Ejercicios pendientes Clase 9

P3 Sea $f(x) = \cos(x)$. Luego,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(h+x)-\cos(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(h)\cos(x) - \operatorname{sen}(x)\operatorname{sen}(h) - \cos(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x)[\cos(h)-1] - \operatorname{sen}(x)\operatorname{sen}(h)}{h}$$

$$= \lim_{h\to 0} \cos(x)\frac{[\cos(h)-1]}{h} - \lim_{h\to 0} \frac{\operatorname{sen}(x)\operatorname{sen}(h)}{h}$$

$$= \cos(x) \underbrace{\cancelto{0}{\lim_{h\to 0} \frac{\cos(h)-1}{h}}}_{*} - \operatorname{sen}(x) \cancelto{1}{\lim_{h\to 0} \frac{\operatorname{sen}(h)}{h}} = -\operatorname{sen}(x)$$

$$[\cos(x)]' = -\operatorname{sen}(x)$$

$$\operatorname{Dom}(f') = \mathbb{R}$$

$$* \quad \lim_{h\to 0} \frac{\cos(h)-1}{h} \cdot \frac{\cos(h)+1}{\cos(h)+1} = \lim_{h\to 0} \frac{\cos^2(h)-1}{h(\cos(h)+1)}$$

$$= \lim_{h\to 0} \frac{-\operatorname{sen}^2(h)}{h(\cos(h)+1)} \cdot \frac{h}{h}$$

$$= \lim_{h\to 0} \frac{-h}{\operatorname{Cos}(h+1)} \cdot \lim_{h\to 0} \left(\frac{\operatorname{sen}(h)}{h}\right)^2$$

$$= \cancelto{0}{\lim_{h\to 0} \frac{-h}{\operatorname{Cos}(h+1)}} \cdot \left\{ \cancelto{1}{\lim_{h\to 0} \frac{\operatorname{sen}(h)}{h}} \right\}^2 = 0$$