<u>EET 3086C – Circuit Analysis</u>

Summer 2024

<u>Experiment # 4</u>

Sinusoidal Response of RLC Circuits

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Objective

The objective of this experiment is to observe the amplitude and phase changes in an RLC circuit under a sinusoidal forcing function. This will involve both theoretical analysis and simulation using PSpice to understand the behavior of the circuit components (resistors, inductors, and capacitors) when subjected to a sinusoidal input signal.

List of Equipment/Parts/Components

1. **Trainer**
   - Model: Elenco XK-150
2. **Resistors (all 5% tolerance)**
   - 100 Ω
   - 510 Ω
   - 1.0 kΩ (x2)
3. **Capacitor/Inductor**
   - 330 µH
   - 1.0 nF
4. **Digital Programmable Multimeter & LCR Meter**
   - Model: HM8012/HM8018
5. **Oscilloscope**
   - Model: Rigol DS1102D
6. **Function Generator**
   - Model: HM8030-6
7. **Computer with PSpice Software**

Theoretical Background Research

In the analysis of AC circuits, particularly those involving resistors (R), inductors (L), and capacitors (C), understanding the sinusoidal steady-state response is crucial. The study involves examining how these components respond to sinusoidal inputs, which are fundamental in many electrical engineering applications.

Characteristics of Sinusoids

A sinusoidal function, which can be represented as $v(t) = V_m \sin(\omega t + \theta)$, where V_m is the amplitude, ω is the angular frequency, and θ is the phase angle, is periodic with a period $\mathrm{T} = 1/f$, where f is the frequency of the sinusoid. The angular frequency is given by $\omega = 2\pi f$ [1].

Phasors and Their Importance

Phasors are a complex representation of sinusoids, converting differential equations into algebraic equations, thus simplifying the analysis of AC circuits. A phasor is a complex number that represents the amplitude and phase of a sinusoidal function. For instance, a sinusoidal voltage $v(t) = V_m\, cos(\omega t + \theta)$ can be represented in phasor form as $V = V_m \angle \theta$ [1].

Phasors are represented in the complex plane, with the real part on the horizontal axis and the imaginary part on the vertical axis. They are particularly useful because they allow the use of Ohm's law, Kirchhoff's voltage and current laws, and other circuit analysis techniques directly in the frequency domain [2].

Impedance and Admittance

Impedance (Z) is the AC equivalent of resistance and is a complex quantity representing both the resistance and the reactance (combination of inductance and capacitance). The impedance of a resistor R, inductor L, and capacitor C are given by:

- Resistor: $Z_R = R$
- Inductor: $Z_L = j\omega L$
- Capacitor: $Z_C = \frac{1}{j\omega C}$

Where j is the imaginary unit and ω is the angular frequency. Admittance (Y) is the reciprocal of impedance and is also a complex quantity, representing how easily a circuit or component will allow current to flow [2].

AC Circuit Analysis Techniques

1. **Kirchhoff's Laws in Phasor Form**: Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) can be applied in the phasor domain similar to how they are applied in the time domain. The laws state that the sum of phasor voltages around a closed loop is zero (KVL) and the sum of phasor currents entering a node is zero (KCL) [1].

2. **Impedance Combinations**: Impedances in series and parallel can be combined using the same rules as resistances in DC circuits. For series, $Z_{eq} = Z_1 + Z_2 + \cdots + Z_n$, and for parallel, $\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$ [2].

3. **Thevenin's and Norton's Theorems**: These theorems are also applicable in the AC analysis using phasors and impedances. They help in simplifying complex networks to a single voltage source and impedance (Thevenin) or a current source and parallel impedance (Norton) [2].

Phase Relationships and Power Calculations

In AC circuits, the phase difference between voltage and current is crucial for power calculations. The power factor, which is the cosine of the phase angle between the voltage and current, indicates the efficiency of power usage. Real power (P), reactive power (Q), and apparent power (S) are calculated as follows:

- Real Power: $P = V_{eff} I_{eff} \cos\left(\theta_v - \phi_i\right) W$
- Reactive Power: $Q = V_{eff} I_{eff} \sin(\theta_v - \phi_i)\ VAR$
- Apparent Power: $S = V_{eff} I_{eff}\ VA$

Where V_{eff} and I_{eff} are the rms values of voltage and current, respectively, and $\theta_v - \phi_i$ is the phase angle between them [2].

Procedure

The procedure for this experiment is adopted from the following reference [3]:

Masood Ejaz, Lab Experiments Manual for EET 3086C – Circuit Analysis, Experiment 4, Valencia College ECET Department, pp. 12-14.

Results & Observations

Component Measurements

This section presents the measured values of components used in the bench setup of the experiment, including resistors, capacitors, and inductors. These measurements are essential for identifying potential sources of error, as the actual component values can deviate from the nominal values used in the theoretical calculations and simulations.

Figure 1 – R_1 Measured value 97.29 Ω

Figure 2 – R_2 Measured value 504.7 Ω

Figure 3 – C_1 Measured value 997.2 pF

Figure 4 – R_3 Measure value 984.8 Ω

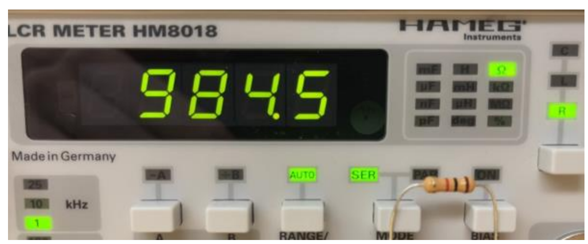


Figure 5 – R_4Measured value 984.5 Ω

Figure 6 – L_1Measured value 329.7 μH

The LCR meter was used to measure the resistance, capacitance, and inductance values, respectively. Each component's value was recorded, and the measurements are summarized in a table below. This table includes the nominal value, the measured value, and the calculated percent error for each component. The percent error is calculated using the formula:

$$\%\text{Error} = \left(\frac{|\text{Measured Value} - \text{Nominal Value}|}{\text{Nominal Value}}\right) \times 100$$

Sample Percent Error Calculation:

$$\%\text{Error for } R_2 = \left(\frac{|504.7\,\Omega - 510\,\Omega|}{510\,\Omega}\right) \times 100 = \left(\frac{|-5.3|}{510}\right) \times 100 = 1.04\,\%$$

This approach helps quantify the potential impact of component tolerances on the experimental results.

Table of Measured Values and Percent Error

The following table (Table 1) lists the nominal values, measured values, and percent error for each component used in the experiment. This detailed account ensures that any deviations from expected results can be attributed to identifiable sources of error in the component values.

Table 1

Component	Nominal Value	Measured Value	% Error
R_1	$100\,\Omega$	$97.29\,\Omega$	2.71 %
R_2	$510\,\Omega$	$504.7\,\Omega$	1.04 %
R_3	$1\,k\Omega$	$984.8\,\Omega$	1.52 %
R_4	$1\,k\Omega$	$984.5\,\Omega$	1.55 %
L_1	$330\,\mu H$	$329.7\,\mu H$	0.09 %
C_1	$1.0\,nF$	$997.2\,pF$	0.28 %

Theoretical Calculations: Nodal Analysis and Voltage Divider

Calculation of Impedances

The circuit analysis begins with the calculation of the impedances for the inductor and capacitor at the given frequency (100kHz):

$$\omega = 2\pi f = 2\pi(100k\text{Hz}) = 628.32\ kRad/Sec$$

- **Inductor Impedance ($\boldsymbol{Z_L}$)**:

$$Z_L = j\omega L = j(628.32\ kRad/Sec)(330\mu H) = j207.35\ \Omega$$

- **Capacitor Impedance ($\boldsymbol{Z_C}$)**:

$$Z_c = \frac{1}{j\omega c} = \frac{1}{j(628.32\ kRad/Sec)(1nF)} = -j1.59\ k\Omega$$

Nodal Analysis

Nodal analysis is employed to find the voltages at nodes V_1 and V_2 (Figure 7).

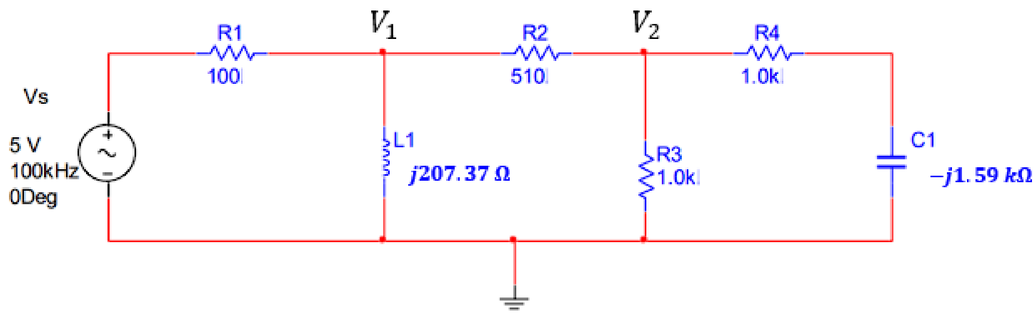


Figure 7 - Phasor domain RLC circuit for the Experiment.

1. **Equation for V_1:**

$$\frac{V_1 - 5}{100\Omega} + \frac{V_1}{j207.35\Omega} + \frac{V_1 - V_2}{510\Omega} = 0$$

$$\left(\frac{1}{100} + \frac{1}{j207.35} + \frac{1}{510}\right) V_1 - \frac{1}{510} V_2 = 0.05\ (1)$$

2. **Equation for V_2:**

$$\frac{V_2 - V_1}{510\Omega} + \frac{V_2}{1k\Omega} + \frac{V_2}{1 - j1.59\Omega} = 0$$

$$-\frac{1}{510} V_1 + \left(\frac{1}{510} + \frac{1}{1000} + \frac{1}{1000 - j1590}\right) V_2 = 0\ (2)$$

These equations are arranged in matrix form for solution:

- **Matrix Representation:**

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{100} + \frac{1}{j207.35} + \frac{1}{510}\right) & \left(-\frac{1}{510}\right) \\ \left(-\frac{1}{510}\right) & \left(\frac{1}{510} + \frac{1}{1000} + \frac{1}{1000 - j1590}\right) \end{bmatrix}^{-1} * \begin{bmatrix} 0.05 \\ 0 \end{bmatrix}$$

- **Solutions:**

$$V_1 = 3.9 + j1.68V\ or\ 4.25\angle 23.3°\ V$$
$$V_2 = 2.49 + j0.677V\ or\ 2.55\angle 15.39°\ V$$
$$V_1 = V_L(j\omega) = 4.25 < 23.3°\ V$$
$$v_L(t) = 4.25\cos(628319t + 23.3°)\ V$$

Voltage Across the Capacitor

Finally, V_C is calculated using the voltage divider rule applied to V_2 and the capacitor:

- **Voltage Divider for V_C:**

$$V_c = \frac{Z_c}{Z_4 + Z_c} V_2 = \frac{-j1.59k\Omega}{1k - j1.59k\Omega}(2.49 + j.677\ V) = 2.07 - j.623V$$

$$V_c(j\omega) = 2.16 < -16.78°$$

$$v_c(t) = 2.16\cos(628319t - 16.78°)\ V$$

PSpice Simulation

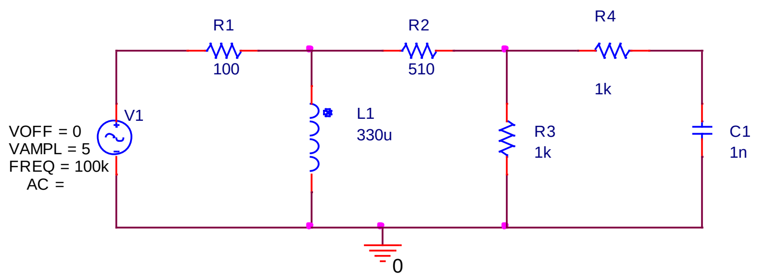


Figure 8 – PSpice circuit schematic.

Transient Analysis

The circuit, as depicted in the schematic, was constructed and analyzed using PSpice software. A transient analysis was performed to study the behavior of the circuit under a sinusoidal voltage source with a frequency of 100 kHz. The main focus of this analysis was to compare the voltage across the source with the voltage across the capacitor and inductor and to determine their phase differences.

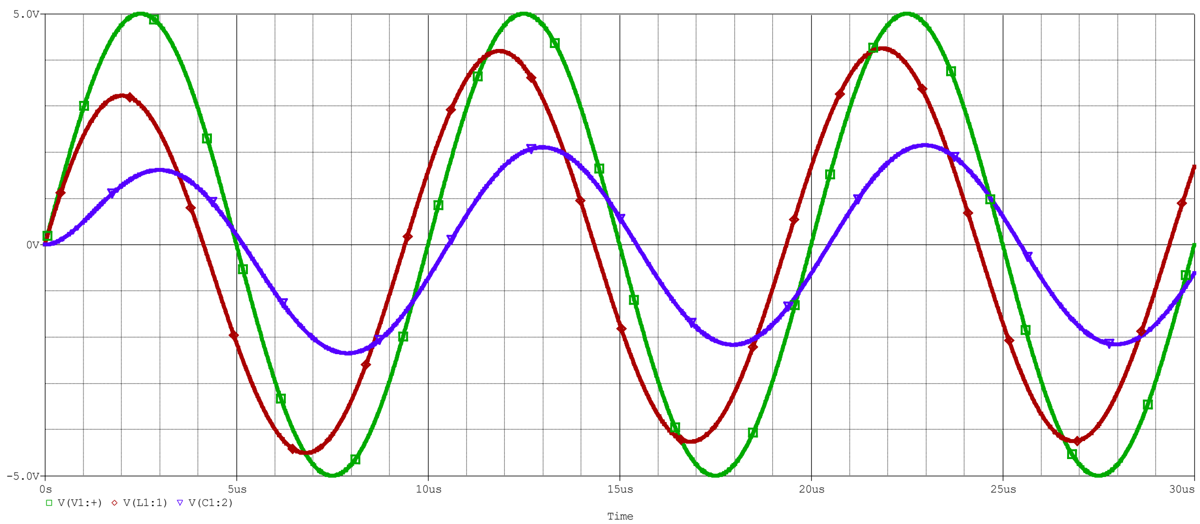


Figure 9 – PSpice Waveform comparison of the voltage across the source, inductor, and capacitor in an RLC circuit.

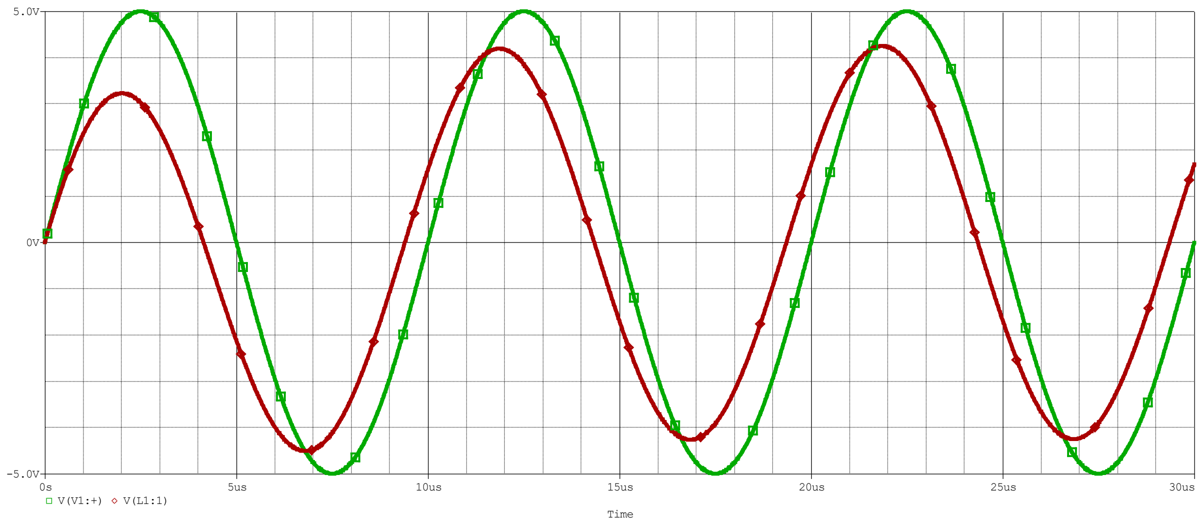


Figure 10 - Waveform comparison of the voltage across the source and inductor in an RLC circuit from the PSpice simulation.

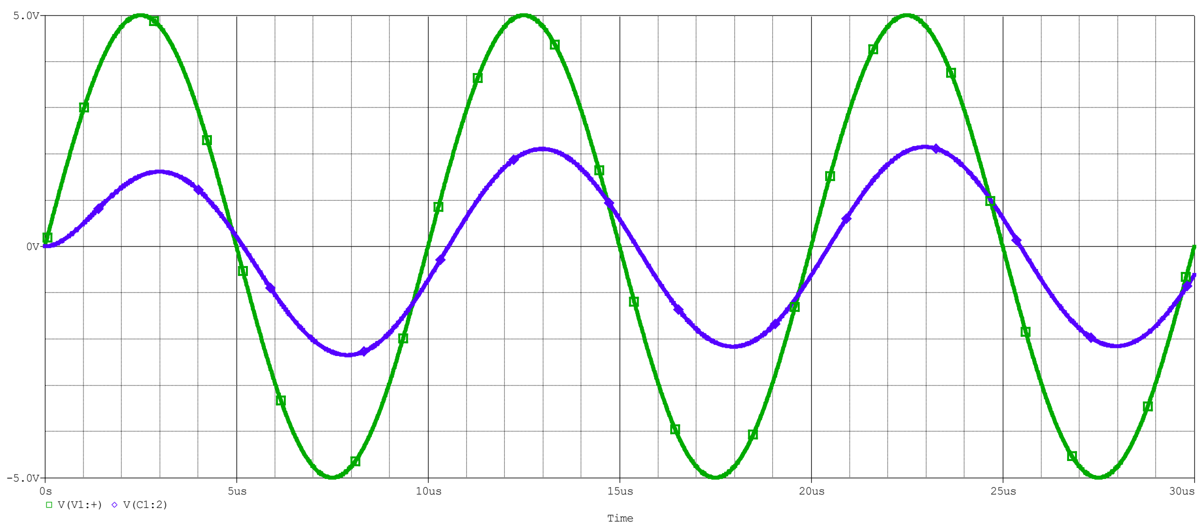


Figure 11 - Waveform comparison of the voltage across the source and Capacitor in an RLC circuit from the PSpice simulation.

The phase difference between the voltage across the source and the components was determined by identifying the time difference at the local maxima of their respective waveforms. This time difference represents the delay between the peaks of the voltage waveforms, which can be translated into a phase angle. The calculation of the phase angle (θ) from the time difference (Δt) is done using the formula:

$$\theta = \left(\frac{\Delta t}{T}\right) \times 360^\circ$$

where T is the period of the sinusoidal wave, given by $10\ \mu s$ for a $100\ kHz$ frequency.

Results from the Simulation

The following key observations were made from the PSpice simulation:

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	22.500u	21.852u	648.035n
CURSOR 1	V(V1:+)	5.0000	4.5806	419.397m
CURSOR 2	V(L1:1)	3.9093	4.2479	-338.671m

Figure 12 - PSpice Voltage/Inductor Phase difference.

- **Voltage Source to Inductor Phase Difference**:
  - Time Difference (Δt): 648.035 ns
  - Calculated Phase Angle:

$$\theta = \left(\frac{648.035 \text{ ns}}{10 \text{ µ}s}\right) \times 360^\circ = 23.37^\circ$$

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	22.500u	22.965u	-465.025n
CURSOR 1	V(V1:+)	5.0000	4.7840	216.023m
CURSOR 2	V(C1:2)	2.0580	2.1526	-94.601m

Figure 13 - PSpice Voltage/Capacitor Phase difference.

- **Voltage Source to Capacitor Phase Difference**:
  - Time Difference (Δt): -465.025 ns
  - Calculated Phase Angle:

$$\theta = \left(\frac{-465.025 \text{ ns}}{10 \text{ µ}s}\right) \times 360^\circ = -16.74^\circ$$

Results from the Bench Test: Oscilloscope Observations

Oscilloscope Measurements

The oscilloscope provided real-time voltage waveforms of the circuit components during the bench test. This setup allowed for accurate visual confirmation and measurement of the phase differences between the voltage source and both the inductor and capacitor.

The image below displays the RLC circuit assembled on a breadboard for bench testing. This setup includes resistors, a 330 µH inductor, and a 1 nF capacitor, powered by a 5V peak-to-peak, 100 kHz sinusoidal source. This configuration allows for direct observation of voltage responses and phase shifts across the circuit components, providing practical insights that complement theoretical calculations and simulations.

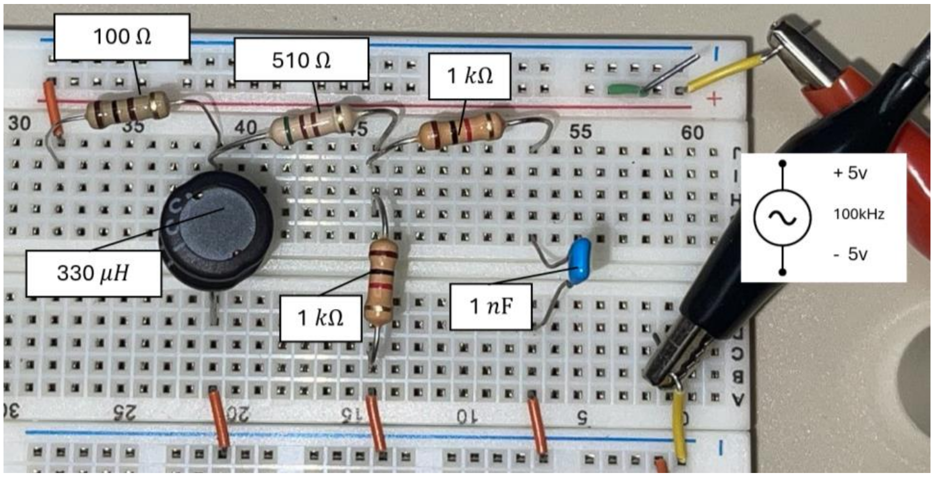


Figure 14 - Bench RLC circuit for the Experiment.

Using the oscilloscope, the time differences at the zero crossings of the waveforms were carefully measured, and phase angles were calculated accordingly. The phase difference is crucial as it illustrates the extent to which one waveform is leading or lagging behind another in time, shedding light on the reactive behavior of the circuit elements involved.

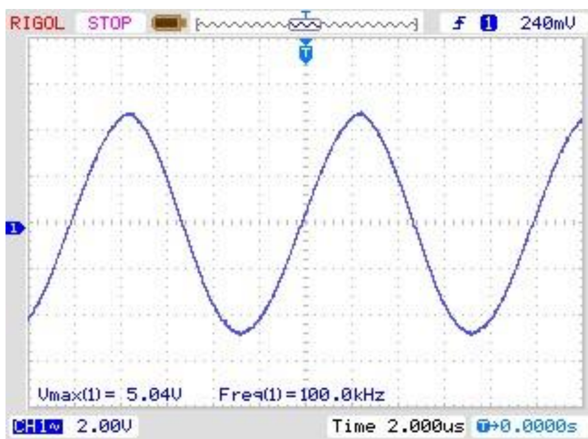


Figure 15 – Oscilloscope verification of AC voltage source within an acceptable amplitude and frequency.

The key observations derived from the oscilloscope measurements during the bench test are detailed below. These measurements focus on the phase differences between the voltage source and both the inductor and the capacitor. By capturing the time delay at the zero crossings of these waveforms, we calculated the phase angles, providing critical insights into how one waveform is leading or lagging another, which is pivotal for analyzing the reactive properties of the circuit under test conditions.

Voltage Source to Inductor Phase Difference:

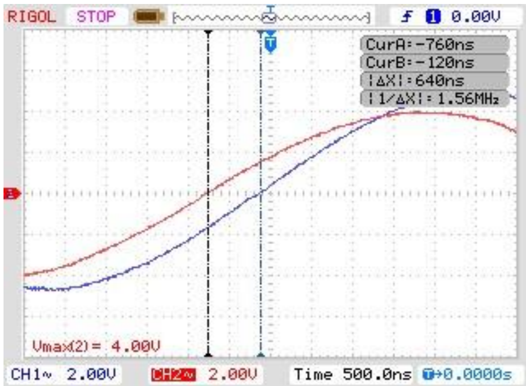


Figure 16 - Oscilloscope trace showing the zero crossings of the voltage source (blue) and the inductor (red). The cursor indicates a time difference of 640 ns.

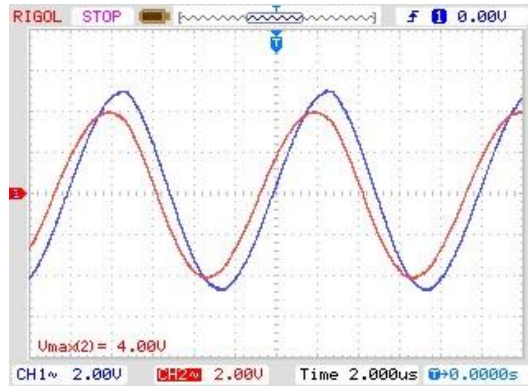


Figure 17 - Continuous waveform display from the oscilloscope capturing the sinusoidal outputs of the voltage source and the inductor, illustrating their periodic behavior and phase alignment over multiple cycles.

- The oscilloscope captured a time difference of 640 ns between the peak of the voltage source and the peak of the voltage across the inductor.
- **Calculated Phase Angle:**

$$\theta = \left(\frac{640\text{ ns}}{10\ \mu s}\right) \times 360^\circ \approx 23.04^\circ$$

Voltage Source to Capacitor Phase Difference

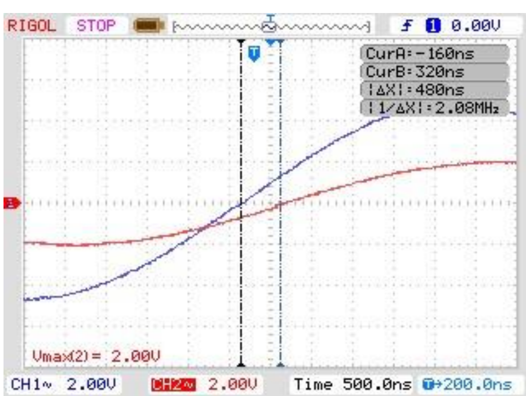


Figure 18 - Oscilloscope trace showing the zero crossings. The cursor indicates a time difference of 480 ns.

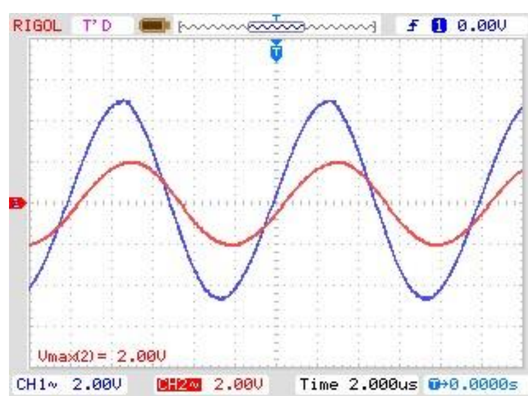


Figure 19 - Sinusoidal outputs of the voltage source and the capacitor.

- A time difference of 480 ns was noted between the peak of the voltage source and the peak of the voltage across the capacitor.
- **Calculated Phase Angle**

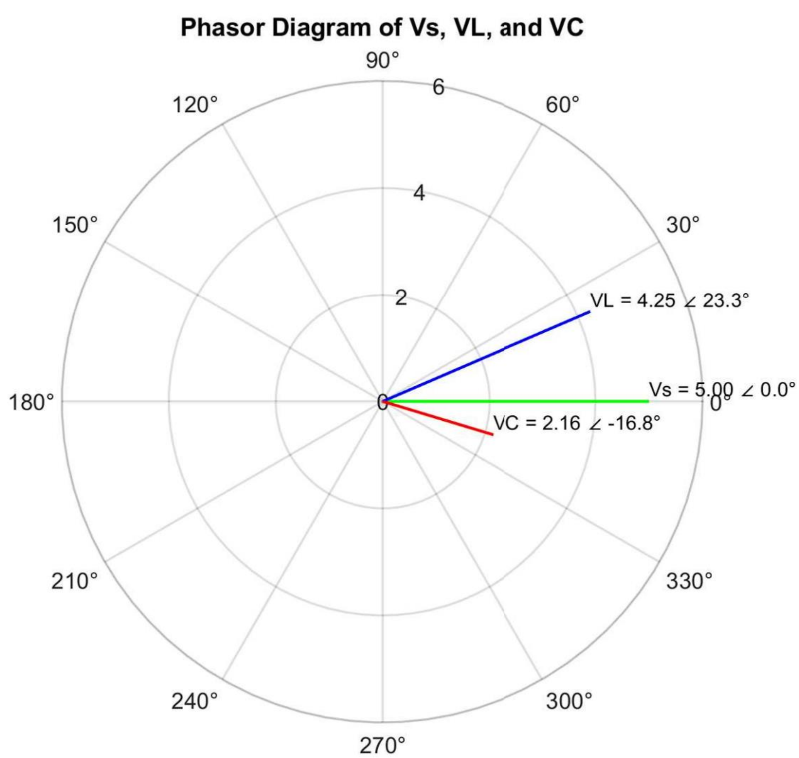
$$\theta = \left(\frac{480\ \text{ns}}{10\ \mu s}\right) \times 360° \approx 17.28°$$

Table 2 – Peaks and Phase angles.

	V_L *(peak Volts)*	Phase angle θ_L	V_C *(peak Volts)*	Phase angle θ_C
Theory	$4.25\ V$	$23.30°$	$2.16\ V$	$-16.78°$
Simulation	$4.25\ V$	$23.37°$	$2.15\ V$	$-16.74°$
Bench	$4.0\ V$	$23.04°$	$2.0\ V$	$-17.28°$
% Error	5.88 %	1.11 %	7.41 %	2.98 %

Answers to Lab Questions and Exercises

The diagram below illustrates the phasor representation of voltages across various components of the RLC circuit, specifically for the voltage source (Vs), the inductor (VL), and the capacitor (VC). Each phasor is marked with its magnitude and phase angle, demonstrating the relative phase relationships and magnitudes visually on a polar coordinate system. This phasor diagram is a crucial tool for understanding the dynamic interactions within the circuit, highlighting how each component influences the overall voltage behavior in response to sinusoidal input.



Discussion

In the conducted experiment, the primary objective was to evaluate the behavior of an RLC circuit by comparing theoretical calculations, simulation outputs, and bench measurements. The experiment focused on measuring the voltage across the inductor and the capacitor and their respective phase angles relative to the voltage source.

The theoretical analysis provided a basis for expected results using ideal component values, which were closely mirrored in the simulation results. These simulations incorporated precise mathematical models to emulate the circuit’s behavior under ideal conditions. However, when transitioning from simulation to physical bench tests, slight deviations were observed.

For the inductor (V_L), both theoretical predictions and simulation results indicated a peak voltage of 4.25 V and a phase angle of approximately 23.30°. The practical bench measurements, however, recorded a slightly lower peak voltage of 4.0 V, resulting in a -5.88% deviation from the expected value. The phase angle measured on the bench was closely aligned with the theoretical and simulation results, demonstrating the accuracy of the phase behavior prediction but highlighting a discrepancy in voltage magnitude.

Similar observations were made for the capacitor (V_C), where the theoretical and simulation analyses suggested a peak voltage of 2.16 V. The bench test results showed a lower voltage of 2.0 V, translating to a -7.41% error compared to the theoretical value. The phase angle for the capacitor also showed a small deviation, with bench measurements slightly differing from the expected results.

The discrepancies observed between the theoretical predictions, simulation results, and actual bench measurements can be attributed to a variety of factors that influence the circuit's performance. Component tolerances are a significant factor; the resistors, capacitors, and inductors used in the experiment may not perfectly match their nominal values, thus affecting the overall behavior of the circuit. Additionally, potential inaccuracies could arise from the measurement equipment itself. Factors such as oscilloscope resolution and probe calibration can impact the precision of the readings obtained during the experiment. Environmental factors also play a role; variations in temperature and other conditions can alter the characteristics of sensitive components like capacitors and inductors, further influencing the experimental outcomes. These elements collectively contribute to the variations seen in the practical implementation of the circuit as compared to its theoretical model.

Conclusion

The experiment conducted provided insightful data regarding the behavior of an RLC circuit under various conditions and successfully demonstrated the close alignment between theoretical predictions, simulated models, and practical measurements, albeit with minor discrepancies primarily attributable to real-world factors. The theoretical and simulation results were largely confirmed by the bench tests, validating the expected voltage magnitudes and phase angles across the circuit components. The small variances observed were primarily due to component tolerances, the precision of measurement instruments, and environmental factors impacting component performance.

While the theoretical and simulation results predicted slightly higher voltage magnitudes, the practical results underscored the impact of non-ideal factors inherent in physical circuit assembly and testing. The phase angles calculated from the practical tests closely matched those predicted theoretically, confirming the accuracy of the phase relationships as modeled in the simulations.

For future work, it is recommended to employ more precise measuring tools and perhaps better-calibrated components to minimize the error margin and enhance the reliability of the results. Additionally, conducting the experiment under controlled environmental conditions could further reduce discrepancies related to variable factors such as temperature and humidity. This experiment underscores the importance of accounting for practical variables in the design and analysis of electrical circuits, which is crucial for applications that demand high fidelity and precision.

References

[1] Notash, “Chapter 10 (Part I) - AC Circuit Steady State Analysis,” EET 3086C Course Material, Valencia College, 2020.

[2] Notash, “Chapter 10 (Part II) - Phasors,” EET 3086C Course Material, Valencia College, 2020.

[3] Masood Ejaz, Lab Experiments Manual for EET 3086C – Circuit Analysis, Experiment 4, Valencia College ECET Department, pp. 12-14.

Appendix A

Thevenin Impedance:

$$Z_{th} = ((R_1||Z_{L1}) + R_2)||R_3 + R_4$$

$$Z_{th} = \left(\left((100\Omega \parallel j207.35\Omega) + 510\Omega\right) \parallel 1.0\ k\Omega\right) + 1.0\ k\Omega$$

$$Z_{th} = 1371.9 + j15.47\ \Omega\ or\ 1.37\angle 0.645°k\Omega$$

Solving for V_{TH} using single node equation and voltage divider:

$$Node\ V_1: -\left(\frac{5 - V_1}{100\Omega}\right) + \frac{V_1}{j207.35\Omega} + \frac{V_1}{1510\Omega} = 0$$

$$\left(\frac{1}{100} + \frac{1}{j207.35} + \frac{1}{1510}\right)V_1 = 0.05$$

$$V_1 = 3.89 + j1.76V$$

$$V_{TH} = \frac{1k\Omega}{1.51k\Omega}(3.89 + j1.76V) = 2.58 + j1.17V \text{ or } 2.83\angle 24.3°V$$

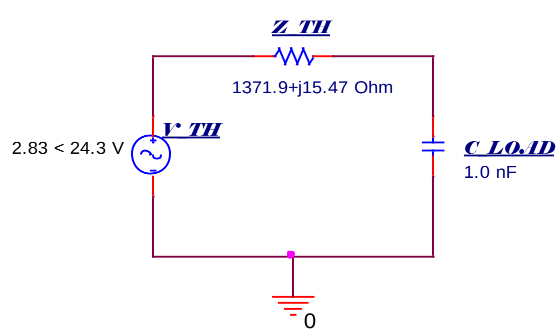


Figure 20 - Thevenin RLC circuit with load C_1.

Appendix B

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% MATLAB Code for Phasor Diagram

% Magnitude and Phase Angles
Vs_mag = 5; % Magnitude of the voltage source in volts
Vs_angle = 0; % Angle in degrees

VL_mag = 4.25; % Magnitude of the voltage across the inductor in volts
VL_angle = 23.30; % Angle in degrees

VC_mag = 2.16; % Magnitude of the voltage across the capacitor in volts
VC_angle = -16.78; % Angle in degrees

% Conversion of angles from degrees to radians for MATLAB functions
Vs_angle_rad = deg2rad(Vs_angle);
VL_angle_rad = deg2rad(VL_angle);
VC_angle_rad = deg2rad(VC_angle);

% Calculation of the complex numbers for phasors
Vs_phasor = Vs_mag * exp(1i * Vs_angle_rad);
VL_phasor = VL_mag * exp(1i * VL_angle_rad);
VC_phasor = VC_mag * exp(1i * VC_angle_rad);

% Plotting
figure;
ax = polaraxes; % Create a polar axes
hold on; % Allows multiple plots on the same figure
polarplot([0, angle(Vs_phasor)], [0, abs(Vs_phasor)], 'g', 'LineWidth', 2); % Voltage
source phasor in blue
polarplot([0, angle(VL_phasor)], [0, abs(VL_phasor)], 'b', 'LineWidth', 2); %
Inductor voltage phasor in red
polarplot([0, angle(VC_phasor)], [0, abs(VC_phasor)], 'r', 'LineWidth', 2); %
Capacitor voltage phasor in green
hold off;

% Annotations
rlim([0 max([Vs_mag, VL_mag, VC_mag])+1]); % Adjust radial limits to fit all phasors
thetaticks(0:30:360); % Set theta ticks every 30 degrees

% Adding labels with magnitude and phase angle
text(angle(Vs_phasor), Vs_mag, sprintf('Vs = %.2f ∠ %.1f°', Vs_mag, Vs_angle), ...
    'VerticalAlignment', 'bottom', 'FontSize', 14); % Increased font size for labels
text(angle(VL_phasor), VL_mag, sprintf('VL = %.2f ∠ %.1f°', VL_mag, VL_angle), ...
    'VerticalAlignment', 'bottom', 'FontSize', 14);
text(angle(VC_phasor), VC_mag, sprintf('VC = %.2f ∠ %.1f°', VC_mag, VC_angle), ...
    'VerticalAlignment', 'bottom', 'FontSize', 14);

title('Phasor Diagram of Vs, VL, and VC');

% Enhance the visibility
ax.LineWidth = 1.5; % Set line width for better visibility
ax.FontSize = 16; % Set general font size for labels and ticks
```