Module 3 Lesson 5 Notes 1

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1 Known Power Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \tag{1}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \tag{2}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \tag{3}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \tag{4}$$

2 Substitution

3 Algebra

$$\frac{x}{1-x} = x \cdot \frac{1}{1-x} = x \sum_{n=0}^{\infty} x^n = x\left(1 + x + x^2 + x^3 + \dots\right) = \left(x + x^2 + x^3 + x^4 + \dots\right) = \sum_{n=0}^{\infty} x^{n+1}$$

$$\boxed{-1 < x < 1}$$

$$\frac{x}{3+x} = \frac{\frac{x}{3}}{1 - \frac{-x}{3}} = \frac{x}{3} \sum_{n=0}^{\infty} \left(\frac{-x}{3}\right)^n = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^{n+1}$$

$$\boxed{-3 < x < 3}$$

4 Differentiation and Integration

Let $f(x) = \sum\limits_{n=0}^{\infty} a_n (x-a)^n$ be a power series with a radius of convergence R. We then know that the function f is defined on the interval $(a - \mathbf{R}, a + \mathbf{R})$ and may or may not be defined at the end points $a - R$ or $a + R$ Under those conditions, we have the following two theorems:

1.) f is differentiable on $(a - \mathbf{R}, a + \mathbf{R})$ and $f^{'}(x) = \sum\limits_{n=0}^{\infty} n a_n (x-a)^{n-1}$

2.) for c and d on $(a - \mathbf{R}, a + \mathbf{R})$ we have $\int\limits_c^d f(x)dx = \sum\limits_{n=0}^{\infty} a_n \int\limits_c^d (x-a)^n \, dx$