Design Structure Matrix

Can be used to structure the problem

Example: $\min f(x) = x_1^2 + x_2^2$

$$\text{s.t.} \quad g_1(\bar{x}) = x_1 + x_2 + g_2(\bar{x}) \leq 3$$

$$g_2(\bar{x}) = x_1 - 2x_2 + g_1(\bar{x}) \leq 3$$

$$-5 \leq x_1, x_2 \leq 5$$

	f	g_1	g_2
f			
g_1			X
g_2		X	

f does not need the value of g_1 or g_2

g_1 needs the value of g_2 } coupled
g_2 needs the value of g_1 }

Reorder the analyses:

Example: $\min f(x) = g_1(x) + 2$

$$g_1(x) = x_1 + 1 \leq 2$$

$$0 \leq x_1 \leq 1$$

	f	g_1
f		X
g_1		

run g_1 before f $\Longrightarrow$ reorder

	g_1	f
g_1		
f	X	

f needs g_1 but it has already been calculated

Penalty Functions

Consider the original problem

$$\min f(\bar{x})$$

$$\text{s.t} \quad g_j(\bar{x}) \leq 0 \quad , \quad j = 1, \dots, m$$

$$h_\ell(\bar{x}) = 0 \quad , \quad \ell = 1, \dots, n$$

we want to punish solutions that violate the constraints

Reformulate the problem with penalty functions:

$$\min \; F(x) = f(x) + \left[\sum_{j=1}^{m} w_j \cdot G_j + \sum_{\ell=1}^{n} v_\ell \cdot L_\ell \right]$$

where $$G_j = \left(\max\left(0, g_j(x)\right)\right)^\beta \quad , \quad \beta = 1, 2, \dots$$

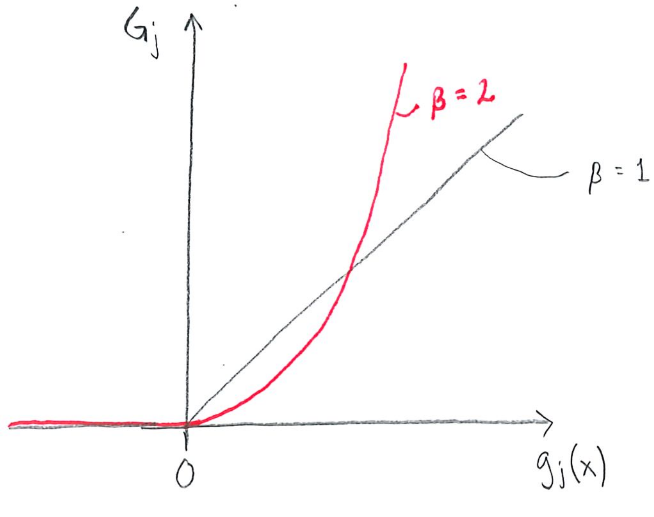
$$\underbrace{\max\left(0, \underbrace{g_j(x)}_{<0}\right)}_{=0}$$ <0 when the constraint is ok!

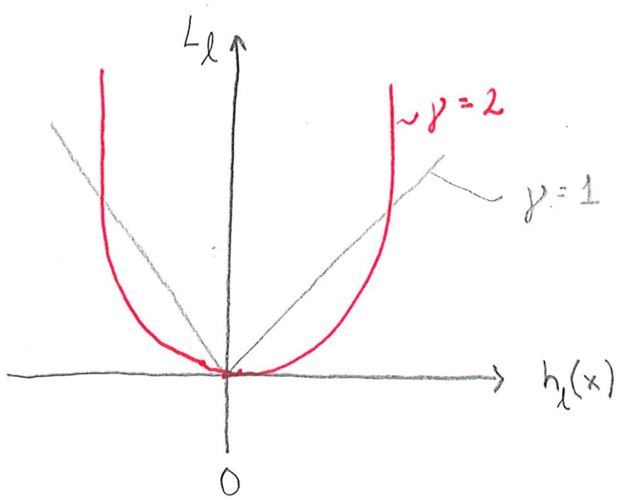
$$\underbrace{\max\left(0, \underbrace{g_j(x)}_{>0}\right)}_{>0}$$ >0 when the constraint is violated

$$L_\ell = |h_\ell(x) - 0|^\gamma = \underbrace{|h_\ell(x)|}_{\text{abs}(h_\ell)}{}^\gamma \quad , \quad \gamma = 1, 2, 3 \dots$$

The factors w_j and v_ℓ might be needed if the $f(x)$ and $g(x)$ are of different magnitudes

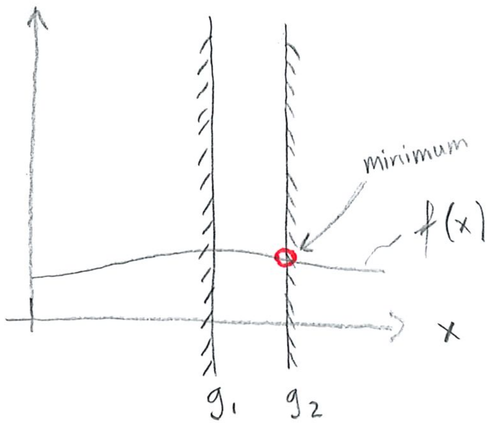
Graphical Explanation:

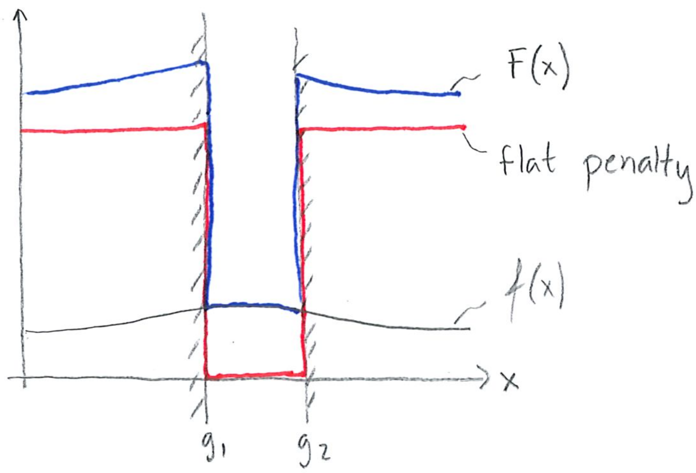




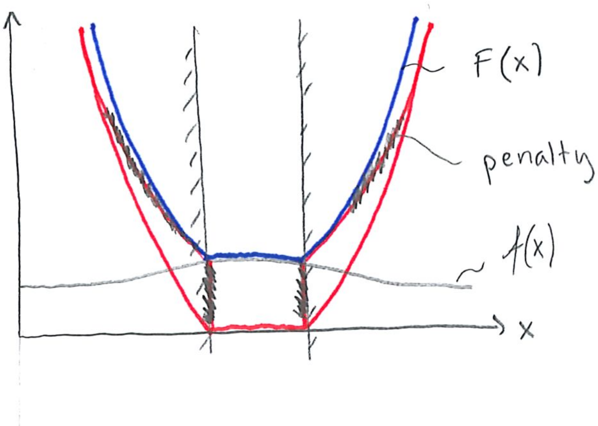
* The constraints should guide the algorithm to the feasible region

* Just adding high constant values create plateus
  $\Rightarrow$ difficult for the optimization to know where OK designs are





Difficult for the algorithm to find the feasible region



The penalties guide the optimization to the feasible region

Objective Function

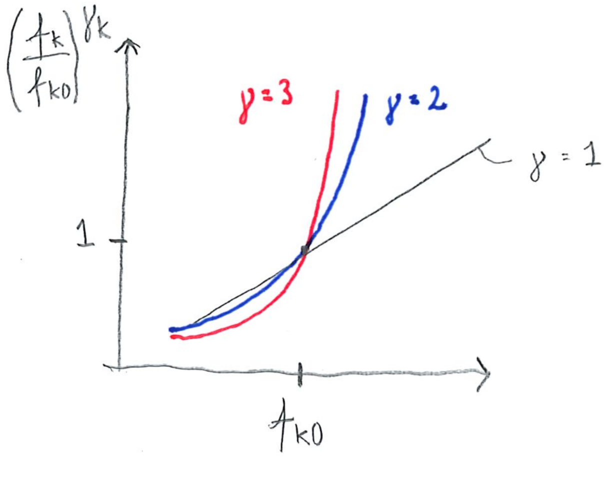
* Should express the preferences of the decision maker
* Multiple objectives are often conflicting

Ex: $$F = \left(\frac{f_1}{f_{10}}\right)^{\gamma_1} + \left(\frac{f_2}{f_{20}}\right)^{\gamma_2} + \ldots + \left(\frac{f_k}{f_{k0}}\right)^{\gamma_k}$$

reference values used for normalization

F is an aggregation of the objectives

Each objective is normalized with a value from a reference design



The exponent γ_k should express the relative importance of each objective