Application of Integrals Test Review

Name your file First_Last_AssignmentName.

Name

$$x = 2(\theta - \sin\theta)$$
$$y = 2(1 - \cos\theta)$$

1) **Calculator Inactive** Given the parametric equations:

a) Determine: $\frac{dx}{d\theta}$, $\frac{dy}{d\theta}$, and $\frac{dy}{dx}$.

$$\frac{dx}{d\theta} = 2(1-\cos\theta) \qquad \frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta} \; //$$

$$\frac{dy}{d\theta} = 2(\sin\theta)$$

b) Using the slope found above, determine the equation of the tangent line at $\theta = \pi$.

$$\frac{dy}{dx}\text{at } \theta=\pi = \frac{\sin(\pi)}{1-\cos(\pi)} = 0$$

$$x = 2(\pi - \sin(\pi)) = 2\pi$$
$$y = 2(1-\cos(\pi)) = 4$$

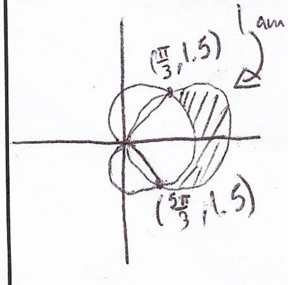
$$\boxed{y = 4}$$

c) Set up, but do not evaluate the integral which represents the length of the curve over the interval $[0,2\pi]$. Express the integrand as a function of θ.

$$\int_0^{2\pi} \sqrt{1+\left(\frac{\sin\theta}{1-\cos\theta}\right)^2}\, d\theta$$

2) **Calculator Active**

Determine the area: inside the curve $r = 3\cos\theta$ and outside the curve $r = 1+\cos\theta$



I am guessing that it is this area

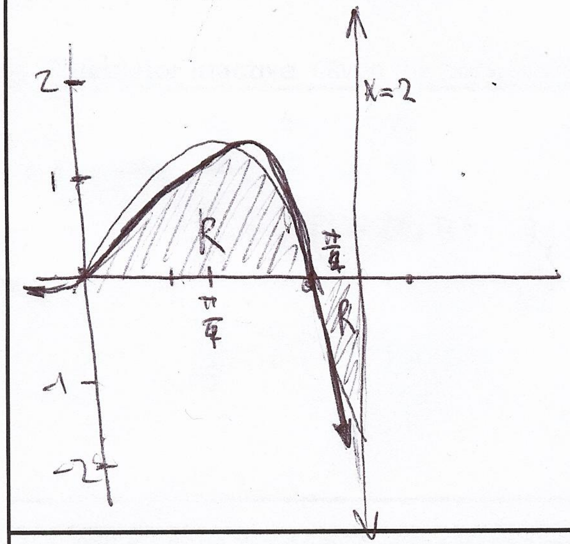
$$\frac{1}{2}\left(\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (3\cos\theta)^2 dx - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1+\cos\theta)^2 dx\right) = \pi \approx 3.14.15926536$$

obtained by calculator

3) **Calculator Active**

Let **R** be the region bounded by the graph of $y = xe^x \cos x$, the origin, and the line $x = 2$. Make sure you show the set up of all integrals. (Hint: This graph will cross the x-axis and you will have to work with 2 parts.)

a) Sketch the graph of **R**. Show the scale.



b) Find the area of region **R**.

$$\int_{\frac{\pi}{2}}^{2} xe^x \cos(x)\, dx \approx -1.0884 \approx R_2$$

$$|R_2| + |R_1| = R$$

$$\boxed{R = 2.4613}$$

$$\int_{0}^{\frac{\pi}{2}} xe^x \cos(x)\, dx \approx 1.3729 = R_1$$

c) Find the volume when the region **R** is revolved about the **x-axis.**

$$\pi \int_{\frac{\pi}{2}}^{2} (xe^x \cos(x))^2\, dx \approx 12.907 = V_1$$

$$\pi \int_{0}^{\frac{\pi}{2}} (xe^x \cos(x))^2\, dx \approx 4.938 = V_2$$

$$V = |V_1| + |V_2| \approx \boxed{17.845}$$

d) Determine the volume if **R** is the base, and every cross section perpendicular to the x-axis is an isosceles right triangle whose hypotenuse is on the base.

$\sqrt{a^2 + a^2} = h$ $\quad h = \sqrt{2a^2}$

$h = a\sqrt{2}$ $\quad \frac{a^2}{2} = A$

$\frac{h\sqrt{2}}{2} = a$

$\frac{\left(\frac{h^2 \cdot 2}{4}\right)}{2} = A$

$$\frac{1}{4}\left(\int_{0}^{\frac{\pi}{2}} (xe^x \cos(x))^2\, dx + \int_{\frac{\pi}{2}}^{2} (e^x x \cos(x))^2\, dx\right) \quad A = \frac{h^2}{4}$$

$.393 + 1.027$ $\quad$ ~~Ave~~ $\boxed{\text{Volume} \approx 1.42}$

e) Determine the length of the curve, $y = xe^x \cos x$, over the interval [0, 2].

$$\frac{d}{dx}\left[e^x x \cos(x)\right]$$

$$-e^x(x\sin(x) + (-x-1)\cos(x))$$

$$\int_0^2 \sqrt{1 + \left(-e^x(x\sin(x) + (-x-1)\cos(x))\right)^2}\, dx$$

$$\boxed{\approx 9.679}$$