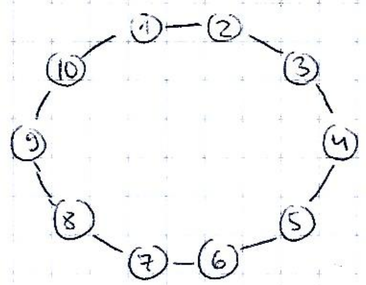


all edge weights = 1

k	o_k	d_k	w_k
1	10	3	3
2	2	5	3
3	4	7	3
4	6	9	3
5	8	1	3
6	5	8	1
7	6	7	1

for each demand, exactly 2 routings are possible!

		length!
1:	10 - 1 - 2 - 3	3
	10 - 9 - 8 - 7 - 6 - 5 - 4 - 3	7
2:	2 - 3 - 4 - 5	3
	2 - 1 - 10 - 9 - 8 - 7 - 6 - 5	7
3:	4 - 5 - 6 - 7	3
	4 - 3 - 2 - 1 - 10 - 9 - 8 - 7	7
4:	6 - 7 - 8 - 9	3
	6 - 5 - 4 - 3 - 2 - 1 - 10 - 9	7
5:	8 - 9 - 10 - 1	3
	8 - 7 - 6 - 5 - 4 - 3 - 2 - 1	7
6:	5 - 6 - 7 - 8	3
	5 - 4 - 3 - 2 - 1 - 10 - 9 - 8	7
7:	6 - 7	1
	6 - 5 - 4 - 3 - 2 - 1 - 10 - 9 - 8 - 7	9

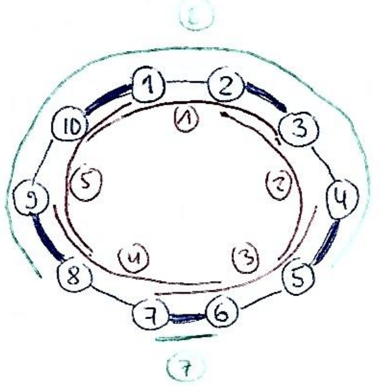
N.B.: the shortest paths of demands 1,...,5 induce C_5 in the edge intersection graph (which requires a span of 9)

the long path of demand 6+7 shares an edge with all shortest paths of demands 1,...5, inducing a 5-wheel in the edge intersection graph (which requires span 10 and, if both demands 6+7 are routed along the long path, even span 11).

there are 2 possible routings with
- capacity 7
- capacity 8
- capacity 9

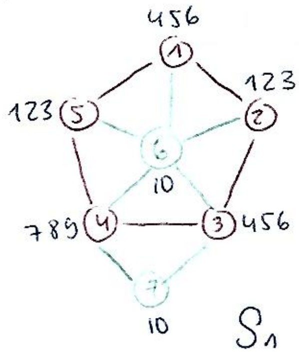
the optimal span is 9 (using the shortest path routing)

$\mathcal{P}_1$



with cap = 7

$I(\mathcal{P}_1)$:

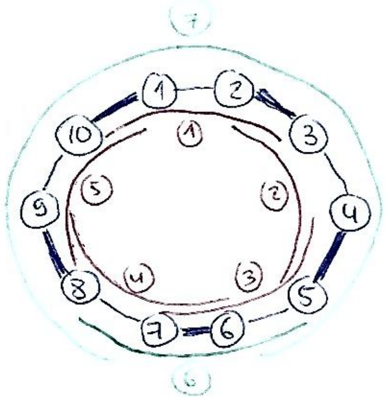


has $\omega = 7$
(all maximal cliques have this weight)

has $\chi_I = 10 = 9 + 1$

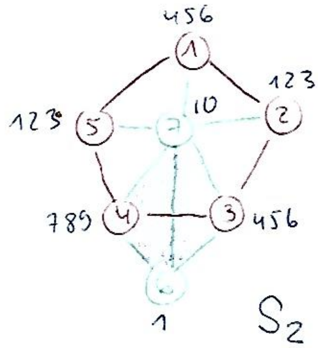
C_5 ⑥

$\mathcal{P}_2$



with cap = 7

$I(\mathcal{P}_2)$:

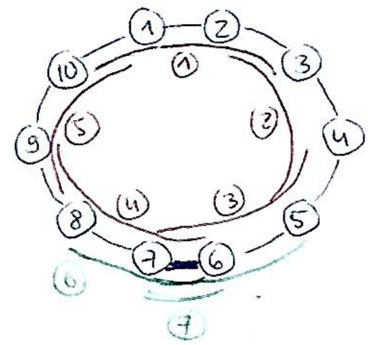


has $\omega = 8$
(induced by non-edge clique {3, 4, 6, 7})

has $\chi_I = 10 = 9 + 1$

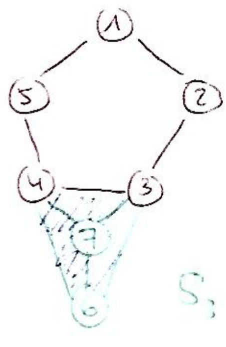
C_5 ⑦

$\mathcal{P}_3$: shortest paths routing



with cap = 8

$I(\mathcal{P}_3)$:

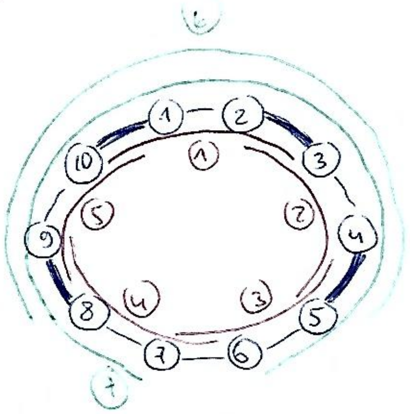


has $\omega = 8$
(induced by non-edge clique {3, 4, 6, 7})

has $\chi_I = 9$

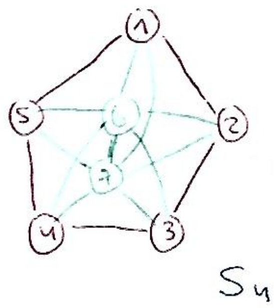
C_5

$\mathcal{P}_4$:



with cap = 8

$I(\mathcal{P}_4)$:



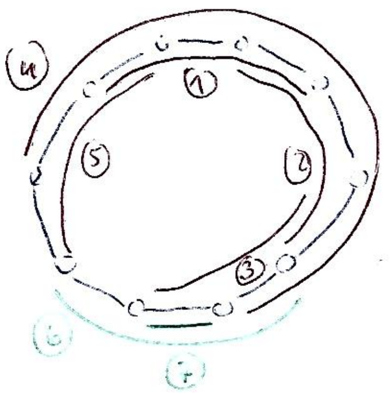
has $\omega = 8$
(all 5 maximal cliques have this weight)

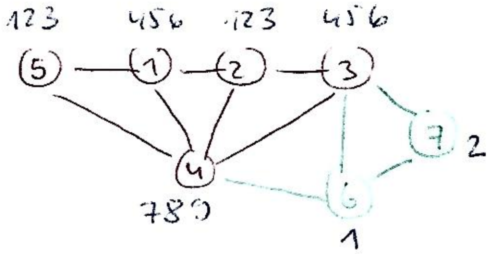
has $\chi_I = 9 + 2 = 11$

C_5 ⑥+⑦

cap = 9 routings

$\mathcal{P}_5$



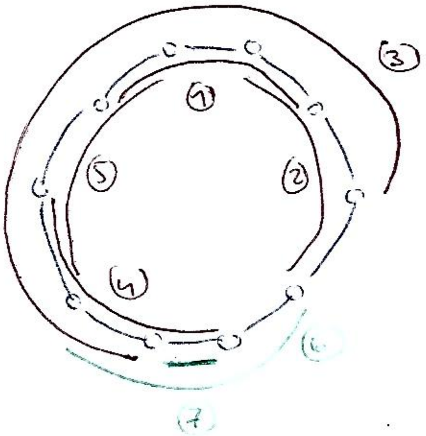


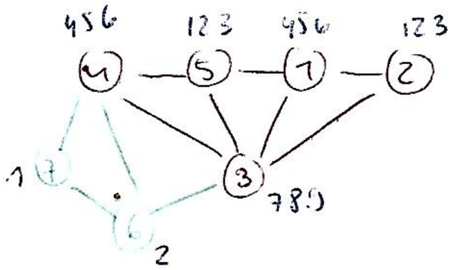
cap = 9

$\omega = 9$

$\chi_I = 9$

$\mathcal{P}_6$





cap = 9

$\omega = 9$

$\chi_I = 9$

---

no further optimal solution (all other routings require already a higher capacity) run of this instance $G = C_{10}$ and studied set D of demands:

- with $\bar{l}_k = 5 \ \forall k \in D$ and $\bar{s} = 7$:

$\bar{l}_k = 5$ allows only to route each demand along the shortest path, thus P_3 is the only possible routing (with cap = 8)
$\Rightarrow$ instance infeasible due to $s_R(G, D) > \bar{s}$

- with $\bar{l}_k = 5 \ \forall k \in D$ and $\bar{s} = 8$:

P_3 is still only possible routing (with cap = 8) and $\chi_I(I(P_3), w) = 9$
$\Rightarrow$ instance infeasible due to $\chi_I(G, D) > \bar{s}$

- with $\bar{l}_k = 5 \ \forall k \in D$ and $\bar{s} = 9$:

P_3 is still only possible routing (with cap = 8) and $\chi_I(I(P_3), w) = 9$
second run of MCF with P_3 forbidden is infeasible $\Rightarrow lb = 9$
$\Rightarrow$ instance feasible, (P_3, S_3) only and optimal solution

- with $\bar{l}_k = 7 \ \forall k \in D$ and $\bar{s} = 9$:

- P_1 is only most balanced routing (with cap = 7) as P_2 is not possible (due to $\bar{l}_7 = 7$)
- due to $\chi_I(I(P_1), w) = 10$ and $\bar{s} = 9$, the first run of the EPF does not give a feasible solution
- the second run of MCF with P_1 forbidden results in P_3 (with cap = 8) as P_4 is not possible (again due to $\bar{l}_7 = 7$)
- the second run of the EPF finds (P_3, S_3) with span = 9, Q stays $\emptyset$
- the next run of MCF with P_1, P_3 forbidden finds P_5 or P_6 (with cap = 9) and increased $lb = 9$

$\Rightarrow$ instance feasible, (P_3, S_3) returned as optimal solution

- with $\bar{l}_k = 7 \ \forall k \in D$ and $\bar{s} = 10$:

- P_1 is still only possible routing with cap = 7
- first run of EPF finds (P_1, S_1) with span = 10, Q stays empty
- second run of MCF with P_1 forbidden results again in P_3 (with cap = 8)
- second run of EPF finds (P_3, S_3) with span = 9, Q stays empty (as $lb = 8$ and $\omega(I(P_3), w) = 8$)
- next run of MCF with P_1, P_3 forbidden finds P_5 or P_6 (with cap = 9) and increases $lb = 9$

$\Rightarrow$ instance feasible, (P_3, S_3) returned as optimal solution

- with $\bar{l}_e = 9 \ \forall e \in D$ and $\bar{s} = 10$:

- first run of MCF finds $\mathcal{P}_1$ or $\mathcal{P}_2$ (both with cap $= 7$)
- first run of EPF finds $(\mathcal{P}_1, S_1)$ or $(\mathcal{P}_2, S_2)$, both with span $= 10$; for $(\mathcal{P}_1, S_1)$: Q stays empty; for $(\mathcal{P}_2, S_2)$: $\{3,4,6,7\}$ enters Q

- second run of MCF results in the other routing with cap $= 7$
- second run of EPF finds $(\mathcal{P}_3, S_3)$ with span 9 (as $\mathcal{P}_3 \subseteq (\mathcal{P}_1 \cup \mathcal{P}_2)$); and $\{3,4,6,7\}$ induced by shortest paths enters Q

- third run of MCF finds $\mathcal{P}_4$ (as $\mathcal{P}_3$ is already forbidden!) with cap $= 8$
  we should now recall the cliques in Q!
- third run of EPF finds the other of $(\mathcal{P}_1, S_1)$ and $(\mathcal{P}_2, S_2)$ with span $= 10$ (as $\mathcal{P}$ is forbidden), hence $(\mathcal{P}_3, S_3)$ stays the currently best solution found and $ub = 9$

- next run of MCF (with $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4$ forbidden) finds one of $\mathcal{P}_5, \mathcal{P}_6$ (with cap $= 9$) $\Rightarrow lb = 9$
  $\Rightarrow$ instance feasible, $(\mathcal{P}_3, S_3)$ returned as optimal solution