On lower bounds for the spectrum width for the routing and spectrum assignment problem

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Abstract

- The routing and spectrum assignment problem.
- Lower bound comparison.
- Frameworks.

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   - Implementation
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RSA problem

Notation :

- Network G = (V ;E)
- Set $\mathcal{D}$ of demands between pairs o_k and d_k of nodes in G with w_k of required frequency slots.
- Optical spectrum $S = \{1, \ldots, \bar{s}\}$
- P_k route from o_k to d_k valued by w_k for each $k \in \mathcal{D}$.
- Routing of $\mathcal{D}$: $\mathcal{P} = \{P_k : k \in \mathcal{D}\}$
- Spectrum assignment : $\mathcal{S} = \{S_k : k \in \mathcal{D}\}$.

RSA problem

There are 3 conditions in the RSA problem :

- Spectrum continuity
- Spectrum contiguity
- Non-overlapping spectrum

Edge intersection graph

The spectrum assignment can be interpreted as an interval coloring of the edge intersection graph :

- Each path $P_k \in \mathcal{P}$ becomes a node k of $I(\mathcal{P})$ and two nodes k and k' are joined by an edge if the corresponding paths $P_k, P_{k'}$ in G are in conflict as they share an edge.
- Any interval coloring in this graph $I(\mathcal{P})$ weighted with the demands w_k correctly solves the spectrum assignment.

Given G and $\mathcal{D}$, the minimum spectrum width of any solution equals $\chi_I(G, \mathcal{D}) = \min\{\chi_I(I(\mathcal{P}), \mathbf{w}) : \mathcal{P}$ possible routing of demands $\mathcal{D}$ in G$\}$.

Lower bound chain

Establishing a chain of lower bounds for $\chi_I(G, \mathcal{D})$:

- $s_R(G, \mathcal{D}) = \min\{w_{\mathcal{P}}(e) : \mathcal{P}$ possible routing of demands $\mathcal{D}$ in G$\}$.
- $\omega(G, \mathcal{D}) = \min\{\omega(I(\mathcal{P}), \mathbf{w}) : \mathcal{P}$ possible routing of demands $\mathcal{D}$ in G$\}$.
- $\chi(G, \mathcal{D}) = \min\{\chi(I(\mathcal{P}), \mathbf{w}) : \mathcal{P}$ possible routing of demands $\mathcal{D}$ in G$\}$.

So, we have

$$s_R(G, \mathcal{D}) \leq \omega(G, \mathcal{D}) \leq \chi(G, \mathcal{D}) \leq \chi_I(G, \mathcal{D}). \tag{1}$$

Example

Consider the following instance of the RSA problem with the optical network G shown in Fig. 1 and the following set $\mathcal{D}$ of demands :

k	$o_k \to d_k$	w_k	routing P_k
1	$a \to c$	1	$a \to b \to c$
2	$c \to e$	3	$c \to b \to d \to e$
3	$e \to f$	3	$e \to d \to f$
4	$f \to g$	3	$f \to d \to g$
5	$g \to h$	3	$g \to d \to h$
6	$h \to a$	3	$h \to d \to b \to a$

As the network G is a tree, there is a unique routing $\mathcal{P}$ as indicated above.

Example

Since the load of all edges incident to node d equals 6, $s_R(G, \mathcal{D}) = 6$ follows. The edge intersection graph $I(\mathcal{P})$ of the routing is also shown in Fig. 1. The nodes $1, 2, 6$ form a clique of weight 7, hence we have $\omega(G, \mathcal{D}) = 7$.

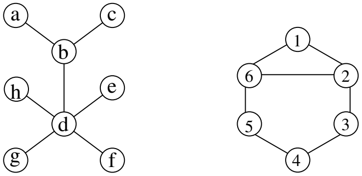


FIGURE 1 – The network G and $I(\mathcal{P})$ of the routing used in Example 1.

Example

An optimal weighted coloring of $I(\mathcal{P})$ using 8 colors is shown in Fig. 2, but any interval coloring of $I(\mathcal{P})$ needs at least 9 colors, see again Fig. 2. Hence, there are instances of the RSA problem with a gap between any two parameters from the chain.

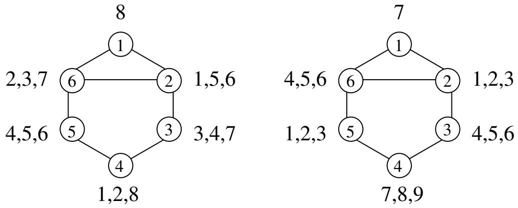


FIGURE 2 – Minimal weighted coloring and interval coloring of $I(\mathcal{P})$ in Example 1

Superperfect graphs

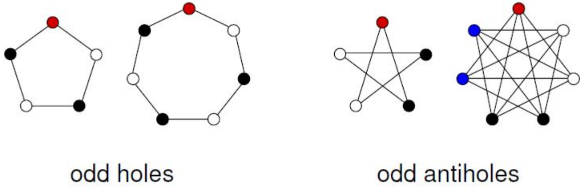
- A graph is superperfect if and only if weighted clique number and interval chromatic number coincide for all possible non-negative integral node weights.
- Every superperfect graph is perfect and perfect graphs G satisfy $\omega(G, \mathbf{w}) = \chi_I(G, \mathbf{w})$ for every $(0, 1)$-weighting $\mathbf{w}$ of its nodes (as in this case any interval coloring is a usual coloring).

Non-superperfect graphs

The basics minimal non-superperfect graphs

The basic minimal non-superperfect are as follows :

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,



Non-superperfect graphs

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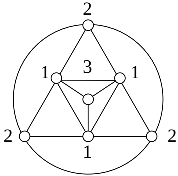
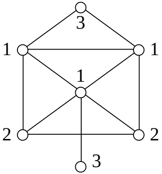
- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ and $\overline{A}_2$,





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- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
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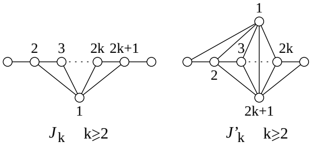
- the graphs J_k for $k \geq 2$ and J'_k for $k \geq 3$,

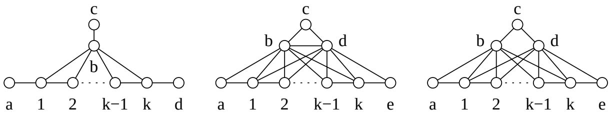


Non-superperfect graphs

The basics minimal non-superperfect graphs

The basic minimal non-superperfect are as follows :

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ and $\overline{A}_2$,
- the graphs J_k for $k \geq 2$ and J'_k for $k \geq 3$,
- the complements of D_k for $k \geq 2$ and of E_k, F_k for $k \geq 1$.



D_k k⩾2 E_k k⩾1 F_k k⩾1

Non-superperfect graphs

The basics minimal non-superperfect graphs

The basic minimal non-superperfect are as follows :

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ and $\overline{A}_2$,
- the graphs J_k for $k \geq 2$ and J'_k for $k \geq 3$,
- the complements of D_k for $k \geq 2$ and of E_k, F_k for $k \geq 1$.

The complete list of all minimal non-superperfect graphs is not yet known.

Plan

If the network is a path

- There exists exactly one (o_k, d_k)-path P_k in P for every traffic demand.
- The RSA problem is reduced to the spectrum assignment part.
- The edge intersection graph $I(\mathcal{P})$ of the (unique) routing $\mathcal{P}$ of the demands is an *interval graph*.

If the network is a path

Corollary

If the network G is a *path*, then $s_R(G, \mathcal{D}) = \omega(G, \mathcal{D})$ holds for any set $\mathcal{D}$ of demands.

Corollary

If the network G is a *path*, then $\omega(G, \mathcal{D}) = \chi(G, \mathcal{D})$ holds for any set $\mathcal{D}$ of demands.

If the network is a path

Theorem (Kerivin and Wagler)

If $\mathcal{P}$ is a set of paths in a *path*, then $I(\mathcal{P})$ can contain the graphs

- J_k for all $k \geq 2$, J'_k for all $k \geq 3$,
- $\overline{E}_2$,

but none of the other minimal non-comparability non-superperfect graphs.

If the network is a tree

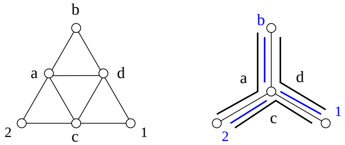
- There exists exactly one (o_k, d_k)-path P_k in P for every traffic demand.
- The RSA problem is reduced to the spectrum assignment part.
- The edge intersection graph $I(\mathcal{P})$ of the (unique) routing $\mathcal{P}$ of the demands is an *EPT graph*.

If the network is a tree

Cliques in edge intersection graphs of paths in a tree : For any claw $K = K_{1,3}$ in T, let

$$\mathcal{P}(K) = \{P \in \mathcal{P} : P \text{ contains two edges of } K\},$$

then $\mathcal{P}(K)$ corresponds to a clique in $I(\mathcal{P})$, called a *claw-clique*.



Theorem (Golumbic and Jamison)

Any maximal clique of the edge intersection graph of paths in a tree T corresponds to an edge-clique $\mathcal{P}(e)$ for some edge e of T or to a claw-clique $\mathcal{P}(K)$ for some claw $K = K_{1,3}$ in T.

If the network is a tree

Superperfection of edge intersection graphs of paths in a tree :

Theorem (Kerivin and Wagler)

If $\mathcal{P}$ is a set of paths in a *tree*, then $I(\mathcal{P})$ can contain $\overline{A}_1$, $\overline{A}_2$ and

- odd holes C_{2k+1} for $k \geq 2$, but no odd antiholes $\overline{C}_{2k+1}$ for $k \geq 3$,
- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- the graphs $\overline{D}_2$, $\overline{D}_3$, $\overline{E}_1$, $\overline{E}_2$, $\overline{E}_3$, $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, but none of $\overline{D}_k$, $\overline{E}_k$, $\overline{F}_k$ for $k \geq 4$.

If the network is a cycle

- There exist exactly two (u, v)-paths P_{uv} in C for every traffic demand and the number of possible routings is $2^{|\mathcal{D}|}$.
- The edge intersection graphs of paths in a cycle are *circular-arc-graphs*

There are only two types of cliques in a circular-arc graph :

- subsets $\mathcal{P}'$ of paths in a cycle $C = (V, E)$ that all share a same edge $e \in E$ (edge-cliques).
- pairwise intersect in some edge and whose union covers all edges in E, but whose intersection is empty, that we call *cycle-cliques*.

If the network is a cycle

For illustration, see a path representation of $\overline{D}_2$ in Fig. 3.

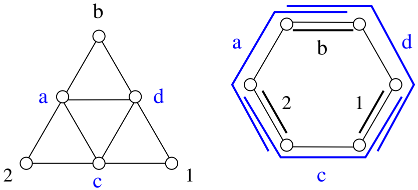


FIGURE 3 – The graph $\overline{D}_2 = I(\mathcal{P})$ with $\mathcal{P}$ in a cycle ; $1cd$, $2ac$ and abd are edge-cliques, acd is a cycle-clique.

If the network is a cycle

Superperfection of edge intersection graphs of paths in a cycle :

Theorem

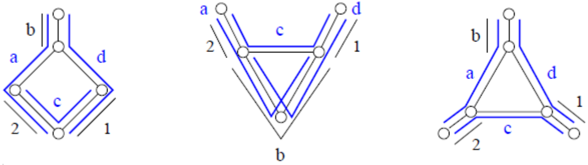
If $\mathcal{P}$ is a set of paths in a *cycle*, then *circular-arc graph* $I(\mathcal{P})$ can contain $\overline{A}_1$ but not $\overline{A}_2$ and

- all odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{D}_2$, $\overline{D}_3$, $\overline{D}_4$, but not the graphs $\overline{D}_k$ for $k \geq 5$,
- $\overline{E}_1$ and $\overline{E}_2$, but not the graphs $\overline{E}_k$ for $k \geq 3$,
- $\overline{F}_2$, but not $\overline{F}_1$ neither the graphs $\overline{F}_k$ for $k \geq 3$.

If the network is a 1-tree

- Graphs obtained from a tree by adding one edge.
- Present characteristics from trees and cycles.

Cliques in edge intersection graphs of paths in a 1-tree :
Edge-cliques, claw-cliques, cycle-cliques and, in addition, pan-cliques, bull-cliques and net-cliques.



If the network is a 1-tree

Superperfection of edge intersection graphs of paths in a 1-tree :
If the network is a 1-tree, then all minimal non-superperfect graphs occuring in $I(\mathcal{P})$ when the network is a tree or a cycle can clearly be present.

Theorem

If $\mathcal{P}$ is a set of paths in a *1-tree*, then $I(\mathcal{P})$ can contain $\overline{A}_1$, $\overline{A}_2$ and

- all odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{D}_2$, $\overline{D}_3$, $\overline{D}_4$, but not the graphs $\overline{D}_k$ for $k \geq 5$,
- the graphs $\overline{E}_1$, $\overline{E}_2$, $\overline{E}_3$, $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, but none of $\overline{E}_k$, $\overline{F}_k$ for $k \geq 4$.

General Case

For modern optical networks we can not assure that specific structures will describe the graph G.

Theorem (Kerivin and Wagler)

All minimal non-comparability non-superperfect graphs can occur in edge intersection graphs $\mathrm{I}(\mathcal{P})$ of sets $\mathcal{P}$ of paths in optical networks G.

General Case

Summary :
$s_R(G, \mathcal{D}) \leq \omega(G, \mathcal{D}) \leq \chi(G, \mathcal{D}) \leq \chi_I(G, \mathcal{D})$.

Problem

How large can be the large between any two values in the chain ?

Plan

Minimum cost multi commodity flow problem

To compute the $s_R(G, \mathcal{D})$, we can use the min-cost multi-commodity flow problem.

Parameters :

G'(V',E') : V' = V and E' = {l,l' : e ∈ E} the directed arcs where l and l' are in opposites directions and share the same vertices of e.

$\mathcal{D}$: Set of demands.

Variables :

Cap : the maximal sum of the flow passing by the pair of arcs l and l'.

$$f_k(l) = \begin{cases} 1 & \text{if } l \in P_k \\ 0 & \text{otherwise} \end{cases}$$

Model

$$Minimize\ z = Cap$$

s.t.

$$\sum_{l \in \delta^{-}(o_k)} f_k(l) = 1 \qquad \forall\ k \in \mathcal{D} \tag{2}$$

$$\sum_{l \in \delta^{+}(d_k)} f_k(l) = 1 \qquad \forall\ k \in \mathcal{D} \tag{3}$$

$$\sum_{l \in \delta^{-}(v)} f_k(l) = \sum_{l \in \delta^{+}(v)} f_k(l) \qquad \forall\ k \in \mathcal{D}\ \ \forall\ v \neq o_k, d_k \tag{4}$$

$$\sum_{k \in \mathcal{D}} w_k f_k(l) + \sum_{k \in \mathcal{D}} w_k f_k(l') \leq Cap \qquad \forall\ pairs\ l, l' \in E' \tag{5}$$

$$\sum_{l \in E'} f_k(l)\bar{l}_l \leq \bar{l}_k \qquad \forall\ k \in \mathcal{D} \tag{6}$$

$$f_k(l) \in \{0, 1\} \qquad \forall\ k \in \mathcal{D}\ \forall\ arcs\ l \in E' \tag{7}$$

Results

- Minimum capacity that has to be installed on the links of the network for the most balanced routing of the demands.
- Solution for the routing problem.
- Lower bound for the RSA problem.

Edge-Path formulation

To do the assignment of slots of each demand, we can use the Edge-Path formulation.

Parameters :

G=(V,E) : Graph of the original network.

$\mathcal{D}$: Set of demands.

$\bar{\mathcal{P}}$: Set of pre-computed paths.

Edge-Path formulation

Variables :

χ_I : Interval chromatic number.

$$y_{kp} = \begin{cases} 1 & \text{if } P_k \ \in \mathcal{P} \\ 0 & \text{otherwise} \end{cases}$$

z_{ks} : for each demand $k \in \mathcal{D}$, z is the slot where the spectrum assignment ends

$$x_{ke} = \begin{cases} 1 & \text{if } e \in P_k \\ 0 & \text{otherwise} \end{cases}$$

$$t_{kes} = \begin{cases} 1 & \text{if } s \in S_k, \ e \in P_k \\ 0 & \text{otherwise} \end{cases}$$

Model

$$Minimize\ z = \chi_I$$

s.t.

$$\sum_{P \in \bar{\mathcal{P}}_k} y_{kp} = 1 \qquad \forall\ k \in \mathcal{D} \tag{8}$$

$$\sum_{P \in \bar{\mathcal{P}}_k : e \in P} y_{kp} = x_{ke} \qquad \forall\ k \in \mathcal{D},\ e \in E \tag{9}$$

$$\sum_{e \in e} x_{ke} \bar{l}_e \leq \bar{l}_k \qquad \forall\ k \in \mathcal{D} \tag{10}$$

$$\sum_{s \in S : s >= w_k} z_{ks} = 1 \qquad \forall\ k \in \mathcal{D} \tag{11}$$

$$\sum_{s=0}^{w_k} z_{ks} = 0 \qquad \forall\ k \in \mathcal{D} \tag{12}$$

Model

$$\sum_{i=0}^{w_k} z_{k,s+i} \ + x_{ke} \ <= \ t_{kes} + 1 \qquad \forall \ k \in \mathcal{D}, \ e \in E, s \in S \tag{13}$$

$$\sum_{s \in S} t_{kes} \ = \ w_k x_{ke} \qquad \forall \ k \in \mathcal{D}, \ e \in E \tag{14}$$

$$\sum_{k \in \mathcal{D}} t_{kes} \ \leq \ 1 \qquad \forall \ e \in E, \ s \in S \tag{15}$$

$$\sum_{s \in S} (s+1) z_{ks} \ \leq \ \chi_I \qquad \forall \ k \in \mathcal{D}, \ s \in S \tag{16}$$

$$y_{kp} \in \{0,1\} \qquad \forall \ k \in \mathcal{D}, \ \forall \ P \in \bar{\mathcal{P}} \tag{17}$$

$$x_{ke} \in \{0,1\} \qquad \forall \ k \in \mathcal{D}, \ \forall \ e \in E \tag{18}$$

$$z_{ks} \in \{0,1\} \qquad \forall \ k \in \mathcal{D}, \ \forall \ s \in S \tag{19}$$

$$t_{kes} \in \{0,1\} \qquad \forall \ k \in \mathcal{D}, \ \forall \ e \in E, \forall \ s \in S \tag{20}$$

Framework for column generation for an edge-path formulation

Input : an instance $(G, \mathcal{D}, \bar{s})$

Output : a solution $(\mathcal{P}^*, \mathcal{S}^*)$ with span $\chi_I(G, \mathcal{D})$ or a certificate for infeasibility

1. Initialize
   - an upper bound by $b_{up} = \bar{s}$,
   - a set of considered paths by $\bar{\mathcal{P}} = \emptyset$,
   - a set of previously used routings by $\mathcal{R} = \emptyset$,
   - a set of non-edge cliques by $\mathcal{Q} = \emptyset$.

Framework for column generation for an edge-path formulation

2. Compute a min-cost multi-commodity flow f in an auxiliary network G_f (constructed from G by replacing every edge $e = uv$ of G by a pair of arcs $a = uv, \bar{a} = vu$). If no feasible solution has been found then
   - return “instance infeasible (due to transmission reach constraints)”

   Else (set a lower bound) : let $b_{low} = s_R(G, \mathcal{D})$. If $b_{low} > \bar{s}$ then
   - return "instance infeasible (as $s_R(G, \mathcal{D}) > \bar{s}$)"

Framework for column generation for an edge-path formulation

3. Determine from this flow f the according routing $\mathcal{P}_f$ and let $\bar{\mathcal{P}} = \bar{\mathcal{P}} \cup \mathcal{P}_f$.
   Launch the edge-path formulation with $\bar{\mathcal{P}}$ as set of paths enhanced by
   - forbidden clique constraints for all $Q \in \mathcal{Q}$

$$\sum_{P_k \in \mathcal{P}_Q} x_{P_k} \leq |Q| - 1$$

   - forbidden routing constraints for all $\mathcal{P} \in \mathcal{R}$

$$\sum_{P_k \in \mathcal{P}} x_{P_k} \leq |\mathcal{D}| - 1$$

   as minimum violation problem where the use of frequency slots
   - within $[1, b_{low}]$ does not cause any costs
   - within $[b_{low} + 1, b_{up}]$ is penalized

   and the objective is to minimize penalties.

Framework for column generation for an edge-path formulation

If no feasible solution has been found : let $\mathcal{R} = \mathcal{R} \cup \{\mathcal{P}_f\}$ and go to Step 5.

4. Evaluate solution $(\mathcal{P}, \mathcal{S})$ found :

   If $(\mathcal{P}, \mathcal{S})$ has objective function value $z^* = 0$ then
   - return $(\mathcal{P}, \mathcal{S})$ as optimal solution

   Else (we have $z^* > 0$ and try to improve b_{up}) :
   - let $\mathcal{R} = \mathcal{R} \cup \{\mathcal{P}_f, \mathcal{P}\}$ and identify max-used slot $s^* = s_{max}(\mathcal{S})$,
   - if $b_{up} > s^*$ then let $b_{up} = s^*$, keep $(\mathcal{P}, \mathcal{S})$ as currently best solution $(\mathcal{P}^*, \mathcal{S}^*)$

   If $(\mathcal{P}^*, \mathcal{S}^*)$ does not yet exist, keep $(\mathcal{P}, \mathcal{S})$ as currently best solution $(\mathcal{P}^*, \mathcal{S}^*)$

Framework for column generation for an edge-path formulation

Determine from $(\mathcal{P}, \mathcal{S})$ the subset $\mathcal{D}_c \subset \mathcal{D}$ of critical demands k whose channel $S_k \in \mathcal{S}$ uses frequency slots within $[b_{low} + 1, b_{up}]$.

For each critical demand $k \in \mathcal{D}_c$:

- construct the subgraph H_k of $I(\mathcal{P})$ induced by $N[k]$
- find in H_k all cliques Q of weight $w(Q) > b_{low}$ and include them in $\mathcal{Q}$ as triple $(\mathcal{P}_Q, E_Q, w(Q))$ with

$$\mathcal{P}_Q = \{P_k \in \mathcal{P} : k \in Q\}$$

and E_Q subset of edges of G where paths from $\mathcal{P}_Q$ meet.

Framework for column generation for an edge-path formulation

5. Relaunch the multi-commodity flow as feasibility problem
   - with capacity $\leq b_{low}$ for all pairs of arcs and enhanced by
   - forbidden routing constraints associated with $\mathcal{P} \in \mathcal{R}$

$$\sum_{k \in \mathcal{D}} \sum_{a \in A^k_{\mathcal{P}}} f_k(a) \leq \sum_{k \in \mathcal{D}} |A^k_{\mathcal{P}}| - 1$$

   where $A^k_{\mathcal{P}}$ denotes the subset of arcs with $f_k(a) > 0$ in $\mathcal{P}$
   - forbidden clique constraints associated with $Q \in \mathcal{Q}$

$$\sum_{k \in Q} \sum_{a \in A^k_Q} f_k(a) \leq \sum_{k \in Q} |A^k_Q| - 1$$

   with A^k_Q subset of arcs a corresponding to edges in E_Q with $f_k(a) > 0$.

   If a feasible solution f has been found, then continue with Step 3.

Framework for column generation for an edge-path formulation

6. Improvement of b_{low} :

   If $\mathcal{Q} \neq \emptyset$ then
   - let $b_{low} = \min\{w(Q) : Q \in \mathcal{Q}\}$,
   - remove from $\mathcal{Q}$ all cliques Q of (new) weight $w(Q) = b_{low}$

   Else (i.e. if $\mathcal{Q} = \emptyset$) let $b_{low} = b_{low} + 1$
   (this corresponds to the special case that all critical configurations are non-superperfect graphs or no feasible solution has been found so far)

   Test for termination :

   If (we now have) $b_{low} > b_{up}$ then
   - return "instance infeasible (as $\chi_I(G, \mathcal{D}) > \bar{s}$)"
     (this corresponds to the case that no feasible solution has been found)

   If (we now have) $b_{low} = b_{up}$ and $(\mathcal{P}^*, \mathcal{S}^*)$ exists then
   - return $(\mathcal{P}^*, \mathcal{S}^*)$ as optimal solution

   Else continue with Step 5 (with new b_{low})

Plan

Implementation

- We forbid in the min-cost multi commodity flow problems the routings previously found by the min-cost multi commodity flow problem and the routings previously found by the edge-path formulation.
- We don't launch feasibility problems.
- We don't look for cliques just in the neighborhood of critical demands but in all demands.
- The clique constraint does not consider only the arcs with flow equal to 1 in the routing but also the other arc in the pair that forms an edge.

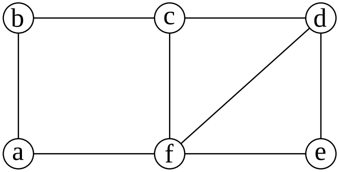
Implementation

$$f_k(l) + f_k(l') \;\leq\; 1 \qquad \forall\; k \in \mathcal{D} \;\; \forall\; pairs\; l, l' \in E' \tag{22}$$

$$\sum_{l \in \delta^-(v)} f_k(l) \;\leq 1 \qquad \forall\; k \in \mathcal{D} \;\; \forall\; v \neq o_k \tag{23}$$

Test instance 1

Given an optical network G (with all edges of length 1), an optical spectrum $S = \{1, \ldots, \bar{s}\}$ with $\bar{s} = 5$, and a set $\mathcal{D}$ of demands $k = (o_k, d_k, \bar{l}_k, w_k)$:



k	$o_k \to d_k$	$\bar{l}_k$	w_k
1	$a \to c$	4	2
2	$a \to d$	4	1
3	$b \to f$	4	2
4	$b \to e$	4	1
5	$d \to f$	4	3

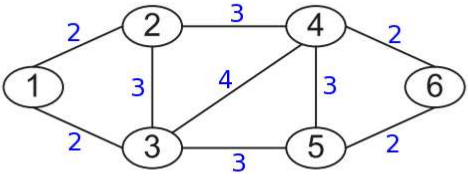
Test instance 1

The solution found by the solver is :

===== SOLUTION =====
This solution becomes feasible and this iteration will be stopped
Output - RSA Problem -
Interval chromatic number = 4
Routing :
From demand : 1 Between nodes : 1 to 3 : : 1 - 6 - 5 - 4 - 3 : : with length 4
Slots : 1 - 3
From demand : 2 Between nodes : 1 to 4 : : 1 - 2 - 3 - 4 : : with length 3
Slots : 0 - 1
From demand : 3 Between nodes : 2 to 6 : : 2 - 3 - 6 : : with length 2
Slots : 1 - 3
From demand : 4 Between nodes : 2 to 5 : : 2 - 1 - 6 - 5 : : with length 3
Slots : 3 - 4
From demand : 5 Between nodes : 4 to 6 : : 4 - 6 : : with length 1
Slots : 0 - 3

Test instance 2

Given an optical network G (with indicated edge lengths), an optical spectrum $S = \{1, \ldots, \bar{s}\}$ with $\bar{s} = 8$, and a set $\mathcal{D}$ of demands $k = (o_k, d_k, \bar{l}_k, w_k)$:



k	$o_k \to d_k$	$\bar{l}_k$	w_k
1	$1 \to 6$	$\bar{l}_1$	3
2	$1 \to 5$	$\bar{l}_2$	1
3	$2 \to 5$	$\bar{l}_3$	3
4	$2 \to 6$	$\bar{l}_4$	1
5	$3 \to 6$	$\bar{l}_5$	3
6	$4 \to 1$	$\bar{l}_6$	2

To test your implementation,

make a first run with $\bar{l}_k = 6$ for all k,

make a second run with $\bar{l}_k = 7$ for all k,

Test instance 2

First run with $\bar{l}_k = 6$ for all k,
The solution found by the solver is :

The first iteration of MCMCF is impossible for this instance
Problem is infeasible

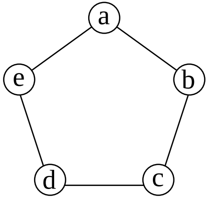
Test instance 2

Second one is with $\bar{l}_k = 7$ for all k. The solution found by the solver is :

===== SOLUTION =====
This RSA solution matches the lowerBound_
Output - RSA Problem -
Interval chromatic number = 6
Routing :
From demand : 1 Between nodes : 1 to 6 : : 1 - 3 - 5 - 6 : : with length 7
Slots : 1 - 4
From demand : 2 Between nodes : 1 to 5 : : 1 - 3 - 5 : : with length 5
Slots : 0 - 1
From demand : 3 Between nodes : 2 to 5 : : 2 - 4 - 5 : : with length 6
Slots : 3 - 6
From demand : 4 Between nodes : 2 to 6 : : 2 - 4 - 6 : : with length 5
Slots : 0 - 1
From demand : 5 Between nodes : 3 to 6 : : 3 - 4 - 6 : : with length 6
Slots : 2 - 5
From demand : 6 Between nodes : 4 to 1 : : 4 - 2 - 1 : : with length 5
Slots : 1 - 3

Test instance 3

Given an optical network G (with all edges of length 1), an optical spectrum $S = \{1, \ldots, \bar{s}\}$ with $\bar{s} = 6$, and a set $\mathcal{D}$ of demands $k = (o_k, d_k, \bar{l}_k, w_k)$:



k	$o_k \to d_k$	$\bar{l}_k$	w_k
1	$a \to c$	3	2
2	$b \to d$	3	2
3	$c \to e$	3	2
4	$d \to a$	3	2
5	$e \to b$	3	2

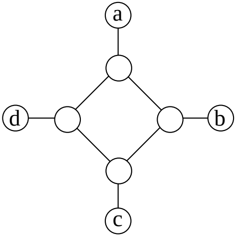
Test instance 3

The solution found by the solver is :

===== SOLUTION =====
This solution becomes feasible and this iteration will be stopped
Output - RSA Problem -
Interval chromatic number = 6
Routing :
From demand : 1 Between nodes : 1 to 3 : : 1 - 2 - 3 : : with length 2
Slots : 2 - 4
From demand : 2 Between nodes : 2 to 4 : : 2 - 3 - 4 : : with length 2
Slots : 4 - 6
From demand : 3 Between nodes : 3 to 5 : : 3 - 4 - 5 : : with length 2
Slots : 2 - 4
From demand : 4 Between nodes : 4 to 1 : : 4 - 5 - 1 : : with length 2
Slots : 4 - 6
From demand : 5 Between nodes : 5 to 2 : : 5 - 1 - 2 : : with length 2
Slots : 0 - 2

Test instance 4

Given an optical network G (with all edges of length 1), an optical spectrum $S = \{1, \ldots, \bar{s}\}$ with $\bar{s} = 5$, and a set $\mathcal{D}$ of demands $k = (o_k, d_k, \bar{l}_k, w_k)$:



k	$o_k \to d_k$	$\bar{l}_k$	w_k
1	$a \to c$	4	1
2	$b \to d$	4	1
3	$c \to a$	4	1
4	$d \to b$	4	1

Test instance 4

The solution found by the solver is :

===== SOLUTION =====
This solution becomes feasible and this iteration will be stopped
Output - RSA Problem -
Interval chromatic number = 4
Routing :
From demand : 1 Between nodes : 1 to 3 : : 1 - 5 - 8 - 7 - 3 : : with length 4
Slots : 0 - 1
From demand : 2 Between nodes : 2 to 4 : : 2 - 6 - 5 - 8 - 4 : : with length 4
Slots : 1 - 2
From demand : 3 Between nodes : 3 to 1 : : 3 - 7 - 6 - 5 - 1 : : with length 4
Slots : 3 - 4
From demand : 4 Between nodes : 4 to 2 : : 4 - 8 - 7 - 6 - 2 : : with length 4
Slots : 2 - 3

Plan

Conclusion

- Our objective with this project is to study the lower bounds for the width for the routing and spectrum assignment problem.
- Difficulties
- Learning

Perspectives

- Theoretical studies about the mathematics models.
- Improvements in the actual implementation
- Add in the implementation the theoretical background.