Module 4 Review Assignment

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$$x = e^t \ \& \ y = \cos t$$

1. **Find the total distance traveled on the close interval** $[0, 2]$

$$\int_0^2 \sqrt{1 + \left(\frac{-\sin t}{e^t}\right)^2} e^t dt \approx 6.558$$

2. **Find the speed of the particle at** $t = 2$

$$-\frac{\sin(t)}{e^t} \Rightarrow \frac{-\sin 2}{e^2} \approx -0.123$$

3. **Find** $\frac{dy}{dx}$

$$-\frac{\sin(t)}{e^t}$$

4. **Find** $\frac{d^2y}{dx^2}$

$$\frac{\frac{d}{dt}\left[-\sin(t)e^{-t}\right]}{e^t} \Rightarrow \frac{-\left(\cos(t)e^{-t} + \left(-e^{-t}\right)\sin(t)\right)}{e^t} = -\frac{e^{-t}\left(\cos(t) - \sin(t)\right)}{e^t}$$

5. **Let** $p(t)$ **be the distance, in meters, from the point** $(0, 1)$ **at time** t**.** $\vec{p} = \langle 2t, \cos(t) \rangle$

   a

$$-\frac{\sin(t)}{2} \rightarrow \frac{-\frac{\cos(t)}{2}}{2} = -\frac{1}{4}\cos(t)$$

   b

$$-\frac{1}{4}\cos\left(\cos\frac{5\pi}{6}\right) = -\frac{1}{4}\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{8}$$

6. **Using** $\vec{p} = \langle 2t, \cos(t) \rangle$ **explain what the acceleration of the particle means in relation to its position at** $t = \frac{5\pi}{6}$
   It means that at the time $\frac{5\pi}{6}$ the velocity of the particle is changing at a rate of $\frac{\sqrt{3}}{8}$

7. **Determine the position vector given the following:**

$$\vec{a}(t) = \left\langle \sqrt{t}, t+1 \right\rangle$$
$$\vec{v}(0) = \langle 1, 2 \rangle$$
$$\vec{p}(0) = \langle -1, 5 \rangle$$

$$\int \vec{a}(t)\ dt \to \int \vec{v}(t)\ dt \longrightarrow \vec{p}(t)$$

$$\frac{2}{3}t^{\frac{3}{2}} + c \Rightarrow \frac{2}{3}0^{\frac{3}{2}} + c = 1 \to c = 1 \qquad \frac{t^2}{2} + t + c \Rightarrow \frac{0^2}{2} + 0 + c = 2 \to c = 2$$

$$\frac{2}{3} \cdot \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + t + c \Rightarrow \frac{4}{15}0^{\frac{5}{2}} + 0 + c = -1 \to c = -1 \quad \frac{t^3}{6} + \frac{t^2}{2} + 2t + c = 5 \Rightarrow \frac{0^3}{6} + \frac{0^2}{2} + 2(0) + c = 5 \to c = 5$$

$$\vec{p}(t) = \left\langle \frac{4}{15}t^{\frac{5}{2}} + t - 1\ ,\ \frac{t^3}{6} + \frac{t^2}{2} + 2t + 5 \right\rangle$$

Then use it to determine the value of $\vec{p}(7)$