Q 12

data

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$$ | EDO 2ª ordem

$$\downarrow$$

$$L y'' + R y' + \frac{1}{C} y = E_0 \operatorname{sen} wt$$

$E_0 = 1\,V$
$R = 0{,}025$
$C = 0{,}25$
$L = 1$
$w^2 = 3{,}5$

$$y = y_h(t) + y_p(t)$$

I) $y_h(t) = ?$

Eq. característica:

$$\lambda^2 + 0{,}025\lambda + 24 = 0$$

$$\lambda_1 = j2$$
$$\lambda_2 = -j2$$

$$y_h(t) = K_1 \cos(2t) + K_2 \operatorname{sen}(2t)$$

II) $y_p(t) = ?$

p/ $G(x) = \operatorname{sen}(wt)$

Solução $\Rightarrow$ $y_p(t) = A\cos(wt) + B\operatorname{sen}(wt)$

$$y'(t) = -A\operatorname{sen}(wt).w + B.w\cos(wt)$$

$$y''(t) = -Aw^2\cos(wt) - Bw^2\operatorname{sen}(wt)$$

data

$$\ddot{} - Aw^2\cos(wt) - Bw^2(\operatorname{sen} wt) - 0{,}025Aw\operatorname{sen}(wt) + 0{,}025Bw\cos(wt) + 4A\cos(wt) + 4B\operatorname{sen}(wt) = \operatorname{sen}(wt)$$

$$\cos(wt)\left[-A.3{,}5 + 0{,}025\sqrt{3{,}5}B + 4A\right] + \operatorname{sen}(wt)\left[-B3{,}5 - 0{,}025\sqrt{3{,}5}A + 4B\right]$$

$$\begin{cases} 0{,}5A = -0{,}025\sqrt{3{,}5}\,B \\ 0{,}5B = 1 + 0{,}025\sqrt{3{,}5}\,A \end{cases}$$

$$0{,}5B = 1 + 0{,}025\sqrt{3{,}5}\,A$$

$$A = \frac{-0{,}025\sqrt{3{,}5}B}{0{,}5} \xrightarrow{\text{em (II)}} 0{,}5B = 1 - \frac{0{,}025^2 \cdot 3{,}5 \cdot B}{0{,}5}$$

$$B = \frac{1}{0{,}5} = \boxed{2}$$

$$\therefore$$

$$A = 0{,}1$$

$$y_p(t) = 0{,}1\cos(wt) + 2\operatorname{sen}(wt)$$

$$y(t) = q(t) = 0{,}1\cos(wt) + 2\operatorname{sen}(wt) + \cos(2wt) + \operatorname{sen}(2wt)$$

$$w = \sqrt{3{,}5} \Rightarrow$$

$$\boxed{q(t) = 0{,}1\cos(1{,}87t) + 2\operatorname{sen}(1{,}87t) + K_1\cos(2t) + \operatorname{sen}K_2(2t)}$$

descobrindo K_1 e K_2:

$q(0) = 0 \rightarrow K_1 = 0$

$i(0) = 0 \rightarrow -K_1.2.\operatorname{sen}(2t) + 2K_2\cos(2t)$

$2K_2 = 0$

$K_2 = 0$ //