Root-Squaring for Root-Finding

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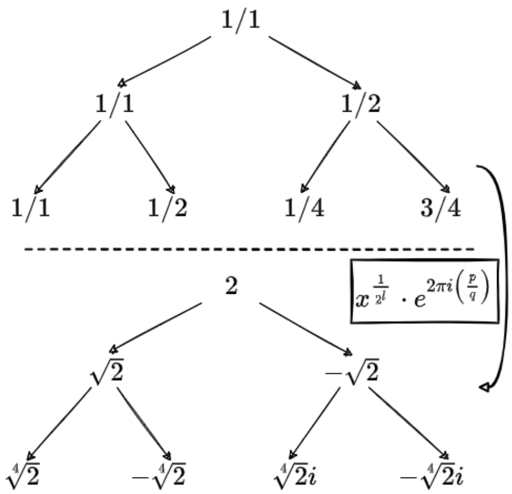

Figure 1: The upper tree depicts the steps of CIRCLE_ROOTS_RATIONAL_FORM(p,q,l) in Alg.1 for $l = 2$, $p = 1$, and $q = 1$. The lower tree depicts the steps of ROOTS(r, t, u, l) in Alg.2 for $r = 2$, $l = 2$, $p = 1$, and $q = 1$

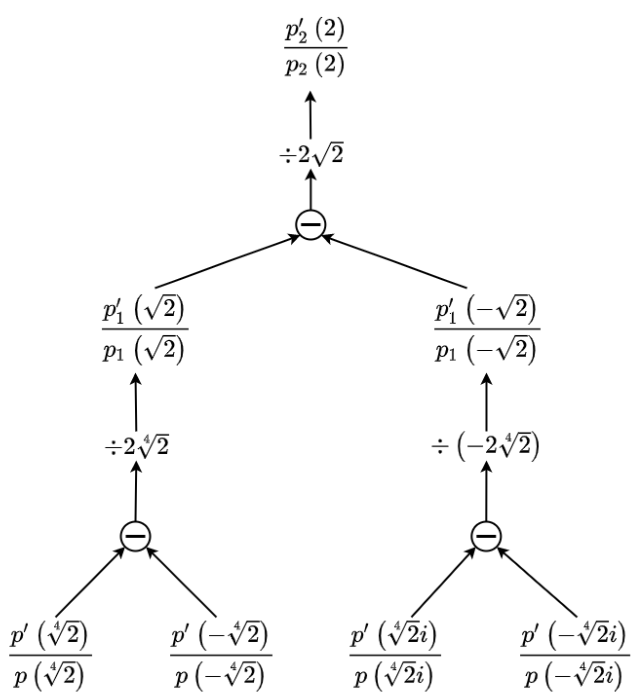

Figure 2: The steps of DLG_RATIONAL_RORM(p, p', r, t, u, l) in Alg.3 for $r = 2$, $l = 2$, $t = 1$, and $u = 1$.

Algorithm 1 CIRCLE_ROOTS_RATIONAL_FORM(p, q, l)

if $p\%q == 0$ **then**
  $r, s := (1,1)$
else
  $r, s := (p,2q)$
end if
if r%s == 0 **then**
  $t, u := (1,2)$
else
  $t, u := (2r + s, 2s)$
end if
if $l == 1$ **then**
  return $[(r, s),(t, u)]$
else if l != 0 **then**
  left := CIRCLE_ROOTS_RATIONAL_FORM($r, s, l - 1$)
  right := CIRCLE_ROOTS_RATIONAL_FORM($t, u, l - 1$)
  return left $\cup$ right
else
  return $[(p, q)]$
end if

Algorithm 2 ROOTS(r, t, u, l)

root_tree = CIRCLE_ROOTS_RATIONAL_FORM(p, q, l)
circ_root = [exp $\left(2 \cdot \pi \cdot i \cdot \frac{r}{s}\right)$ for r, s in root_tree]
roots =[$\sqrt[2^l]{r}$·root for root in circ_root]

ABSTRACT

We revisited the classical root-squaring formula of Dandelin-Lobachevsky-Graeffe for polynomials and found new interesting applications to root-finding.

CCS CONCEPTS

• **Computing methodologies → Hybrid symbolic-numeric methods**.

KEYWORDS

symbolic-numeric computing, root finding, polynomial algorithms, computer algebra

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Algorithm 3 DLG_Rational_Form(p, p', r, t, u, l)

root := Roots(r, t, u, l)
for $r_i \in$ root **do**
  base_step[i] := $\frac{p'(r_i)}{p(r_i)}$
end for
diff[0] := base_step
for $i \leq l$ **do**
  for $j \leq 2^{l-i-1}$ **do**
    diff[$i+1$][j]:=$\frac{1}{2}\frac{\text{diff}[i][2j]-\text{diff}[i][2j+1]}{\text{root}[2j]}$
    root = roots($r, t, u, l-1-i$)
  end for
end for
return derivs[l][0]

Algorithm 4 DLG(p, p', l, x, ϵ)

angle := $\frac{1}{2\pi i}\log(x)$
Set t, u := Angle_As_Integer_Ratio(angle, ϵ)
$r := |x|$
return DLG_Rational_Form(p, p', r, t, u, l)

1 INTRODUCTION

2 RELATED WORKS

3 BACKGROUND

4 MOTIVATING EXAMPLE

5 ALGORITHM DESIGN

6 THEORETICAL ANALYSIS

Lemma 6.1. *The (relative) condition number operator satisifies the following properties:*

(1) $\kappa(f(x)) = |x\log'(f(x))|$
(2) $\kappa(f(x)-g(x)) = |x\frac{\kappa(f(x))-\kappa(g(x))}{f(x)-g(x)}|$
(3) $\kappa(\frac{f(x)}{g(x)}) = ||\kappa(f(x))| - |\kappa(g(x))||$
(4) $\kappa(f(g(x))) = ||\kappa(f(x))\cdot\kappa(g(x))||$

Theorem 6.2. *The condition number for $\frac{p_l'}{p_l}$ using*

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7 EXPERIMENTAL RESULTS

8 CONCLUSION

REFERENCES