelligence. In 1902 he collected suitable data: test scores and class marks of the pupils at two schools in England. In 1904 he published two papers in the *American Journal of Psychology*; both of them were landmarks. The first[2] was his *credo*, in which he explained and defended his statistical methods. Here for the first time a distinction was made between the statistical treatment appropriate to measurements that can be made accurately, such as the heights and weights and limb measurements of the orthodox statisticians, and the statistical treatment required for measurements that must always contain a large amount of error, such as examination marks, scores at psychological tests, and subjective estimates of temperamental characteristics—'statisticoids' he called them. This paper is the book of Genesis of much that is called psychometrics today. Spearman's second paper[3] was the first publication describing experimental work using the method of dealing with 'statisticoids' that has since developed into factor analysis.

This second paper not only uses the word 'factor' for the first time with its present-day meaning, but it contains a list of factor loadings that Spearman had calculated for eleven measurements. He shows, as an example, a table of the inter-correlations of six sets of measurements arranged as Table I, from which some of his factor loadings were calculated. His first attempt at factor analysis was surprisingly good. His table has been factorised by modern methods, and the six factor loadings obtained agree very well with those in his list.

It is unfortunate that in this 1904 paper Spearman gives no indication at all of the method he used. Apparently he was interested only in obtaining numerical data to support his theory of the nature of intelligence and he did not regard his computational methods as being of any importance. It is certain, however, that in the early days he had devised more than one method of factorising. Among the Spearman documents in the library of the National Institute of Industrial Psychology there is a letter written in 1909 in which he states that there are various formulae which can be used, depending upon the degree of exactness aimed at, but that the results are practically the same in any case.

Only one of these methods has been preserved, and for that we have to thank Sir Cyril Burt. In a paper in 1909[4] he has factorised a table of correlations by using a formula obtained from Spearman, and he has given this formula in a footnote. It is

$$m_s = \frac{a_s}{\sqrt{(2\Sigma - a_s)}} \cdot \sqrt{\frac{n-2}{n-1-na/2\Sigma}} \qquad \ldots .(3)$$

Here m_s is the factor loading of the measurement or test s, a_s is the sum of a column of a table such as Table I (where the communalities are omitted), 2Σ is the sum of all the correlation coefficients in the table; and n is the number of tests or measurements.

This formula is of interest as it is the first factorisation formula to be published. The second term is a correction factor which tends to unity as the number of tests becomes large. The first term of this formula can be used alone without much loss of accuracy. It will be noted that it has a close resemblance to equation (2) and was doubtless obtained by a similar process of reasoning. Actually this early formula is remarkably accurate; in fact it is more accurate than some of the methods most frequently used today. As an illustration of its accuracy an artificial table of the 'correlation coefficients' of six imaginary tests having factor loadings of 8, 7, 6, 5, 4, and 3 has been factorised by this early formula, and by a modern method using guessed communalities. An artificial table is necessary to get 'correlation coefficients' free from error. The factor loadings obtained are as in Table III.

TABLE III

True loadings	0·8	0·7	0·6	0·5	0·4	0·3
Loadings obtained with Spearman's formula (without correction factor)	0·762	0·684	0·596	0·511	0·417	0·318
Loadings obtained with Spearman's formula (with correction factor)	0·798	0·705	0·603	0·508	0·406	0·304
Loadings obtained with a modern method (centroid)	0·756	0·703	0·620	0·532	0·447	0·337

Sir Cyril Burt, again, was responsible for the next factorisation formula to be published. In a memorandum published in 1917[5] he gives in a footnote a formula identical with formula (2) of this article but does not explain how it has been obtained. This footnote suggests that the factor loadings of his tables were obtained not from this formula but from a modification of it. There are several possible modifications of the formula—the first term of Spearman's formula, formula (3) of this article, is one of them. No doubt at this time ideas were rather fluid, and everyone had his own. I know of no positive evidence of when the device of using guessed communalities first came into use, but as formula (2) cannot be used without modification except with guessed communalities it is likely that this device was in use at this time. The fact that Sir Cyril Burt had quoted this formula suggests that he had been using it, if not for the factor loadings of that memorandum then for some previous work.

No new factorisation formula was published till 1927[6], when Spearman published a particularly ingenious formula which will factorise an error-free single-factor table of inter-correlations with perfect accuracy. It is

$$m_s = \sqrt{(a_s^2 - A_s)}/\sqrt{(2\Sigma - 2a_s)}$$

where A_s is the sum of the squares of the correlation coefficients in column s and the other symbols have the same meaning as in his earlier formula. Probably no one uses it today, as it is rather laborious to work out and the increased accuracy is lost with tables of actual correlation coefficients that are not free from error.

The Beginning of Multiple Factor Analysis

In the early days Spearman and his colleagues were interested in one factor only, that associated with intelligence and which they called the general factor *g*. That other factors could be present in their tables of correlations was known from the beginning. In the work that formed the basis of Spearman's second paper in 1904[3] he had found 'a community purely specific' between Latin translation, Latin grammar, French prose, and French dictation. He even made an attempt to evaluate this 'community.' In the early days such communities, or group factors as they came to be called later, were a nuisance to be avoided while the purpose of factorisation was to provide data for research on *g*.

About 1915 there was a change of outlook and psychologists engaged in factor analysis began to take an interest in group factors for their own sake. In Sir Cyril Burt's memoranda previously mentioned[5] considerable space is devoted to the probability of other factors being present in his measurements and an attempt was made to evaluate them. In 1916 Sir Godfrey Thomson[7] published a paper showing that it was theoretically possible that *g* was the resultant of a complex of group factors. In 1916 the *British Journal of Psychology*[8] published the doctoral thesis of Miss N. Carey, one of Spearman's students. In it she presented a technique for testing a table of correlations for the presence of factors other than *g* and for identifying which tests or measurements have loadings, though not for working out the actual loadings. Her method does, however, contain the germ of the idea that resulted in one of the first techniques of multiple factor analysis, which in its final form became the so-called 'hollow staircase' method.

Dr Carey's idea was to divide the tests or measurements into groups, all the tests in each group being suspected of containing some particular factor in addition to the general factor *g*. Then a table like Table I was made up in which the tests of each group were placed together. If there are three such groups of tests the table can be divided into nine divisions. The factor loadings to be expected in such a table are as shown in Table IV.

TABLE IV

—	$g_2g_1+a_2a_1$	$g_3g_1+a_3a_1$	g_4g_1	g_5g_1	g_6g_1	g_7g_1	g_8g_1	g_9g_1
$g_1g_2+a_1a_2$	—	$g_3g_2+a_3a_2$	g_4g_2	g_5g_2	g_6g_2	g_7g_2	g_8g_2	g_9g_2
$g_1g_3+a_1a_3$	$g_2g_3+a_2a_3$	—	g_4g_3	g_5g_3	g_6g_3	g_7g_3	g_8g_3	g_9g_3
g_1g_4	g_2g_4	g_3g_4	—	$g_5g_4+b_5b_4$	$g_6g_4+b_6b_4$	g_7g_4	g_8g_4	g_9g_4
g_1g_5	g_2g_5	g_3g_5	$g_4g_5+b_4b_5$	—	$g_6g_5+b_6b_5$	g_7g_5	g_8g_5	g_9g_5
g_1g_6	g_2g_6	g_3g_6	$g_4g_6+b_4b_6$	$g_5g_6+b_5b_6$	—	g_7g_6	g_8g_6	g_9g_6
g_1g_7	g_2g_7	g_3g_7	g_4g_7	g_5g_7	g_6g_7	—	$g_8g_7+c_8c_7$	g_9g_7 c_9c_7
g_1g_8	g_2g_8	g_3g_8	g_4g_8	g_5g_8	g_6g_8	$g_7g_8+c_7c_8$	—	$g_9g_8+c_9c_8$
g_1g_9	g_2g_9	g_3g_9	g_4g_9	g_5g_9	g_6g_9	$g_7g_9+c_7c_9$	$g_8g_9+c_8c_9$	—

Dr Carey suggested that the presence of the group factors *a*, *b*, and *c* in Table IV is indicated by the numerical pattern of the correlation coefficients of the table and by the average of the correlation coefficients in each of the divisions. Methods for calculating the loadings of such group factors came much later. However, it was from a table such as this that Dr W. Stephenson, a colleague and former pupil of Spearman, proved the existence of *v*, the first of the factors appearing in mental tests to be accepted by psychologists as having an importance comparable with Spearman's *g*.

It will be obvious that the loadings of the general factor for all the tests in Table IV could be calculated from those divisions that contain only the general factor. For instance,

$$g_1^2 = g_1g_4 \cdot g_1g_7/g_4g_7 = r_{14}r_{17}/r_{47}$$

and a number of other estimates of g_1 could be made, which could be averaged. Since the correlations in the other divisions are of the form $r_{12} = g_1g_2 + a_1a_2$, if the g loadings have been calculated terms like a_1a_2 can be found by difference. These can be assembled into a new table and the loadings of the a factor found by a single factor method. This looks simple, but considerable refinement was necessary before a practicable method was devised. The technique known as the 'bi-factor' and the 'hollow staircase' method devised by Dr K. J. Holzinger,[9] which is still in use today, works on this principle.

The next, and up to date the most important, phase in the development of factor analysis began in the early nineteen-thirties and was due mainly to the work of Dr L. L. Thurstone.[10] He not only introduced an entirely new factorising technique but also a new conception of the nature of factors. His technique, known as the 'centroid' method, is the one most widely used today.

In the first stage a table like Table I is factorised by a single factor method as though it contains one factor only. From the factor loadings all the products, like m_1m_2, m_2m_3, etc. of Table I, are calculated. The original table is then treated as though it contains two factors, that is as though

$$r_{12} = a_1a_2 + b_1b_2 \quad \text{and} \quad r_{23} = a_2a_3 + b_2b_3, \text{ etc.}$$

The terms b_1b_2, b_2b_3, etc., are found by difference and are assembled into a new table. The new table is factorised by a single factor method and the operation repeated, and so on. In this way each of the correlation coefficients is expanded into an infinite convergent series so that

$$r_{12} = a_1a_2 \pm b_1b_2 \pm c_1c_2 \pm d_1d_2 \pm \ldots$$

and similarly for the other correlation coefficients. If, say, three factors are present, the fourth term of the series will fall to zero or, in practice, to a value judged not to be significant. Unlike similar quantities arising in other techniques, the quantities a, b, c, d, etc., need not have any factual interpretation. Indeed, since assumptions have been made that are justifiable only if the original correlation coefficients are regarded as mere numbers, there is no reason why a, b, c, d, etc., should have any meaning, although in some psychological work the first of them is often a rough approximation to the g loading.

If, say, three factors are present, the correlation coefficients are each resolved into three terms, so that

$$\begin{aligned} r_{12} &= a_1a_2 \pm b_1b_2 \pm c_1c_2 \\ r_{13} &= a_1a_3 \pm b_1b_3 \pm c_1c_3 \\ r_{23} &= a_2a_3 \pm b_2b_3 \pm c_2c_3 \end{aligned}$$

and so on.

If there are n tests in the table there are $\frac{1}{2}n(n - 1)$ such equations. The whole set of equations, however, is indeterminate; there are an infinite number of solutions. The values for the a's, b's, and c's obtained by Thurstone's method are only one possible solution. The great importance of Dr Thurston's contribution to factor analysis is his device by which any one solution can be transformed into *any* possible solution involving the same number of factors. A meaningless set of factor loadings can be transformed into a set of loadings for which there is an acceptable factual interpretation. Factor loadings are vector quantities, that is to say, they have direction (or a quality analogous to direction) as well as magnitude. Forces are a common example of vector quantities, and Dr Thurstone's original device was a graphical method very similar to the parallelogram of forces. Many more refined graphical and non-graphical methods have been developed from his original idea.

It is Dr Thurstone's device for changing from a meaningless set of factor loadings to a set of loadings that can have a factual interpretation that has led to a new conception of the nature of factors. It has shown that it is always possible that any particular set of tests or measurements can be factorised in more than one way, each set of loadings having an acceptable and even a perfectly obvious factual interpretation. Dr Thurstone's own illustration is one of the best. He took a number of cylinders of varying shape and size and made a number of measurements, such as the length of the greatest diagonal, the area of the ends, and the area of the curved surfaces. He factorised these sets of measurements and showed that there were two quite different sets of factor loadings, each having a perfectly obvious factual interpretation. One set of factor loadings corresponded very closely with *length* and *diameter*, the other with *bulk* and *shape*, shape varying from disc-like to rod-like.

Present-day Views

During the past twenty years there has been a gradual change of outlook upon the nature of factors, which has been due to the work of others besides Dr Thurstone. There is no space to mention everyone here, but Sir Godfrey Thomson's contribution cannot be omitted. He was interested in factor analysis from the very beginning, and in 1916 he published a paper[7] which showed that Spearman's conception of the nature of *g* was not the only one possible. He did not show that Spearman was necessarily wrong, but he broadened the outlook of psychologists using factor analysis and started a new train of ideas which he has continued to develop. The ideas that he has embodied in his Sampling Theory[11] have given those who inclined to Spearman's views much food for thought. Probably his contributions, more than those of anyone, have made psychologists appreciate that factor loadings are statistical conceptions, like means and standard deviations, and are subject to the same limitations. A value for the mean height of a man is of very little use without a definition of the population to which it applies. In the same way the *g* loading, say, of a particular test can be misleading without a definition of the *population of tests* to which it applies. If a non-verbal intelligence test is factorised in a battery of other non-verbal intelligence tests the *g* loading obtained is likely, quite apart from experimental errors, to be slightly different from the loading that would be obtained if it were factorised in a battery containing both verbal and non-verbal tests. If the battery were more varied—if, for instance, it contained tests of attainment in those scholastic subjects known to demand intelligence—the *g* loading obtained would be still more different.

Until about the time of the appearance of Dr Thurstone's work factor analysis had been used only by a small band of enthusiasts. During the past twenty years many psychologists and some statisticians in all parts of the world have become interested in factor analysis and a great many new techniques have appeared. They all depend upon the principles that have already been outlined in this article, and consist mainly in refinements of methods. Reference to all these contributions to factor analysis is clearly impossible in so short an account as this.

Most treatises on factor anlysis, however, contain some reference to Hotelling's 'principal components,' and this article would not be complete if it did not mention the work of Dr H. Hotelling.[12] But first it should be said that Karl Pearson had to some extent anticipated Hotelling's work thirty years earlier.[13] Pearson used the same mathematical device as Hotelling, but he used it for a different purpose. Pearson's object was to provide an improved technique for prediction, such as making the best estimate of height from arm and leg measurements; he does not appear to have had any thought of analysing his variables. There is no reason to suppose that Dr Hotelling was aware of this previous work.

Principal components have some resemblance to factors but the underlying idea is different. Like factors the loadings of the principal components are calculated from the inter-correlations of the tests, but unlike factors there is no question of choosing an acceptable solution from a number of possible solutions; a resolution into principal components is an exact mathematical operation and the result is unique. To obtain this mathematically unique solution certain assumptions have to be made, which in practice mean that either unities or reliabilities are entered in the table of inter-correlations in place of the communalities. Principal components have the undeniable advantages of a unique solution and the elimination of all guesswork, but they have the disadvantage that about half the loadings are negative and that there is nothing corresponding to the specific factor of factor analysis.

From the mathematical and statistical point of view principal components are far more satisfactory than factors, but from the psychologist's point of view they are of little use; it is only in a few rare instances that they have found an application in psychological work. The obstacles are the inevitable negative loadings and the absence of anything corresponding to a specific factor. Neither factors nor components are of any value to psychologists unless they have a factual interpretation, and it is often quite impossible to attach any meaning to a negative loading. Without the possibility of a specific factor it is necessary to assume that any cause operating to produce a variance in any one of the tests, or other measurements, must also influence the variances of all the others in the battery. In most psychological work this is usually an absurd assumption. If, as an illustration, there is a test of arithmetic in the battery, one cause operating to affect the variance of the scores is the ability to add correctly; it would be absurd to assume that it also affects the scores of any test that does not contain arithmetical questions.

It has been stated that factor analysis is a subject of controversy between the mathematical statisticians and the psychologists. This is not quite the whole truth. There is only a controversy between those who wish to be controversial. Factor analysis is not a purely statistical technique; there is always a certain amount of guesswork in it which no fair-minded person would attempt to deny; but factor analysis is chiefly a research tool, and there is of necessity a lot of guesswork in all research, from astronomy to nuclear physics. Factor analysis is certainly a very treacherous tool in inexperienced hands. Dr Eysenck[14] in a previous article in this journal has described some of the pitfalls that beset the unwary, but he has also told how leading psychologists, working independently and in very different fields, have obtained remarkably similar results. Undoubtedly there have been many controversies about deductions made from the results of work employing factor analysis, which have focused attention on the weaknesses of the method and have made some people feel very dubious about it. If some mathematical statisticians have suggested that psychologists would be well advised to use the more trustworthy orthodox statistical procedures in place of factor analysis whenever possible, that is not being controversial; it is offering well-meant advice. Unfortunately, however, it is advice that often cannot be taken. There are problems, some of them are mentioned in Dr Eysenck's article, in which factor analysis is the only tool that the psychologists can use.

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