Root-Squaring for Root-Finding

Pedro Soto
The Graduate Center, CUNY
New York, New York, USA
psoto@gradcenter.cuny.edu

Soo Go
The Graduate Center, CUNY
New York, New York, USA
sgo@gradcenter.cuny.edu

ABSTRACT

We revisited the classical root-squaring formula of Dandelin-Lobachevsky-Graeffe for polynomials and found new interesting applications to root-finding.

CCS CONCEPTS

• **Computing methodologies → Hybrid symbolic-numeric methods**.

KEYWORDS

symbolic-numeric computing, root finding, polynomial algorithms, computer algebra

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1 INTRODUCTION

We revisit the famous methods simaltanously discovered by Dandelin, Lobachevsky, and Graeffe (See [1] for a history of this problem) and and make furhter progress by applying this indentity to the problem of approximating the root radius of a polynomial.

2 RELATED WORKS

3 BACKGROUND

4 MOTIVATING EXAMPLE

5 ALGORITHM DESIGN

6 THEORETICAL ANALYSIS

THEOREM 6.1. *Algorithm. 3 performs* $q \log q$ *floating point subtractions, divisions, and multiplications and* $q \log q$ *applications of* sin *and* cos*, where* $q = 2^l$*; furthermore, Algorithm. 3 performs at most* $q \log q$ *integer additions, "multiplications-by-2", and* $\%2^\epsilon$ *(*i.e., *mod* 2^ϵ *) operations.*

7 EXPERIMENTAL RESULTS

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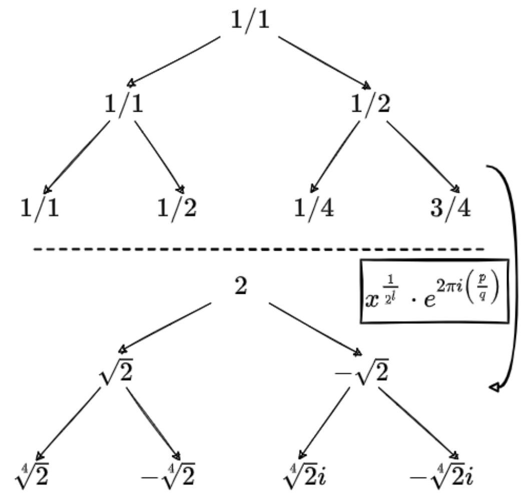

Figure 1: The upper tree depicts the steps of CIRCLE_ROOTS_RATIONAL_FORM(p, q, l) in Alg.1 for $l = 2$, $p = 1$, and $q = 1$. The lower tree depicts the steps of ROOTS(r, t, u, l) in Alg.?? for $r = 2$, $l = 2$, $p = 1$, and $q = 1$

Algorithm 1 CIRCLE_ROOTS_RATIONAL_FORM(p, q, l)

if $p\%q == 0$ **then**
  $r, s := (1,1)$
else
  $r, s := (p,2q)$
end if
if r%s $== 0$ **then**
  $t, u := (1,2)$
else
  $t, u := (2r + s, 2s)$
end if
if $l == 1$ **then**
  return $[(r, s),(t, u)]$
else if $l \mathrel{!=} 0$ **then**
  left := CIRCLE_ROOTS_RATIONAL_FORM($r, s, l - 1$)
  right := CIRCLE_ROOTS_RATIONAL_FORM($t, u, l - 1$)
  return left ∪ right
else
  return $[(p, q)]$
end if

8 CONCLUSION

REFERENCES

[1] Alston S. Householder. 1959. Dandelin, Lobacevskii, or Graeffe. *The American Mathematical Monthly* 66, 6 (1959), 464–466. http://www.jstor.org/stable/2310626

Table 1: Experimental Data for Chebyshev.

DEGREE	l	$e = -\log(\lvert x \rvert)$	MPMATH PRECISION	RELATIVE ERROR r_d	RELATIVE ERROR r_1	RUNTIME	MPSOLVE ROOT RADIUS
20	4	616	332	0.15	0.09	0.17	$[0.00982, 1.0]$
40							
80							
160							
320							

Algorithm 3 DLG_RATIONAL_FORM(p, p', r, t, u, l)

root := ROOTS(r, t, u, l)
for $r_i \in$ root **do**
  base_step[i] := $\frac{p'(r_i)}{p(r_i)}$
end for
diff[0] := base_step
for $i \leq l$ **do**
  for $j \leq 2^{l-i-1}$ **do**
    diff[$i+1$][j]:= $\frac{1}{2} \frac{\text{diff}[i][2j] - \text{diff}[i][2j+1]}{\text{root}[2j]}$
    root = roots($r, t, u, l - 1 - i$)
  end for
end for
return diff[l][0]

Algorithm 4 DLG(p, p', l, x, ϵ)

angle := $\frac{1}{2\pi i} \log(x)$
$u := 2^{\epsilon}$
$t :=$ (angle $\cdot\, u$)%1
$r := |x|$
return DLG_RATIONAL_FORM(p, p', r, t, u, l)