‘MATH+ECON+CODE’ MASTERCLASS ON COMPETITIVE EQUILIBRIUM: WALRASIAN EQUILIBRIUM WITH SUBSTITUTES

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Day 3, May 23 2018: matching with general transfers (1)
Block 9. Code: network algorithms

- Network algorithms

Download the NYC subway data from the Gihtub repository (subrepository `mec_equil\data\NYC_subway`). The '`arcs`' file lists for each arc represented by a line, the origin node (column 1), the destination node (column 2) and the length of the arc (column 3). You can ignore the other colums. The '`nodes`' file lists for each node represented by line, the name of the node (column 1). You can ignore the other columns.
Your origin point z^0 will be the "14 St - Union Sq" station in Manhattan (node #452), and your destination point z^d will be the "59 St (R/N)" station in Brooklyn (node #471).

The shortest path problem is given by

$$\min_{\mu \geq 0} \sum_{a \in \mathcal{A}} d_a \mu_a$$
$$s.t.\ \nabla^\intercal \mu = s$$

where $s_z = 1\left\{z = z^d\right\} - 1\left\{z = z^o\right\}$, and d_a is the lenght of arc a.

Q1. Compute the above problem using Gubori. (Contact us for instruction to install if needed).
Q2. Given the solution μ returned by Gurobi, write down an algorithm to list each nodes of the shortest path.

Q3. Recover your previous answers by writing your own implementation of the Bellman-Ford algorithm.

Consider now an approximate shortest path provided by an entropic regularization of the previous problem ($T > 0$)

$$\min_{\mu \geq 0} \sum_{a \in \mathcal{A}} d_a \mu_a + T \sum_{a \in \mathcal{A}} \mu_a \ln \mu_a$$
$$s.t.\ \nabla^{\mathsf{T}} \mu = s$$

whose primal formulation is

$$\max \sum_z s_z p_z - T \sum_{a \in \mathcal{A}} \exp\left(\frac{(\nabla p - d)_a}{T}\right)$$

Q4. Using one or several methods we've seen (Gradient descent, Newton, or CU), compute the solution μ_a to regularized problem for $T = 1$.

If $a = xy$, then the probability of moving to y conditionally on being at x is given by

$$\mu_{y|x} = \frac{\mu_{xy}}{\sum_{z:xz\in\mathcal{A}} \mu_{xz}} = \frac{\exp\left(\frac{p_y - p_x - d_{xy}}{T}\right)}{\sum_{z:xz\in\mathcal{A}} \exp\left(\frac{p_z - p_x - d_{xz}}{T}\right)}$$

$$= \frac{\exp\left(\frac{p_y - d_{xy}}{T}\right)}{\sum_{z:xz\in\mathcal{A}} \exp\left(\frac{p_z - d_{xz}}{T}\right)}$$

Q5 (optional). Simulate these random trajectories from the origin until they reach the destination point. Report the distribution of the distance travelled.