Question 1: Money in Utility

Derive parts of the MIU setup and log linearize a few of them.

(a) (from Walsh Section 2, Question 5) Suppose our utility function is replaced by

$$u(c_t, m_t, 1-n_t) = \frac{\left[ac_t^{1-b} + (1-a)m_t^{1-b}\right]^{\frac{1-\Phi}{1-b}}}{1-\Phi}\left(\frac{(1-n_t)^{1-\eta}}{1-\eta}\right) \tag{1}$$

Derive the first-order conditions for the household's optimal money holdings.

(b) Using the consumer's dynamic problem, derive euler-equations for these preferences to price bonds, money holdings, and capital.

(c) (Optional) Derive the log-linearized marginal utility of consumption and euler equations in (34) and (39) of the money in utility notes.[1]

[1] Hint: Look at Walsh Appendix 2.7 if you are having difficulty on the log-linearization

Question 2: Neoclassical Growth/RBC

A planner maximizes the representative household's welfare with stochastic productivity,

$$\max_{\{c_t,n_t,k_{t+1}\}} \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \frac{\left(c_t^{\theta}(1-n_t)^{1-\theta}\right)^{1-\gamma}}{1-\gamma}\right] \tag{2}$$

$$\text{s.t. } c_t + k_{t+1} = e^{z_t}k_t^{\alpha}n_t^{1-\alpha} + (1-\delta)k_t \tag{3}$$

$$z_{t+1} = \rho z_t + \sigma\epsilon_{t+1},\ \epsilon_{t+1} \sim N(0,1) \tag{4}$$

(a) Take the FONC for the choice variables to derive an intertemporal Euler condition along with a condition on the labor supply. Combine with the resource constraint and the stochastic process of z_t to characterize the system by 4 stochastic difference equations.

(b) Assume the parameters are: $\beta = 0.987$, $\theta = 0.357$, $\delta = .012$, $\alpha = .4$, $\gamma = 2.0$, $\rho = .95$, and $\sigma = 0.007$

Conduct the following simulations/calculations with Dolo.jl (or equivalent)

1. Starting from the non-stochastic steady state k^*, simulate a path of k_t, c_t, z_t, n_t using draws from the random z_t process for 40 periods. Display all of these variables: k_t, c_t, z_t, n_t
2. Calculate the impulse response functions to a technology shock to z_t (i.e., a 1 standard deviation shock to ϵ_t

(c) (Optional) start from $k_0 = k^*/2$ and $z_0 = 0$, simulate the transition dynamics for 40 periods.