Aprendizagem 2021/22

Homework I – Group XXX

I. Pen-and-paper

1)

	y_1	y_2	C
x_1	A	0	1
x_2	B	1	1
x_3	A	1	1
x_4	A	0	1
x_5	B	0	0
x_6	B	0	0
x_7	A	1	0
x_8	B	1	0

$$d(x_1, x_2) = \text{Hamming}(x_1, x_2) + \frac{1}{2}$$

$$= (\underbrace{1}_{A \neq B} + \underbrace{1}_{1 \neq 0}) + 1/2$$

$$= \frac{5}{2}$$

...

$$\text{Weighted mode} = \frac{1}{d_1} + \frac{1}{d_2} + \dots$$

$d(x_1, x_2)$	x_1 (+)	x_2 (+)	x_3 (+)	x_4 (+)	x_5 (−)	x_6 (−)	x_7 (−)	x_8 (−)		
x_1		$5/2$	$3/2$	$1/2$	$3/2$	$3/2$	$3/2$	$5/2$	+	← $\boxed{x_1+}$ $\frac{2}{3} + \frac{2}{1} = \frac{8}{3}$ $\boxed{x_1-}$ $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2$
x_2	$5/2$		$3/2$	$5/2$	$3/2$	$3/2$	$3/2$	$1/2$	−	← $\boxed{x_2+}$ $\frac{2}{3}$ $\boxed{x_2-}$ $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{1}{3} = \frac{7}{3}$
x_3	$3/2$	$3/2$		$3/2$	$5/2$	$5/2$	$1/2$	$3/2$	−	← $\boxed{x_3+}$ $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2$ $\boxed{x_3-}$ $2 + 2/3 = 8/3$
x_4	$1/2$	$5/2$	$3/2$		$3/2$	$3/2$	$3/2$	$5/2$	+	← $\boxed{x_4+}$ $2 + 2/3 = 8/3$ $\boxed{x_4-}$ $2/3 + 2/3 + 2/3 = 2$
x_5	$3/2$	$3/2$	$5/2$	$3/2$		$1/2$	$5/2$	$3/2$	−	← $\boxed{x_5+}$ $2/3 + 2/3 + 2/3 = 2$ $\boxed{x_5-}$ $2 + 2/3 = 8/3$
x_6	$3/2$	$3/2$	$5/2$	$3/2$	$1/2$		$5/2$	$3/2$	−	← $\boxed{x_6+}$ $2/3 + 2/3 + 2/3 = 2$ $\boxed{x_6-}$ $2 + 2/3 = 8/3$
x_7	$3/2$	$3/2$	$1/2$	$3/2$	$5/2$	$5/2$		$3/2$	+	← $\boxed{x_7+}$ $2/3 + 2/3 + 2 + 2/3 = 4$ $\boxed{x_7-}$ $2/3$
x_8	$5/2$	$1/2$	$3/2$	$5/2$	$3/2$	$3/2$	$3/2$		+	← $\boxed{x_8+}$ $2 + 2/3 = 8/3$ $\boxed{x_8-}$ $2/3 + 2/3 + 2/3 = 2$

⇓

$$\text{recall} = \frac{TP}{TP + FN} = \frac{\overbrace{2}^{x_1, x_4}}{2 + \underbrace{2}_{x_7, x_8}} = \frac{1}{2}$$

2)

	y_1	y_2	y_3	C
x_1	A	0	1.2	1
x_2	B	1	0.8	1
x_3	A	1	0.5	1
x_4	A	0	0.9	1
x_5	B	0	1	0
x_6	B	0	0.9	0
x_7	A	1	1.2	0
x_8	B	1	0.8	0
x_9	B	0	0.8	1

- $\{y_1, y_2\} \perp\!\!\!\perp \{y_3\} \Rightarrow P(y_1, y_2, y_3 | c) = P(y_1, y_2 | c) \times P(y_3 | c)$

- Bayesian Theorem: $P(c|y) = \dfrac{P(y|c)P(c)}{P(y)}$

Learn a Bayesian Classifier $\Rightarrow$ Compute it's parameters

P(C)

$P(C=1) = 5/9 \quad P(C=0) = 4/9$

$P(y_1, y_2 | C)$

$P(y_1 = A, y_2 = 0 | C=1) = 2/5 = 0.4$

$P(y_1 = A, y_2 = 1 | C=1) = 1/5 = 0.2$

$P(y_1 = B, y_2 = 0 | C=1) = 1/5 = 0.2$

$P(y_1 = B, y_2 = 1 | C=1) = 1/5 = 0.2$

$P(y_1 = A, y_2 = 0 | C=0) = 0$

$P(y_1 = A, y_2 = 1 | C=0) = 1/4 = 0.25$

$P(y_1 = B, y_2 = 0 | C=0) = 2/4 = 0.5$

$P(y_1 = B, y_2 = 1 | C=0) = 1/4 = 0.25$

$P(y_3 | C)$

$\mu_1 / \sigma_1 \rightarrow$ mean/standard deviation of observations where $C=1$

$\mu_0 / \sigma_0 \rightarrow$ mean/standard deviation of observations where $C=0$

$$\mu_1 = \frac{1.2 + 0.8 + 0.5 + 0.9 + 0.8}{5} = 0.84 \qquad \sigma_1 = \sqrt{\frac{(1.2-0.84)^2 + (0.8-0.84)^2 + (0.5-0.84)^2 + (0.9-0.84)^2 + (0.8-0.84)^2}{5-1}} \simeq 0.251$$

$$\mu_0 = \frac{1 + 0.9 + 1.2 + 0.8}{4} = 0.975 \qquad \sigma_0 = \ldots \simeq 0.171$$

$$P(y_3 | C=1) \sim N(y_3 | \mu_1, \sigma_1^2) = \frac{1}{0.251\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2 \times 0.063}(y_3 - 0.84)^2\right)$$

$$\simeq 1.5894\, e^{-7.9365(y_3 - 0.84)^2}$$

$$P(y_3 | C=0) \sim N(y_3 | \mu_0, \sigma_0^2) = \frac{1}{0.171\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2 \times 0.2924}(y_3 - 0.975)^2\right)$$

$$\simeq 2.333\, e^{-17.1(y_3 - 0.975)^2}$$

3) under MAP assumption, $P(c=1 \mid \theta_i) = \dfrac{P(\theta_i \mid c=1)P(c=1)}{P(\theta_i)}$

$\theta_1 = (A, 1, 0.8)$ $\theta_2 = (B, 1, 1)$ $\theta_3 = (B, 0, 0.9)$

$$P(\theta_1 \mid c=1) = P(A,1 \mid c=1)\, P(0.8 \mid c=1)$$
$$= 0.2 \times N(0.8 \mid \mu_1, \sigma_1^2)$$
$$= 0.2 \cdot 1.56934$$
$$= 0.31387$$

$$P(\theta_1 \mid c=0) = P(A,1 \mid c=0)\, P(0.8 \mid c=0)$$
$$= 0.25 \times N(0.8 \mid \mu_0, \sigma_0^2)$$
$$\simeq 0.25 \cdot 1.38191$$
$$= 0.34548$$

$$P(\theta_2 \mid c=1) = P(B,1 \mid c=1)\, P(1 \mid c=1)$$
$$= 0.2 \times N(1 \mid \mu_1, \sigma_1^2)$$
$$= 0.2 \cdot 1.29717$$
$$= 0.25943$$

$$P(\theta_2 \mid c=0) = P(B,1 \mid c=0)\, P(1 \mid c=0)$$
$$= 0.25 \times N(1 \mid \mu_0, \sigma_0^2)$$
$$\simeq 0.25 \cdot 2.3082$$
$$= 0.577$$

$$P(\theta_3 \mid c=1) = P(B,0 \mid c=1)\, P(0.9 \mid c=1)$$
$$= 0.2 \times N(0.9 \mid \mu_1, \sigma_1^2)$$
$$= 0.2 \cdot 1.54463$$
$$= 0.30893$$

$$P(\theta_3 \mid c=0) = P(B,0 \mid c=0)\, P(0.9 \mid c=0)$$
$$= 0.5 \times N(0.9 \mid \mu_0, \sigma_0^2)$$
$$\simeq 0.5 \cdot 2.1190$$
$$= 1.0595$$

$$P(c=1 \mid \theta_1) = \frac{P(\theta_1 \mid c=1)P(c=1)}{P(\theta_1)} = \frac{0.31387 \cdot 5/9}{P(\theta_1)} = \frac{0.17434}{P(\theta_1)}$$

$$P(c=0 \mid \theta_1) = \frac{P(\theta_1 \mid c=0)P(c=0)}{P(\theta_1)} = \frac{0.34548 \cdot 4/9}{P(\theta_1)} = \frac{0.15355}{P(\theta_1)}$$

$$P(c=1 \mid \theta_1) = \frac{0.17434}{0.17434 + 0.15355}$$
$$= 0.53170$$

$$P(c=1 \mid \theta_2) = \frac{P(\theta_2 \mid c=1)P(c=1)}{P(\theta_2)} = \frac{0.25943 \times 5/9}{P(\theta_2)} = \frac{0.14413}{P(\theta)}$$

$$P(c=0 \mid \theta_2) = \frac{P(\theta_2 \mid c=0)P(c=0)}{P(\theta)} = \frac{0.577 \times 4/9}{P(\theta_2)} = \frac{0.25644}{P(\theta_2)}$$

$$P(c=1 \mid \theta_2) = \frac{0.14413}{0.14413 + 0.25644} = 0.35981$$

$$P(c=1 \mid \theta_3) = \frac{P(\theta_3 \mid c=1)P(c=1)}{P(\theta_3)} = \frac{0.30893 \times 5/9}{P(\theta_3)} = \frac{0.17163}{P(\theta_3)}$$

$$P(c=0 \mid \theta_3) = \frac{P(\theta_3 \mid c=0)P(c=0)}{P(\theta_3)} = \frac{1.0595 \times 4/9}{P(\theta_3)} = \frac{0.47089}{P(\theta_3)}$$

$$P(c=1 \mid \theta_3) = \frac{0.17163}{0.17163 + 0.47089} = 0.26712$$

Therefore, the posterior probabilities of c = Positive (given $\theta_1, \theta_2, \theta_3$) are:

$$P(c=1 \mid \theta_1) = 0.53170$$
$$P(c=1 \mid \theta_2) = 0.35981$$
$$P(c=1 \mid \theta_3) = 0.26712$$

4) $\theta = 0.5$

$$\left.\begin{aligned} P(c=1 \mid \theta_1) &= 0.53170 > 0.5 \Rightarrow \text{Classify as Positive } ✓ \\ P(c=1 \mid \theta_2) &= 0.35981 < 0.5 \Rightarrow \text{Classify as Negative } ✗ \\ P(c=1 \mid \theta_3) &= 0.26712 < 0.5 \Rightarrow \text{Classify as Negative } ✓ \end{aligned}\right\} \text{Accuracy} = 2/3$$

$\theta = 0.3$

$$\left.\begin{aligned} P(c=1 \mid \theta_1) &= 0.53170 > 0.3 \Rightarrow \text{Classify as Positive } ✓ \\ P(c=1 \mid \theta_2) &= 0.35981 > 0.3 \Rightarrow \text{Classify as Positive } ✓ \\ P(c=1 \mid \theta_3) &= 0.26712 < 0.3 \Rightarrow \text{Classify as Negative } ✓ \end{aligned}\right\} \text{Accuracy} = 1$$

$\theta = 0.7$

$$\left.\begin{aligned} P(c=1 \mid \theta_1) &= 0.53170 < 0.7 \Rightarrow \text{Classify as Negative } ✗ \\ P(c=1 \mid \theta_2) &= 0.35981 < 0.7 \Rightarrow \text{Classify as Negative } ✗ \\ P(c=1 \mid \theta_3) &= 0.26712 < 0.7 \Rightarrow \text{Classify as Negative } ✓ \end{aligned}\right\} \text{Accuracy} = 1/3$$

Therefore, the decision threshold of $\theta = 0.3$ optimizes test accuracy.