Module 3 Lesson 3 Notes

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1 P-Series

$\frac{1}{n^p}$ p determines convergence, if $p > 1$ the series will converge

2 Nth Term Test

if $\sum_{n=1}^{\infty} a_n$ and $\lim_{n \to \infty} [a_n] \neq 0$ the series diverges, note that this test only proves that it diverges when it does not equal to 0, when it equals to 0 it does not prove anything.

3 The Integral Test

If $\sum_{n=1}^{\infty} a_n$ where $a_n = f(n)$ for f is continious, decreasing and $f(n) \geq 0$
Then:

$$\sum_{n=1}^{\infty} a_n \text{ and } \int_{1}^{\infty} f(n)\, dn$$

Either both converge or both diverge

4 Direct Comparison Test

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1} \qquad \frac{1}{n^2+1} < \frac{1}{n^2} \text{ Therefore it converges}$$

$$\sum_{n=1}^{\infty} \frac{1}{n-3} \qquad \frac{1}{n-3} > \frac{1}{n} \text{ Therefore it diverges}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n-11}} \qquad \frac{1}{\sqrt{n-11}} > \frac{1}{\sqrt{n}} \text{ Therefore it diverges}$$

$$\sum_{n=1}^{\infty} \left(\frac{4}{5+n}\right)^n \qquad \left(\frac{4}{5+n}\right)^n < \left(\frac{4}{5}\right)^n \text{ Therefore it converges}$$

$$\sum_{n=1}^{\infty} \frac{n^2}{n^5-4} \qquad \frac{1}{n^3-\frac{4}{n^2}} > \frac{1}{n^3} \text{ Needs another test}$$

$$\sum_{n=1}^{\infty} \frac{n}{n^2+2} \qquad \frac{1}{n+\frac{2}{n}} < \frac{1}{n} \text{ Needs another test}$$

5 Limit Comparison Test

Given $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ where $a_n > 0$ and $b_n > 0$

$$\lim_{n\to\infty}\left[\frac{a_n}{b_n}\right] = L$$

- If $L > 0$ and finite, either both converge or both diverge
- If b_n converges and $L = 0$ both converge
- If b_n diverges and $L = \infty$ both diverge

Example: $\sum_{n=1}^{\infty} \frac{n^2}{n^5-4} \qquad \sum_{n=1}^{\infty} \frac{1}{n^3}$

$$\lim_{n\to\infty}\left[\frac{n^2}{n^5-4} : \frac{1}{n^3}\right] = \lim_{n\to\infty} \frac{n^5}{n^5-4} = 1$$

Example: $\sum_{n=1}^{\infty} \frac{2n}{n^2+2} \qquad \sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n\to\infty}\left[\frac{2n}{n^2+2} : \frac{1}{n}\right] = \lim_{n\to\infty} \frac{2n^2}{n^2+2} = 2$$

6 Alternating Series Test

$a_n > 0$ then
$\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$
converge if:

1. $\lim_{n\to\infty} a_n = 0$ $\Rightarrow$ nth term test
2. $a_{n+1} < a_n$ for all n $\Rightarrow$ The terms decrease

7 Alternating Series Estimation Theorem

$$\text{If } \sum_{n=1}^{\infty} |a_n| \text{ converges, then } \sum_{n=1}^{\infty} a_n \text{ also converges}$$

$$\sum_{n=1}^{\infty} a_n \text{ is } \underline{\text{absolutely convergent}} \text{ if } \sum_{n=1}^{\infty} |a_n| \text{ also converges}$$

$$\sum_{n=1}^{\infty} a_n \text{ is } \underline{\text{conditionally convergent}} \text{ if } \sum_{n=1}^{\infty} |a_n| \text{ diverges}$$

example:

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \text{ meets the alternating series test but } \sum_{n=1}^{\infty} |-1|^n \frac{1}{n} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges}$$

8 Ratio Test

$$\lim_{n\to\infty}\left[\frac{a_{n+1}}{a_n}\right]$$

Converges	Diverges
$\sum_{n=1}^{\infty}\frac{2}{n^2+1} \quad \lim_{n\to\infty}\left[\frac{a_{n+1}}{a_n}\right]$	$\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{3}{2}\right)^n \quad \lim_{n\to\infty}\left[\frac{a_{n+1}}{a_n}\right]$
$\lim_{n\to\infty}\left[\frac{2}{(n+1)^2+1}\cdot\frac{n^2+1}{2}\right]$	$\lim_{n\to\infty}\left[\frac{3^{n+1}}{(n+1)\,2^{n+1}}\cdot\frac{2^n n}{3^n}\right]$
$\lim_{n\to\infty}\left[\frac{n^2+1}{(n+1)^2+1}\right]$	$\lim_{n\to\infty}\left[\frac{3^n 3n2^n}{(n+1)2^n 2\cdot 3^n}\right]$
$\lim_{n\to\infty}\left[\frac{n^2+1}{n^2+2n+2}\right]=1$	$\lim_{n\to\infty}\left[\frac{3n}{2n+2}\right]=\frac{2}{3}$

For any series $\sum_{n=1}^{\infty} a_n$ find $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$

1. If $L < 1$, the series converges absolutely
2. If $L > 1$ (or ∞), the series diverges
3. If $L = 1$, no conclusion can be made

9 Root Test

For any series $\sum_{n=1}^{\infty} a_n$ find $\lim_{n\to\infty}\left[\sqrt[n]{|a_n|}\right] = L$

1. If $L < 1$, the series converges absolutely
2. If $L > 1$ (or ∞), the series diverges
3. If $L = 1$, no conclusion can be made