Module 3 Review Assignment

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1

$$
\begin{aligned}
a_n =& \frac{(1+n)!}{n!} \\
a_n =& \frac{(n+1)(n)(n-1)\cdots}{(n)(n-1)\cdots} \\
a_n =& n+1 \\
& \lim_{\infty}[n+1] = \infty
\end{aligned}
$$

2

$$
\begin{aligned}
& \sum_{n=0}^{\infty} \frac{(-2)^{n+3}}{5^{n+2}} \\
& \sum_{n=0}^{\infty} \frac{(-2)^n (-2)^3}{5^n (5)^2} \\
\frac{(-2)^3}{(5)^2} & \sum_{n=0}^{\infty} \left(\frac{-2}{5}\right)^n \\
-\frac{8}{25} & \sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{5}\right)^n \\
-\frac{8}{25} & \cdot \frac{5}{7} = \boxed{-\tfrac{40}{175}}
\end{aligned}
$$

3

$$
\sum_{n=1}^{\infty} \frac{1}{n(n+2)} \to \frac{A}{n} + \frac{B}{n+2} = \frac{1}{n(n+2)} \to A(n+2) + Bn = 1 \;\Rightarrow\; A = \frac{1}{2}\; B = -\frac{1}{2}
$$

$$
\frac{1}{2}\lim_{N\to\infty}\left[\sum_{n=0}^{N}\left(\frac{1}{n+2} - \frac{1}{n}\right)\right] = \frac{1}{2}\left(1 + \frac{1}{2} + \lim_{N\to\infty}\left[\frac{1}{N+2}\right]\right) = \frac{3}{4}
$$

4

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2\cdot 3^n)}{2^{2n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n(-1)(n+2)\cdot 3^n}{2^{2n}\cdot 2} = \frac{1}{2}\sum_{n=1}^{\infty}(-1)^n \frac{(n+2)\cdot 3^n}{2^{2n}}$$

$$\lim_{n\to\infty} \frac{(n+2\cdot 3^n)}{2^{2n}} = 0$$

$$\frac{(1+2)3}{2^2} \quad \frac{(2+2)3^2}{2^4}$$

$$\underset{b_1}{\frac{9}{4}} \quad \underset{b_2}{\frac{\cancel{4}\cdot 9}{\cancel{4}\cdot 4}} \qquad \frac{(3+2)3^3}{2^6} = \frac{5\cdot 27}{64} = \underset{b_3}{\frac{135}{64}}$$

$$a_n = \frac{(-1)^{n+1}(n+2)\,3^n}{2^{2n}}$$

$$b_n = \frac{(n+2)\,3^n}{2^{2n}}$$

a_1 cancels a_2 and from a_3 on, b_n converges.

5

a) $\sum_{n=1}^{\infty} \left(\sqrt{2}\right)^n$

geometric series

$r = \sqrt{2}$

$|r| > 1$

the series diverges by the nth term test

b) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+\ln(n)}$

$\lim_{n\to\infty} \left[\frac{n^{\frac{1}{2}}}{n^2} : \frac{1}{n^2}\right]$

$\lim_{n\to\infty} \left[\frac{n^{\frac{1}{2}+2}}{n^2}\right]$ $\rightarrow$ By the limit comparison test, both converge and by direct comparison test $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+\ln(n)}$ converges absolutely.

$\sum_{n=1}^{\infty} e^{-1}n! = \sum_{n=1}^{\infty} \frac{n!}{e^n}$

$\lim_{n\to\infty} \left[\sqrt[n]{\left|\frac{n!}{e^n}\right|}\right] = \infty$

by the Root test $\sum_{n=1}^{\infty} e^{-1}n!$ diverges.