Module 3 Lesson 5 Assignment

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a) Write the first four non-zero terms of the power series of $f(x)$ centered at zero, in terms of a.

$$\begin{aligned}
f^0(x) &= (-1)^0\, a^0 f(x)\\
f^{'}(x) &= (-1)^1\, a^1 f(x)\\
f^{''}(x) &= (-1)^2\, a^2 f^{'}(x)\\
f^{'''}(x) &= (-1)^3\, a^3 f^{''}(x)\\
f^4(x) &= (-1)^4\, a^4 f^{'''}(x)
\end{aligned}$$

$$f(x) = 1 - ax + \frac{(ax)^2}{2!} - \frac{(ax)^3}{3!} + \cdots + (-1)^n \cdot \frac{(ax)^n}{n!}$$

b) Write $f(x)$ as a familiar function in terms of a.

$$f(x) = e^{-ax}$$

c) How many terms of the power series are necessary to approximate $f(0.2)$ with an error less than 0.001 with $a = 2$? Justify your answer.

$$f(x) = 1 - 2x + \frac{(2x)^2}{2!} - \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} - \frac{(2x)^5}{5!}$$

$$\frac{8 \cdot (.2)^3}{6} \approx .01067 \qquad \frac{16 \cdot (.2)^4}{24} \approx .001067 \qquad \frac{48 \cdot (.2)^5}{120} \approx .00008$$

It takes five terms to approximate $f(0.2)$ with an error less than 0.001 when $a = 2$

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a)

$$\frac{d}{dx}[\tan(x)] = \sec^2(x)$$

$$\tan(x) \approx x + \frac{x^3}{3} + \frac{2x^5}{15} \rightarrow \sec(x) \approx 1 + x^2 + \frac{2}{3}x^4$$

b)

$$\frac{x + \frac{1}{3}x^3 + \frac{2}{15}x^5}{x} \rightarrow \frac{x}{x} + \frac{1}{3} \cdot \frac{x^3}{x} + \frac{2}{15} \cdot \frac{x^5}{x} \rightarrow 1 + \frac{x^2}{3} + \frac{2x^4}{15}$$

c)

$$1+\frac{0^2}{3}+\frac{2\cdot 0^4}{15}=1$$

d) The limit found in part c is exact since all the terms that come after the one contain an x multipliying the entire numerator and every x is substituted by a zero, thus it will be practically equivalent to $1+\sum_{n=1}^{\infty} 0x^n$