

1 Length of a vector

$$\int_{\alpha}^{\beta} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \frac{dx}{dt} dt$$

2 Derivatives

$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

$$\text{First Derivative} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx}$$

$$\text{Second Derivative} = \frac{d}{dt} \left[\frac{dy}{dx} \right]$$

3 Example

Find the length of the following vector from time $t = 1$ to $t = 5$

$$\vec{r}(t) = \langle e^t, \cos(t) \rangle$$

$$\text{Find out slope at time } t \Rightarrow \frac{-\sin(t)}{e^t}$$

$$\text{Find the length} \Rightarrow \int_1^5 \sqrt{1 + \left(\frac{-\sin(t)}{e^t} \right)^2} e^t dt$$

Using the calculator, we obtain that the length is ≈ 145.825