Module 2 Lesson 1 Assignment

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1 $\sum_{n=1}^{\infty} \frac{4^n+3^n}{7^n}$

$$\sum_{n=1}^{\infty} \frac{4^n + 3^n}{7^n}$$
$$\sum_{n=1}^{\infty} \frac{4^n}{7^n} + \frac{3^n}{7^n}$$
$$\sum_{n=1}^{\infty} \frac{4^n}{7^n} + \sum_{n=1}^{\infty} \frac{3^n}{7^n}$$
$$\sum_{n=1}^{\infty} \left(\frac{4}{7}\right)^n + \sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n$$

Bboth infinite sums are geometric and the ratio r is in the $1 > r > 0$ range. Therefore both sums converge to

$$\frac{1}{1 - \frac{4}{7}} + \frac{1}{\frac{3}{7}}$$

2 $\sum\limits_{n=1}^{\infty} \frac{2}{n(n+2)}$

In this infinite sum it looks like partial fraction decomposition will be necessary to solve the sum

$$\sum_{n=1}^{\infty} \frac{2}{n\,(n+2)}$$

As seen on the right side, the expression needs to be changed in order to be able to find the infinite sum

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2}$$

This looks like a telescoping serie which can be solved by

$$\lim_{N\to\infty} \left[\frac{1}{1} - \frac{1}{N+2}\right]$$
$$\lim_{N\to\infty} \left[\frac{1}{1}\right] - \lim_{N\to\infty} \left[\frac{1}{N+2}\right]$$
$$1-0$$
$$1$$

$$\frac{A}{n} + \frac{B}{n+2} = \frac{2}{n(n+2)}$$

$$\frac{A(n+2)+B(n)}{n(n+2)} = \frac{2}{n(n+2)}$$

$$A(n+2)+B(n) = 2$$

Here to find A, n needs to equal 0

$$\begin{aligned} A(n+2)+B(n) &= 2 \\ A(0+2)+B(0) &= 2 \\ 2A &= 2 \\ A &= 1 \end{aligned}$$

And to find B, n needs to equal -2

$$\begin{aligned} A(n+2)+B(n) &= 2 \\ A(-2+2)+B(-2) &= 2 \\ -2B &= 2 \\ B &= -1 \end{aligned}$$

When substituted, the resulting expression is:

$$\frac{1}{n} - \frac{1}{n+2}$$

3 $\sum_{n=1}^{\infty} e^{-n}$

This is a geometric series that converges since $0 < r < 1$

$$\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n = \frac{1}{1-\frac{1}{e}} = \left(\frac{e-1}{e}\right)^{-1} = \frac{e}{e-1}$$