Root-Squaring for Root-Finding

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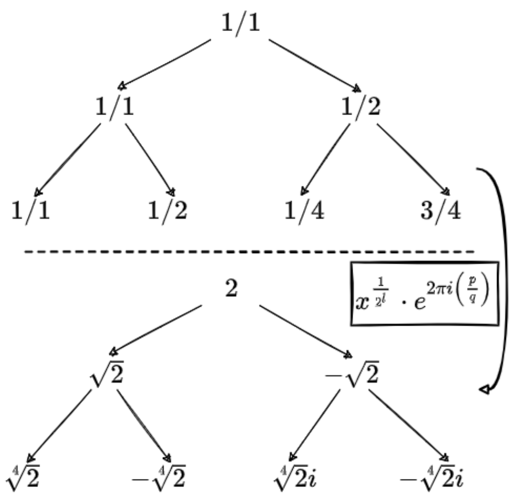

Figure 1: The upper tree depicts the steps of CIRCLE_ROOTS_RATIONAL_FORM(p,q,l) in Alg.1 for $l = 2$, $p = 1$, and $q = 1$. The lower tree depicts the steps of ROOTS(r, t, u, l) in Alg.2 for $r = 2$, $l = 2$, $p = 1$, and $q = 1$

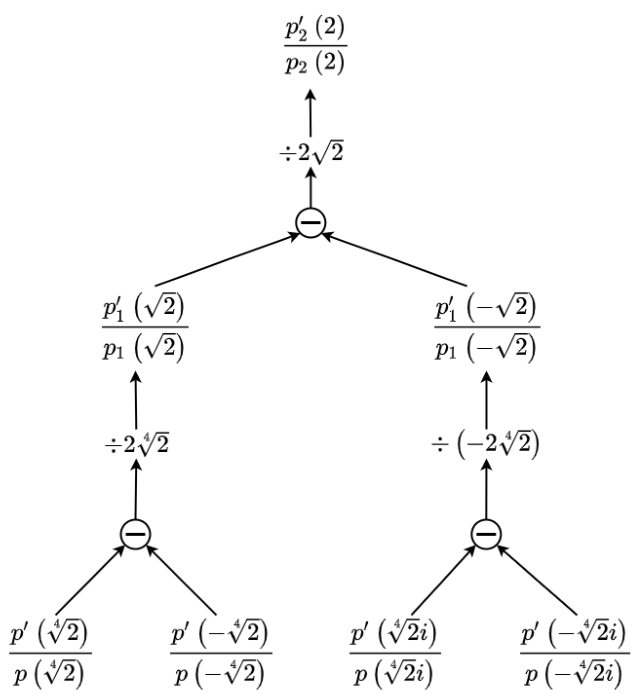

Figure 2: The steps of DLG_RATIONAL_RORM(p, p', r, t, u, l) in Alg.3 for $r = 2$, $l = 2$, $t = 1$, and $u = 1$.

Algorithm 1 CIRCLE_ROOTS_RATIONAL_FORM(p, q, l)

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  if $p\%q == 0$ **then**
    $r, s := (1,1)$
  else
    $r, s := (p,2q)$
  end if
  if r%s == 0 **then**
    $t, u := (1,2)$
  else
    $t, u := (2r + s, 2s)$
  end if
  if $l == 1$ **then**
    return $[(r, s),(t, u)]$
  else if $l\ != 0$ **then**
    left := CIRCLE_ROOTS_RATIONAL_FORM($r, s, l - 1$)
    right := CIRCLE_ROOTS_RATIONAL_FORM($t, u, l - 1$)
    return left $\cup$ right
  else
    return $[(p, q)]$
  end if

Algorithm 2 ROOTS(r, t, u, l)

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  root_tree = CIRCLE_ROOTS_RATIONAL_FORM(p, q, l)
  circ_root = [exp $\left(2 \cdot \pi \cdot i \cdot \frac{r}{s}\right)$ for r, s in root_tree]
  roots =[$\sqrt[2^l]{r}$·root for root in circ_root]

ABSTRACT

We revisited the classical root-squaring formula of Dandelin-Lobachevsky-Graeffe for polynomials and found new interesting applications to root-finding.

CCS CONCEPTS

• **Computing methodologies → Hybrid symbolic-numeric methods**.

KEYWORDS

symbolic-numeric computing, root finding, polynomial algorithms, computer algebra

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Algorithm 3 DLG_RATIONAL_FORM(p, p', r, t, u, l)

```
root := ROOTS(r, t, u, l)
for r_i ∈ root do
    base_step[i] := p'(r_i) / p(r_i)
end for
diff[0] := base_step
for i ≤ l do
    for j ≤ 2^(l−i−1) do
        diff[i + 1][j] := 1/2 (diff[i][2j] − diff[i][2j+1]) / root[2j]
        root = roots(r, t, u, l − 1 − i)
    end for
end for
return derivs[l][0]
```

Algorithm 4 DLG(p, p', l, x, ϵ)

```
angle := 1/(2πi) log(x)
u := 2^ε
t := (angle · u)%1
r := |x|
return DLG_RATIONAL_FORM(p, p', r, t, u, l)
```

1 INTRODUCTION

2 RELATED WORKS

3 BACKGROUND

4 MOTIVATING EXAMPLE

5 ALGORITHM DESIGN

6 THEORETICAL ANALYSIS

LEMMA 6.1. *The (relative) condition number operator satisifies the following properties:*

(1) $\kappa(f(x)) = |x \log'(f(x))|$
(2) $\kappa(f(x) - g(x)) = |x \frac{\kappa(f(x)) - \kappa(g(x))}{f(x) - g(x)}|$
(3) $\kappa(\frac{f(x)}{g(x)}) = ||\kappa(f(x))| - |\kappa(g(x))||$
(4) $\kappa(f(g(x))) = ||\kappa(f(x)) \cdot \kappa(g(x))||$

THEOREM 6.2. *The condition number for $\frac{p_l'}{p_l}$ using*

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7 EXPERIMENTAL RESULTS

8 CONCLUSION

REFERENCES