EET 3086C – Circuit Analysis

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Experiment # 3

Transient Response of RLC Circuits

Performed By:

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Submitted to:

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Objective

The objective of this experiment is to analyze the transient response of series and parallel RLC circuits. By conducting theoretical calculations, simulations using PSpice, and practical bench measurements, the goal is to understand the behavior of the circuits under different damping conditions, determine the resonant and neper frequencies, and compare the observed results with theoretical predictions. This experiment aims to enhance the understanding of RLC circuit dynamics, including the identification of over-damped, under-damped, and critically damped responses.

List of Equipment/Parts/Components

1. **Trainer**
   - Model: Elenco XK-150
2. **Resistors (all 5% tolerance)**
   - 1.0 kΩ (x2)
   - 2.0 kΩ
   - 3.0 kΩ
3. **Capacitor/Inductor**
   - 10 nF
   - 100 nF
   - 100 mH
4. **Digital Programmable Multimeter & LCR Meter**
   - Model: HM8012/HM8018
5. **Oscilloscope**
   - Model: Rigol DS1102D
6. **Function Generator**
   - Model: Hameg HM8030-6
7. **Computer with PSpice Software**

Theoretical Background Research

An RLC circuit consists of a resistor (R), inductor (L), and capacitor (C) connected in series or parallel. These circuits exhibit complex behavior characterized by oscillations and damping, which are essential in various applications such as filters, oscillators, and communication systems [1].

Characteristic Equation

The characteristic equation is derived from the differential equations governing the circuit's behavior. For an RLC circuit, Kirchhoff's Voltage Law (KVL) or Kirchhoff's Current Law (KCL) is applied, leading to a second-order differential equation. The standard form of the differential equation for a series RLC circuit is:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

For a parallel RLC circuit, the differential equation is:

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

These equations can be expressed in a general second-order differential form:

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0$$

The characteristic equation is then derived by solving the homogeneous equation associated with the differential equation. For example, considering the series RLC circuit, the characteristic equation is:

$$Ls^2 + Rs + \frac{1}{C} = 0$$

Similarly, for the parallel RLC circuit, the characteristic equation is:

$$Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0$$

These characteristic equations are quadratic and can be solved using the quadratic formula:

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the context of RLC circuits, the general form of the characteristic equation is:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

where $\alpha = \frac{R}{2L}$ (series) or $\alpha = \frac{1}{2RC}$ (parallel) is the neper frequency (damping factor) and $\omega_0 = \frac{1}{\sqrt{LC}}$ is the resonant frequency [1].

Damping Types

The damping of an RLC circuit is determined by the relationship between the damping factor (α) and the resonant frequency (ω_0):

1. **Overdamped ($\alpha > \omega_0$)**: The characteristic equation yields two real and distinct roots. The system returns to equilibrium without oscillating. The voltage or current response can be expressed as:

   $$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

   where $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ and $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$ [1].

2. **Critically Damped ($\alpha = \omega_0$)**: The characteristic equation yields two real and repeated roots. The system returns to equilibrium as quickly as possible without oscillating. The response is given by:

   $$v(t) = (A_1 + A_2 t)e^{-\alpha t}$$

   where $s_1 = s_2 = -\alpha$ [1].

3. **Underdamped ($\alpha < \omega_0$)**: The characteristic equation yields two complex conjugate roots. The system oscillates while gradually returning to equilibrium. The response is represented by:

   $$v(t) = e^{-\alpha t}(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$$

   where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ [1].

These types of damping significantly influence the behavior of RLC circuits in practical applications, affecting their stability and performance [2].

Roots of the Characteristic Equation

The roots of the characteristic equation determine the behavior of the RLC circuit. For the quadratic characteristic equation:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

the solutions (roots) are given by:

- For the overdamped case ($\alpha > \omega_0$), the roots are real and distinct.
- For the critically damped case ($\alpha = \omega_0$), the roots are real and repeated.

- For the underdamped case ($\alpha < \omega_0$), the roots are complex conjugates.

$$s = -\alpha \pm j\omega_d$$

The type of damping affects the transient response of the circuit, which can be observed through oscillatory or non-oscillatory behavior. Understanding these responses is crucial for designing circuits that meet specific performance criteria [2].

Settling Time

Settling time, t_s, is an important parameter in the transient response analysis of RLC circuits. It refers to the time required for the system's response to ultimately disappear (or "damp out") or to reach approximately less than 1% of its maximum absolute value (v_m). The settling time depends on the damping type and the specific values of the circuit components.

For overdamped and critically damped systems, the response does not oscillate, and the settling time is relatively straightforward to determine based on the exponential terms in the solution. For underdamped systems, the response oscillates, and the settling time is influenced by the damping factor (α) and the damped natural frequency (ω_d). Understanding and minimizing the settling time is crucial for applications requiring fast and stable responses [1].

Procedure

The procedure for this experiment is adopted from the following reference [3]:

Masood Ejaz, Lab Experiments Manual for EET 3086C – Circuit Analysis, Experiment 3, Valencia College ECET Department, pp. 8-11.

Results & Observations

Component Values and Measurements

In this experiment, the theoretical, simulation, and hands-on bench work measurements were performed using specific components whose values were crucial for achieving accurate results. The numerical values of the components were used in both the simulation and the theoretical calculations. However, the actual values of all components were recorded during the bench measurements. This approach was taken to account for potential sources of error and deviations in the results.

Recording the actual values of the components is essential for several reasons. It allows for accurate error analysis by providing a precise understanding of any discrepancies between theoretical, simulation, and experimental results. This helps identify whether deviations are due to component tolerances or other factors. Ensuring that components are within their specified tolerance ranges improves the reliability of the experiment and highlights any variations due to manufacturing processes. Overall, this meticulous approach ensures that all components used are within their tolerance limits and provides a more comprehensive analysis of the experimental results.

Series RLC circuit components:

Figure 1 – R_1 Measured Value 980.8 *Ω*

Figure 2 – R_2 measured value 994 *Ω*

Figure 3 – L Measured value $101.8\ mH$

Figure 4 – C measured value $9.367\ nF$

Table 1

Component	Nominal Value	Measured Value	% Error
R_1	$1\ k\Omega$	$994\ \Omega$	$0.60\ \%$
R_2	$1\ k\Omega$	$980.8\ \Omega$	$1.92\ \%$
L	$100\ mH$	$101.8\ mH$	$1.80\ \%$
C	$10\ nF$	$9.367\ nF$	$6.33\ \%$

Sample Percent Error Calculation:

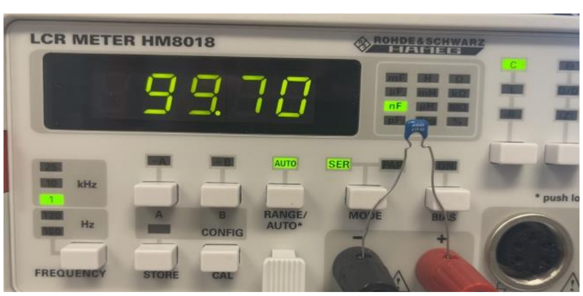
$$\%\text{Error} = \left(\frac{|\text{Measured Value} - \text{Nominal Value}|}{\text{Nominal Value}}\right) \times 100$$

$$\%\text{Error for } R_2 = \left(\frac{|980.8\ \Omega - 1000\ \Omega|}{1000\ \Omega}\right) \times 100 = \left(\frac{|-19.2|}{1000}\right) \times 100 = 1.92\ \%$$

Parallel *RLC* circuit components:

Figure 5 – R_2 *Measured Value* $2.002\ k\Omega$

Figure 6 – R_3 *Measured Value* $2.938\ k\Omega$



Note: The same $1.0\ k\Omega$ resistor (R_1) and 100 mH inductor (L) were used in both Circuit 1 and Circuit 2. The measured values for these components remained consistent across both circuits.

Figure 7 – C Measured Value $99.7\ nF$

Table 2

Component	Nominal Value	Measured Value	% Error
R_1	$1\ k\Omega$	$994\ \Omega$	0.60 %
R_2	$2\ k\Omega$	$2002\ \Omega$	0.10 %
R_3	$3\ k\Omega$	$2938\ \Omega$	2.07 %
L	$100\ mH$	$101.8\ mH$	1.80 %
C	$100\ nF$	$99.7\ nF$	0.30 %

Note: The numerical values were only recorded to account for any deviations in the recorded results from the theoretical and simulation results.

Theoretical Calculations for Series *RLC* (Circuit 1)

In the series RLC circuit, the component values and their configurations are used to determine key parameters such as the neper frequency (α), resonant frequency (ω_0), and the damping type. These parameters help to predict the transient response of the circuit.

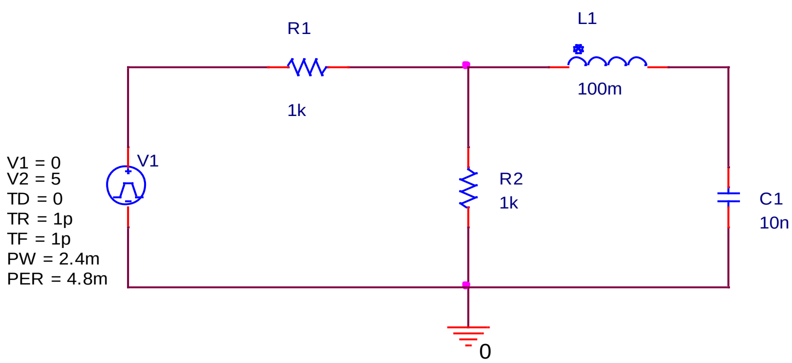


Figure 8 - Series RLC (Circuit 1)

Calculations:

1. Equivalent Resistance:

$$R_{\mathrm{eq}} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{1\ \mathrm{k\Omega} \cdot 1\ \mathrm{k\Omega}}{1\ \mathrm{k\Omega} + 1\ \mathrm{k\Omega}} = 500\ \Omega$$

2. Neper Frequency (α):

$$\alpha = \frac{R_{\mathrm{eq}}}{2L} = \frac{500\ \Omega}{2 \cdot 100\ \mathrm{mH}} = 2500\ \mathrm{s}^{-1}$$

3. Resonant Frequency (ω_0):

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100\ \mathrm{mH} \cdot 10\ \mathrm{nF}}} = 31623\ \mathrm{rad/s}$$

4. Characteristic Roots (s_1 and s_2):

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - 31623^2}$$

$$s_{1,2} = -2500 \pm j31524$$

5. Pulse Width (PW) and Period (PER):

$$PW = \frac{6}{\alpha} = \frac{6}{2500} = 2.4\ ms$$

$$PER = 2 \times PW = 2 \times 2.4\ ms = 4.8\ ms$$

6. Damping Type:

Since $\alpha < \omega_0$, the circuit is underdamped.

7. Step Response:

$$v_C(t) = V_{C(\infty)} + e^{-\alpha t}(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$$

8. Source-Free Response:

$$v_C(t) = e^{-\alpha t}(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$$

9. Initial Conditions:
   - Step response:

$$v_C(0) = 0\ \text{V (initial capacitor voltage, uncharged)}$$

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} V_{\text{in}} = \frac{1\ \text{k}\Omega}{2\ \text{k}\Omega} \cdot 5\ \text{V} = 2.5\ \text{V}$$

   - Source-Free response:

$$v_C(0) = 2.5\ \text{V (initial voltage across capacitor)}$$

$$v_C(\infty) = 0\ \text{V (capacitor fully discharged)}$$

$$i_C(0) = 0\ \text{A}$$

10. Solving for Constants (Step Response):

$$V_{C(\infty)} = \frac{R_2}{R_1 + R_2} V_{\text{in}} = \frac{1\ \text{k}\Omega}{2\ \text{k}\Omega} \cdot 5\ \text{V} = 2.5\ \text{V}$$

$$v_C(0) = 2.5 + e^0 B_1$$

$$B_1 = -2.5\ \text{V}$$

$$\frac{I_c}{C} = -\alpha B_1 + \omega_d B_2$$

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = 0$$

$$0 = -2500 \cdot (-2.5) + \omega_d B_2$$

$$B_2 = -0.1983\ \text{V}$$

11. Solving for constants (Source-Free):

$$v_C(0) = 2.5\text{ V}$$

$$v_C(0) = e^0 B_1$$

$$B_1 = 2.5\text{ V}$$

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = 0$$

$$0 = -2500 \cdot 2.5 + \omega_d B_2$$

$$B_2 = 0.1983\text{ V}$$

12. Complete Solutions:

$$v_C(t)(Step) = 2.5 + e^{-2500t}(-2.5\,cos(31524t) - 0.1983\,sin(31524t))\ V$$

$$v_C(t)(Source - Free) = e^{-2500t}(2.5\,cos(31524t) + 0.1983\,sin(31524t))\ V$$

PSpice Simulation for Circuit 1

The PSpice simulation was performed to analyze the transient response of the series RLC circuit. The objective was to obtain the maximum voltage (first peak) time and value, as well as the steady-state value for both the step and source-free responses. A 5V square wave with a pulse width (PW) of 2.4 ms and a period (PER) of 4.8 ms was used for the simulation, consisting of two pulses (Figure 9).

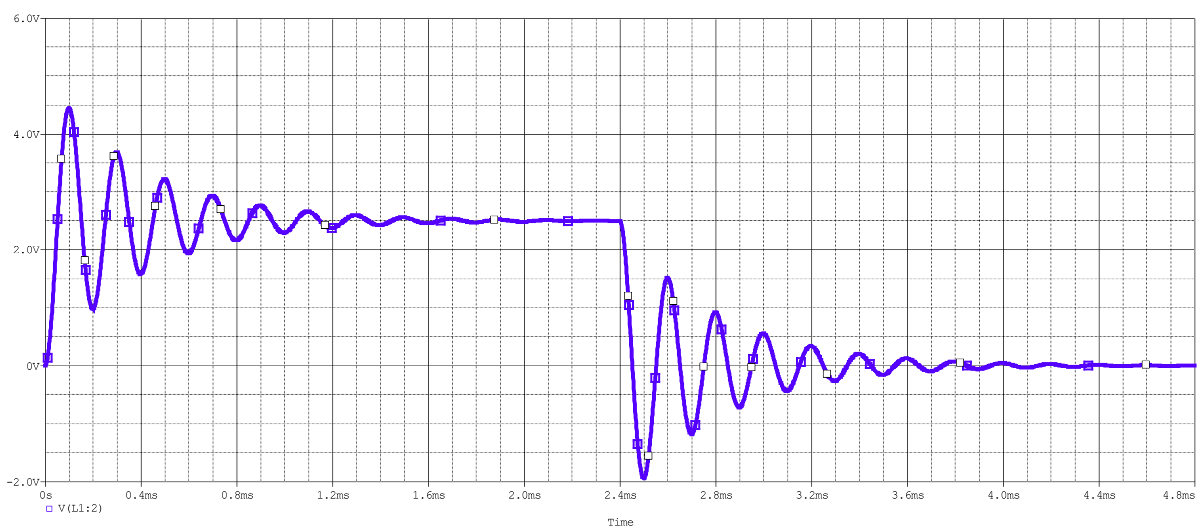


Figure 9 - Transient Analysis (Series RLC)

The purpose of the simulation was to observe the behavior of the circuit during transient conditions, both when a step input is applied and when the input is removed (source-free). This allows for measuring key parameters such as the first peak time and value, and the steady-state value. Additionally, the simulation aimed to verify theoretical calculations by comparing the simulated results with the theoretical models, thereby validating their accuracy. Any discrepancies between the theoretical and practical behaviors of the circuit could be identified and analyzed.

Circuit 1 PSpice Step response analysis:

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	99.503u	0.000	99.503u
CURSOR 1,2	V(L1:2)	4.4491	0.000	4.4491

Figure 10 - First Peak time and Value.

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	2.4000m	0.000	2.4000m
CURSOR 1,2	V(L1:2)	2.4939	0.000	2.4939

Figure 11 - Steady-State Value.

Circuit 1 PSpice Source-Free response analysis:

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	2.4995m	0.000	2.4995m
CURSOR 1,2	V(L1:2)	-1.9436	0.000	-1.9436

Figure 12 - First Peak time and Value.

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	4.8000m	0.000	4.8000m
CURSOR 1,2	V(L1:2)	6.1178m	0.000	6.1178m

Figure 13 - Steady-State Value.

MATLAB simulation circuit 1:

The MATLAB simulation was utilized to validate the theoretical and PSpice simulation results for the series RLC circuit (Circuit 1). It provided a platform for visualizing the transient behavior of the circuit, verifying analytical solutions, and highlighting key parameters such as the first peak time and value, and the steady-state value. By comparing MATLAB plots with PSpice simulation results and theoretical calculations, any discrepancies could be identified and analyzed. This comparison helped ensure the accuracy of the derived formulas and improved understanding of the circuit's response to sudden changes in voltage, both when a step input is applied and when the input is removed.

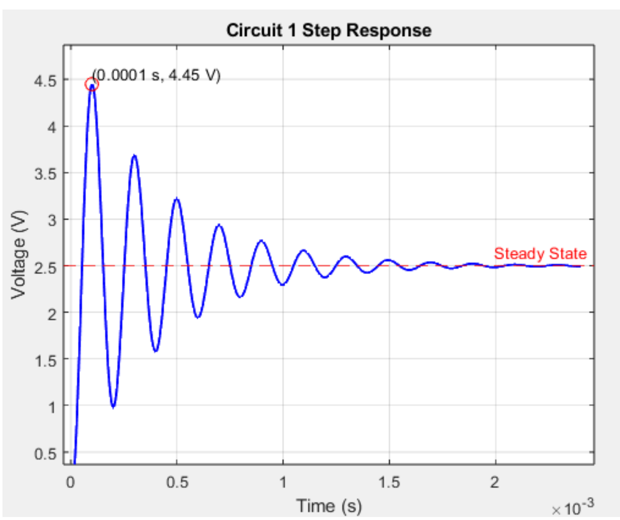


Figure 14 - MATLAB Graph for Step Response.

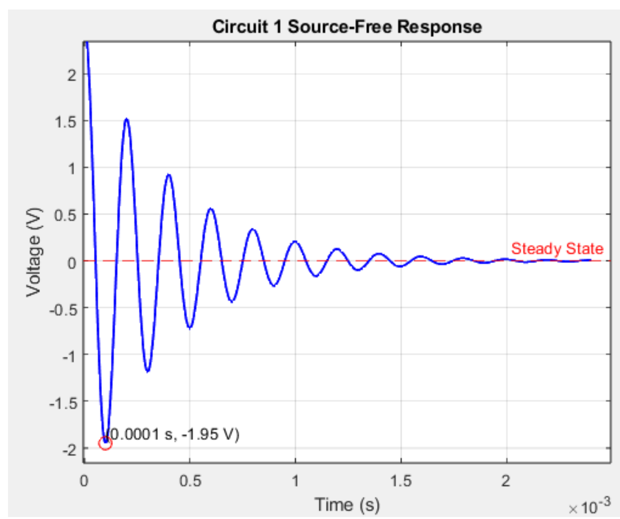


Figure 15- MATLAB Graph for source-free response.

Bench Measurements for Circuit 1

The bench measurements for Circuit 1 were conducted to validate the theoretical and simulated results through practical implementation. This involved constructing the series RLC circuit on a breadboard, placing the probes accurately, and capturing the transient response using an oscilloscope.

The series RLC circuit was constructed on a breadboard using the specified components: R_1, R_2, L, and C. The exact component values used in the construction were previously recorded to account for any deviations (Figure 16).

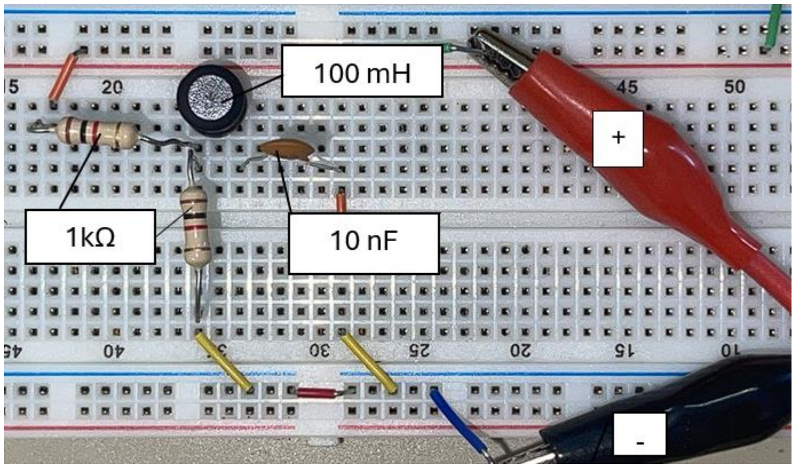


Figure 16 -Circuit 1 on Breadboard

Probes were placed at appropriate points in the circuit to measure the voltage across the capacitor C. The placement of the probes was crucial for obtaining accurate measurements of the step and source-free responses (Figure 17).

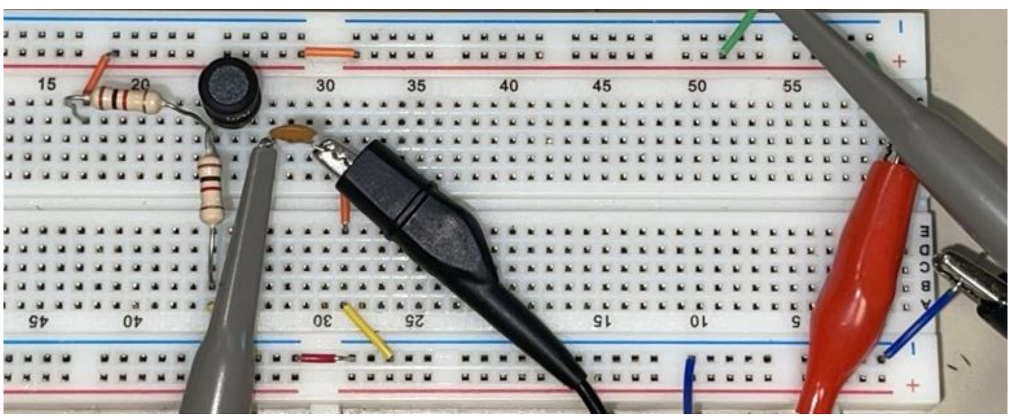


Figure 17 - Series RLC probe placements.

The oscilloscope was used to capture the detailed transient response of the circuit, providing a high-resolution view of the voltage changes over time. The step response measurements illustrated how the circuit reacted to the sudden application of a 5V square wave, capturing the voltage at its initial rise and subsequent stabilization. The first peak highlighted the maximum voltage reached immediately after the step input was applied, while the steady-state value showed the voltage level the circuit stabilized at after the transient effects subsided. For the source-free response, the measurements documented the first negative peak, demonstrating how the circuit responded to the sudden removal of the input voltage, and the steady-state value in the source-free condition, illustrating the final voltage level as the circuit fully discharged.

The frequency of the input signal used in the bench measurements was determined by the inverse of the period of the square wave. Given a period of 4.8 ms, the frequency was calculated as:

$$f = \frac{1}{\text{Period}} = \frac{1}{4.8 \times 10^{-3}\ \text{s}} = 208.33\ \text{Hz}$$

Figure 18 - Function Generator Square wave source at 208.3 Hz

Bench Step Response for series *RLC*:

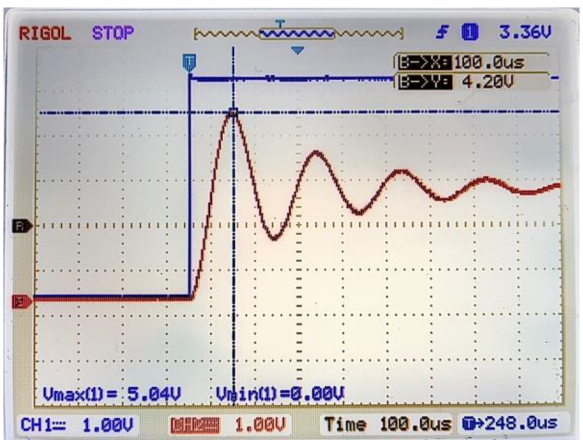


Figure 19 - Step First Peak time and Value.

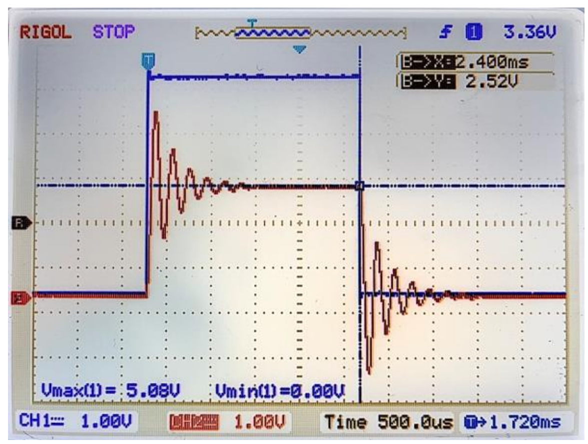


Figure 20 - Step Steady state time and value.

Bench Source-Free Response for series *RLC*:

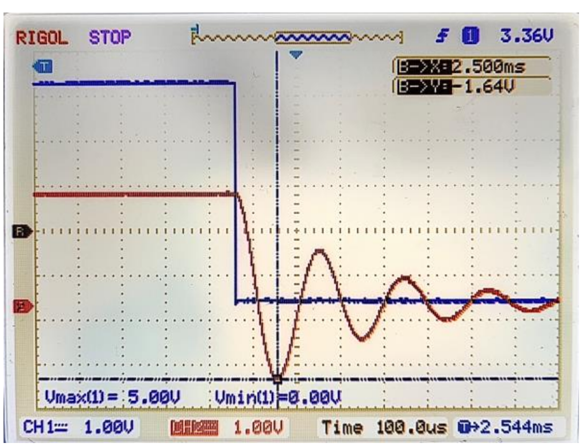


Figure 21 – Source-Free First Peak time and Value.

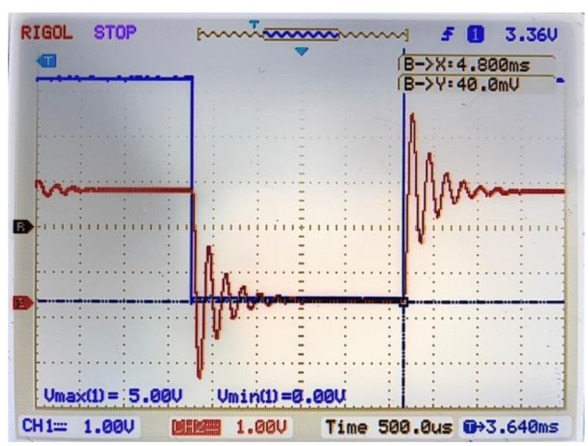


Figure 22 – Source-Free Steady state time and value.

To comprehensively analyze and validate the transient response of Circuit 1, the following table compares the results obtained from bench measurements, MATLAB simulations, and PSpice simulations. The table includes key parameters such as the first peak time and value, as well as the steady-state values for both the step and source-free responses. This comparison helps in identifying any discrepancies between the different methods and provides a clear understanding of the circuit's behavior under various conditions.

Table 3

	Step Response			Source-Free Response		
	First Peak time	First Peak Value	Steady-State Value	First Peak time	First Peak Value	Steady-State Value
Simulation	$99.5\ \mu s$	$4.45\ V$	$2.49\ V$	$99.5\ \mu s$	$-1.95\ V$	$6.1\ mV$
MATLAB	$100\ \mu s$	$4.45\ V$	$2.50\ V$	$100\ \mu s$	$-1.95\ V$	$0\ V$
Bench	$100\ \mu s$	$4.20\ V$	$2.52\ V$	$100\ \mu s$	$-1.64\ V$	$40\ mV$
% Error	0 %	5.62 %	0.80 %	0 %	15.90 %	N/A

The comparison between MATLAB simulations and bench measurements reveals some discrepancies, particularly in the first peak values and steady-state values. The percent errors for the step response first peak value and the source-free response first peak value are 5.62% and 15.90%, respectively. The percent errors for the steady-state values in the step and source-free responses are 0.80% and unquantifiable for the source-free response due to the very small expected steady-state value (0 V in MATLAB). The significant deviation in the source-free response steady-state value on the bench can be attributed to its small magnitude, which makes it sensitive to even minor variations and potential sources of error in practical measurements. Despite these discrepancies, the overall comparison helps validate the results and understand the transient response behavior of Circuit 1 under various conditions.

Theoretical Calculations for Parallel RLC (Circuit 2)

In the parallel RLC circuit, the component values and their configurations are used to determine key parameters such as the neper frequency (α), resonant frequency (ω_0), and the damping type. These parameters help to predict the transient response of the circuit.

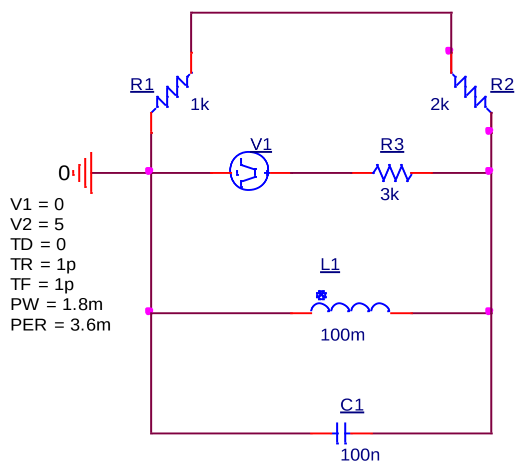


Figure 23 – Parallel RLC (Circuit 2)

Calculations:

1. Equivalent Resistance:

$$R_{\text{eq}} = R_3 \parallel (R_1 + R_2) = 3k\Omega \parallel (1k\Omega + 2k\Omega) = 1500\Omega$$

2. Neper Frequency (α):

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 1.5\,\mathrm{k\Omega} \cdot 100\,\mathrm{nF}} = 3333\,\mathrm{s}^{-1}$$

3. Resonant Frequency (ω_0):

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100\,\mathrm{mH} \cdot 100\,\mathrm{nF}}} = 10000\,\mathrm{rad/s}$$

4. Characteristic Roots (s_1 and s_2):

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$s_{1,2} = -3333 \pm \sqrt{3333^2 - 10000^2}$$

$$s_{1,2} = -3333 \pm j9428$$

5. Pulse Width (PW) and Period (PER):

$$\mathrm{PW} = \frac{6}{\alpha} = \frac{6}{3333} = 1.8\,\mathrm{ms}$$

$$\mathrm{PER} = 2 \times \mathrm{PW} = 2 \times 1.8\,\mathrm{ms} = 3.6\,\mathrm{ms}$$

6. Damping Type:

Since $\alpha < \omega_0$, the circuit is underdamped.

7. Step Response:

$$v_C(t) = e^{-\alpha t}(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$$

8. Source-Free Response:

$$v_C(t) = e^{-\alpha t}(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$$

9. Initial Conditions:
   - For the step response:

$$v_C(0) = 0\,\mathrm{V}$$

$$v_C(\infty) = 0\,\mathrm{V}$$

   - For the source-free response:

$$v_C(0) = 0\,\mathrm{V}$$

10. Solving for Constants:
    - For Step Response:

$$v_C(0) = e^0 B_1$$

$$B_1 = 0\,\mathrm{V}$$

By differentiating and solving:

$$\frac{I_c}{C} = -\alpha B_1 + \omega_d B_2$$

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{1.667\text{ mA}}{100\text{ nF}} = 16670$$

The derivative simplifies to:

$$16670 = \omega_d B_2$$

$$B_2 = 1.768\text{ V}$$

- For source-free response:

$$v_C(0) = e^0 B_1$$

$$B_1 = 0\text{ V}$$

By differentiating and solving:

$$\frac{I_c}{C} = -\alpha B_1 + \omega_d B_2$$

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-1.667\text{ mA}}{100\text{ nF}} = -16670$$

The derivative simplifies to:

$$-16670 = \omega_d B_2$$

$$B_2 = -1.768\text{ V}$$

11. Complete solutions:

$$v_C(t)(Step) = e^{-3333t}(1.768\sin(9428t))\text{ V}$$

$$v_C(t)(Source - Free) = e^{-3333t}(-1.768\sin(9428t))\text{ V}$$

12. Voltage Across R_2:

$$v_{R_2}(t) = v_C(t) \cdot \frac{R_2}{R_1 + R_2}$$

$$v_{R_2}(t) = \frac{2}{3} v_C(t)$$

$$v_{R_2}(t)(Step) = \frac{2}{3} e^{-3333t}(1.768\sin(9428t))\text{ V}$$

$$v_{R_2}(t)(Source - Free) = \frac{2}{3} e^{-3333t}(-1.768\sin(9428t))\text{ V}$$

PSpice simulation for Circuit 2

The PSpice simulation was conducted to analyze the transient response of the parallel RLC circuit. The goal was to determine the maximum voltage (first peak) time and value, as well as the steady-state value for both the step and source-free responses. A 5V square wave with a pulse width (PW) of 1.8 ms and a period (PER) of 3.6 ms was used for the simulation, consisting of two pulses (Figure 24).

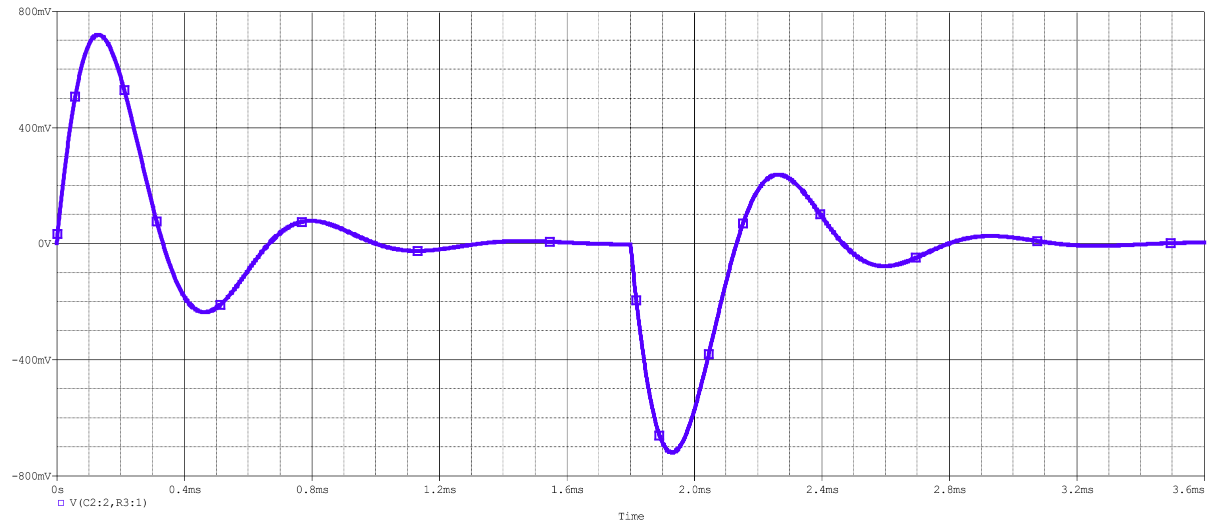


Figure 24 - Transient Analysis (Parallel RLC)

The purpose of this simulation was to observe the circuit's behavior during transient conditions, both when a step input is applied and when the input is removed (source-free). This allowed for measuring key parameters such as the first peak time and value, and the steady-state value. The simulation aimed to verify theoretical calculations by comparing the simulated results with the theoretical models, thereby validating their accuracy. Identifying and analyzing any discrepancies between the theoretical and practical behaviors of the circuit was an essential aspect of this process.

Circuit 2 PSpice Step response analysis:

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	129.851u	0.000	129.851u
CURSOR 1,2	V(R7:2,R7:1)	719.018m	0.000	719.018m

Figure 25 - First Peak time and Value.

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	1.8000m	0.000	1.8000m
CURSOR 1,2	V(R7:2,R7:1)	-2.7899m	0.000	-2.7899m

Figure 26 - Steady state Value

Circuit 2 PSpice Source Free response analysis:

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	1.9299m	0.000	1.9299m
CURSOR 1,2	V(R7:2,R7:1)	-720.199m	0.000	-720.199m

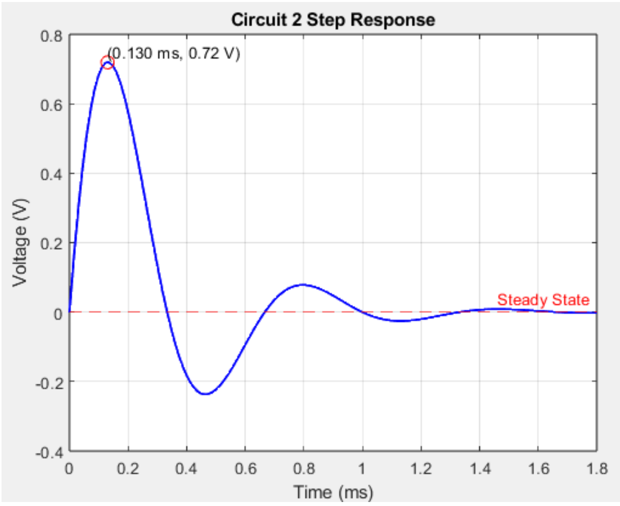
Figure 27 - First Peak time and Value.

Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	3.6000m	0.000	3.6000m
CURSOR 1,2	V(R7:2,R7:1)	2.7882m	0.000	2.7882m

Figure 28 - Steady state Value

MATLAB Simulation Circuit 2:

The MATLAB simulation was utilized to validate the theoretical and PSpice simulation results for the parallel RLC circuit (Circuit 2). It provided a platform for visualizing the transient behavior of the circuit, verifying analytical solutions, and highlighting key parameters such as the first peak time and value, and the steady-state value. By comparing MATLAB plots with PSpice simulation results and theoretical calculations, any discrepancies could be identified and analyzed. This comparison helped ensure the accuracy of the derived formulas and improved understanding of the circuit's response to sudden changes in voltage, both when a step input is applied and when the input is removed. The MATLAB simulation also allowed for precise determination of peak times and values, aiding in the validation of practical measurements and enhancing the overall analysis of the circuit's performance.



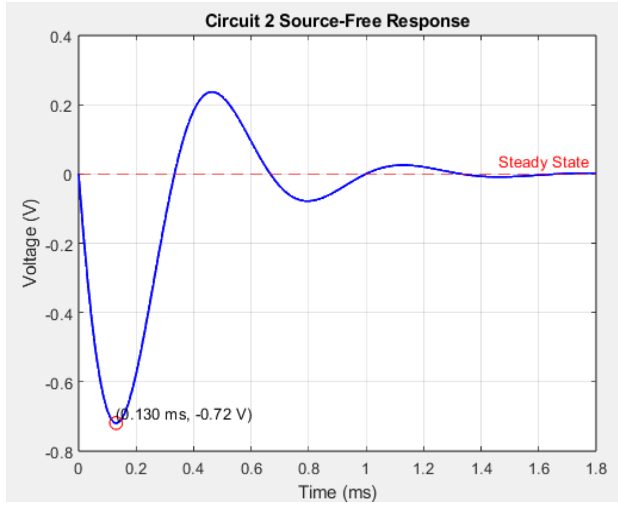


Figure 29 - MATLAB Graph for Sep Response.

Figure 30 - MATLAB Graph for Source-Free Response.

Bench Measurements for Circuit 2

The bench measurements for Circuit 2 were conducted to validate the theoretical and simulated results through practical implementation. This involved constructing the parallel RLC circuit on a breadboard, placing the probes accurately, and capturing the transient response using an oscilloscope.

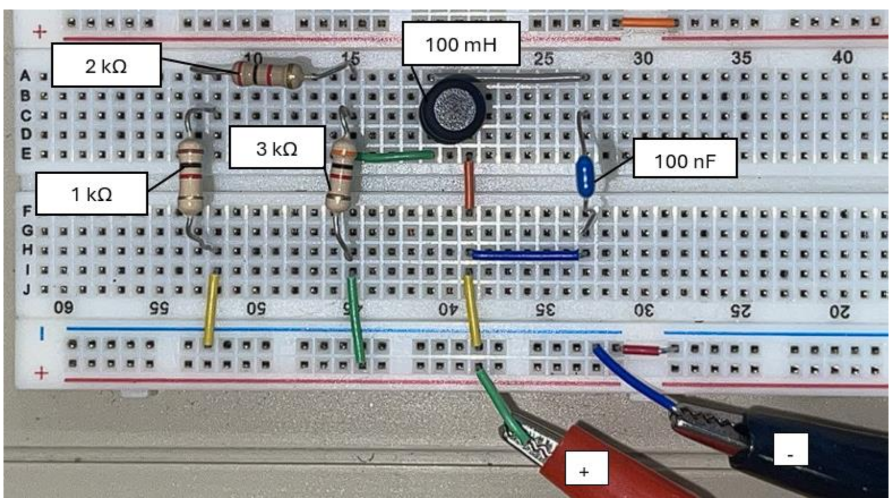


Figure 31 - Circuit 2 on Breadboard

During the bench setup, it was necessary to switch the 1 kΩ resistor and the 2 kΩ resistor so that the 2 kΩ resistor was referenced to ground. This adjustment was made because the available probes were non-differential probes, which require one end to be grounded. The probes were placed to measure the voltage across R_2 accurately, ensuring that the correct transient response could be captured.

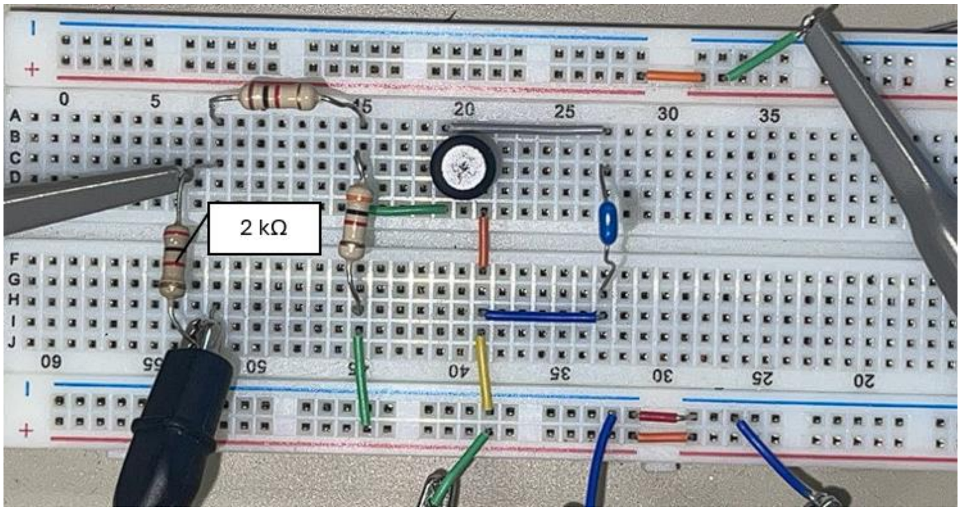


Figure 32 – Parallel RLC probe placements.

The oscilloscope was used to capture the detailed transient response of the circuit, providing a high-resolution view of the voltage changes over time. The step response measurements illustrated how the circuit reacted to the sudden application of a 5V square wave, capturing the voltage at its initial rise and subsequent stabilization. The first peak highlighted the maximum voltage reached immediately after the step input was applied, while the steady-state value showed the voltage level the circuit stabilized at after the transient effects subsided. For the source-free response, the measurements documented the first negative peak, demonstrating how the circuit responded to the sudden removal of the input voltage, and the steady-state value in the source-free condition, illustrating the final voltage level as the circuit fully discharged.

The frequency of the input signal used in the bench measurements was determined by the inverse of the period of the square wave. Given a period of 3.6 ms, the frequency was calculated as:

$$f = \frac{1}{\text{Period}} = \frac{1}{3.6 \times 10^{-3}\ \text{s}} \approx 277.78\,\text{Hz}$$

Figure 33 - Function Generator at 277.7 Hz.

Bench Step Response for Parallel RLC:

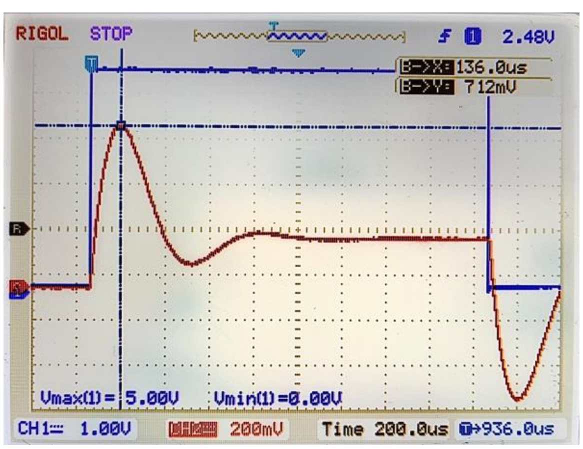


Figure 34 - Step First Peak time and value.

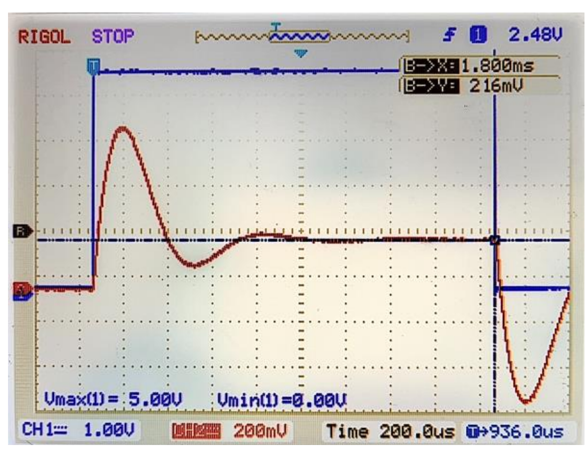


Figure 35 - Step Steady state time and value.

Bench Source-Free Response for Parallel RLC:

Figure 36 - Source-Free First Peak time and value.

Figure 37 – Source-Free Steady state time and value.

To comprehensively analyze and validate the transient response of Circuit 2, the following table compares the results obtained from bench measurements, PSpice, and MATLAB simulations. The table includes key parameters such as the first peak time and value, as well as the steady-state values for both the step and source-free responses. This comparison helps in identifying any discrepancies between the different methods and provides a clear understanding of the circuit's behavior under various conditions. The results also consider the adjustments made during the bench setup, such as switching the resistor placements for proper probe referencing.

Table 4

	Step Response			Source-Free Response		
	First Peak time	First Peak Value	Steady-State Value	First Peak time	First Peak Value	Steady-State Value
Simulation	$129.8\ \mu s$	$719\ mV$	$-2.8\ mV$	$129.9\ \mu s$	$-720.2\ mV$	$2.8\ V$
MATLAB	$130\ \mu s$	$720\ mV$	$0\ V$	$130\ \mu s$	$-720\ mV$	$0\ V$
Bench	$136\ \mu s$	$712\ mV$	$216\ mV$	$136\ \mu s$	$-504\ mV$	$0\ V$
% Error	4.62 %	1.11 %	N/A	4.62 %	30.0 %	N/A

The comparison between MATLAB simulations and bench measurements for Circuit 2 reveals several discrepancies. The percent errors for the first peak times and values are relatively low, indicating good agreement between MATLAB and bench measurements for these parameters. However, the steady-state value for the step response on the bench was observed to be 216 mV instead of the expected 0 V. This discrepancy likely caused the first negative peak of the source-free response to be -504 mV rather than the expected -720 mV. Interestingly, the sum of the step response steady state (216 mV) and the first negative peak of the source-free response (-504 mV) adds up to 720 mV, matching the expected peak value.

This unexpected steady-state value in the step response might have resulted from several factors, including residual charge in the capacitor or non-idealities in the components. The residual voltage could have persisted in the circuit, affecting the subsequent source-free response. Understanding and addressing these factors is essential for improving the accuracy of practical measurements and ensuring that the observed behavior aligns closely with theoretical predictions.

Discussion

The objective of this experiment was to analyze the transient response of RLC circuits, comparing theoretical calculations, PSpice simulations, and practical bench measurements. The discussion highlights the comparison characteristics of the circuits built in the lab, addressing the accuracy of measurements, sources of error, and any discrepancies observed.

The theoretical results provided the expected transient behavior of the series and parallel RLC circuits, with precise calculations for neper frequency (α), resonant frequency (ω_0), and characteristic roots (s_1, s_2). These theoretical predictions were validated using MATLAB and PSpice simulations, which closely matched the calculated values. The practical measurements, however, revealed some deviations, particularly in the steady-state values and peak responses.

For Circuit 1, the step response and source-free response were closely aligned between MATLAB and PSpice simulations. The first peak time, peak value, and steady-state value showed minimal discrepancies. The bench measurements, however, indicated a slightly lower first peak value and a higher steady-state value than predicted. The source-free response also exhibited a deviation in the first negative peak value, which can be attributed to residual voltage.

For Circuit 2, the theoretical and simulated results predicted a specific transient behavior. However, practical bench measurements required an adjustment due to the non-differential probes available, necessitating a switch in resistor placements. The step response on the bench showed a steady-state value of 216 mV instead of the expected 0 V, which influenced the source-free response, resulting in a first negative peak of -504 mV instead of -720 mV. Despite this, the sum of the step steady state value and the first negative peak value matched the expected peak value, indicating a possible residual charge in the capacitor or non-ideal component behavior.

Several sources of error were identified in this experiment. Component tolerances could have affected the transient response, as the actual values of resistors, capacitors, and inductors may deviate from their nominal values due to manufacturing tolerances. Measurement inaccuracies, particularly the use of non-differential probes and the need for resistor placement adjustments, introduced potential errors in the bench measurements of Circuit 2. Residual voltage in the circuit could lead to deviations in the steady-state values and subsequent transient responses. Additionally, theoretical and PSpice simulations assume ideal components and perfect conditions, which do not account for real-world imperfections and noise.

The accuracy of measurements varied across the different methods used. MATLAB and PSpice simulations showed high accuracy, with minimal discrepancies between them. Bench measurements, however, exhibited larger percent errors, particularly in the steady-state values and peak responses. This highlights the challenges in achieving precise measurements in practical scenarios, where non-idealities and external factors come into play. The percent errors calculated for Circuit 1 and Circuit 2 indicated that while the first peak times and values were reasonably accurate, the steady-state values, especially for the source-free response, showed significant deviations. These errors underscore the importance of accounting for real-world conditions and the limitations of theoretical predictions.

The technical reasons for the observed variations include non-ideal component behavior, as real components do not behave as ideal elements, leading to discrepancies between theoretical predictions and practical measurements. Measurement limitations, such as the use of non-differential probes and the need for adjustments in resistor placements, impacted the accuracy of bench measurements, particularly in Circuit 2. Residual effects, such as residual voltage and charge in the circuit components, influenced the transient responses, causing deviations from expected values. Additionally, environmental factors like external noise, temperature variations, and other conditions may have affected the measurements, leading to discrepancies.

Conclusion

This experiment successfully demonstrated how RLC circuits behave when configured in series and parallel, providing valuable insights into their transient responses under different conditions. The comparison of theoretical calculations, MATLAB simulations, PSpice simulations, and practical bench measurements revealed a strong correlation among the methods, despite some discrepancies. The observed deviations in practical measurements underscored the importance of accounting for real-world factors and limitations in component behavior and measurement techniques.

The series RLC circuit (Circuit 1) exhibited a close match between theoretical predictions and simulated results, with practical measurements showing minor discrepancies primarily in the steady-state values. The parallel RLC circuit (Circuit 2) required specific adjustments in the bench setup due to non-differential probes, which impacted the accuracy of the measurements. The significant deviation in the steady-state value for the step response of Circuit 2 emphasized the need for precise component placement and measurement techniques.

The experiment achieved its objective of demonstrating the transient responses of RLC circuits in series and parallel configurations, enhancing the understanding of the factors influencing these responses. Future work could focus on improving measurement accuracy by using differential probes and minimizing residual voltages in the circuit. Additionally, exploring the effects of component tolerances and environmental factors on transient responses could further enhance the reliability of practical measurements. Overall, this experiment reinforced the importance of combining theoretical analysis, simulations, and practical measurements to gain a comprehensive understanding of electronic circuit behavior.

References

[1] Prof. Ali Notash, "Chapter 9 - RLC Circuits," EET 3086C, Dept. of Electrical & Computer Engineering Technology (ECET), Valencia College, pp. 2-16.

[2] William H. Hayt, Jr., Jack E. Kemmerly, Jamie D. Phillips, and Steven M. Durbin, "Engineering Circuit Analysis," Ninth Edition, McGraw-Hill Education, 2019, pp. 330-331.

[3] Masood Ejaz, Lab Experiments Manual for EET 3086C – Circuit Analysis, Experiment 3, Valencia College ECET Department, pp. 8-11.

Appendix A

```
% MATLAB Code for Step and Source-Free Responses

% Time vector adjusted to half a period for Circuit 1 and Circuit 2
t1 = linspace(0, 0.0024, 1000); % For Circuit 1 (half of 4.8 ms)
t2 = linspace(0, 0.0018, 1000); % For Circuit 2 (half of 3.6 ms)

% Circuit 1: Step Response
vC1_step = 2.5 + exp(-2500*t1) .* (-2.5 * cos(31524 * t1) - 0.1983 * sin(31524 * t1));
[~, peak_idx1_step] = max(vC1_step);
peak_time1_step = t1(peak_idx1_step);
peak_value1_step = vC1_step(peak_idx1_step);

figure;
plot(t1, vC1_step, 'b', 'LineWidth', 1.5);
hold on;
plot(peak_time1_step, peak_value1_step, 'ro', 'MarkerSize', 8);
yline(2.5, '--r', 'Steady State');
text(peak_time1_step, peak_value1_step, sprintf('(%0.4f s, %0.2f V)', peak_time1_step, peak_value1_step),
'VerticalAlignment', 'bottom');
title('Circuit 1 Step Response');
xlabel('Time (s)');
ylabel('Voltage (V)');
grid on;
hold off;

% Circuit 1: Source-Free Response
vC1_source_free = exp(-2500*t1) .* (2.5 * cos(31524 * t1) + 0.1983 * sin(31524 * t1));
[~, peak_idx1_source_free] = min(vC1_source_free);
peak_time1_source_free = t1(peak_idx1_source_free);
```

```
peak_value1_source_free = vC1_source_free(peak_idx1_source_free);

figure;
plot(t1, vC1_source_free, 'b', 'LineWidth', 1.5);
hold on;
plot(peak_time1_source_free, peak_value1_source_free, 'ro', 'MarkerSize', 8);
yline(0, '--r', 'Steady State');
text(peak_time1_source_free, peak_value1_source_free, sprintf('(%0.4f s, %0.2f V)', peak_time1_source_free,
peak_value1_source_free), 'VerticalAlignment', 'bottom');
title('Circuit 1 Source-Free Response');
xlabel('Time (s)');
ylabel('Voltage (V)');
grid on;
hold off;

% Circuit 2: Step Response
vR2_step = (2/3) * exp(-3333*t2) .* (1.768 * sin(9428 * t2));
[~, peak_idx2_step] = max(vR2_step);
peak_time2_step = t2(peak_idx2_step) * 1e3; % Convert to ms
peak_value2_step = vR2_step(peak_idx2_step);

figure;
plot(t2 * 1e3, vR2_step, 'b', 'LineWidth', 1.5);
hold on;
plot(peak_time2_step, peak_value2_step, 'ro', 'MarkerSize', 8);
yline(0, '--r', 'Steady State');
text(peak_time2_step, peak_value2_step, sprintf('(%0.3f ms, %0.2f V)', peak_time2_step, peak_value2_step),
'VerticalAlignment', 'bottom');
title('Circuit 2 Step Response');
xlabel('Time (ms)');
ylabel('Voltage (V)');
grid on;
hold off;

% Circuit 2: Source-Free Response
vR2_source_free = (2/3) * exp(-3333*t2) .* (-1.768 * sin(9428 * t2));
[~, peak_idx2_source_free] = min(vR2_source_free);
peak_time2_source_free = t2(peak_idx2_source_free) * 1e3; % Convert to ms
peak_value2_source_free = vR2_source_free(peak_idx2_source_free);

figure;
plot(t2 * 1e3, vR2_source_free, 'b', 'LineWidth', 1.5);
hold on;
plot(peak_time2_source_free, peak_value2_source_free, 'ro', 'MarkerSize', 8);
yline(0, '--r', 'Steady State');
text(peak_time2_source_free, peak_value2_source_free, sprintf('(%0.3f ms, %0.2f V)', peak_time2_source_free,
peak_value2_source_free), 'VerticalAlignment', 'bottom');
title('Circuit 2 Source-Free Response');
xlabel('Time (ms)');
ylabel('Voltage (V)');
grid on;
hold off;
```