# EET 3086C – Circuit Analysis

Summer 2024

Experiment # 2

***Transient Response of First-Order RC and RL Circuits***

Performed By:

**Pedro Cabrera**

Submitted to:

**Professor Ali Notash**

**Department of Electrical & Computer Engineering Technology (ECET – B.S.)**
**School of Engineering, Technology, and Advanced Manufacturing (ETAM)**

**Valencia College**

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# Objective

The objective of this experiment is to design and analyze the transient response of first-order RC and RL circuits. By observing the step and natural responses of these circuits, the experiment aims to enhance understanding of how capacitors and inductors behave in transient conditions. This involves deriving theoretical equations, conducting simulations, and comparing the results with practical measurements. The goal is to validate the theoretical models and understand the discrepancies between simulated and observed data, thus gaining deeper insights into the transient behavior of RC and RL circuits.

# List of Equipment/Parts/Components

1. **Breadboard**
   - Model: Elenco 9830
2. **Resistors (all 5% tolerance)**
   - 1.0 kΩ (x2)
   - 2.0 kΩ
   - 100 Ω (x3)
3. **Capacitor/ Inductor**
   - 10 nF
   - 1 mH
4. **Digital Programmable Multimeter & LCR Meter**
   - Model: HM8012/HM8018
5. **Oscilloscope**
   - Model: Rigol DS1102D
6. **Function Generator**
   - Model: Hameg HM8030-6
7. **Computer with PSpice Software**

# Theoretical Background Research

**Capacitors and Inductors**

**Capacitors:**
A capacitor is a passive element consisting of two parallel conducting plates separated by a dielectric material. It stores energy in the electric field created between the plates. Capacitance $C$ is defined as the ability to store charge per unit voltage. The relationship between the current through a capacitor and the voltage across it is given by:

$$I_C(t) = C\left(\frac{dV_{C(t)}}{dt}\right)$$

indicating that the current is proportional to the rate of change of voltage across the capacitor.[1]

**Inductors:**
An inductor is a passive element designed to store energy in a magnetic field. It consists of a coil of conducting wire and its core property is inductance $L$, which describes the ability to store magnetic energy. The fundamental relationship for an inductor is:

$$V_L(t) = L\left(\frac{dI_{L(t)}}{dt}\right)$$

where the voltage across the inductor is proportional to the rate of change of current through it. [1]

**Response to a DC Source:**
When a capacitor is directly connected to a DC source, it initially allows current to flow as it charges up to the source voltage. Once fully charged, the capacitor behaves as an open circuit, blocking any further DC current. Conversely, an inductor connected to a DC source initially resists changes in current,

behaving as an open circuit at the moment of connection. Over time, the inductor allows continuous current to flow as if it were a short circuit, with the voltage across it dropping to zero [2].

**RC and RL Circuits**

**RC Circuits:**
In an RC circuit, the capacitor and resistor influence the circuit's transient response. The time constant $\tau$ for an RC circuit is given by:

$$\tau = R_{eq} \cdot C$$

where $R_{eq}$ is the equivalent resistance and $C$ is the capacitance. This time constant determines how quickly the capacitor charges and discharges, which is critical in understanding the step response of the circuit [2].

The general formula for the voltage across a capacitor in an RC circuit during charging is:

$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]e^{-\frac{t}{\tau}}$$

where:

- $V_C(\infty)$ is the final steady-state voltage.
- $V_C(0^+)$ is the initial voltage at $t = 0$ after the switch is closed.
- $\tau = R_{eq}C$ is the time constant [3].

**RL Circuits:**
An RL circuit includes a resistor and an inductor. The time constant $\tau$ for an RL circuit is:

$$\tau = \frac{L}{R_{eq}}$$

where $L$ is the inductance and $R_{eq}$ is the equivalent resistance. This time constant determines the rate at which the inductor reaches its steady-state response after a change in voltage or current [2].

The corresponding formula for the current through an inductor in an RL circuit is:

$$I_L(t) = I(\infty) + [I(0^+) - I(\infty)]e^{-\frac{t}{\tau}}$$

where:

- $I_L(\infty)$ is the final steady-state current.
- $I_L(0^+)$ is the initial current at $t = 0$ after the switch is closed.
- $\tau = \frac{L}{R_{eq}}$ is the time constant [3].

# Procedure

The procedure for this experiment is adopted from the following reference [4]:

Masood Ejaz, Lab Experiments Manual for EET 3086C – Circuit Analysis, Experiment 2, Valencia College ECET Department, pp. 5-7

# Results & Observations

### Component Values and Measurements

In this experiment, the theoretical, simulation, and hands-on bench work measurements were performed using specific components whose values were crucial for achieving accurate results. The numerical values of the components were used in both the simulation and the theoretical calculations. However, the actual values of all components were recorded during the bench measurements. This approach was taken to account for potential sources of error and deviations in the results.

Recording the actual values of the components is essential for several reasons. It allows for accurate error analysis by providing a precise understanding of any discrepancies between theoretical, simulation, and experimental results. This helps identify whether deviations are due to component tolerances or other factors. Ensuring that components are within their specified tolerance ranges improves the reliability of the experiment and highlights any variations due to manufacturing processes. Overall, this meticulous approach ensures that all components used are within their tolerance limits and provides a more comprehensive analysis of the experimental results.

**Components for RC Circuit:**


*Figure 1 - Measured value of capacitor* $C_1$ ($10.02\ nF$).


*Figure 2 - Measured value of resistor* $R_1$ ( $989.7\ \Omega$).


*Figure 3 - Measured value of resistor* $R_2$($1997\ \Omega$).


*Figure 4 - Measured value of resistor* $R_3$($2003\ \Omega$).

**Note:** The numerical values of the components were used in both the simulation and the calculations. The actual values of all components were recorded during the bench measurements to account for potential sources of error and deviations in the results.

**Table 1**

| Component | Nominal Value | Measured Value | Percent Error (%) |
|---|---|---|---|
| $C_1$ | $10\ nF$ | $10.02\ nF$ | 0.02 % |
| $R_1$ | $1k\Omega$ | 989.7 Ω | 1.03 % |
| $R_2$ | $2\ k\Omega$ | 1997 Ω | 0.15 % |
| $R_3$ | $2\ k\Omega$ | 2003 Ω | 0.15 % |

**Sample Percent Error (%) Calculation:**

$$\% \; Error = \left( \frac{|Measured \; Value - \; Nominal \; Value|}{Nominal \; Value} \right) \times 100$$

$$\% \; Error \text{ for } R_1 = \left| \frac{989.7 - 1000}{1000} \right| \times 100\%$$

$$\% \; Error \; = \left| \frac{-10.3}{1000} \right| \times 100\% = 1.03\%$$

**Components for RC Circuit:**


*Figure 5 - Measured value of Inductor $L_1$* ($1.002 \; mH$).


*Figure 7 - Measured value of resistor* $R_2$ ( $100.1 \; \Omega$).


*Figure 6 - Measured value of resistor* $R_1$ ( $98.80 \; \Omega$).


*Figure 8 - Measured value of resistor* $R_3$ ( $99.96 \; \Omega$).

**Table 2**

| Component | Nominal Value | Measured Value | Percent Error (%) |
|---|---|---|---|
| $L_1$ | $1 \; mH$ | $1.002 \; mH$ | 0.2 % |
| $R_1$ | $100 \; \Omega$ | $98.8 \; \Omega$ | 1.2 % |
| $R_2$ | $100 \; \Omega$ | $100.1 \; \Omega$ | 0.1 % |
| $R_3$ | $100 \; \Omega$ | $99.96 \; \Omega$ | 0.04 % |

**RC Circuit**

**Theoretical Calculations for Step and Natural Response**

For the RC circuit, the theoretical calculations involve determining the step response and the natural response of the circuit. These calculations are crucial for understanding the behavior of the circuit when subjected to a sudden change in input voltage (step response) and when left to naturally respond without any external input (natural response).

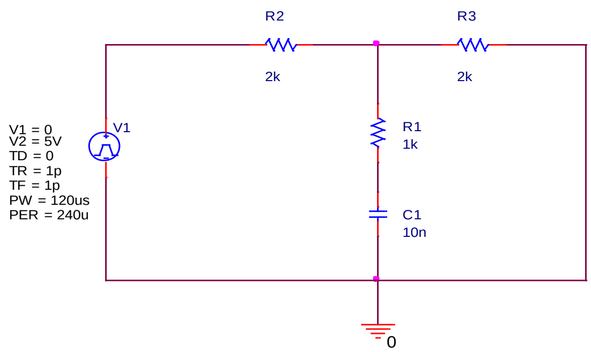


*Figure 9 - RC Circuit Schematic.*

**Component Values & Time Constant:**

The values of the components used in the RC circuit are critical in determining the time constant τ, which affects both the step and natural responses of the circuit. The capacitor value is given as $C = 10nF$. The equivalent resistance $R_{eq}$ is calculated as follows:

$$R_{eq} = 1k\Omega + \left(\frac{2k\Omega \parallel 2k\Omega}{2}\right)$$

$$R_{eq} = 1k\Omega + 1k\Omega$$

$$R_{eq} = 2k\Omega$$

The time constant $\tau$ is then calculated using the formula $\tau = R_{eq} \cdot C$:

$$\tau = 2k\Omega \cdot 10nF$$

$$\tau = 20\mu s$$

Step Response Initial Voltage:

$$V_C(0^+) = 0\ V$$

**As the capacitor is initially uncharged. **

Step Response Final Voltage:

Using Voltage Divider:

$$V_C(\infty^+) = \frac{5V \cdot 2k\Omega}{4k\Omega}$$

$$V_C(\infty) = 2.5\ V$$

Step Response Equation:

$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]e^{-\frac{t}{\tau}}$$

$$V_C(t) = 2.5 + [0 - 2.5]e^{-\frac{t}{20\mu s}}$$

$$V_C(t) = 2.5 - 2.5e^{-\frac{t}{20\mu s}}$$

$$Or$$

$$V_C(t) = 2.5\left(1 - e^{-\frac{t}{20\mu s}}\right)V$$

The step response of the RC circuit describes how the voltage across the capacitor $V_C(t)$ changes when a sudden voltage is applied.

**Natural Response:**

Natural Response Initial Voltage:

Using Voltage Divider:

$$V_C(0^+) = \frac{5V \cdot 2k\Omega}{4k\Omega} = 2.5V$$

Natural Response Final Voltage:

The final voltage across the capacitor after a long time, when fully discharged, is:

$$V_C(\infty) = 0V$$

Natural Response Equation:

$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]e^{-\frac{t}{\tau}}$$

$$V_C(t) = 0 + [2.5 - 0]e^{-\frac{t}{20\mu s}}$$

$$V_C(t) = 2.5e^{-\frac{t}{20\mu s}}\ V$$

**Theoretical Calculations at Specific Times**

The theoretical voltage values for the capacitor at different times ($0, \tau, 2\tau,$ and $3\tau$) are calculated as follows:

**Step Response at different $t$:**

$$V_C(t) = 2.5\left(1 - e^{-\frac{t}{20\mu s}}\right)$$

At $t = 0$:

$$V_C(0) = 2.5\left(1 - e^{-\frac{0}{20\mu s}}\right) = 2.5(1 - 1) = 0\ V$$

At $t = \tau$:

$$V_C(20\mu s) = 2.5\left(1 - e^{-\frac{20\mu s}{20\mu s}}\right) = 2.5(1 - e^{-1}) = 2.5(1 - 0.3679) = 2.5 \times 0.6321 = 1.58\ V$$

At $t = 2\tau$:

$$V_C(40\mu s) = 2.5\left(1 - e^{-\frac{40\mu s}{20\mu s}}\right) = 2.5(1 - e^{-2}) = 2.5(1 - 0.1353) = 2.5 \times 0.8647 = 2.162\ V$$

At $t = 3\tau$:

$$V_C(60\mu s) = 2.5\left(1 - e^{-\frac{60\mu s}{20\mu s}}\right) = 2.5(1 - e^{-3}) = 2.5(1 - 0.0498) = 2.5 \times 0.9502 = 2.376\ V$$

**Natural Response at different *t*:**

At $t = 0$:

$$V_C(0) = 2.5e^{-\frac{0}{20\mu s}} = 2.5 \times 1 = 2.5\ V$$

At $t = \tau$:

$$V_C(20\mu s) = 2.5e^{-\frac{20\mu s}{20\mu s}} = 2.5e^{-1} = 2.5 \times 0.3679 = 0.92\ V$$

At $t = 2\tau$:

$$V_C(40\mu s) = 2.5e^{-\frac{40\mu s}{20\mu s}} = 2.5e^{-2} = 2.5 \times 0.1353 = 0.34\ V$$

At $t = 3\tau$:

$$V_C(60\mu s) = 2.5e^{-\frac{60\mu s}{20\mu s}} = 2.5e^{-3} = 2.5 \times 0.0498 = 0.125\ V$$

**RC Circuit**

**PSpice Simulation**

To validate the theoretical calculations, a PSpice simulation was performed for the RC circuit. The simulation involved setting up the RC circuit in PSpice and conducting a transient analysis to observe the voltage across the capacitor over time. (Figure 10)

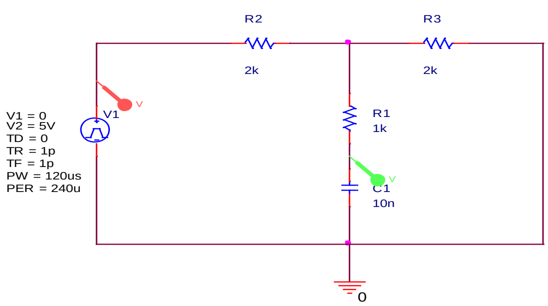


*Figure 10 – PSpice RC Circuit simulation with probe Placements.*

**Note:** Since the components we are measuring across are directly connected to ground we do not require the use of differential probes.

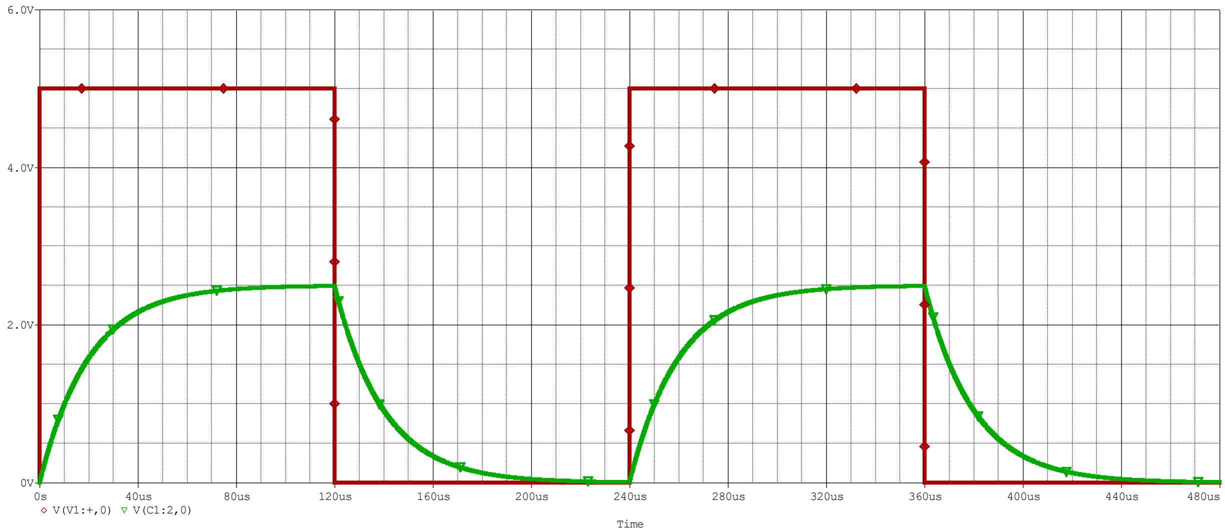


*Figure 11 - PSpice Transient Analysis Displaying two periods.*

The **transient analysis** was configured to run for a sufficient duration to capture the complete charging and discharging behavior of the capacitor. The voltage across the capacitor $V_C(t)$ was measured at various time intervals.

Using **cursor measurements**, the voltage values at the specific times of interest ($0, \tau, 2\tau$, and $3\tau$) were recorded.

**Step Response:**

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 0.000 | 0.000 | 0.000 |
| CURSOR 1,2 | V(C1:2,0) | 0.000 | 0.000 | 0.000 |
| | V(V1:+,0) | 0.000 | 0.000 | 0.000 |

*Figure 12 - Measuring Voltage Across $C_1$ at $t = 0$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 40.000u | 0.000 | 40.000u |
| CURSOR 1,2 | V(C1:2,0) | 2.1625 | 0.000 | 2.1625 |
| | V(V1:+,0) | 5.0000 | 0.000 | 5.0000 |

*Figure 14 - Measuring Voltage Across $C_1$ at $t = 2\tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 19.961u | 0.000 | 19.961u |
| CURSOR 1,2 | V(C1:2,0) | 1.5785 | 0.000 | 1.5785 |
| | V(V1:+,0) | 5.0000 | 0.000 | 5.0000 |

*Figure 13 - Voltage Across $C_1$ at $t = \tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 60.136u | 0.000 | 60.136u |
| CURSOR 1,2 | V(C1:2,0) | 2.3764 | 0.000 | 2.3764 |
| | V(V1:+,0) | 5.0000 | 0.000 | 5.0000 |

*Figure 15 -Voltage Across $C_1$ at $t = 3\tau$*

**Natural Response:**

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 120.000u | 0.000 | 120.000u |
| CURSOR 1,2 | V(C1:2) | 2.4938 | 0.000 | 2.4938 |
| | V(V1:+) | 4.9949 | 0.000 | 4.9949 |

*Figure 16 - Voltage Across $C_1$ at $t = 0$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 139.991u | 0.000 | 139.991u |
| CURSOR 1,2 | V(C1:2) | 917.850m | 0.000 | 917.850m |
| | V(V1:+) | 0.000 | 0.000 | 0.000 |

*Figure 17 - Voltage Across $C_1$ at $t = \tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 160.000u | 0.000 | 160.000u |
| CURSOR 1,2 | V(C1:2) | 337.500m | 0.000 | 337.500m |
| | V(V1:+) | 0.000 | 0.000 | 0.000 |

*Figure 18 - Voltage Across $C_1$ at $t = 2\tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 180.000u | 0.000 | 180.000u |
| CURSOR 1,2 | V(C1:2) | 124.084m | 0.000 | 124.084m |
| | V(V1:+) | 0.000 | 0.000 | 0.000 |

*Figure 19 - Voltage Across $C_1$ at $t = 3\tau$*

**RC Circuit Bench Measurements**

To validate the theoretical calculations and PSpice simulation, the RC circuit was constructed on a physical breadboard, and measurements were taken using a function generator and an oscilloscope.

The function generator was set up to produce a square wave with an amplitude of 5 V and an offset function to ensure the lower voltage was 0 V and the upper voltage was 5 V. Despite this setup, the function generator output was observed to fluctuate approximately $\pm 0.5\ V$, which introduced some instability in the input signal. This fluctuation was monitored and verified using the oscilloscope to ensure that the input signal was within the acceptable range for the lab (Figure 20).

**Calculations for the Frequency of the Function Generator**

The frequency of the function generator was calculated based on the pulse width and period of the square wave. The pulse width was given as $6\tau$ [4], meaning that the full period of the square wave was $12\tau$. Given that $\tau = 20\mu s$, the full period $T$ can be calculated as follows:

$$T = 12\tau = 12 \times 20\mu\text{s} = 240\mu\text{s}$$

The frequency $f$ of the function generator is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{240\mu\text{s}} = \frac{1}{240 \times 10^{-6}\text{s}} = 4.167\text{kHz}$$

Thus, the function generator frequency was determined to be approximately 4.167 kHz (Figure 21).

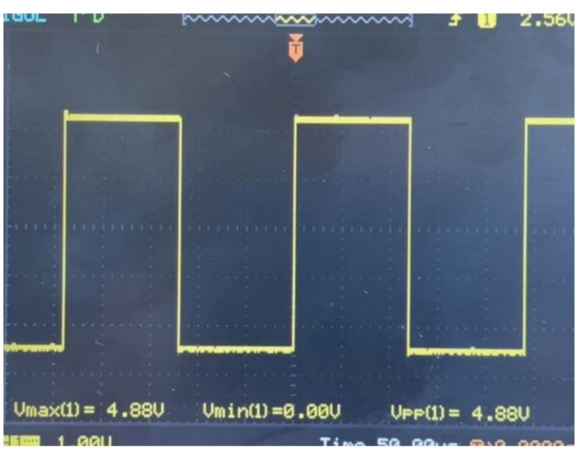


*Figure 20 - Function Generator Waveform on Oscilloscope*

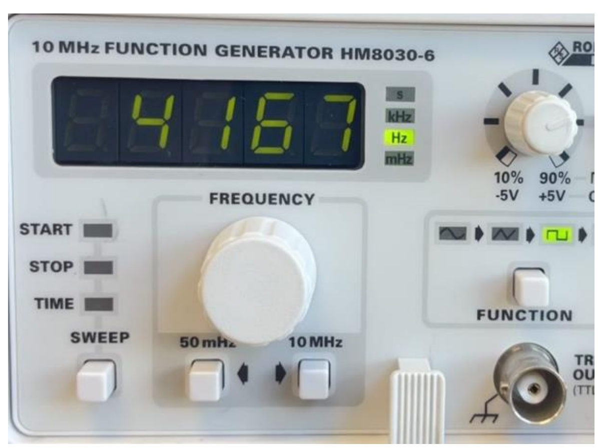


*Figure 21 - Function Generator set to 4167 Hz*

The **circuit setup on the breadboard** involved connecting all components (resistors and capacitors) as per the RC circuit configuration. This physical setup was essential for performing hands-on measurements and validating the theoretical and simulation results.

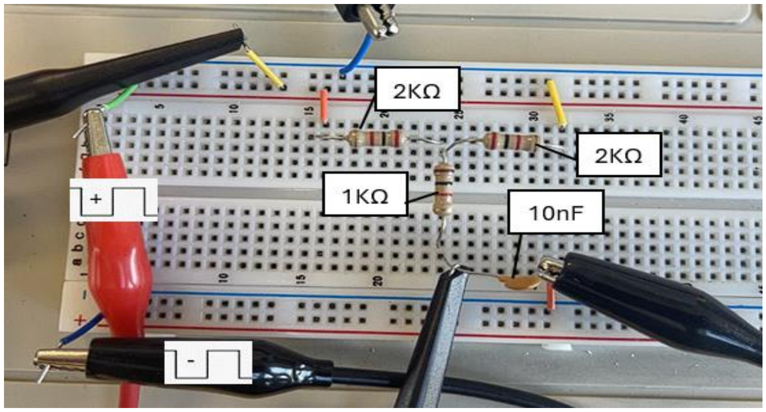


*Figure 22 - RC Circuit on Breadboard with Probe connections.*

Following this, measurements were taken at different times ($0, \tau, 2\tau$, and $3\tau$) using the oscilloscope cursors to record the voltage across the capacitor. The First Four Measurements are for the Step response followed by the Natural Response.

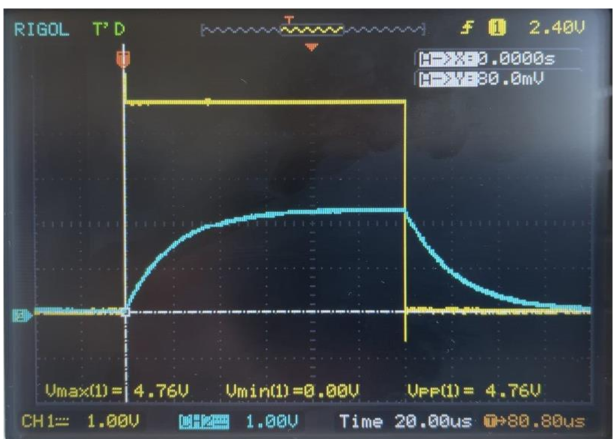


*Figure 23 - Voltage Across $C_1$ at $t = 0$.*

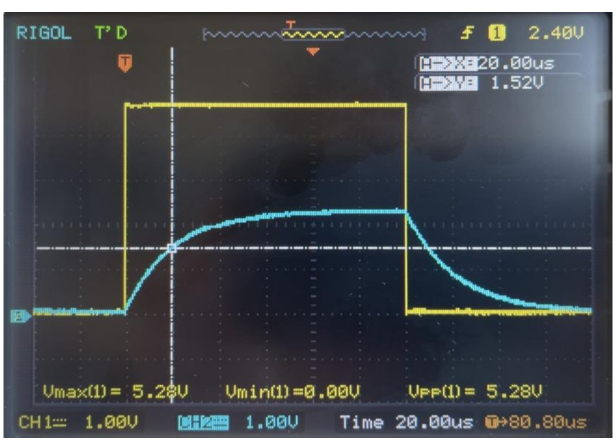


*Figure 24 - Voltage Across $C_1$ at $t = \tau$.*

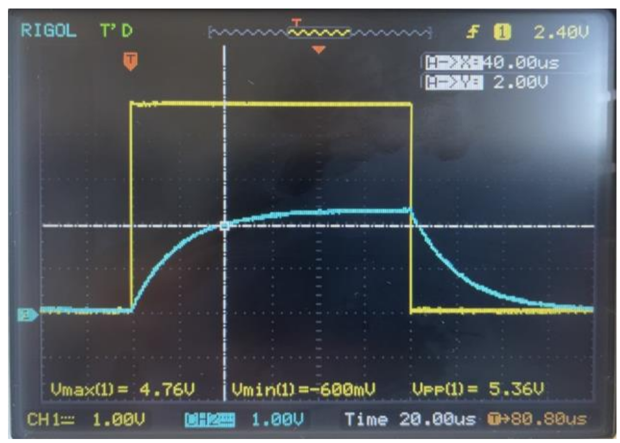


*Figure 25 - Voltage Across $C_1$ at $t = 2\tau$.*

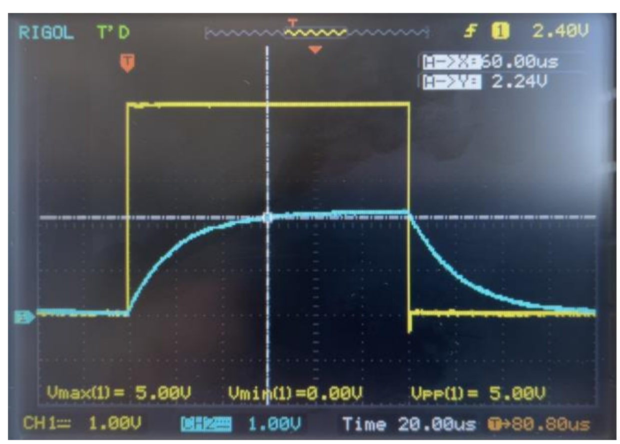


*Figure 26 - Voltage Across $C_1$ at $t = 3\tau$*

The Following four images represent the voltage at $C_1$ during the natural response:

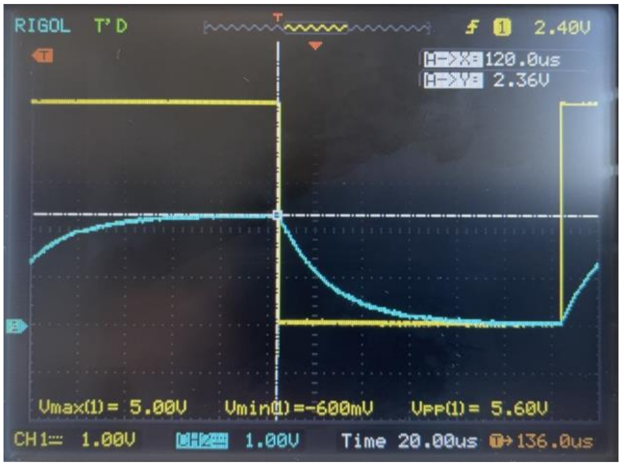


*Figure 27 - Voltage Across $C_1$ at $t = 0$.*

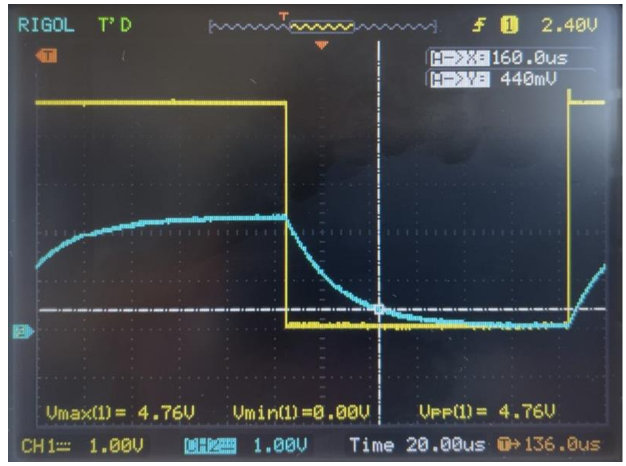


*Figure 29 - Voltage Across $C_1$ at $t = 2\tau$.*

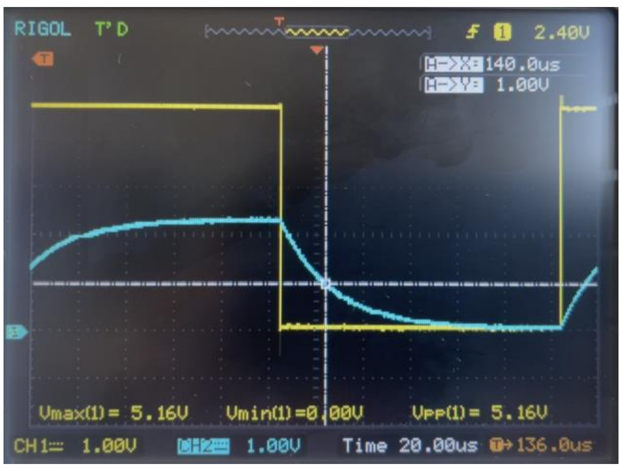


*Figure 28 - Voltage Across $C_1$ at $t = \tau$.*

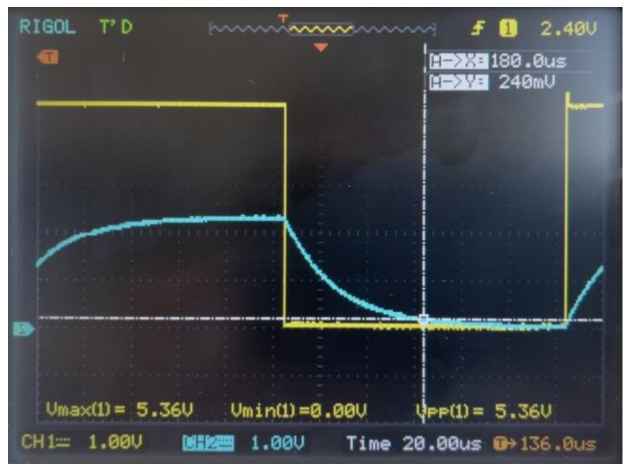


*Figure 30 - Voltage Across $C_1$ at $t = 3\tau$*

The following table presents the voltage values obtained from the theoretical calculations, PSpice simulations, and bench measurements for the RC circuit. This comprehensive overview highlights the consistency and any deviations observed across the different methods.

Please refer to the table below for a detailed comparison of the step and natural responses at the specified times (Table 3):

**Table 3**

| | $V_C(t)$ [step response] | | | | $V_C(t)$ [natural response] | | | |
|---|---|---|---|---|---|---|---|---|
| *Time* | *Theory* | *Simulation* | *Lab* | *Error %* | *Theory* | *Simulation* | *Lab* | *Error %* |
| $t = 0$ | $0\ V$ | $0\ V$ | $0.08\ V$ | $-$ | $2.5\ V$ | $2.5\ V$ | $2.36\ V$ | $5.6\ \%$ |
| $t = \tau$ | $1.58\ V$ | $1.58\ V$ | $1.52\ V$ | $3.8\ \%$ | $0.92\ V$ | $0.92\ V$ | $1\ V$ | $8.7\ \%$ |
| $t = 2\tau$ | $2.162\ V$ | $2.162\ V$ | $2\ V$ | $7.5\ \%$ | $0.34\ V$ | $0.337\ V$ | $.44\ V$ | $29.4\ \%$ |
| $t = 3\tau$ | $2.376\ V$ | $2.376\ V$ | $2.24\ V$ | $5.7\ \%$ | $0.125\ V$ | $0.124\ V$ | $0.24\ V$ | $92\ \%$ |

**Analysis of Percent Error**

The table above presents the percent error between the theoretical and bench measurements for both the step and natural responses of the RC circuit. Notably, as the voltage values become smaller, the percent error tends to become much greater. This is particularly evident in the natural response measurements at $t = 2\tau$ and $t = 3\tau$, where the percent errors are 29.4% and 92%, respectively. This increased error can be attributed to the sensitivity of the measurements at lower voltages, where even small absolute differences between theoretical and measured values result in high percent errors. Additionally, noise and fluctuations in the measurement equipment, such as the function generator and oscilloscope, can have a more pronounced impact on smaller voltage values, further contributing to the higher percent errors observed in these cases.

**RL Circuit**

**Theoretical Calculations for Step and Natural Response**

For the RL circuit, the theoretical calculations involve determining the step response and the natural response of the circuit. These calculations are crucial for understanding the behavior of the circuit when subjected to a sudden change in input voltage (step response) and when left to naturally respond without any external input (natural response).

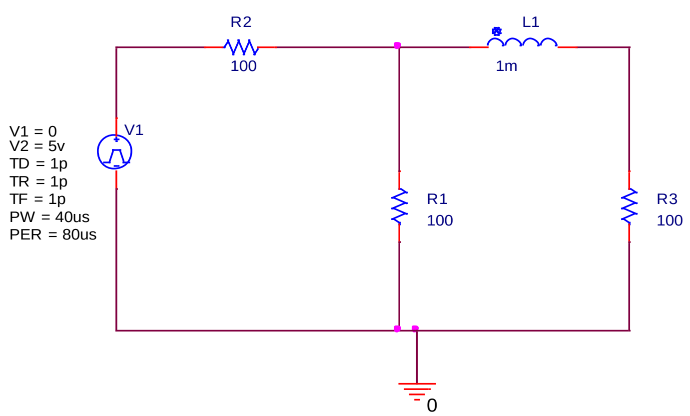


*Figure 31 - RL Circuit Schematic PSpice.*

**Component Values & Time Constant:**

The values of the components used in the RL circuit are critical in determining the time constant $\tau$, which affects both the step and natural responses of the circuit. The inductor value is given as $L = 1\ mH$ . The equivalent resistance $R_{eq}$ is calculated as follows:

$$R_{eq} = (R_1 \parallel R_2) + R_3$$

$$R_{eq} = (100 \parallel 100) + 100$$

$$R_{eq} = 50 + 100 = 150$$

The time constant $\tau$ is then calculated using the formula $\tau = \frac{L}{R_{eq}}$:

$$\tau = \frac{1mH}{150\Omega}$$

$$\tau = 6.67\mu s$$

Total Current:

Using Ohms Law compressing the circuit to a single resistor network:

$$I_T = \frac{5V}{150\Omega} = 33.33 \text{ mA}$$

Current Through $L_1$ and $R_3$:

This was calculated through a current divider using the following equation:

$$I_{R3} = I_T\left(\frac{R_2}{R_3 + R_2}\right)$$

$$I_{R3} = 33.33 \text{ mA} \times \left(\frac{100\Omega}{200\Omega}\right) = 33.33 \text{ mA} \times 0.5 = 16.67 \text{ mA}$$

Step Response Current:

Initial Current:

$$I(0^+) = 0 \text{ A}$$

**As the Inductor is initially uncharged. **

Steady-state current:

$$I(\infty) = 16.67 \text{ mA}$$

Step Response current Equation:

$$I_L(t) = I(\infty) + [I(0^+) - I(\infty)]e^{-\frac{t}{\tau}}$$

$$I_L(t) = 16.67 + [0 - 16.67]e^{-\frac{t}{6.67\mu s}}$$

$$I_L(t) = 16.67 - 16.67e^{-\frac{t}{6.67\mu s}}\text{mA}$$

Step Response Voltage:

$$V_L(t) = L\left(\frac{dI(t)}{dt}\right)$$

The derivative of the current must first be found:

$$\frac{dI_L(t)}{dt} = -\frac{1}{6.67\mu} * \left(-16.67e^{-\frac{t}{6.67\mu s}}\right) mA/s$$

$$\frac{dI_L(t)}{dt} = 2500e^{-\frac{t}{6.67\mu s}} \text{A/s}$$

$$V_L(t) = 1\text{mH}\left(2500e^{-\frac{t}{6.67\mu s}} A/s\right)$$

**Note:** The units for the derivative of the current are A/s, since it is the rate of change in current with respect to time.

Final Step Response voltage:

$$V_L(t) = 2.5e^{-\frac{t}{6.67\mu s}} \text{V}$$

Natural Response:

$$I(0^+) = 16.67 \text{ mA}$$

The final current across the inductor after a long time, when fully discharged, is:

$$I(\infty) = 0 \text{ A}$$

Natural Response equation:

$$I_L(t) = I(\infty) + [I(0^+) - I(\infty)]e^{-\frac{t}{\tau}}$$

$$I_L(t) = 0 + [16.67 - 0]e^{-\frac{t}{6.67\mu s}}$$

$$I_L(t) = 16.67e^{-\frac{t}{6.67\mu s}} \text{ mA}$$

Natural Response voltage across the Inductor:

$$V_L(t) = L\left(\frac{dI_L(t)}{dt}\right)$$

$$\frac{dI_L(t)}{dt} = -\frac{1}{6.67\mu s}\left(16.67e^{-\frac{t}{6.67\mu s}}\right) m\ A/s$$

$$\frac{dI_L(t)}{dt} = -2500e^{-\frac{t}{6.67\mu s}} \text{ A/s}$$

Final Natural Response voltage:

$$V_L(t) = 1\text{mH}\left(-2500e^{-\frac{t}{6.67\mu s}}\right)$$

$$V_L(t) = -2.5e^{-\frac{t}{6.67\mu s}} \text{ V}$$

**Theoretical Calculations at Specific Times**

The theoretical voltage values for the inductor at different times ($0, \tau, 2\tau$, and $3\tau$) are calculated as follows:

**Step Response at different $t$:**

At $t = 0$:

$$V_L(0) = 2.5e^{-\frac{0}{6.67\mu s}} = 2.5e^0 = 2.5\text{ V}$$

At $t = \tau$:

$$V_L(6.67\mu s) = 2.5e^{-\frac{6.67\mu s}{6.67\mu s}} = 2.5e^{-1} = 2.5 \times \frac{1}{e} = 2.5 \times 0.3679 = 0.92\text{ V}$$

At $t = 2\tau$:

$$V_L(13.34\mu s) = 2.5e^{-\frac{13.34\mu s}{6.67\mu s}} = 2.5e^{-2} = 2.5 \times e^{-2} = 2.5 \times 0.1353 = 0.34\text{ V}$$

At $t = 3\tau$:

$$V_L(20.01\mu s) = 2.5e^{-\frac{20.01\mu s}{6.67\mu s}} = 2.5e^{-3} = 2.5 \times e^{-3} = 2.5 \times 0.0498 = 0.1245\text{ V}$$

**Natural Response at different *t*:**

At $t = 0$:

$$V_L(0) = -2.5e^{-\frac{0}{6.67\mu s}} = -2.5e^0 = -2.5\text{ V}$$

At $t = \tau$:

$$V_L(6.67\mu s) = -2.5e^{-\frac{6.67\mu s}{6.67\mu s}} = -2.5e^{-1} = -2.5 \times \frac{1}{e} = -2.5 \times 0.3679 = -0.92\text{ V}$$

At $t = 2\tau$:

$$V_L(13.34\mu s) = -2.5e^{-\frac{13.34\mu s}{6.67\mu s}} = -2.5e^{-2} = -2.5 \times e^{-2} = -2.5 \times 0.1353 = -0.34\text{ V}$$

At $t = 3\tau$:

$$V_L(20.01\mu s) = -2.5e^{-\frac{20.01\mu s}{6.67\mu s}} = -2.5e^{-3} = -2.5 \times e^{-3} = -2.5 \times 0.0498 = -0.1245\text{ V}$$

**RL Circuit**

**PSpice Simulation**

To validate the theoretical calculations, a PSpice simulation was performed for the RL circuit. The simulation involved setting up the RL circuit in PSpice and conducting a transient analysis to observe the current through the inductor over time.

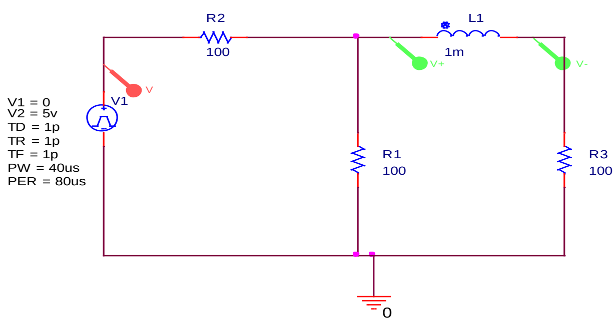


*Figure 32 - PSpice RL Circuit simulation with probe placements.*

**Note:** Since the inductor is not directly connected to ground, we do require the use of differential probes for accurate measurements.

The PSpice simulation provided a detailed insight into the transient behavior of the RL circuit, allowing for an accurate comparison with the theoretical predictions. This simulation played a crucial role in visualizing how the inductor responds to sudden changes in current, capturing the nuances of the circuit's behavior that are difficult to observe through theoretical calculations alone. The transient analysis effectively captured the exponential rise and decay of the inductor current, highlighting the characteristic time-dependent behavior of the RL circuit. Measurements were taken at various time intervals using cursors, similar to the RC circuit analysis, ensuring a comprehensive understanding of the circuit's dynamics over time. This detailed approach not only validated the theoretical models but also provided a practical framework for analyzing real-world circuit behavior, making it an invaluable tool for both learning and application in circuit design.

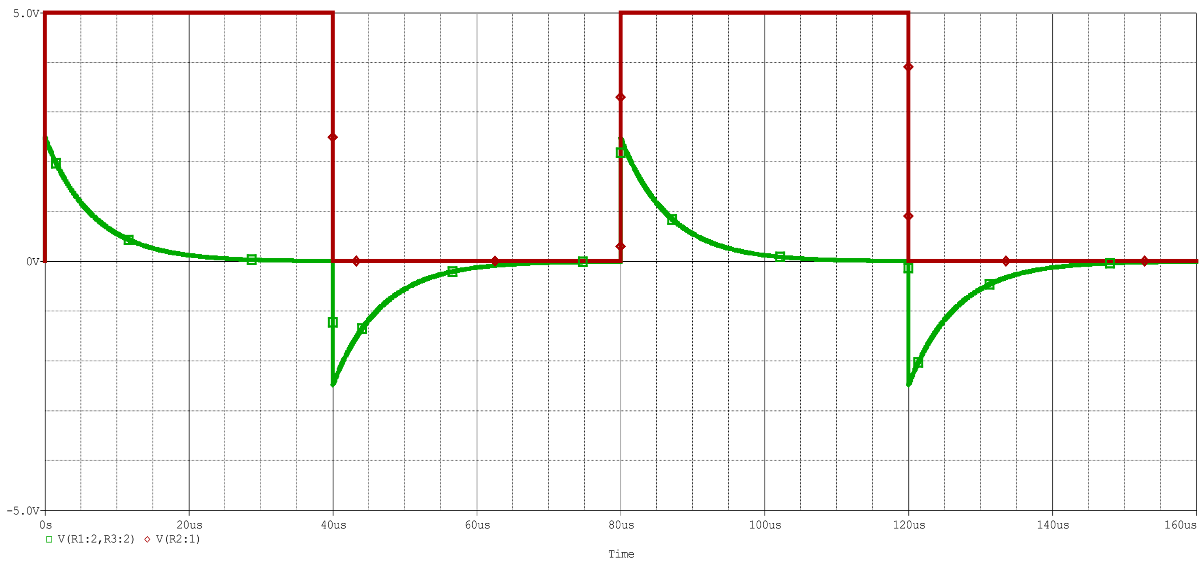


*Figure 33 - PSpice Transient Analysis Displaying two periods.*

The transient analysis was configured to run for a sufficient duration to capture the complete charging and discharging behavior of the inductor. The voltage across the inductor $V_L(t)$ was measured at various time intervals.

Using cursor measurements, the voltage values at the specific times of interest $(0, \tau, 2\tau, \text{and } 3\tau)$ were recorded.

**Step Response:**

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 0.000 | 0.000 | 0.000 |
| CURSOR 1,2 | V(L1:1,L1:2) | 2.5028 | 0.000 | 2.5028 |
| | V(V1:+) | 0.000 | 0.000 | 0.000 |

*Figure 34- Voltage Across $L_1$ at $t = 0$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 6.6667u | 0.000 | 6.6667u |
| CURSOR 1,2 | V(L1:1,L1:2) | 919.697m | 0.000 | 919.697m |
| | V(V1:+) | 5.0000 | 0.000 | 5.0000 |

*Figure 35 - Voltage Across $L_1$ at $t = \tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 13.246u | 0.000 | 13.246u |
| CURSOR 1,2 | V(L1:1,L1:2) | 342.799m | 0.000 | 342.799m |
| | V(V1:+) | 5.0000 | 0.000 | 5.0000 |

*Figure 36 - Voltage Across $L_1$ at $t = \tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 20.000u | 0.000 | 20.000u |
| CURSOR 1,2 | V(L1:1,L1:2) | 124.468m | 0.000 | 124.468m |
| | V(V1:+) | 5.0000 | 0.000 | 5.0000 |

*Figure 37 - Voltage Across $L_1$ at $t = \tau$.*

**Natural Response:**

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 40.000u | 0.000 | 40.000u |
| CURSOR 1,2 | V(L1:1,L1:2) | -2.4943 | 0.000 | -2.4943 |
| | V(V1:+) | 4.9988 | 0.000 | 4.9988 |

*Figure 38 - Voltage Across $L_1$ at $t = 0$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 53.341u | 0.000 | 53.341u |
| CURSOR 1,2 | V(L1:1,L1:2) | -337.632m | 0.000 | -337.632m |
| | V(V1:+) | 0.000 | 0.000 | 0.000 |

*Figure 40 - Voltage Across $L_1$ at $t = \tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 46.686u | 0.000 | 46.686u |
| CURSOR 1,2 | V(L1:1,L1:2) | -914.807m | 0.000 | -914.807m |
| | V(V1:+) | 0.000 | 0.000 | 0.000 |

*Figure 39 - Voltage Across $L_1$ at $t = \tau$.*

| Trace Color | Trace Name | Y1 | Y2 | Y1 - Y2 |
|---|---|---|---|---|
| | X Values | 60.000u | 0.000 | 60.000u |
| CURSOR 1,2 | V(L1:1,L1:2) | -124.159m | 0.000 | -124.159m |
| | V(V1:+) | 0.000 | 0.000 | 0.000 |

*Figure 41 - Voltage Across $L_1$ at $t = \tau$.*

**RL Circuit**

**Bench Measurements**

To validate the theoretical calculations and PSpice simulation, the RL circuit was constructed on a physical breadboard, and measurements were taken using a function generator and an oscilloscope.

**Calculations for the Frequency of the Function Generator**

The frequency of the function generator was calculated based on the pulse width and period of the square wave. The pulse width was given as $6\tau$, meaning that the full period of the square wave was $12\tau$. Given that $\tau = 6.67\mu s$, the full period $T$ can be calculated as follows:

$$T = 12\tau = 12 \times 6.67\mu\text{s} = 80\ \mu\text{s}$$

The frequency $f$ of the function generator is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{80\ \mu\text{s}} = \frac{1}{80 \times 10^{-6}\text{s}} = 12.5\text{kHz}$$

Thus, the function generator frequency was determined to be approximately 12.5 kHz.


*Figure 42 - Function Generator set to 12.5 kHz*

**Breadboard Setup**

To accommodate the limitations of the available probes in the lab, the placement of Resistor 3 and the inductor was adjusted so that the inductor was referenced to ground. This modification allowed the use of the standard probes available in the lab, which are not differential probes.

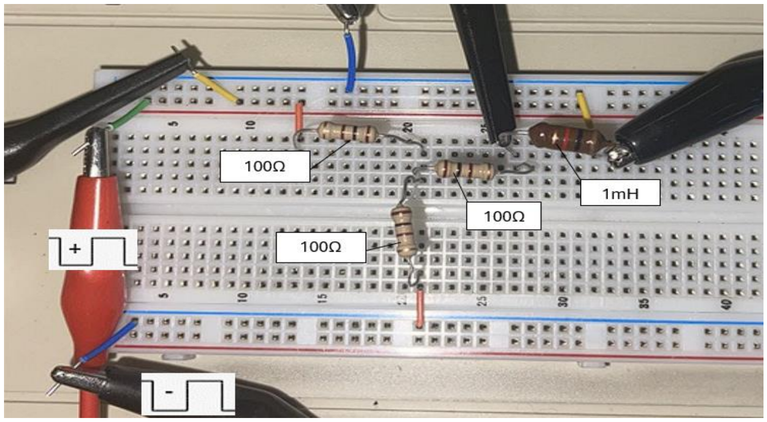


*Figure 43 - RL Circuit on Breadboard*

This image shows the RL circuit constructed on the breadboard, illustrating the connections between the resistors, inductor, and the function generator. The placement of Resistor 3 and the inductor has been adjusted so that the inductor is with reference to ground, allowing for accurate measurements with the standard probes.

Following this, measurements were taken at different times ($0, \tau, 2\tau,$ and $3\tau$) using the oscilloscope cursors to record the voltage across the inductor. These measurements were crucial for comparing the theoretical and simulation results.

Even after calibrating the probe, the inductor affects the circuit in a way that slightly distorts the input signal wave. Inductors resist changes in current, and this property can cause a delay in the current's response to a sudden change in voltage, such as the square wave input. This delay can manifest as a distortion in the waveform observed on the oscilloscope, particularly at the transitions of the square wave where the voltage changes abruptly (Figure 44). This inherent characteristic of inductors needs to be considered when analyzing the circuit's behavior and the accuracy of the measurements.

The following four figures illustrate the step response of the inductor's voltage at different times .

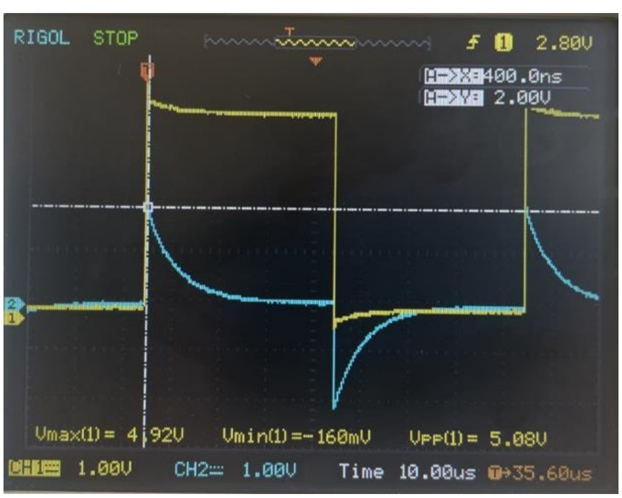


*Figure 44 - Voltage Across $L_1$ at $t = 0$*

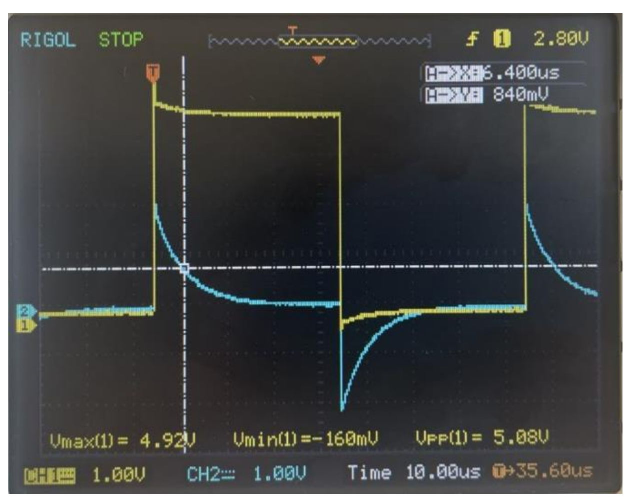


*Figure 45 - Voltage Across $L_1$ at $t = \tau$*

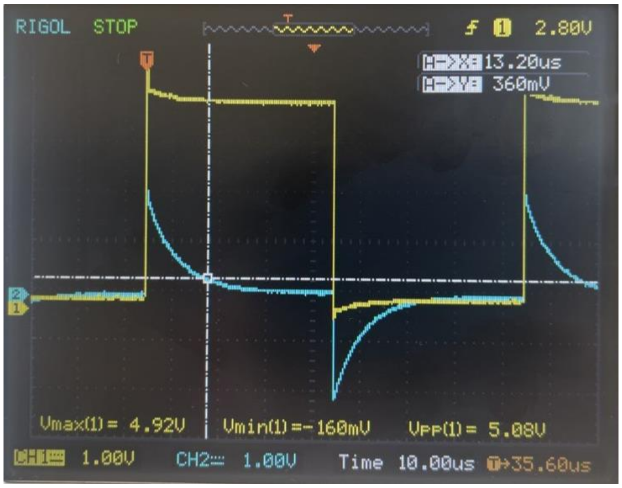


*Figure 46 - Voltage Across $L_1$ at $t = 2\tau$*

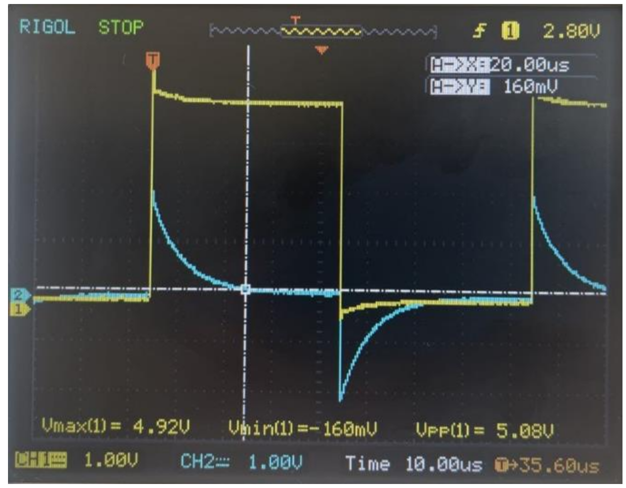


*Figure 47 - Voltage Across $L_1$ at $t = 3\tau$*

The next four figures show the natural response of the inductor's voltage at different times ($0, \tau, 2\tau$, and $3\tau$).

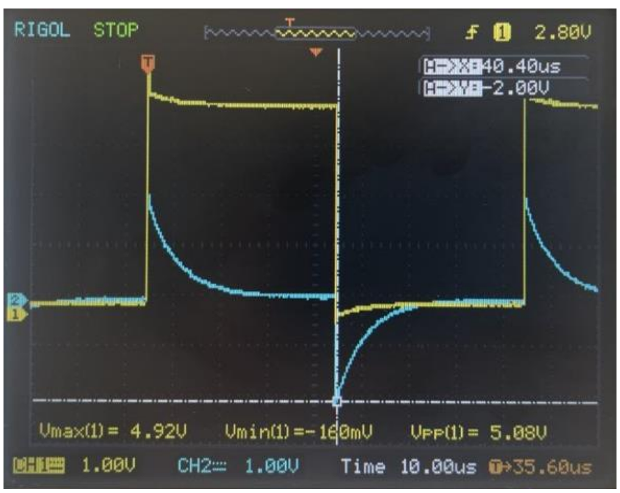


*Figure 48 - Voltage Across $L_1$ at $t = 0$*

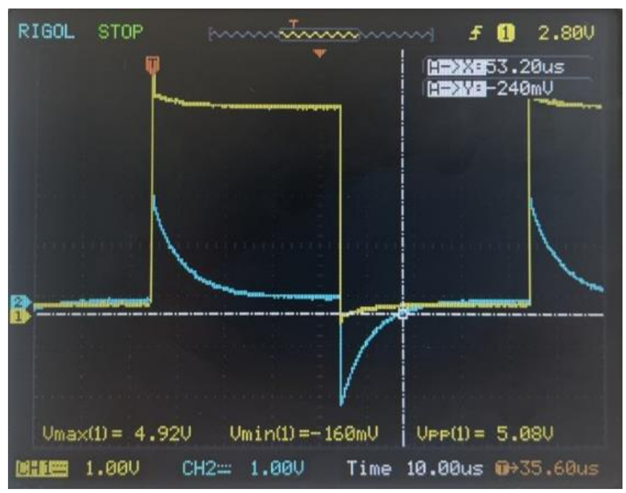


*Figure 50 - Voltage Across $L_1$ at $t = 2\tau$*

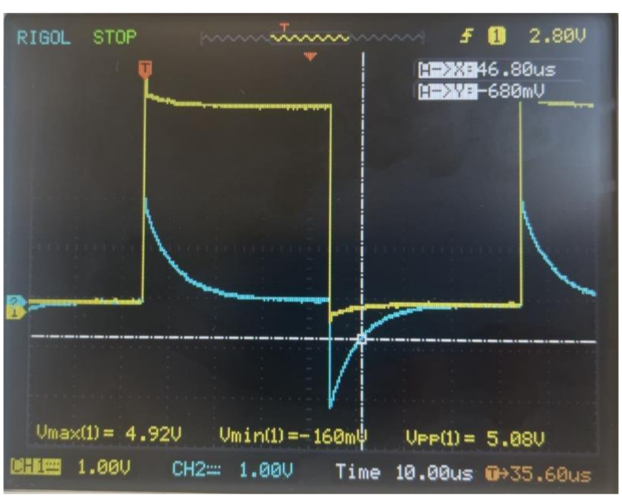


*Figure 49 - Voltage Across $L_1$ at $t = \tau$*

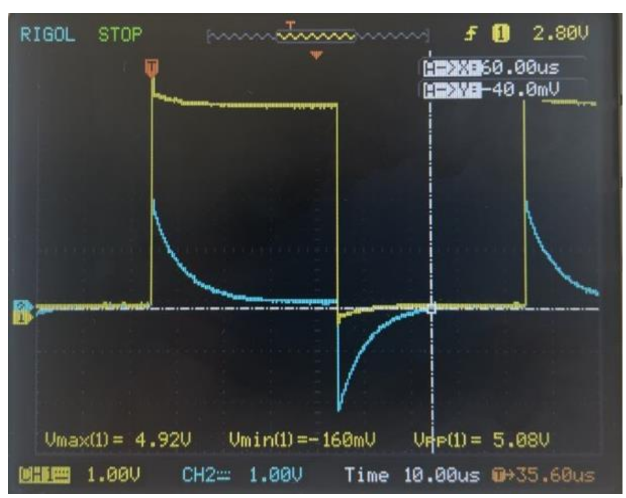


*Figure 51 - Voltage Across $L_1$ at $t = 3\tau$*

It was extremely challenging to get the readings to match the theoretical values for the inductor. Several factors could have contributed to this difficulty. Firstly, the inherent properties of the inductor, such as its resistance to changes in current, can cause delays and distortions in the observed waveform, making precise measurements difficult. Additionally, the function generator's output instability, with fluctuations of approximately ±0.5 V, introduced noise into the circuit, further complicating the measurement process. Calibration issues with the oscilloscope probes and the non-idealities in the components used, such as slight deviations from their nominal values, also played a significant role. These combined factors resulted in measurements that were often inconsistent with the theoretical predictions, highlighting the complexities involved in real-world circuit analysis.

**RL Circuit**

To summarize the findings from the theoretical calculations, PSpice simulations, and bench measurements, the voltage across the inductor $V_L(t)$ at different times $(0, \tau, 2\tau,$ and $3\tau)$ for both the step and natural responses will be compiled into a table. This comparison will provide a clear visual representation of how closely the experimental and simulated results align with the theoretical predictions.

The following table presents the voltage values obtained from the theoretical calculations, PSpice simulations, and bench measurements for the RL circuit. This comprehensive overview highlights the consistency and any deviations observed across the different methods.

Please refer to the table below for a detailed comparison of the step and natural responses at the specified times:

**Table 4**

| | $V_C(t)$ [step response] | | | | $V_C(t)$ [natural response] | | | |
|---|---|---|---|---|---|---|---|---|
| Time | Theory | Simulation | Lab | Error % | Theory | Simulation | Lab | Error % |
| $t = 0$ | $2.5\ V$ | $2.5\ V$ | $2\ V$ | 20 % | $-2.5\ V$ | $-2.5\ V$ | $-2\ V$ | 20 % |
| $t = \tau$ | $0.92\ V$ | $0.92\ V$ | $840\ mV$ | 8.7 % | $-0.92\ V$ | $-0.91\ V$ | $-680\ mV$ | 26.1 % |
| $t = 2\tau$ | $0.34\ V$ | $0.34\ V$ | $360\ mV$ | 5.9 % | $-0.34\ V$ | $-0.34\ V$ | $-240\ mV$ | 29.4 % |
| $t = 3\tau$ | $0.125\ V$ | $0.124\ V$ | $166\ mV$ | 32.8 % | $-0.125\ V$ | $-0.124\ V$ | $-40\ mV$ | 68 % |

Several external factors could have affected the output values in the RL circuit measurements. The probes used in the lab were not differential probes, which could have led to inaccuracies in measuring the voltage across the inductor, especially given the adjustments made to reference the inductor to ground. The inductor itself, with its property of resisting changes in current, introduced delays and distortions that were challenging to measure precisely. Environmental influences, such as electromagnetic interference and temperature variations, could have affected the circuit's behavior and the accuracy of the measurements. Additionally, the function generator exhibited fluctuations of approximately ±0.5 V, introducing noise into the input signal. The small nature of the measured voltages, especially in the millivolt range, further exacerbated these issues, as even minor deviations or noise had a significant impact on the percent error. These combined factors highlight the importance of accounting for real-world conditions when taking measurements.

# Discussion

The experiment aimed to analyze the transient responses in RC and RL circuits, comparing theoretical predictions with simulation and practical measurements. Understanding these transient behaviors is crucial for practical applications in electronics, where components need to react to sudden changes in voltage or current.

The theoretical calculations provided a baseline for understanding the expected behavior of the circuits. The simulation results, obtained using PSpice, closely matched the theoretical values, validating the accuracy of the models. However, the practical bench measurements showed some discrepancies, highlighting the challenges of real-world implementations.

For the RC circuit, the step response and natural response were analyzed at different times ($0, \tau, 2\tau,$ and $3\tau$). While the theoretical and simulation results aligned well, bench measurements exhibited variations. Fluctuations in the function generator output and calibration issues with the oscilloscope probes contributed to these discrepancies. The small voltage levels, especially in the millivolt range, amplified the impact of minor deviations, resulting in higher percent errors.

The RL circuit also showed discrepancies between theoretical predictions and practical measurements. Factors such as the use of non-differential probes, the inductor's inherent delay in responding to current changes, and environmental influences like electromagnetic interference played significant roles in affecting measurement accuracy. These external factors created a complex measurement environment, emphasizing the importance of accounting for real-world conditions.

The primary sources of discrepancies included measurement inaccuracies due to non-differential probes, the inductor's resistance to changes in current causing delays and waveform distortions, function generator output instability adding noise, and environmental factors like electromagnetic interference and temperature variations. The small nature of the measured voltages meant even minor noise or deviations significantly impacted the percent error.

Studying the transient analysis of RC and RL circuits is essential due to their practical implications in various electronic applications. Transient responses determine how quickly a circuit can react to changes, which is critical for signal processing, filtering, and timing applications. Understanding these behaviors enables engineers to design circuits that perform reliably under different conditions.

# Conclusion

This experiment provided valuable insights into the transient responses of RC and RL circuits, revealing both the strengths and limitations of theoretical predictions and practical implementations. The theoretical calculations and PSpice simulations established an expected baseline for the circuits' behavior, which were largely consistent with each other. However, the practical measurements on the bench presented certain challenges and discrepancies.

In the RC circuit, the step and natural responses at various intervals ($0, \tau, 2\tau,$ and $3\tau$) highlighted the differences between ideal conditions and real-world measurements. While the simulation results closely mirrored the theoretical values, the bench measurements showed variations due to several factors. These included the instability of the function generator output, probe calibration issues, and the significant effect of small voltage levels, which led to higher percent errors.

The RL circuit faced similar issues, with noticeable differences between theoretical predictions and practical measurements. The use of non-differential probes, the inductor's natural delay in current response, and environmental factors such as electromagnetic interference significantly influenced the accuracy of the measurements. These real-world conditions underscored the need to account for practical variables when performing circuit analysis.

Several key factors contributed to these discrepancies. Non-differential probes led to measurement inaccuracies, the inductor's resistance to current changes introduced delays and waveform distortions, and fluctuations in the function generator output added noise. Additionally, environmental influences like electromagnetic interference and temperature variations impacted the circuit's performance. These factors, combined with the small nature of the measured voltages, highlighted the importance of considering real-world conditions in circuit design and analysis.

The importance of studying transient responses in RC and RL circuits is evident due to their critical roles in electronic applications such as signal processing, filtering, and timing. Understanding these transient behaviors enables engineers to design circuits that perform reliably under varying conditions, ensuring robustness and accuracy in practical applications.

Future work should aim at reducing measurement inaccuracies by using appropriate differential probes and ensuring stable function generator outputs. Further studies could also delve deeper into the impact of environmental factors, thereby aiding the development of more robust and reliable circuit designs.

In conclusion, while the theoretical and simulation results provided a solid foundation, the practical measurements highlighted the complexities of real-world applications. This experiment emphasized the necessity of accounting for external factors and measurement limitations to achieve accurate and reliable circuit designs, ultimately bridging the gap between theoretical predictions and practical implementations.

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