‘MATH+ECON+CODE’ MASTERCLASS ON MATCHING MODELS, OPTIMAL TRANSPORT AND APPLICATIONS

Alfred Galichon (NYU)

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Day 4, May 24 2018: one-to-many matching
Block 12. Matching function equlibrium algorithms

- Assume that $\mathcal{X}$ is worker's location, and $\mathcal{Y}$ is firm's location. These sets are all set to be a grid of 100 points in a square city $[0,1]^2$. That is, $\mathcal{X} = \mathcal{Y} = \{1/10, 2/10, 3/10, ..., 9/10, 1\} \times \{1/10, 2/10, 3/10, ..., 9/10, 1\}$.
- Assume:
  - there is a mass one $n_x = 1$ of workers at each $x \in \mathcal{X}$.
  - if a firm $y \in \mathcal{Y}$ has coordinates $\left(y^1, y^2\right)$ there is a mass $m_y = y^1 \times y^2$ of passengers at y.

- Assume that if the wage of a worker i of type x working for a firm of type y is w_{xy}, then the payoff of the worker is

$$\begin{aligned} u_i &= \alpha_{xy} + w_{xy} + \varepsilon_{iy} \text{ if matched with firm } y \\ u_i &= \varepsilon_{i0} \text{ if unassigned} \end{aligned}$$

  where $\alpha_{xy} = -0.1 d\left(x, y\right)^2$ and $d\left(x, y\right)^2$ is the squared Euclidian distance between y and z, and $\left(\varepsilon_{iy}, y \in \mathcal{Y}_0\right)$ are i.i.d. Gumbel random utility terms.
- The payoff of a firm j of type y is

$$\begin{aligned} v_j &= \gamma_{xy} - w_{xy} + \eta_{xj} \text{ if matched with } x \\ v_j &= \eta_{0j} \text{ if unassigned} \end{aligned}$$

- **Q1**. Compute:
  1. the total number of matched pairs $\sum_{xy} \mu_{xy}$
  2. the average wage $\left(\sum_{xy} \mu_{xy} w_{xy}\right) / \left(\sum_{xy} \mu_{xy}\right)$.

- **Q2**. Same question if the workers' utilities are replaced by

$$u_i = \alpha_{xy} + 0.8 \times w_{xy} + \varepsilon_{iy} \text{ if matched with firm } y$$
$$u_i = \varepsilon_{i0} \text{ if unassigned}$$

and the firms' utilities are unchanged.

- **Q3**. Same question if the taxation schedule is given by the following progressive taxation schedule with three tax rates

$$\min \{w, 0.8 \times (w - 0.1) + 0.1, 0.6\,(w - 0.2) + 0.18\}.$$

- Now keep the same n_x, m_y and the same α_{xy} and γ_{xy} but change the interpretation to assume that i is a man and j a woman; and if a man i matches with a woman j, they get utilities

$$u_i = 1 - 0.1d\left(x, y\right)^2 + 0.2g + \ln c_x$$
$$v_j = 1 - 0.2d\left(x, y\right)^2 + 0.1g + \ln c_y$$

  where $g \in \{0, 1, 2\}$ is the number of kids, and c_x and c_y are the private consumptions, which are subject to the constraint $c_x + c_y = 2 - 5g$.
- **Q4**. Compute the total number of matched agents at equilibrium $\sum_{xy} \mu_{xy}$, as well as the average number of kids per couples

$$\frac{\sum_{xy} \mu_{xy} g_{xy}}{\sum_{xy} \mu_{xy}}.$$