Module 3 Review Assignment

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1

$$
\begin{aligned}
a_n =& \frac{(1+n)!}{n!} \\
a_n =& \frac{(n+1)(n)(n-1)\cdots}{(n)(n-1)\cdots} \\
a_n =& n+1 \\
& \lim_{\infty} [n+1] = \infty
\end{aligned}
$$

2

$$
\begin{aligned}
& \sum_{n=0}^{\infty} \frac{(-2)^{n+3}}{5^{n+2}} \\
& \sum_{n=0}^{\infty} \frac{(-2)^n (-2)^3}{5^n (5)^2} \\
\frac{(-2)^3}{(5)^2} & \sum_{n=0}^{\infty} \left(\frac{-2}{5}\right)^n \\
-\frac{8}{25} & \sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{5}\right)^n \\
-\frac{8}{25} & \cdot \frac{5}{7} = \boxed{-\tfrac{40}{175}}
\end{aligned}
$$

3

$$
\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{n(n+2)} \to \frac{A}{n} + \frac{B}{n+2} = \frac{1}{n(n+2)} \to A(n+2) + Bn = 1 \Rightarrow A = \frac{1}{2}\ B = -\frac{1}{2} \\
\frac{1}{2} \lim_{N\to\infty} \left[\sum_{n=0}^{N} \left(\frac{1}{n+2} - \frac{1}{n}\right)\right] = \frac{1}{2}\left(1 + \frac{1}{2} + \lim_{N\to\infty}\left[\frac{1}{N+2}\right]\right) = \frac{3}{4}
\end{aligned}
$$

4

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+2\cdot 3^n)}{2^{2n+1}}=\sum_{n=1}^{\infty}\frac{(-1)^n(-1)(n+2)\cdot 3^n}{2^{2n}\cdot 2}=\frac{1}{2}\sum_{n=1}^{\infty}(-1)^n\frac{(n+2)\cdot 3^n}{2^{2n}}$$

$$\lim_{n\to\infty}\frac{(n+2\cdot 3^n)}{2^{2n}}=0$$

$$\frac{(1+2)3}{2^2}\quad\frac{(2+2)3^2}{2^4}$$

$$\underset{b_1}{\frac{9}{4}}\quad\underset{b_2}{\frac{\cancel{4}\cdot 9}{\cancel{4}\cdot 4}}\qquad \frac{(3+2)3^3}{2^6}=\frac{5\cdot 27}{64}=\underset{b_3}{\frac{135}{64}}$$

$$a_n=\frac{(-1)^{n+1}(n+2)\,3^n}{2^{2n}}$$

$$b_n=\frac{(n+2)\,3^n}{2^{2n}}$$

a_1 cancels a_2 and from a_3 on, b_n converges.

5

a) $\sum_{n=1}^{\infty}\left(\sqrt{2}\right)^n$

geometric series

$r=\sqrt{2}$

$|r|>1$

the series diverges by the nth term test

b) $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+\ln(n)}$

$\lim_{n\to\infty}\left[\frac{n^{\frac{1}{2}}}{n^2}:\frac{1}{n^2}\right]$

$\lim_{n\to\infty}\left[\frac{n^{\frac{1}{2}+2}}{n^2}\right]$ $\rightarrow$ By the limit comparison test, both converge and by direct comparison test $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+\ln(n)}$ converges absolutely.

$\sum_{n=1}^{\infty}e^{-1}n!=\sum_{n=1}^{\infty}\frac{n!}{e^n}$

$\lim_{n\to\infty}\left[\sqrt[n]{\left|\frac{n!}{e^n}\right|}\right]=\infty$

by the Root test $\sum_{n=1}^{\infty}e^{-1}n!$ diverges.

6

a) $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$

converges

$|r|<1$

b) $\sum_{n=1}^{\infty}\frac{1}{\ln n}\boxed{\frac{1}{\ln n}>\frac{1}{n}}$

by direct comparison test, $a_n>b_n$ the series diverges

$$c)\ \sum_{n=1}^{\infty}\frac{2n}{n+1}\qquad\frac{2(n+1)}{(n+1)+1}:\frac{2n}{n+1}\rightarrow\frac{\cancel{2}(n+1)^2}{\cancel{2}n(n+2)}=\frac{(n+1)^2}{n^2+2n}$$

$\sum_{n=1}^{\infty}\frac{2n}{n+1}$ diverges by the nth term test

$$\lim_{n\to\infty}\left[\frac{(n+1)^2}{n^2+2n}\right]=1\ 1\neq 0$$

$d)$ $\sum\limits_{n=1}^{\infty}(-1)^n\frac{1}{\ln(n)}$ by the alternating series test, the series converge conditionally.
$e)$ $\sum\limits_{n=1}^{\infty}\frac{1}{n(2n+1)}$ Converges by direct comparison to $\sum\limits_{n=1}^{\infty}\frac{1}{n^2}$

7

$$\sum_{n=1}^{4}(-1)^{n+1}\left(\frac{1}{2}\right)^n = S_n$$

$a)$ $S_4 = +\left(\frac{1}{2}\right) - \left(\frac{1}{4}\right) + \left(\frac{1}{8}\right) - \left(\frac{1}{16}\right) = \frac{4+1}{8} - \frac{4+1}{16} = \frac{10-5}{16} = \frac{5}{16}$
$b)$ $\frac{5}{16} < S_n < \frac{11}{32}$
$c)$ $\frac{\frac{1}{2}}{1-\left(\frac{1}{2}\right)} \cdot \frac{1}{2} = \frac{1}{2}$
$d)$ yes