Normal Distribution Derivation

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1 Intuition

First, to derive this incredibly useful formula, we start by the intuition that it is symmetric and that it is similar to the derivative of the logistic equation, with that intuition, we can come up with the following differential equation:

$$\frac{df}{dx} = -k(x-\mu)f(x) \tag{1}$$

Where k is the constant that defines the rate at which it decreases, $(x-\mu)$ describes the center, and x describes the rate at which the frequencies fall off proportionally to the distance of the score from the mean, and $f(x)$ to the frequencies themselves.

2 Solution to the Intuition

With a differential equation, we can now find a solution:

$$\frac{df}{f} = -k(x-\mu)dx \tag{2}$$

$$\int \frac{1}{f}df = -k\int (x-\mu)dx \tag{3}$$

$$\ln(f) = \left[\frac{-k(x-\mu)^2}{2} + C\right] \tag{4}$$

$$f = e^{-k\frac{(x-\mu)^2}{2}+C} \tag{5}$$

$$f = e^{-k\frac{(x-\mu)^2}{2}}e^C \tag{6}$$

$$f = Ce^{-k\frac{(x-\mu)^2}{2}} \tag{7}$$