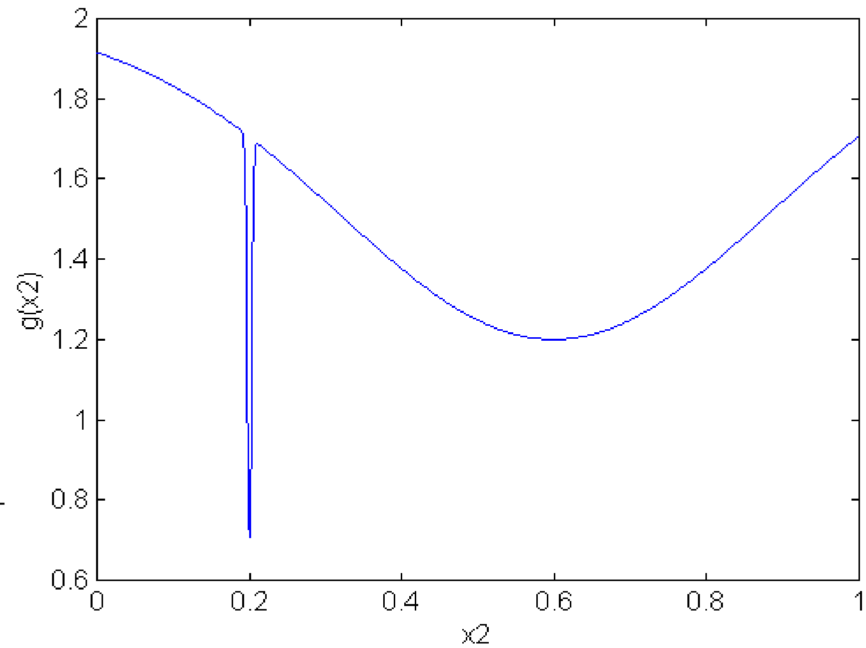
Probabilistic Optimization

TMKT48 Design Optimization

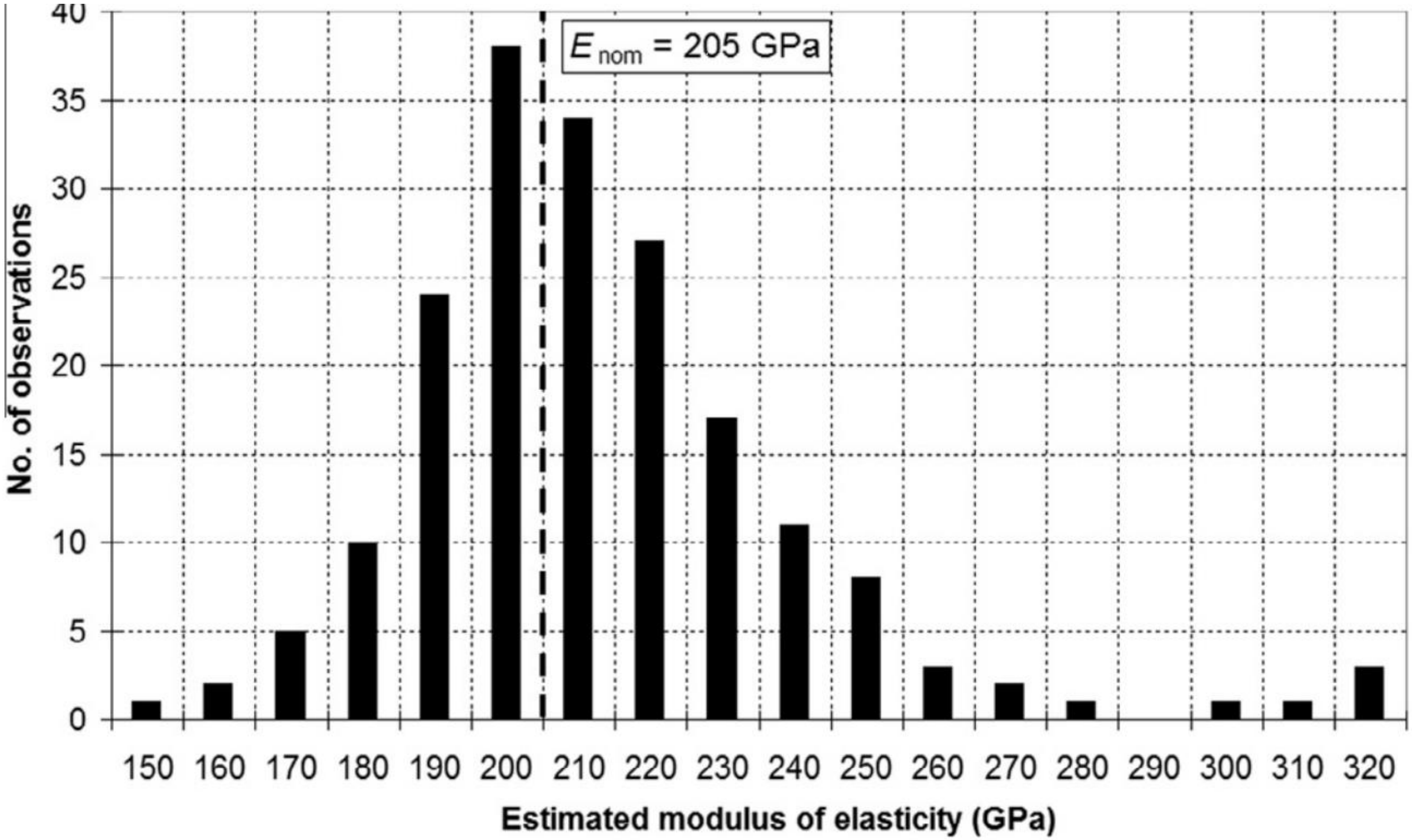
LINKÖPING UNIVERSITY

Deterministic vs Probabilistic

- Most computer models are determininistic
  - You get the same answer if you run the same model with the same settings
- The real world is probabilistic
- This is important for the problem below



Example: Variation in Young's modulus



Sadowski, A. J., Rotter, J. M., Reinke, T., & Ummenhofer, T. (2015). Statistical analysis of the material properties of selected structural carbon steels. *Structural Safety*, *53*, 26-35.

Probabilistic Design (Sannolikhetsbaserad Konstruktion)

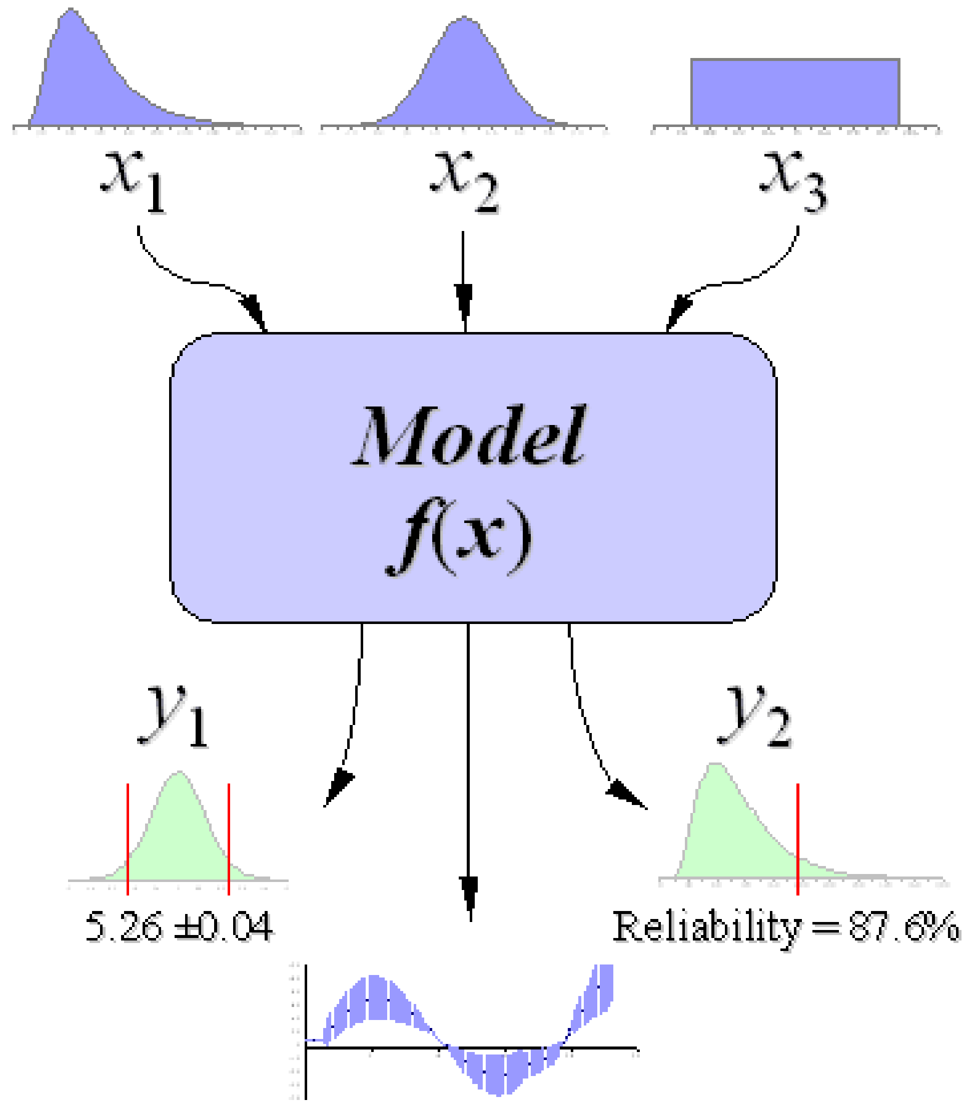
- Real world construction is subject to uncertainties and variations
- Estimate uncertainty in the design and quantify the effects of them
- Based on probability theory and parametric models
- Example:
  - What is the probability to meet all requirements with a specific concept?
  - How much must the requirements be relaxed in order for the probability of success to be sufficiently high?

Probability and Design

- How can we work with uncertainty and probability in design?
- With a simulation model we can simulate the uncertainty and analyze how it affects different properties

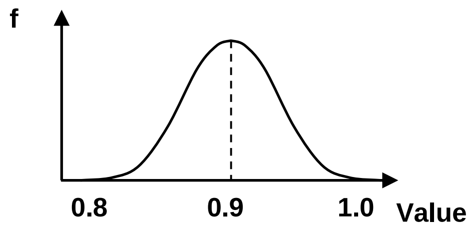
Probabilistic Optimization Process

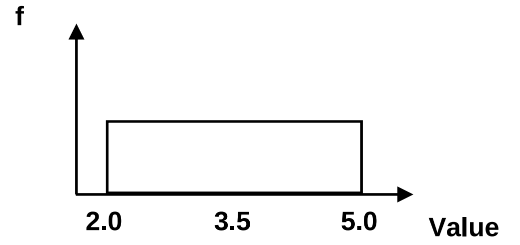
- Estimate uncertainties/probability distributions in
  - Design variables
  - Environmental variables
  - Models
- Estimate the impact that the uncertainties and variations have on the performance
- Calculate the desired statistical entities
  - Mean value
  - Standard Deviation



Assign Probability Distributions

PDF

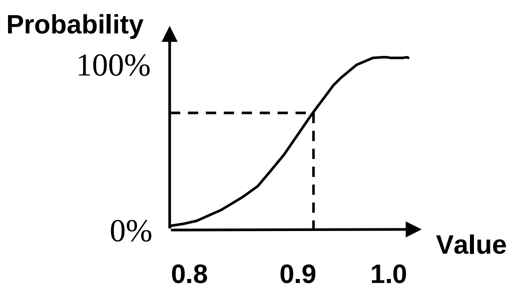


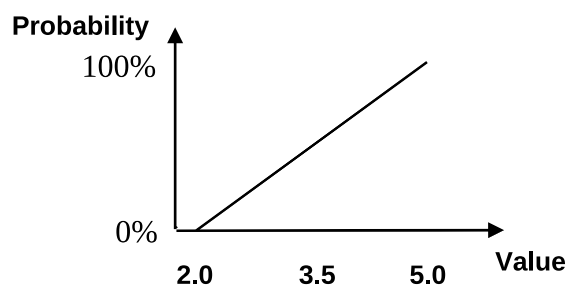


Normal/Gaussian

Uniform

CDF





Estimating Statistical Entities

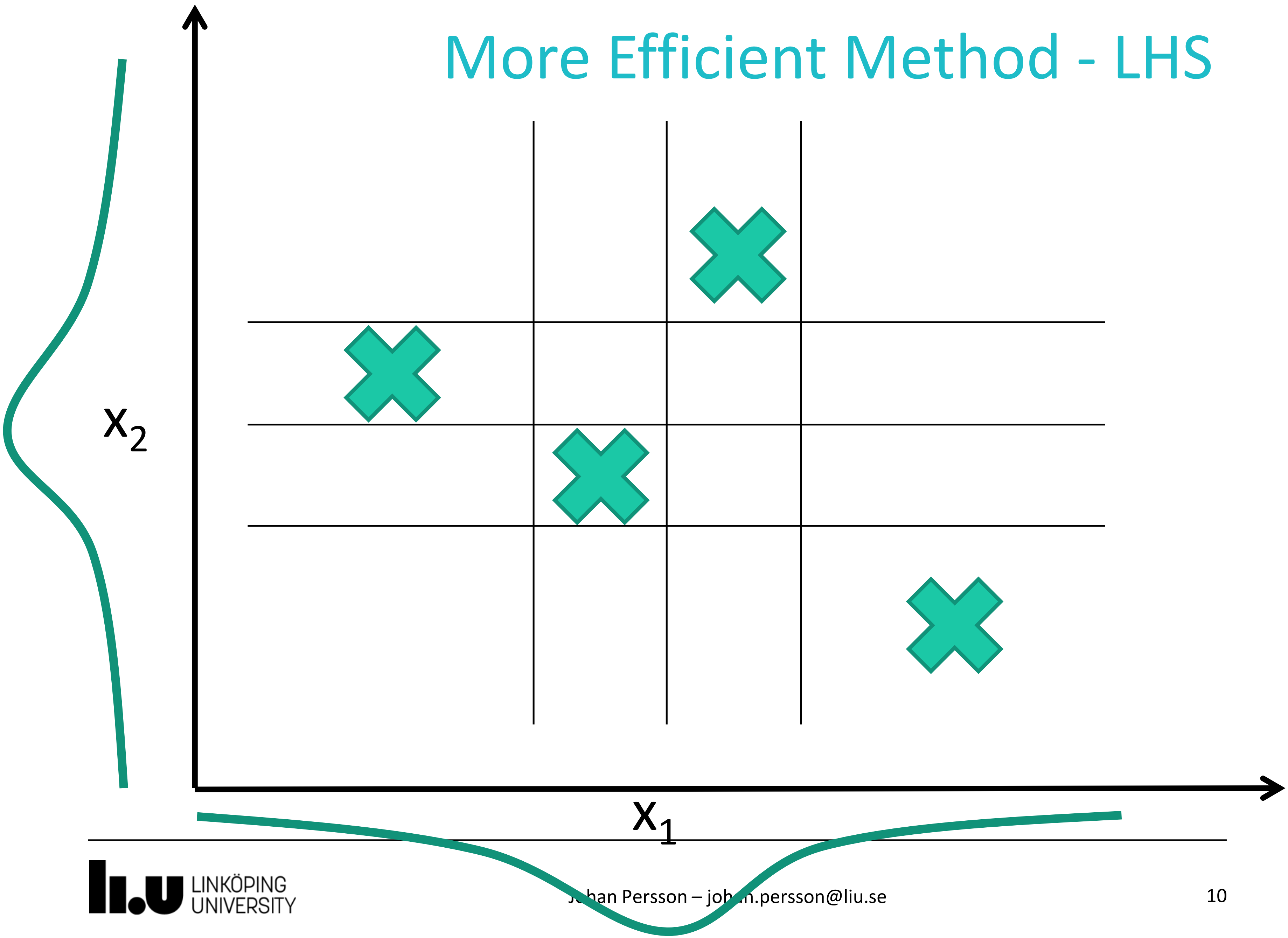
- Monte Carlo Simulation most common
  - It is a sampling method
  - Similar to throwing a pair of dice numerous times
  - Will converge but requires many simulations

Why not Probabilistic Optimization?

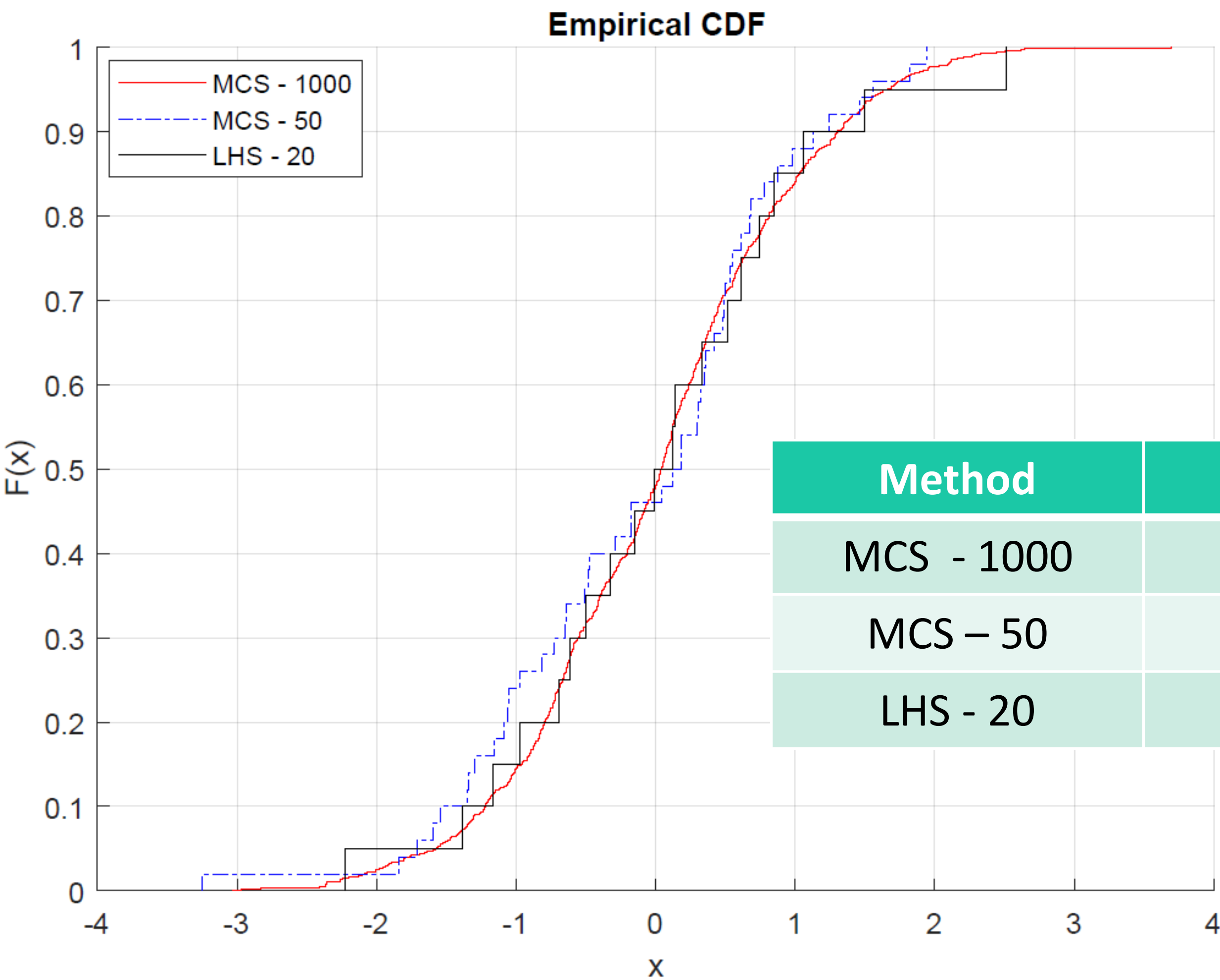
- Extremely computationally expensive
  - Each design is evaluated numerous times

- Solution
  - Replace expensive models with surrogate/metamodels
  - Use efficient methods to estimate statistical entities

More Efficient Method - LHS



Comparison MCS - LHS



Method	Mean	StDev
MCS - 1000	0.009	0.983
MCS – 50	-0.106	1.06
LHS - 20	0.017	1.07

Two Types of Probabilistic Optimization

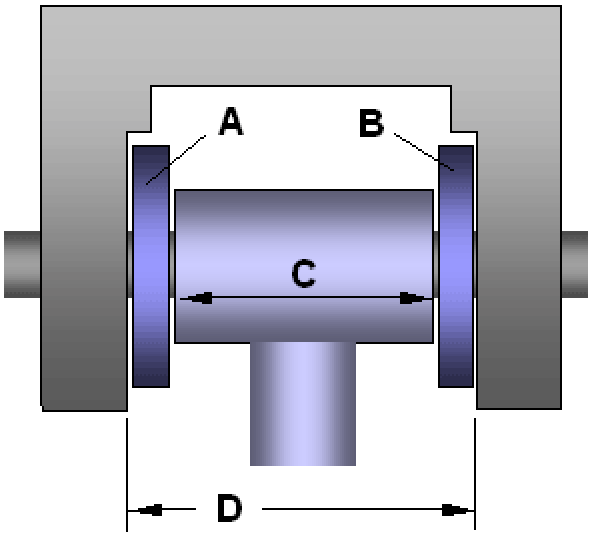
- Robust Design Optimization
  - Optimal design that is insensitive to uncertainties and variations

$$\min F(x) = w_1\mu(f(x)) + w_2\sigma(f(x))$$

- Reliability Based Design Optimization
  - Minimize the probability of failure
  - Maximize the probability of success

$$\max F(x) = P(f(x) < f_{ref})$$

Stochastic model example



Tolerance Model				
Part	**Min**	**Max**	**Random**	
A	1.95	2.05	1.987347	
B	1.95	2.05	1.951695	
C	29.5	30.5	30.14836	
D	34	35	34.72724	
	Clearance: D-(A+B+C)		0.639839	

Probabilistic Design of the motorcycle example

- Study probabilistic design using MATLAB
- Estimate for example the influence of variation of the weight of the driver on the acceleration of the motorcycle.
  - Let the total mass of the motorcycle be Mtot=Mcycle + Mdriver
  - Sample Mdriver from a normal distribution with mue=75 kg and sigma = 10 kg.
  - Calculate Cumulative Distribution Functions of the acceleration to investigate the effect of the variation.

Questions?