Module 3 Review Assignment

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1

$$a_n = \frac{(1+n)!}{n!}$$
$$a_n = \frac{(n+1)\,(n)\,(n-1)\cdots}{(n)\,(n-1)\cdots}$$
$$a_n = n+1$$
$$\lim_{\infty}[n+1] = \infty$$

2

$$\sum_{n=0}^{\infty} \frac{(-2)^{n+3}}{5^{n+2}}$$
$$\sum_{n=0}^{\infty} \frac{(-2)^n\,(-2)^3}{5^n(5)^2}$$
$$\frac{(-2)^3}{(5)^2}\sum_{n=0}^{\infty}\left(\frac{-2}{5}\right)^n$$
$$-\frac{8}{25}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{5}\right)^n$$
$$-\frac{8}{25}\cdot\frac{5}{7} = \boxed{-\frac{40}{175}}$$

3

$$\sum_{n=1}^{\infty} \frac{1}{n\,(n+2)} \to \frac{A}{n}+\frac{B}{n+2}=\frac{1}{n(n+2)} \to A(n+2)+Bn=1 \;\Rightarrow\; A=\frac{1}{2}\; B=-\frac{1}{2}$$
$$\frac{1}{2}\lim_{N\to\infty}\left[\sum_{n=0}^{N}\left(\frac{1}{n+2}-\frac{1}{n}\right)\right]=\frac{1}{2}\left(1+\frac{1}{2}+\lim_{N\to\infty}\left[\frac{1}{N+2}\right]\right)=\frac{3}{4}$$

4

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2\cdot 3^n)}{2^{2n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n(-1)(n+2)\cdot 3^n}{2^{2n}\cdot 2} = \frac{1}{2}\sum_{n=1}^{\infty}(-1)^n \frac{(n+2)\cdot 3^n}{2^{2n}}$$

$$\lim_{n\to\infty} \frac{(n+2\cdot 3^n)}{2^{2n}} = 0$$

$$\frac{(1+2)3}{2^2} \quad \frac{(2+2)3^2}{2^4}$$

$$a_n = \frac{(-1)^{n+1}(n+2)\,3^n}{2^{2n}}$$

$$b_n = \frac{(n+2)\,3^n}{2^{2n}}$$

$$\frac{9}{4} \quad \frac{\cancel{4}\cdot 9}{\cancel{4}\cdot 4} \qquad \frac{(3+2)3^3}{2^6} = \frac{5\cdot 27}{64} = \frac{135}{64}$$

$$b_1 \quad b_2 \qquad\qquad\qquad b_3$$

a_1 cancels a_2 and from a_3 on, b_n converges.

5

a) $\sum_{n=1}^{\infty} \left(\sqrt{2}\right)^n$

geometric series

$r = \sqrt{2}$

$|r| > 1$

the series diverges by the nth term test

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b) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+\ln(n)}$

$$\lim_{n\to\infty} \left[\frac{n^{\frac{1}{2}}}{n^2} : \frac{1}{n^2} \right]$$

$$\lim_{n\to\infty} \left[\frac{n^{\frac{1}{2}+2}}{n^2} \right] \to \text{By the limit comparison test, both converge}$$

and by direct comparison test $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+\ln(n)}$ converges absolutely.

$$\sum_{n=1}^{\infty} e^{-1}n! = \sum_{n=1}^{\infty} \frac{n!}{e^n}$$

$$\lim_{n\to\infty} \left[\sqrt[n]{\left|\frac{n!}{e^n}\right|} \right] = \infty$$

by the Root test $\sum_{n=1}^{\infty} e^{-1}n!$ diverges.

6

a) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

converges

$|r| < 1$

b) $\sum_{n=1}^{\infty} \frac{1}{\ln n} \boxed{\frac{1}{\ln n} > \frac{1}{n}}$

by direct comparison test, $a_n > b_n$ the series diverges

c) $\sum_{n=1}^{\infty} \frac{2n}{n+1} \quad \frac{2(n+1)}{(n+1)+1} : \frac{2n}{n+1} \to \frac{\cancel{2}(n+1)^2}{\cancel{2}n(n+2)} = \frac{(n+1)^2}{n^2+2n}$

$\sum_{n=1}^{\infty} \frac{2n}{n+1}$ diverges by the nth term test

$$\lim_{n\to\infty} \left[\frac{(n+1)^2}{n^2+2n} \right] = 1 \quad 1 \neq 0$$

d) $\sum\limits_{n=1}^{\infty} (-1)^n \frac{1}{\ln(n)}$ by the alternating series test, the series converge conditionally.

e) $\sum\limits_{n=1}^{\infty} \frac{1}{n(2n+1)}$ Converges by direct comparison to $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$