Module 3 Mastery Assignment

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April 30, 2017

Content

We want to approximate the function $f(x)$ that satisfies the following conditions:

- Is a real or a complex-value function.
- It is infinitely differentiable at c

$$P(0) = f(0)$$
$$P(x) = f(0)$$

$$
\begin{aligned}
P^{'}(0) &= f^{'}(0) \\
P(x) &= f(0) + f^{'}(0)\,x
\end{aligned}
$$

$$P''(0) = f''(0)$$
$$P(x) = f(0) + f'(0)\,x + \frac{1}{2}f''(0)\,x^2$$

$$P'''(0) = f'''(0)$$
$$P(x) = f(0) + f'(0)\,x + \frac{1}{2} f''(0)\,x^2 + \frac{1}{2 \cdot 3} f'''(c)\,x^3$$

$$P^{(4)}(0) = f^{(4)}(0)$$

$$P(x) = f(0) + f'(0)\,x + \frac{1}{2}f''(0)\,x^2 + \frac{1}{2\cdot 3}f'''(c)\,x^3 + \frac{1}{2\cdot 3\cdot 4}f^{(4)}(0)\,x^4$$

$$P^{(4)}(0) = f^{(4)}(0)$$

$$P(x) = f(0) + f'(0)\,x + \frac{1}{2} f''(0)\,x^2 + \frac{1}{2\cdot 3} f'''(c)\,x^3 + \frac{1}{2\cdot 3\cdot 4} f^{(4)}(0)\,x^4 + \cdots + \frac{1}{n!} f^{(n)}(0)\,x^n$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \cdot x^n$$

$$P(x) = f(c) \longrightarrow P(c) = f(c)$$

$$P(x) = f(c) + f'(c)(x-c) + \frac{1}{2} f''(c)(x-c)^2 + \frac{1}{2 \cdot 3} f'''(c)(x-c)^3 + \frac{1}{2 \cdot 3 \cdot 4} f^{(4)}(c)(x-c)^4 + \cdots + \frac{1}{n!} f^{(n)}(c)(x-c)^n$$

$$P(c) = f(c) + f'(c)(c-c) + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

Approximating $\sin(x)$

$$
\begin{aligned}
f(x) &= \sin(x) \\
f'(x) &= \cos(x) \\
f''(x) &= -\sin(x) \\
f^{(3)}(x) &= -\cos(x)
\end{aligned}
$$

Taylor Polynomials
Example

$$
\begin{aligned}
f(0) &= \sin(0) = 0 \\
f^{'}(0) &= \cos(0) = 1 \\
f^{''}(0) &= -\sin(0) = 0 \\
f^{(3)}(0) &= -\cos(0) = -1
\end{aligned}
$$

Since it becomes cyclic, we can

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n$$
$$\frac{0}{0!}x^0 + \frac{1}{1!}x^1 + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \frac{1}{5!}x^5 + \frac{0}{6!}x^6 + \frac{-1}{7!}x^7$$
$$x - (3!)^{-1}x^3 + (5!)^{-1}x^5 - (7!)^{-1}x^7$$

A clear pattern begins to form, we can condense it into:

$$\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{x^{(2n-1)}}{(2n-1)!}$$

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