$$F(x,y,z) = (2x+3y)^{\cos(z)}$$

$$\frac{\partial f}{\partial x} = e^{\cos(z)\ln(2x+3y)'} = \frac{2\cos(z) \cdot e^{\cos(z)\cdot \ln(2x+3y)}}{2x+3y}$$

$$\frac{\partial f}{\partial x^2} = \frac{2\cdot\cos(z)\cdot(2x+3y)^{\cos(z)}}{2x+3y}$$

$$\frac{\partial f}{\partial x^2} = 2\cos(z)\cdot(2x+3y)^{\cos(z)}\cdot(2x+3y)^{-1}$$

$$\frac{\partial f}{\partial x^2} = 2\cdot\cos(z)\cdot(2x+3y)^{\cos(z)-1}$$

$$2\cos(z)\cdot\frac{\partial f}{\partial x^2} = \left((2x+3y)^{\cos(z)-1}\right)'$$

$$2\cdot\cos(z)\cdot\frac{\partial f}{\partial x^2} = (\cos(z)-1)\cdot(2x+3y)^{\cos(z)-2}\cdot(2x+3y)'$$

$$\frac{\partial f}{\partial x^2} = 2\cdot\cos(z)\cdot(\cos z-1)\cdot(2x+3y)^{\cos(z)-2}\cdot(2x+3y)'$$

$$\frac{\partial f}{\partial x^2} = 4\cos(z)\cdot(\cos(z)-1)\cdot(2x+3y)^{\cos(z)-2}$$

$f_x = \dfrac{2 \cos(z) \cdot e^{\cos(z) \cdot \ln(2x+3y)}}{2x+3y} = 2\cos(z) \cdot (2x+3y)^{\cos(z)-1}$

$f_y = \left((2x+3y)^{\cos(z)}\right)' \rightarrow$ pela regra da cadeia

$= \cos(z) \cdot (2x+3y)^{\cos(z)-1} \cdot (2x+3y)'$

$f_y = 3\cos(z) \cdot (2x+3y)^{\cos(z)-1}$

$f_z = e^{\cos(z) \cdot \ln(2x+3y)'}$

$f_z = e^{\cos(z) \cdot \ln(2x+3y)} \cdot (\cos(z))' \cdot \ln(2x+3y)$

$f_z = (2x+3y)^{\cos(z)} \cdot \ln(2x+3y) \cdot (-\operatorname{sen}(z) \cdot 1)$

$f_{xy} = \left(2 \cdot \cos(z) \cdot (2x+3y)^{\cos(z)-1}\right)'$

$f_{xy} = 2 \cdot \cos(z) \cdot (\cos(z)-1) \cdot (2x+3y)^{\cos(z)-2} \cdot 3$

$f_{xy} = 6 \cdot \cos(z) \cdot (\cos(z)-1) \cdot (2x+3y)^{\cos(z)-2}$

$f_{yx} = \left(3 \cdot \cos(z) \cdot (2x+3y)^{\cos(z)-1}\right)'$

$f_{yx} = 3 \cdot \cos(z) \cdot (\cos(z)-1) \cdot (2x+3y)^{\cos-2} \cdot (2x+3y)'$

$f_{yx} = 6 \cdot \cos(z) \cdot (\cos(z)-1) \cdot (2x+3y)^{\cos-2}$

$f_{yz} = 3\cos z \cdot (2x+3y)^{\cos(z)-1}$

$f_{yz} = 3\cos z' \cdot (2x+3y)^{\cos(z)-1} + 3\cos z \cdot (2x+3y)^{(\cos -1)'}$

$f_{yz} = 3 \cdot -\text{sen}(z) \cdot (2x+3y)^{\cos(z)-1} + 3 \cdot \cos(z) \cdot (-\text{sen}(z)) \cdot \ln(2x+3y) \cdot (2x+3y)^{\cos z}$

$f_{yz} = -3\,\text{sen}(z)\left((2x+3y)^{\cos(z)-1} + \cos(z) \cdot \ln(2x+3y) \cdot (2x+3y)^{\cos z - 2}\right)$

$f_{zy} = \left((2x+3y)^{\cos(z)} \cdot \ln(2x+3y)\right)' \cdot (-\text{sen}(z))$

$f_{zy} = \left(3 \cdot \cos(z) \cdot (2x+3y)^{\cos(z)-1} \cdot \ln(2x+3y) + 3 \cdot (2x+3y)^{-1} \cdot (2x+3y)^{\cos(z)}\right.$

$f_{zy} = -3\,\text{sen}(z)\left(\cos(z) \cdot \ln(2x+3y) \cdot (2x+3y)^{\cos(z)-1} + (2x+3y)^{\cos(z)-2}\right)$

$f_{xz} = 2 \cdot \cos(z) \cdot (2x+3y)^{\cos(z)-1}$

$f_{xz} = \cos(z)' \cdot (2x+3y)^{\cos(z)-1} + 2 \cdot \cos(z) \cdot (\cos z)' \cdot \ln(2x+3y) \cdot (2x+3y)^{\cos z}$

$f_{xz} = -2 \cdot \text{sen}(z) \cdot (2x+3y)^{\cos(z)-1} + (-2\,\text{sen}(z)) \cdot \cos(z) \cdot \ln(2x+3y) \cdot (2x+3y)^{\cos z - 2}$

$f_{xz} = -2 \cdot \text{sen}(z)\left((2x+3y)^{\cos(z)-1} + \cos(z) \cdot \ln(2x+3y) \cdot (2x+3y)^{\cos(z)-2}\right)$

$f_{zx} = (2x+3y)^{\cos(z)} \cdot \ln(2x+3y) \cdot (-\text{sen}(z))$

$f_{zx} = 2\cos(z) \cdot (2x+3y)^{\cos z - 1} \cdot \ln(2x+3y) + 2 \cdot (2x+3y)^{-1} \cdot (2x+3y)^{\cos z - 1}$

$f_{zx} = -2\,\text{sen}(z) \cdot \left(\ln(2x+3y) \cdot \cos(z) \cdot (2x+3y)^{\cos(z)-1} + (2x+3y)^{\cos(z)-2}\right)$

Dessa forma, podemos afirmar que as derivadas mistas são iguais.

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