Problem Formulation

TMKT48

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Content

- How to go from problem description to objective function
  - Decision Support Matrix
- How to handle constraints during optimization
  - Penalty Functions
- How to handle multiple objectives

How to go from problem description to optimization formulation

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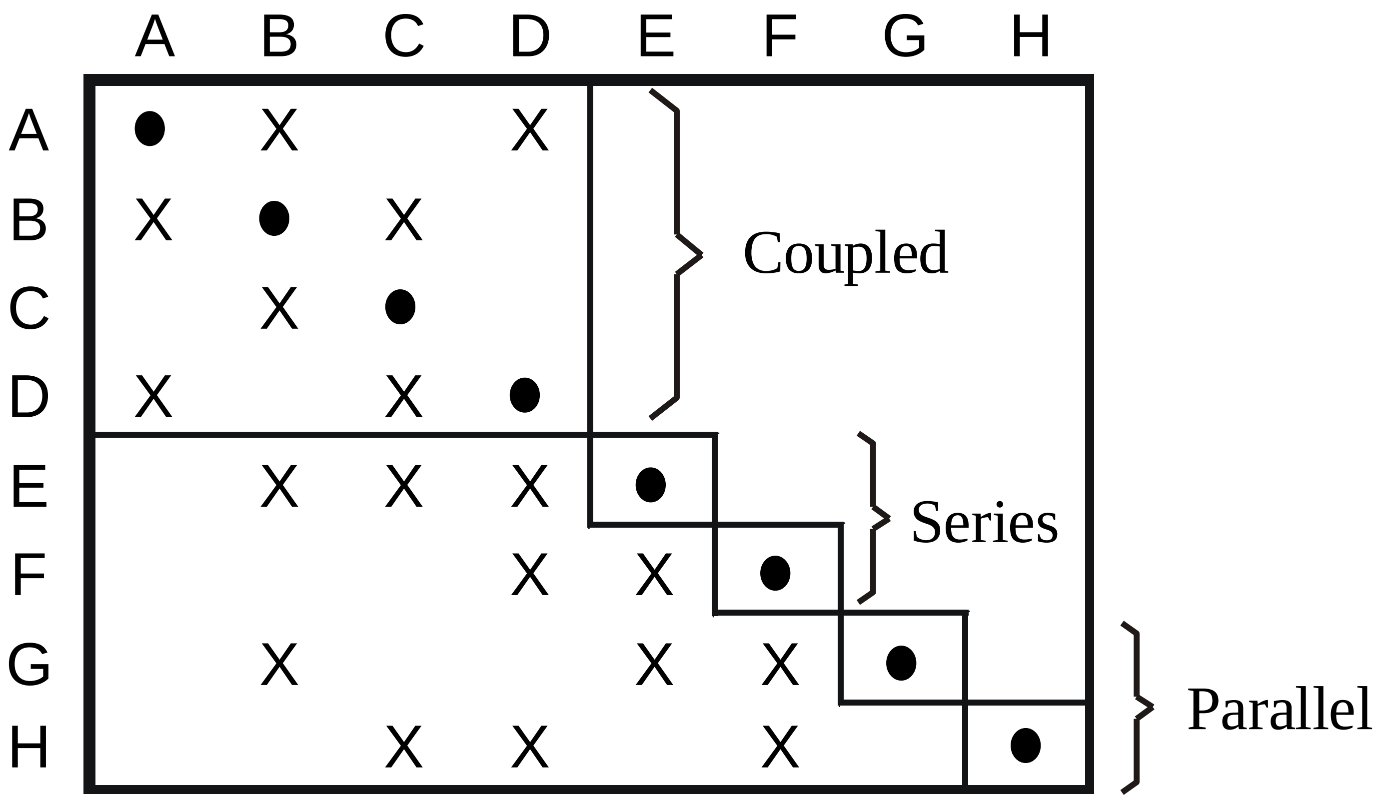
Content

- We would like to go from problem description to optimization formulation
  – Which analyses should be done in which order?
  – Which parameters shall we include in the problem formulation
  – Which characteristics shall we include in the objective functions
- We would also like to document the process

Design Structure Matrix

- A tool that visualizes the couplings between different design parameters and analyses



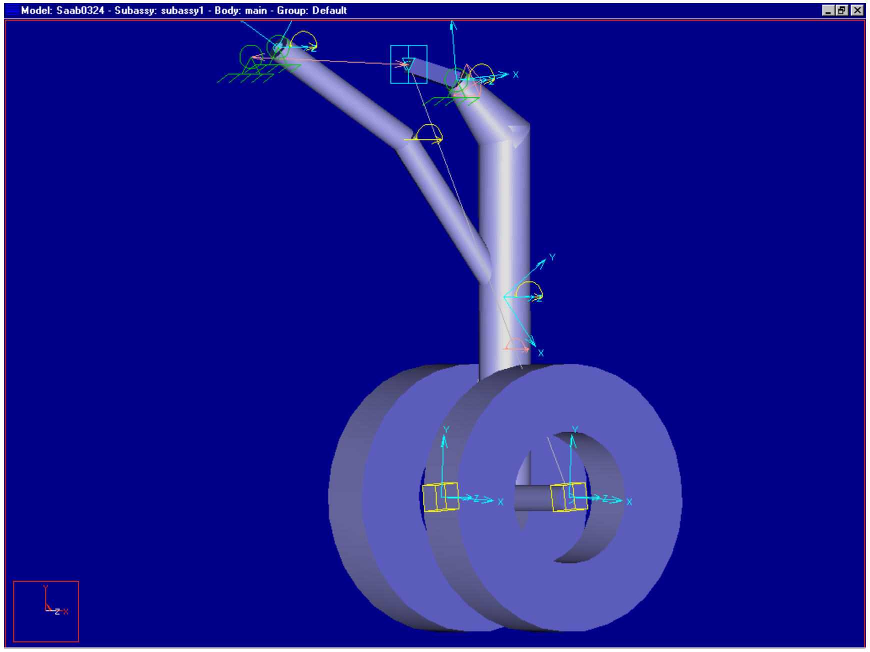
Design Structure Matrix

- Developed by Steward 1981, and further improved by Eppinger et al.
- It visualizes the couplings between different design tasks
- It makes it possible to reorganize the design process and to make it perform more successfully.

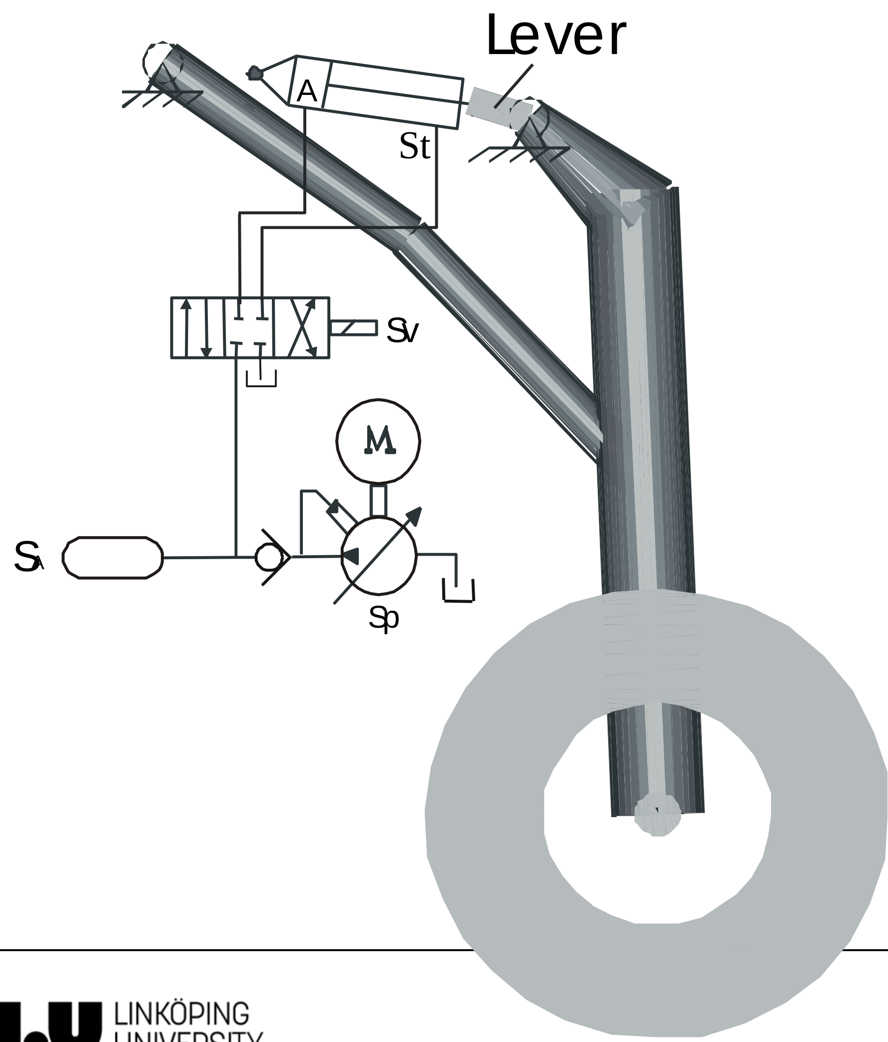
Design Structure Matrix

- Gain a better understanding of the problem.
- Which parameters could be seen as input parameters to the optimization.
- Tells us how the problem can be structured
- Which parameters have to be included in order to avoid sub-optimization.
- It is a good documentation describing how the problem was addressed.

Landing Gear Example



Landing Gear Example



Pump parameters

speed: n_{pump}= 1500rpm

displacement: Dp = 10 cm^3/rev

pressure: p=250 bar

Piston parameters

diameter: D_1 = 19mm

area ratio: α = 0.5

Valve parameters

diameter: sd= 7mm

max. opening: xv_{max}=1.8mm

Landing gear DSM

	Landing gear leg					Movement control								
						Actuation				Supply				
	A	B	C	D	E	F	G	H	I	J	K	L	M	
A	●	X												Wheel diameter
B	X	●												Number of wheels
C	X		●	X										Leg length
D			X	●										Leg diameter
E			X	X	●									Supporting beam
F	X	X	X	X		●	X	X		X		X		Length of lever
G	X	X	X	X		X	●	X		X		X		Angle of lever
H	X	X	X	X		X	X	●		X				Piston Area
I	X		X			X	X		●					Piston stroke
J								X		●				System pressure
K								X	X	X	●			Valve size
L								X	X			●	X	Pump size
M								X	X	X		X	●	Accumulator size

Landing gear problem Objective function

Weight $$f_{weight} = K_1 \cdot s_{Pump} + K_2 \cdot s_{Acumulator} + K_3 \cdot s_{valve} + K_4 \cdot s_{Piston}$$

Cost $$f_{cost} = C_1 \cdot s_{Pump} + C_2 \cdot s_{Acumulator} + C_3 \cdot s_{valve} + C_4 \cdot s_{Piston}$$

Energy consumption $$f_{Energy} = \int_0^{t_{ret}} p_s \cdot q_s \cdot dt$$

$$F = \left(\frac{f_{weight}}{f_{10}} \right)^{\gamma_1} + \left(\frac{f_{\cos t}}{f_{20}} \right)^{\gamma_2} + \left(\frac{f_{energy}}{f_{30}} \right)^{\gamma_3}$$

Constraint Handling

Handle Constraints

- Some algorithms have the capability of handling constraints inbuilt in them
  - Most algorithms in MATLAB's optimization toolbox
- Some algorithms do not
  - The ones given out in this course
    - Complex-RF
    - GA

Penalty Functions

- Original Formulation

$$\min f(x)$$

$$s.t.$$

$$g_i(x) \leq 0, i = 1 \ldots n$$

$$h_j(x) = 0, j = 1 \ldots p$$

- Formulation with Penalty Functions

$$F = f(x) + \left[\sum_{i=1}^{n} w_i G_i + \sum_{j=1}^{p} v_i L_i \right]$$

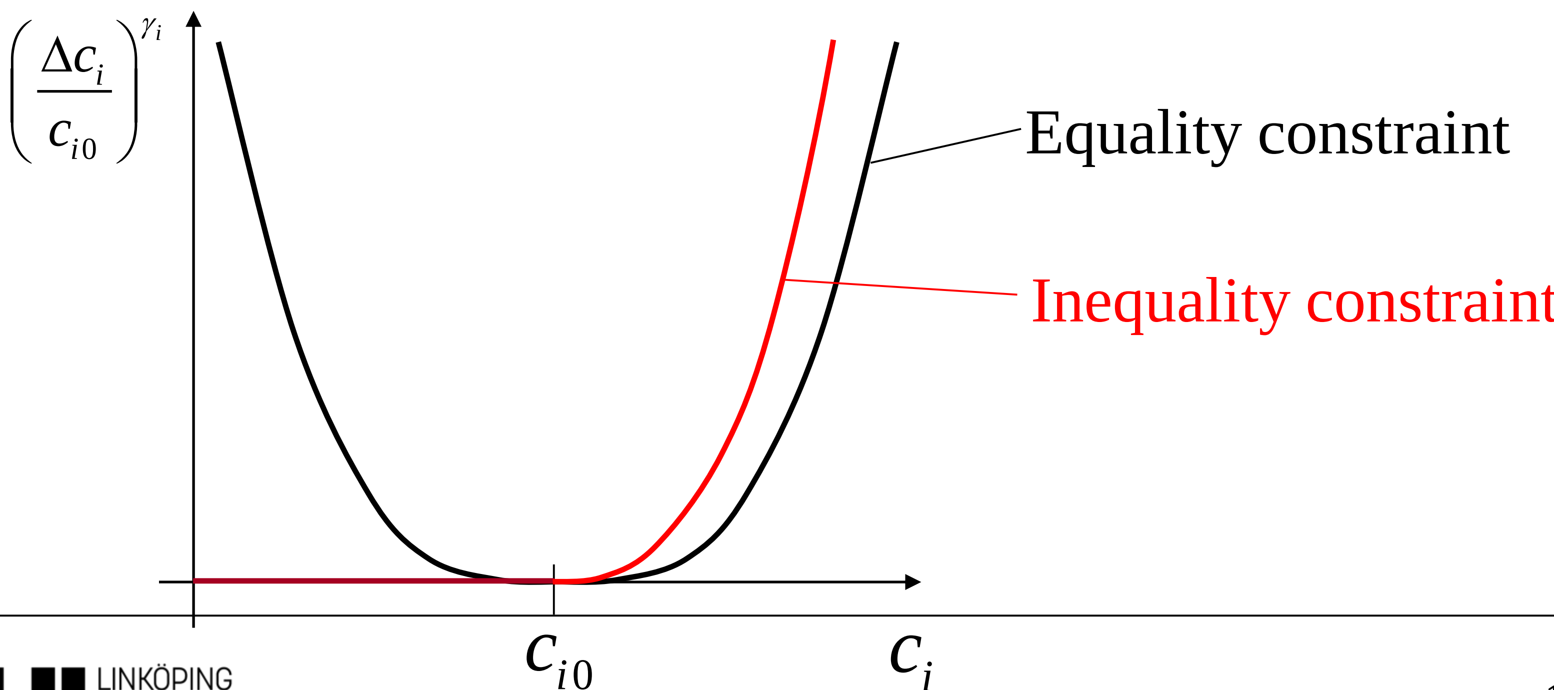
$$G_i = \max\left[0, g_i(x)\right]^{\beta}$$

$$L_j = \left[h_j(x)\right]^{\gamma}$$

Penalty Functions: Illustration

$$F = f(x) + \left[\sum_{i=1}^{n} w_i \left(\frac{\Delta c_i}{c_{i0}} \right)^{\gamma_i} \right]$$

Δc = distance to feasible region



Question:
Why do we add a dynamic penalty instead of a fixed one (e.g. penalty=10^6)?

- Original Formulation

$$s.t.$$

$$g_i(x) \le 0, i = 1 \ldots n$$

$$h_j(x) = 0, j = 1 \ldots p$$

$$F = f(x) + \left[\sum_{i=1}^{n} w_i G_i + \sum_{j=1}^{p} v_i L_i \right]$$

- Formulation with

$$G_i = \max\left[0, g_i(x)\right]^{\beta}$$

We would like to tell the optimization algorithm how much it is violating the constraint and this means that the algorithm will know if a seach direction is good or bad

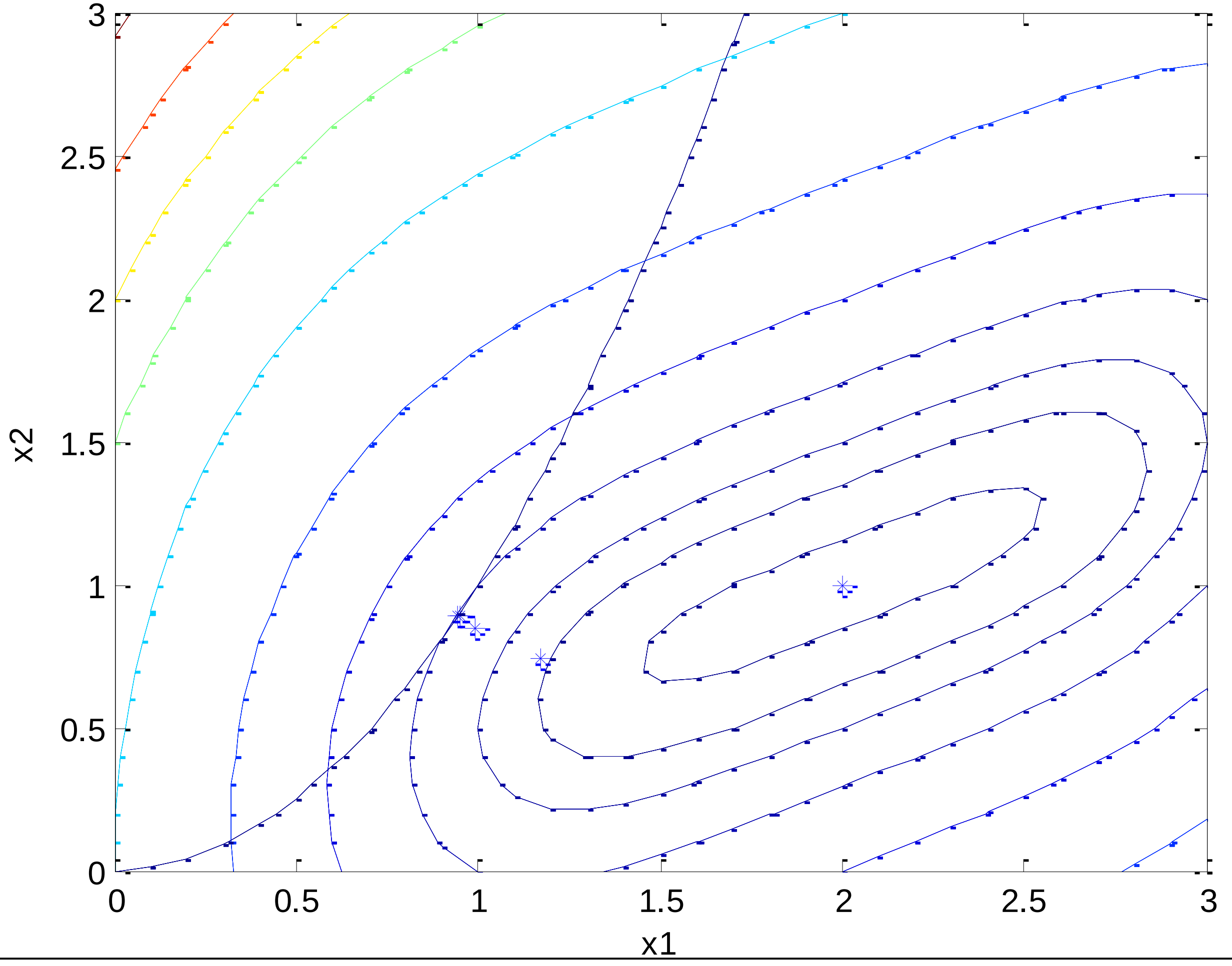
Penalty Functions: Example

$$\min f(x_1, x_2) = (x_1 - 2)^4 + (x_1 - 2x_2)^2$$

$$s.t.$$

$$x_1^2 - x_2 \leq 0$$

Penalty Functions: Example



Penalty Functions: Example

$$\min f(x_1, x_2) = (x_1 - 2)^4 + (x_1 - 2x_2)^2$$

$$s.t.$$

$$x_1^2 - x_2 \leq 0$$

$$p(x_1, x_2) = \max(0, x_1^2 - x_2)$$

Penalty Functions: Example

$$\min f(x_1, x_2) + \mu p(x_1, x_2)$$

where:

$$f(x_1, x_2) = (x_1 - 2)^4 + (x_1 - 2x_2)^2$$

$$p(x_1, x_2) = \left(\max\left(0, x_1^2 - x_2\right)\right)^2$$

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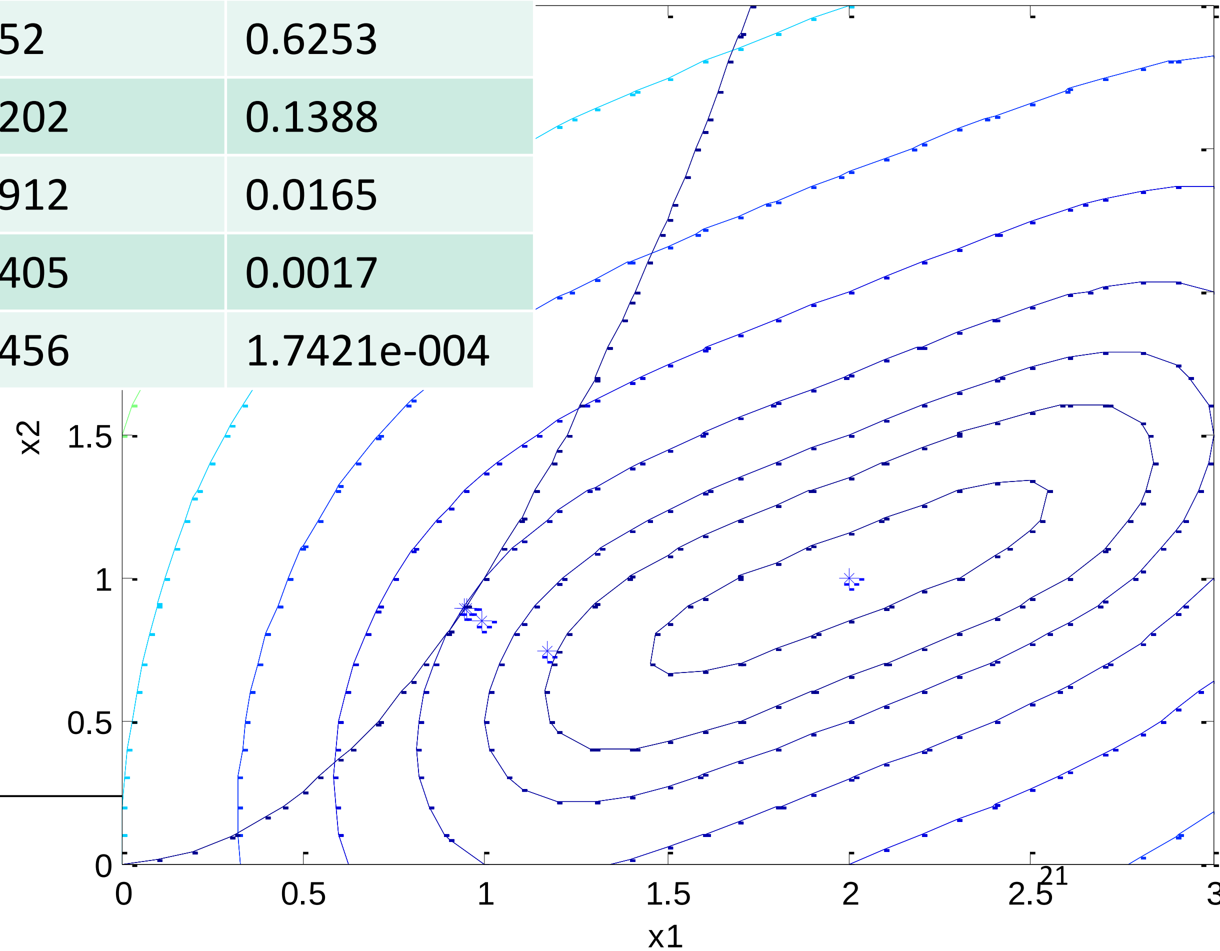
Penalty Functions: Example

μ	$X=(x_1,x_2)$	f(x)	g(x)
0	(2, 1)	0	3
1	(1.169, 0.7407)	0.752	0.6253
10	(0.9906, 08425)	1.5202	0.1388
100	(0.9508, 0.8875)	1.8912	0.0165
1 000	(0.9460, 0.8933)	1.9405	0.0017
10 000	(0.9448, 0.8925)	1.9456	1.7421e-004



Questions?