Probabilistic Design

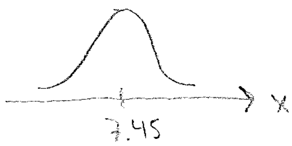
To estimate the uncertainties present in a design problem and quantify the effects of them

Deterministic values are single points estimates,

e.g. $x = 7.45$

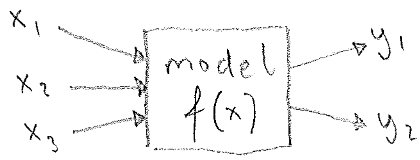
When we talk about probabilistic design the variables are stochastic

e.g.



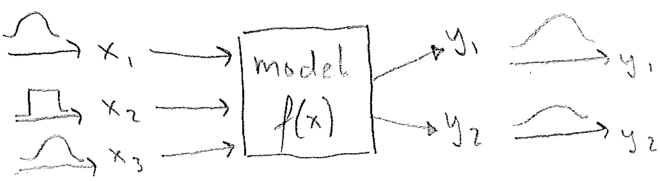
Models in engineering are often deterministic

Fixed values in $\Rightarrow$ Fixed responses out

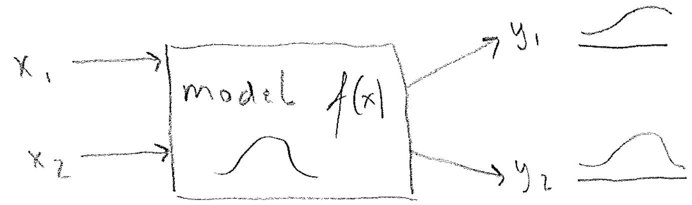


In probabilistic design, the input will have distributions

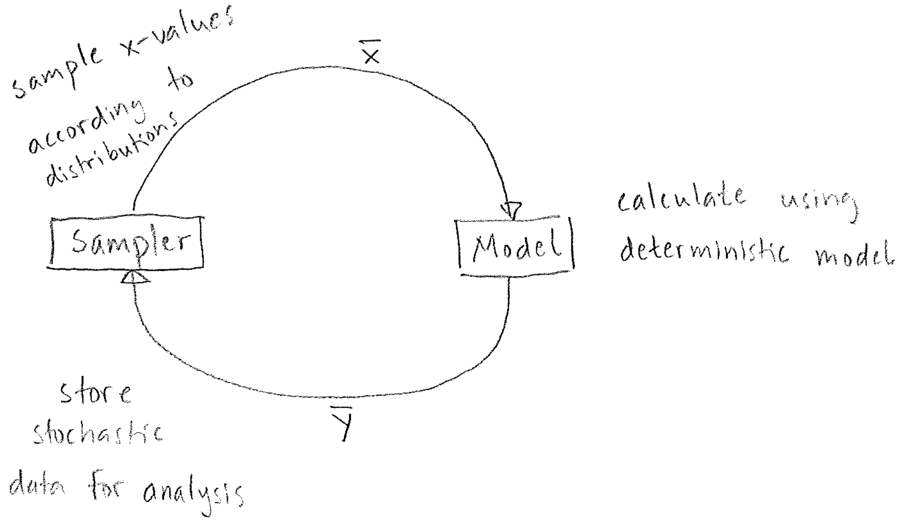
$\Rightarrow$ the outputs will have distributions



The model could also include stochastic elements



In practice, Monte Carlo simulations are used to generate the stochastic output



The more loops, the better statistical data is obtained

Robust Design Optimization

The design should both be good and be insensitive to uncertainties

weights

$$\min F(x) = w_1 \cdot \underbrace{\mu(f(x))}_{\text{mean/expected value}} + w_2 \cdot \underbrace{\sigma(f(x))}_{\text{standard deviation}}$$

Reliabity Based Design Optimization

Optimize the probability of success

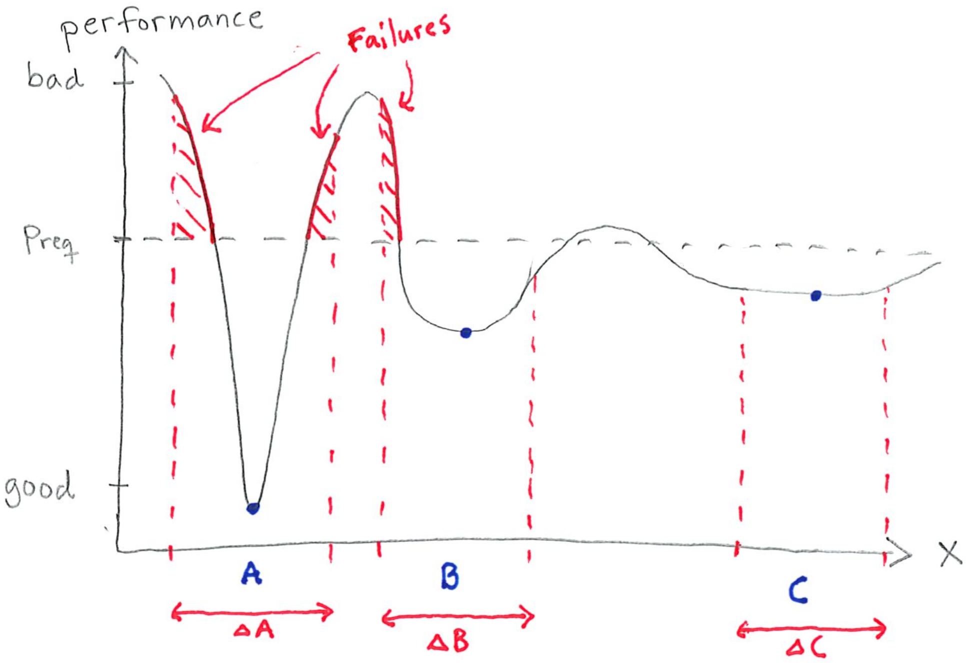
or

Minimize probability of failure

$$\text{Max } F(x) = P\big(\underbrace{f(x) \leq f_{req}}_{\text{probability that } f(x) \text{ is lower than } f_{req}}\big)$$

limit for failure

Probabilistic Optimum



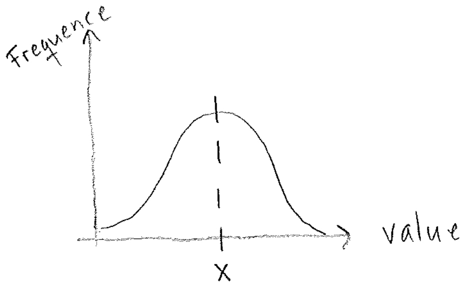
A is the deterministic optimum

B is a robust optimum

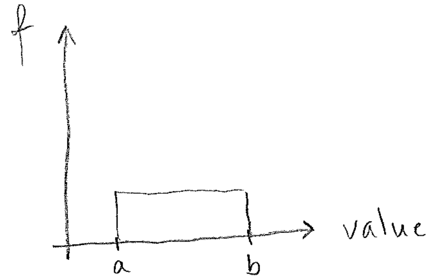
C is a reliable optimum

Some Probability Theory

* Probability function (PDF)



Normal Distribution

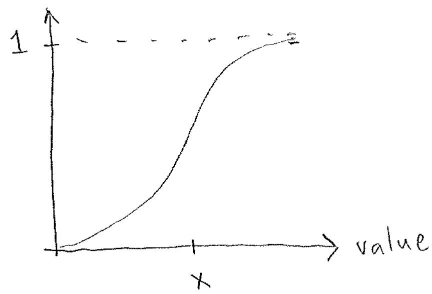


Uniform Distribution

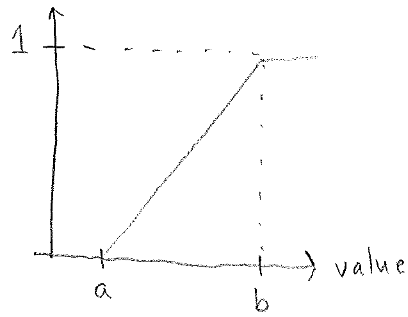
$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{if } x < a,\ x > b \end{cases}$$

* A PDF describes how often a value occurs
* The area under the graph is 1

* Cumulative Distribution Function



Normal Distribution



Uniform Distribution

The probability that the value is below a certain number X

$$F(x) = \int_{-\infty}^{X} f(x)\,dx$$

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } x > b \end{cases}$$