Module 3 Review Assignment

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1

$$\begin{aligned} a_n =& \frac{(1+n)!}{n!} \\ a_n =& \frac{(n+1)\,(n)\,(n-1)\cdots}{(n)\,(n-1)\cdots} \\ a_n =& n+1 \\ & \lim_{\infty}[n+1] = \infty \end{aligned}$$

2

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(-2)^{n+3}}{5^{n+2}} \\ & \sum_{n=0}^{\infty} \frac{(-2)^{n}\,(-2)^{3}}{5^{n}(5)^{2}} \\ & \frac{(-2)^{3}}{(5)^{2}} \sum_{n=0}^{\infty} \left(\frac{-2}{5}\right)^{n} \\ & -\frac{8}{25} \sum_{n=0}^{\infty} (-1)^{n} \left(\frac{2}{5}\right)^{n} \\ & -\frac{8}{25} \cdot \frac{5}{7} = \boxed{-\tfrac{40}{175}} \end{aligned}$$

3

$$\sum_{n=1}^{\infty} \frac{1}{n\,(n+2)} \to \frac{A}{n} + \frac{B}{n+2} = \frac{1}{n(n+2)} \to A(n+2) + Bn = 1 \;\Rightarrow\; A = \frac{1}{2}\; B = -\frac{1}{2}$$

$$\frac{1}{2} \lim_{N\to\infty} \left[\sum_{n=0}^{N} \left(\frac{1}{n+2} - \frac{1}{n}\right)\right] = \frac{1}{2}\left(1 + \frac{1}{2} + \lim_{N\to\infty}\left[\frac{1}{N+2}\right]\right) = \frac{3}{4}$$

4

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2\cdot 3^n)}{2^{2n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n(-1)(n+2)\cdot 3^n}{2^{2n}\cdot 2} = \frac{1}{2}\sum_{n=1}^{\infty}(-1)^n \frac{(n+2)\cdot 3^n}{2^{2n}}$$

$$\lim_{n\to\infty} \frac{(n+2\cdot 3^n)}{2^{2n}} = 0$$

$$\frac{(1+2)3}{2^2} \quad \frac{(2+2)3^2}{2^4}$$

$$\frac{9}{4} \quad \frac{\not{4}\cdot 9}{\not{4}\cdot 4} \qquad \frac{(3+2)3^3}{2^6} = \frac{5\cdot 27}{64} = \frac{135}{64}$$

b_1 b_2 b_3

$$a_n = \frac{(-1)^{n+1}(n+2)\,3^n}{2^{2n}}$$

$$b_n = \frac{(n+2)\,3^n}{2^{2n}}$$

a_1 cancels a_2 and from a_3 on, b_n converges.