Ejercicios pendiente clase 5

Sea f una función tal que

$$x^2-x+2 \leq f(x) \leq \frac{x^2-1}{x-1}$$

Para todo $x \in (0,2)-\{1\}$ ¿Qué puede decir respecto de la existencia del $\lim\limits_{x\to 1} f(x)$?

Sol:

$$x^2-x+2 \leq f(x) \leq \frac{x^2-1}{x-1} \quad / \lim_{x\to 1}$$

$$\underbrace{\lim_{x\to 1}(x^2-x+2)}_{(*)} \leq \lim_{x\to 1} f(x) \leq \underbrace{\lim_{x\to 1} \frac{x^2-1}{x-1}}_{(**)}$$

$(*)$ $\lim\limits_{x\to 1}(x^2-x+2) = 2$

$(**)$ $\lim\limits_{x\to 1}\frac{x^2-1}{x-1} = \lim\limits_{x\to 1}\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = \lim\limits_{x\to 1} x+1 = 2$

Luego,

$$\lim_{x\to 1}(x^2-x+2) \leq \lim_{x\to 1} f(x) \leq \lim_{x\to 1} \frac{x^2-1}{x-1}$$

$$2 \leq \lim_{x\to 1} f(x) \leq 2$$

Por teorema de compresión (o sandwich),

$$\lim_{x\to 1} f(x) = 2 \quad //$$

Determine

$$\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$$

Sol: $\displaystyle \lim_{x \to 0} \frac{1-\cos(x)}{x^2} \cdot \frac{1+\cos(x)}{1+\cos(x)} = \lim_{x \to 0} \frac{1-\cos^2(x)}{x^2(1+\cos(x))}$

$$= \lim_{x \to 0} \frac{\operatorname{sen}^2(x)}{x^2(1+\cos(x))}$$

$$= \lim_{x \to 0} \frac{\operatorname{sen}^2(x)}{x^2} \cdot \lim_{x \to 0} \frac{1}{1+\cos(x)}$$

$$= \lim_{x \to 0} \left[\frac{\operatorname{sen}(x)}{x}\right]^2 \cdot \lim_{x \to 0} \frac{1}{1+\cos(x)}$$

$$= \Big[\overbrace{\lim_{x \to 0} \frac{\operatorname{sen}(x)}{x}}^{1}\Big]^2 \cdot \underbrace{\lim_{x \to 0} \frac{1}{1+\cos(x)}}_{1/2}$$

$$= \frac{1}{2}$$

Determine

$$\lim_{x \to 0} \frac{\sin(x) + x}{x\cos(x)}$$

Sol:

$$\lim_{x \to 0} \frac{\sin(x)+x}{x\cos(x)} = \lim_{x \to 0} \frac{\sin(x)}{x\cos(x)} + \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}\cos(x)}$$

$$= \lim_{x \to 0} \frac{\sin(x)}{x} \cdot \lim_{x \to 0} \frac{1}{\cos(x)} + \lim_{x \to 0} \frac{1}{\cos(x)}$$

$$= 1 \cdot 1 + 1$$

$$= 2 //$$

Determine $\lim\limits_{x \to 0} \dfrac{e^{2x}-1}{x}$

Sol: $\lim\limits_{x \to 0} \dfrac{e^{2x}-1}{x} = \lim\limits_{u \to 0} \dfrac{e^{u}-1}{u/2}$ $\quad u = 2x$

$x \to 0,\ u \to 0$

$= \lim\limits_{u \to 0} 2\,\dfrac{e^{u}-1}{u}$

$= 2 \lim\limits_{u \to 0} \dfrac{e^{u}-1}{u}$

$= 2 \cdot 1$

$= 2_{//}$

Determine

$$\lim_{x \to 0} \frac{\sin(5x)}{x}$$

Sol: $\displaystyle \lim_{x \to 0} \frac{\sin(5x)}{x} = \lim_{u \to 0} \frac{\sin(u)}{u/5}$, $u = 5x$; $x \to 0$, $u \to 0$

$$= \lim_{u \to 0} 5 \frac{\sin(u)}{u}$$

$$= 5 \lim_{u \to 0} \frac{\sin(u)}{u}$$

$$= 5 \cdot 1$$

$$= 5$$

Determine

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x-1}$$

Sol: $$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x-1}=\lim_{x\to\infty}\left[\frac{\left(1+\frac{1}{x}\right)^{x}\left(1+\frac{1}{x}\right)^{x}}{\left(1+\frac{1}{x}\right)}\right]$$

$$=\frac{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\cdot\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}}{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)}$$

$$=\frac{e\cdot e}{1}$$

$$=e^{2}$$ //