

Module 3 Review Assignment

Pedro Gómez Martín

April 19, 2017

1

$$\begin{aligned}a_n &= \frac{(1+n)!}{n!} \\a_n &= \frac{(n+1)(n)(n-1)\cdots}{(n)(n-1)\cdots} \\a_n &= n+1 \\ \lim_{\infty} [n+1] &= \infty\end{aligned}$$

2

$$\begin{aligned}& \sum_{n=0}^{\infty} \frac{(-2)^{n+3}}{5^{n+2}} \\& \sum_{n=0}^{\infty} \frac{(-2)^n (-2)^3}{5^n (5)^2} \\& \frac{(-2)^3}{(5)^2} \sum_{n=0}^{\infty} \left(\frac{-2}{5}\right)^n \\& -\frac{8}{25} \sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{5}\right)^n \\& -\frac{8}{25} \cdot \frac{5}{7} = \boxed{-\frac{40}{175}}\end{aligned}$$

3

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{1}{n(n+2)} &\rightarrow \frac{A}{n} + \frac{B}{n+2} = \frac{1}{n(n+2)} \rightarrow A(n+2) + Bn = 1 \Rightarrow A = \frac{1}{2} \quad B = -\frac{1}{2} \\ \frac{1}{2} \lim_{N \rightarrow \infty} \left[\sum_{n=0}^N \left(\frac{1}{n+2} - \frac{1}{n} \right) \right] &= \frac{1}{2} \left(1 + \frac{1}{2} + \lim_{N \rightarrow \infty} \left[\frac{1}{N+2} \right] \right) = \frac{3}{4}\end{aligned}$$

4

$$\sum_{n=1}^\infty \frac{(-1)^{n+1}(n+2 \cdot 3^n)}{2^{2n+1}} = \sum_{n=1}^\infty \frac{(-1)^n(-1)(n+2) \cdot 3^n}{2^{2n} \cdot 2} = \frac{1}{2} \sum_{n=1}^\infty (-1)^n \frac{(n+2) \cdot 3^n}{2^{2n}}$$
$$\lim_{n \rightarrow \infty} \frac{(n+2 \cdot 3^n)}{2^{2n}} = 0$$

$$\frac{(1+2)3}{2^2} \quad \frac{(2+2)3^2}{2^4}$$
$$a_n = \frac{(-1)^{n+1}(n+2)3^n}{2^{2n}}$$
$$b_n = \frac{(n+2)3^n}{2^{2n}}$$

$$\frac{9}{4} \quad \frac{4 \cdot 9}{4 \cdot 4} \quad \frac{(3+2)3^3}{2^6} = \frac{5 \cdot 27}{64} = \frac{135}{64}$$

$b_1 \quad b_2 \qquad \qquad \qquad b_3$

a_1 cancels a_2 and from a_3 on, b_n converges.