Question 1: Optimal Unemployment Policy

(From Walsh in Section 7.6) Suppose the central bank dislikes inflation variability around a target level π^*. It also prefers to keep unemployment stable around an unemployment target u^*. These objectives can be represented in terms of minimizing

$$V = \lambda(u - u^*)^2 + \frac{1}{2}(\pi - \pi^*)^2$$

where π is the inflation fate and u is the unemployment rate. The economy is described by

$$u = u_n - a(\pi - \pi^e) + v,$$

where u_n is the natural rate of unemployment, and π^e is expected inflation. Expectations are formed by the public before observing the disturbance v. The central bank can set inflation after observing v. Assume $u^* < u_n$.

(a) What is the equilibrium rate of inflation under discretion? What is the equilibrium unemployment rate?

(b) Is equilibrium unemployment under discretion affected by u^*? Explain.

(c) Is equilibrium inflation under discretion affected by u^*? Explain.

(d) How is equilibrium inflation under discretion affected by v? Explain.

(e) What is the equilibrium rate of inflation under commitment? What is the equilibrium unemployment rate under commitment? How are they affected by u^*? Explain.

Question 2: Inflation Bias

(from Walsh in Section 7.6) Consider a version of the linearized New Keynesian IS-curve with persistence in output

$$y_t = (1-\theta)y_n + \theta y_{t-1} + a(\pi_t - \pi_t^e) + e$$

where $\theta \in (0,1)$ and $a > 0$, with y_n the "natural" rate of output and π^e is expected inflation, y_t realized output, y_{t-1} last periods realized output, and e a shock.

Suppose the policymaker has a two-period horizon with objective function given by minimizing the expected loss

$$L = \mathbb{E}\left[L_t + \beta L_{t+1}\right]$$

with the quadratic loss function at each period i,

$$L_i = \frac{1}{2}\left(\lambda(y_i - y_n - k)^2 + \pi_i^2\right)$$

and parameters λ and k.

(a) Derive the optimal policy under commitment

(b) Derive the optimal policy under discretion (i.e., no ability to commit to a policy prior to the first period).

(c) How does the presence of persistence ($\theta > 0$ affect the inflation bias)?

Question 3: Model of Sticky Prices

Consider the following sticky price model. The representative household has preferences given by:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t U\left(C_t,\frac{M_t}{P_t},H_t\right)\right],\quad \beta\in(0,1)\,, \tag{1}$$

where

$$U=\log(C)+\chi\log\left(\frac{M}{P}\right)-\eta H,\quad \chi,\eta>0. \tag{2}$$

The household maximizes utility via its choice of consumption, C_t, labor supply, H_t, money holdings, M_t, and nominal one-period bond holdings, B_t, subject to the budget constraint:

$$P_tC_t+M_t+B_t=W_tH_t+\int_0^1\Pi_{jt}dj+M_{t-1}+(1+i_{t-1})\,B_{t-1}+T_t. \tag{3}$$

Here, P_t is the price level, W_t is the nominal wage rate, Π_{jt} is the profit earned from intermediate good firm j, i_t is the nominal interest rate earned on holdings of B_t, and T_t is a lump-sum transfer, all of which the household takes as given.

On the production side, there are 2 types of firms: final good firms and intermediate good firms. Final good firms buy intermediate goods to produce output according to:

$$Y_t=\left[\int_0^1 Y_{jt}^{\frac{\lambda-1}{\lambda}}dj\right]^{\frac{\lambda}{\lambda-1}},\quad \lambda>1. \tag{4}$$

Final good firms are perfect competitors, and take the price of their output, P_t, as well as the price of intermediate inputs, P_{jt} $\forall j\in[0,1]$, as given. Final goods are sold to households as consumption. The representative final good firm's objective is to maximize profit:

$$\max_{\{Y_{jt}\}_{\forall j}}\left[P_tY_t-\int_0^1 P_{jt}Y_{jt}dj\right]. \tag{5}$$

Each intermediate firm is a monopolist in the production of its good $j\in[0,1]$. It produces goods according to the production function, $Y_{jt}=H_{jt}$. It hires labor, H_{jt}, from a perfectly competitive labor market at the wage W_t.

Intermediate firms set prices in staggered, overlapping, 2-period contracts. A half of firms set their prices in even periods, the other half in odd periods. When intermediate good firm j sets its price, it does so to maximize:

$$\max_{P_{jt}}\left\{\Pi\left(P_{jt},Y_{jt},W_t\right)+E_t\left[\theta_{t+1}\Pi\left(P_{jt},Y_{jt+1},W_{t+1}\right)\right]\right\}, \tag{6}$$

where in any given period, profit is given by:

$$\Pi\left(P_j,Y_j,W\right)=\left(P_j-W\right)Y_j. \tag{7}$$

θ_{t+1} is the discount factor the firm uses when comparing the marginal value of future profit to current profit. When the firm chooses its price, it does so taking wages and the final good firm's demand for its product as given. At date t, in a symmetric equilibrium a half of firms will be charging the same price P_{jt}; to simplify notation denote $P_{jt} = \bar{P}_t$. Hence, the other half of firms will be charging the price $\bar{P}_{t-1}$.

Finally, we specify the central bank as acting according to the money supply rule:

$$\tilde{M}_t = \tilde{M}_{t-1} \exp\left(\mu + v_t\right). \tag{8}$$

Here, $\tilde{M}_t$ is the amount of money the central bank supplies at date t, μ is the trend money growth rate, and v_t is a money shock which has mean zero. Money growth finances transfers to the household, so: $T_t = \tilde{M}_t - \tilde{M}_{t-1}$.

(a) Derive the household's date t FONCs and interpret these.

(b) Derive the final good firm's date t FONC with respect to Y_{jt}. How does the final good firm's demand for Y_{jt} depend on P_{jt}?

(c) Derive the optimal 2-period price, $\bar{P}_t$. If the intermediate good firm could set flexible prices (i.e., set a different price for each date t), how would that price – call it P_t^* – be related to W_t? Using this, interpret the FONC for $\bar{P}_t$.

(d) In symmetric equilibrium, what is the relationship between the aggregate price level, P_t, and the intermediate good prices, $\bar{P}_t$ and $\bar{P}_{t-1}$? Use your answer to part (b) and the fact that final good firms earn zero profit in equilibrium to derive this expression.

(e) Explain how a money supply shock, v_t, would affect output and prices in equilibrium. In particular, will the shock have a persistent effect on output? Discuss or show why or why not.

Question 4: (Optional) Sticky Wages

Consider a simple sticky wage monetary model. In order to have wage setting behaviour, we need some sort of monopoly power in labor supply. To get this, suppose there is a continuum of households populated on the interval from 0 to 1. Each household i supplies a differentiated labor service, ℓ_i, for $i \in [0, 1]$. These labor services are combined to form a supply of aggregate labor, L, via:

$$L_t = \left[\int_0^1 \ell_{it}^{1/\mu} di\right]^{\mu}, \quad \mu > 1. \tag{9}$$

Aggregate labor is produced by perfectly competitive employment agencies which sell L_t to final goods producing firms at the market wage W_t, and hire ℓ_{it} from household i at the wage w_{it-1}. Note that the factor price of differentiated labor is set based on date $t-1$ information. This is the manifestation of the sticky wage assumption; households must choose their wage in effect at date t before observing any date t shocks. Hence, the representative employment agency's problem is

$$\max_{\{\ell_{it}\}} \left[W_t L_t - \int_0^1 w_{it-1}\ell_{it} di\right], \tag{10}$$

subject to the production function above.

(a) Derive the employment agency's demand function for labor input ℓ_{it} and interpret.

(b) Perfectly competitive final good firms hire aggregate labor at the wage W_t to produce output according to the production function:

$$Y_t = L_t^{\alpha} K_t^{1-\alpha}, \quad \alpha \leq 1. \tag{11}$$

Final output is consumed by households, and traded at the market price P_t. Capital is hired at the competitive rental rate, r_t. Derive the representative final good firm's FONC.

(c) Each household is a monopolist in its differentiated labor service, with markup determined by μ. The household derives utility from consumption, holdings of real balances, and leisure. It maximizes:

$$E \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{M_t}{P_t}, \ell_t\right), \tag{12}$$

subject to the budget constraint:

$$P_t c_t + M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + T_t + w_{it-1}\ell_{it} + r_t K_t. \tag{13}$$

For convenience, I do not subscript consumption (c) or money holdings (M) by the household indicator, i, since in a symmetric equilibrium all households will behave the same (but I do subscript the household's wage and labor supply by i to maintain consistency with the presentation of the production side of the economy). The household's capital holdings cannot be changed, so $K_t \equiv 1$ always. Here, B_t represents the household's purchases of nominal one-period bonds at date t; R_t is the gross return on the bond upon maturity at date $t+1$. The monetary authority injects and extracts money from the private sector via lump-sum transfers T_t.

The household chooses date t consumption, real balances, and bond holdings after observing date t shocks. However, it must set a wage in effect at date t, w_{it-1}, before observing any date t shocks. At date t, the household supplies as much labor as is demanded by employment agencies at the posted wage.

Setup the household's problem as a Lagrangian and derive the FONC's with respect to consumption, money holdings, and bond holdings at date t. Simplify to get an intertemporal FONC for bond holdings and an intratemporal FONC for real balances. Interpret these.

(d) Derive the date t FONC with respect to w_{it-1}. Simplify and interpret.