Module 3 Review Assignment

Pedro Gómez Martín

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1

$$
\begin{aligned}
a_n =& \frac{(1+n)!}{n!} \\
a_n =& \frac{(n+1)(n)(n-1)\cdots}{(n)(n-1)\cdots} \\
a_n =& n+1 \\
& \lim_{\infty}[n+1] = \infty
\end{aligned}
$$

2

$$
\begin{aligned}
&\sum_{n=0}^{\infty} \frac{(-2)^{n+3}}{5^{n+2}} \\
&\sum_{n=0}^{\infty} \frac{(-2)^{n}(-2)^{3}}{5^{n}(5)^{2}} \\
\frac{(-2)^3}{(5)^2} &\sum_{n=0}^{\infty} \left(\frac{-2}{5}\right)^n \\
-\frac{8}{25} &\sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{5}\right)^n \\
-\frac{8}{25} &\cdot \frac{5}{7} = \boxed{-\frac{40}{175}}
\end{aligned}
$$

3

$$
\sum_{n=1}^{\infty} \frac{1}{n(n+2)} \rightarrow \frac{A}{n} + \frac{B}{n+2} = \frac{1}{n(n+2)} \rightarrow A(n+2) + Bn = 1 \Rightarrow A = \frac{1}{2} \; B = -\frac{1}{2}
$$

$$
\frac{1}{2} \lim_{N \to \infty} \left[\sum_{n=0}^{N} \left(\frac{1}{n+2} - \frac{1}{n} \right) \right] = \frac{1}{2} \left(1 + \frac{1}{2} + \lim_{N \to \infty} \left[\frac{1}{N+2} \right] \right) = \frac{3}{4}
$$

4

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2\cdot 3^n)}{2^{2n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n(-1)(n+2)\cdot 3^n}{2^{2n}\cdot 2} = \frac{1}{2}\sum_{n=1}^{\infty}(-1)^n \frac{(n+2)\cdot 3^n}{2^{2n}}$$

$$\lim_{n\to\infty} \frac{(n+2\cdot 3^n)}{2^{2n}} = 0$$

$$\frac{(1+2)3}{2^2} \quad \frac{(2+2)3^2}{2^4}$$

$$\underset{b_1}{\frac{9}{4}} \quad \underset{b_2}{\frac{\cancel{4}\cdot 9}{\cancel{4}\cdot 4}} \qquad \frac{(3+2)3^3}{2^6} = \frac{5\cdot 27}{64} = \underset{b_3}{\frac{135}{64}}$$

$$a_n = \frac{(-1)^{n+1}\,(n+2)\,3^n}{2^{2n}}$$

$$b_n = \frac{(n+2)\,3^n}{2^{2n}}$$

a_1 cancels a_2 and from a_3 on, b_n converges.

5

a) $\sum_{n=1}^{\infty} \left(\sqrt{2}\right)^n$

geometric series

$r = \sqrt{2}$

$|r| > 1$

the series diverges by the nth term test

b) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+\ln(n)}$

$\lim_{n\to\infty} \left[\frac{n^{\frac{1}{2}}}{n^2} : \frac{1}{n^2}\right]$

$\lim_{n\to\infty} \left[\frac{n^{\frac{1}{2}+2}}{n^2}\right] \to$ By the limit comparison test, both converge and by direct comparison test $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+\ln(n)}$ converges absolutely.

$\sum_{n=1}^{\infty} e^{-1}n! = \sum_{n=1}^{\infty} \frac{n!}{e^n}$

$\lim_{n\to\infty} \left[\sqrt[n]{\left|\frac{n!}{e^n}\right|}\right] = \infty$

by the Root test $\sum_{n=1}^{\infty} e^{-1}n!$ diverges.

6

a) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

converges

$|r| < 1$

b) $\sum_{n=1}^{\infty} \frac{1}{\ln n} \boxed{\frac{1}{\ln n} > \frac{1}{n}}$

by direct comparison test, $a_n > b_n$ the series diverges

$$c)\ \sum_{n=1}^{\infty} \frac{2n}{n+1} \qquad \frac{2(n+1)}{(n+1)+1} : \frac{2n}{n+1} \to \frac{\cancel{2}(n+1)^2}{\cancel{2}n(n+2)} = \frac{(n+1)^2}{n^2+2n}$$

$\sum_{n=1}^{\infty} \frac{2n}{n+1}$ diverges by the nth term test

$$\lim_{n\to\infty} \left[\frac{(n+1)^2}{n^2+2n}\right] = 1 \quad 1 \neq 0$$

d) $\sum\limits_{n=1}^{\infty}(-1)^n\frac{1}{\ln(n)}$ by the alternating series test, the series converge conditionally.
e) $\sum\limits_{n=1}^{\infty}\frac{1}{n(2n+1)}$ Converges by direct comparison to $\sum\limits_{n=1}^{\infty}\frac{1}{n^2}$

7

$$\sum_{n=1}^{4}(-1)^{n+1}\left(\frac{1}{2}\right)^n=S_n$$

a) $S_4=+\left(\frac{1}{2}\right)-\left(\frac{1}{4}\right)+\left(\frac{1}{8}\right)-\left(\frac{1}{16}\right)=\frac{4+1}{8}-\frac{4+1}{16}=\frac{10-5}{16}=\frac{5}{16}$
b) $\frac{5}{16}<S_n<\frac{11}{32}$
c) $\frac{\frac{1}{2}}{1-\left(\frac{1}{2}\right)}\cdot\frac{1}{2}=\frac{1}{2}$
d) yes

8

a) F *b)* F *c)* T *d)* T

9

$$a)\ \sum_{n=0}^{\infty}\frac{x^n}{3n+1}\qquad\frac{\frac{x^{n+1}}{3(n+1)+1}}{\frac{x^n}{3n+1}}$$

$$\lim_{n\to\infty}\left|\frac{x^{n+1}(3n+1)}{(3(n+1))\,x^n}\right|=\lim_{n\to\infty}\left|\frac{x(3n+1)}{3n+4}\right|=\lim_{n\to\infty}\left[|x|\,\frac{3+\frac{1}{n}}{3+\frac{4}{n}}\right]=|x|\qquad-1<x<1$$

10

$$a)\ \sum_{n=1}^{\infty}\frac{1}{2+3^n}\Rightarrow\lim_{n\to\infty}\left|\frac{\frac{1}{2+3^{n+1}}}{\frac{1}{2+3^n}}\right|=\lim_{n\to\infty}\left|\frac{2+3^n}{2+3^{n+1}}\right|=0$$

$$b)\ \sum_{n=0}^{\infty}\frac{n}{n^3+4}\Rightarrow\lim_{n\to\infty}\left|\frac{\frac{n+1}{(n+1)^3+4}}{\frac{n}{n^3+4}}\right|=1$$

$$\lim_{n\to\infty}\left[\frac{\frac{n}{n^3+4}}{\frac{1}{n^2}}\right]\Rightarrow\lim_{n\to\infty}\left[\frac{n^3}{n^3+4}\right]=\lim_{n\to\infty}\left[\frac{1}{1+\frac{4}{n^3}}\right]=0$$

$$c)\ \sum_{n=0}^{\infty}(-1)^{n+1}\frac{n^2}{3^n}\Rightarrow\lim_{n\to\infty}\left|\frac{\frac{(n+1)^2}{3^{(n+1)}}(-1)^{n+2}}{\frac{n^2}{3^n}(-1)^{n+1}}\right|=\lim_{n\to\infty}\left[\frac{\cancel{3}^n(n+1)^2(-1)^{\cancel{n}+2}}{3\cdot\ \cancel{3}^n\cdot n^2(-1)^{\cancel{n}+1}}\right]=\lim_{n\to\infty}\left[(-1)^2\frac{(n+1)^2}{n^2}\right]=1$$