Post Optimal Analysis

TMKT48 Design Optimization

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Post Optimal Analysis

- Can we trust the optimization results? (Have we found the optimum?)
  - Convergence History
- Choose a solution

- How sensitive is the solution to each variables?
  - Sensitivity Analysis

Convergence History

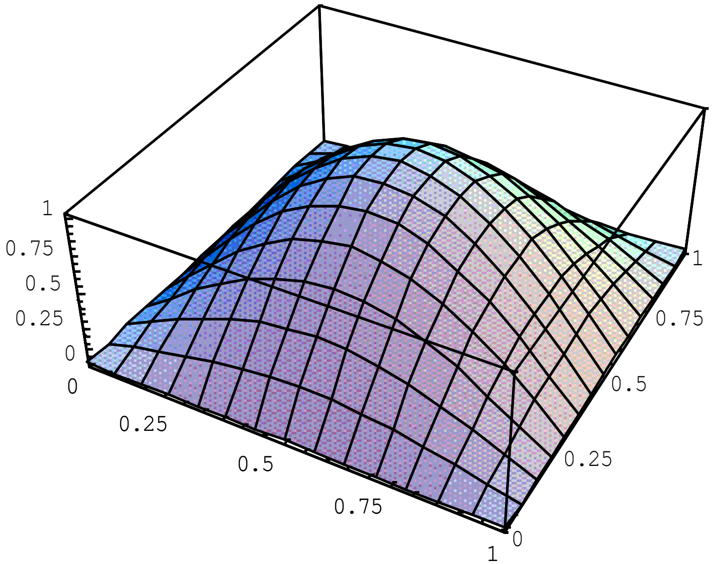
- Plot the evolution of the variables and the objectives for the optimization

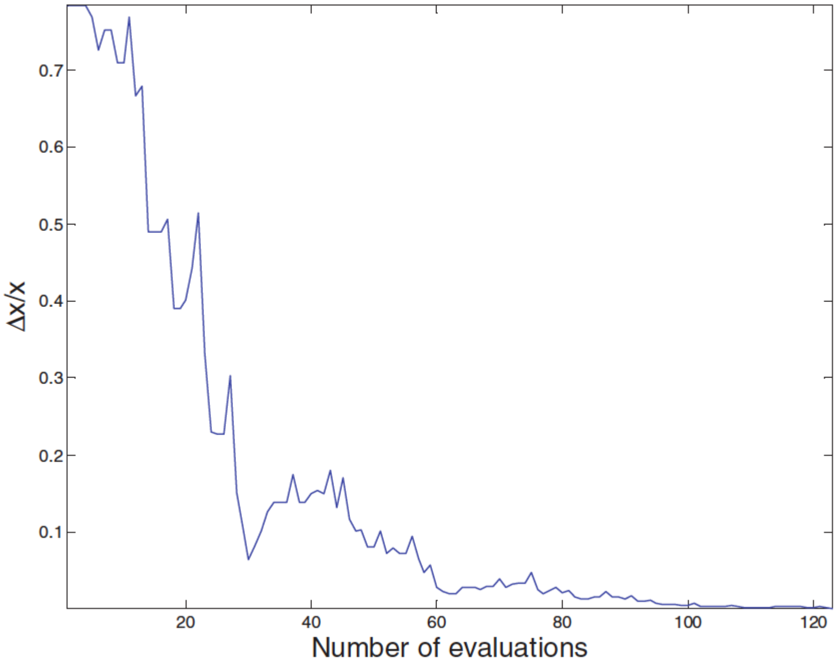
- Have we covered the design space?
  - Balance between
    - Exploitation (investigate current best area)
    - Exploration (try to find better areas)

- Does the convergence speed seem reasonable?

Convergence in Parameters

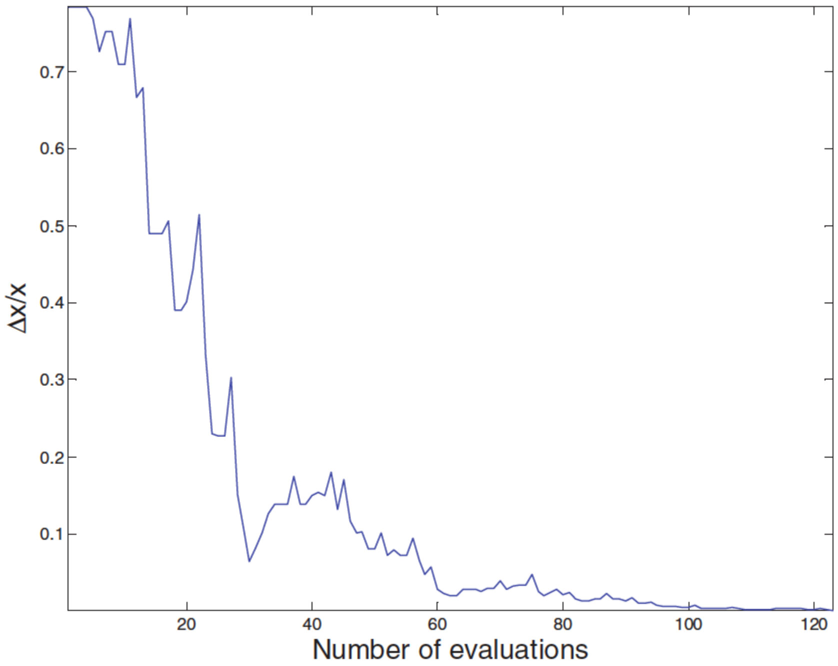
$$f(x_1, x_2) = \sin(\pi x_1)\sin(\pi x_2)$$

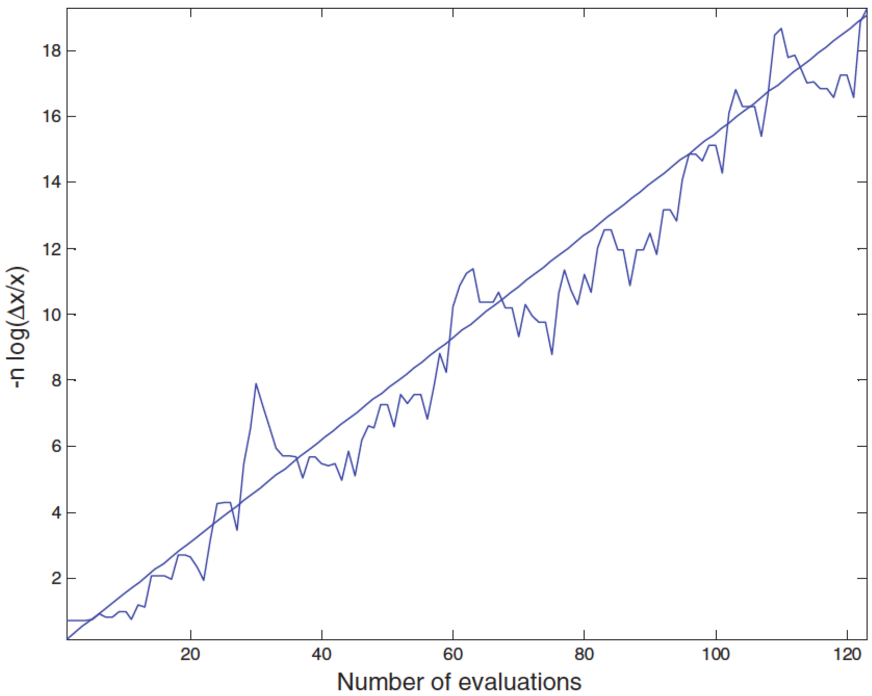




Convergence of optimization variables. This shows the max relative spread dx of all (both) variables

Convergence in Parameters





Convergence of optimization variables. This shows the max relative spread dx of all (both) variables

The convergence expressed as –n log2 max(dx), and a straight line corresponding to the theoretical convergence rate for the Complex algorithm.

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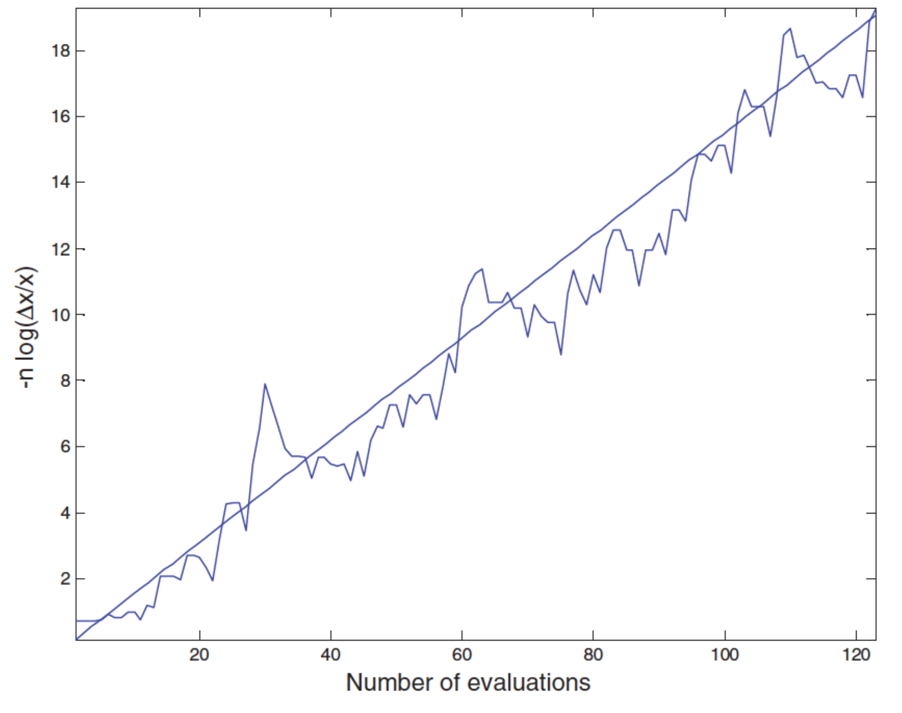
Example 1

- The Convergence Speed seems good

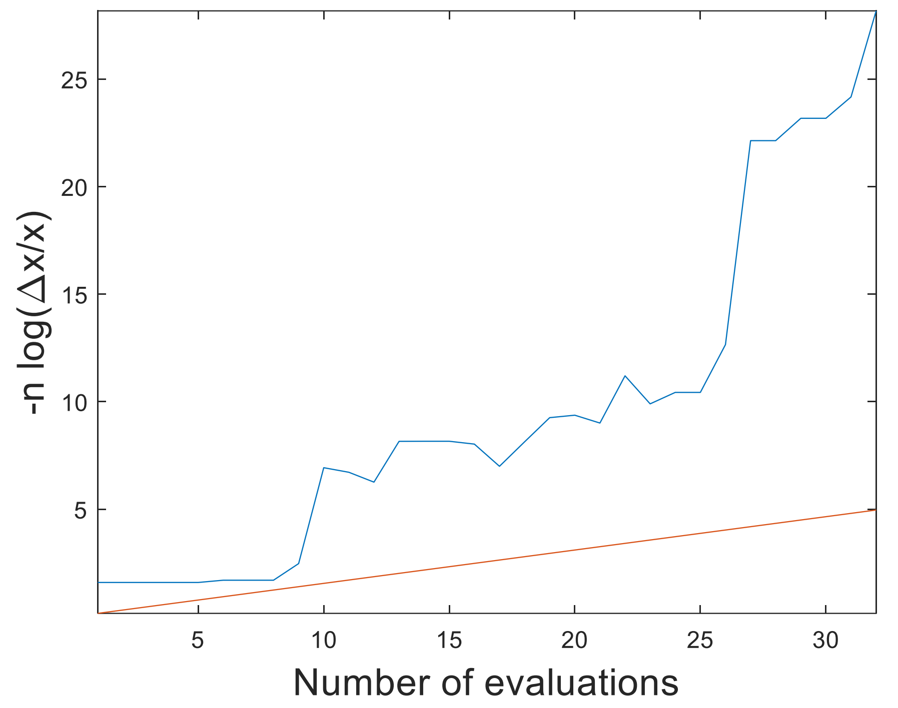
- It is probably quite easy to find the optimum

- The optimization seems OK



Example 2

- The Convergence Speed is way too fast
- The algorithm probably ended up in a bad place
- The optimization seems suspicious.



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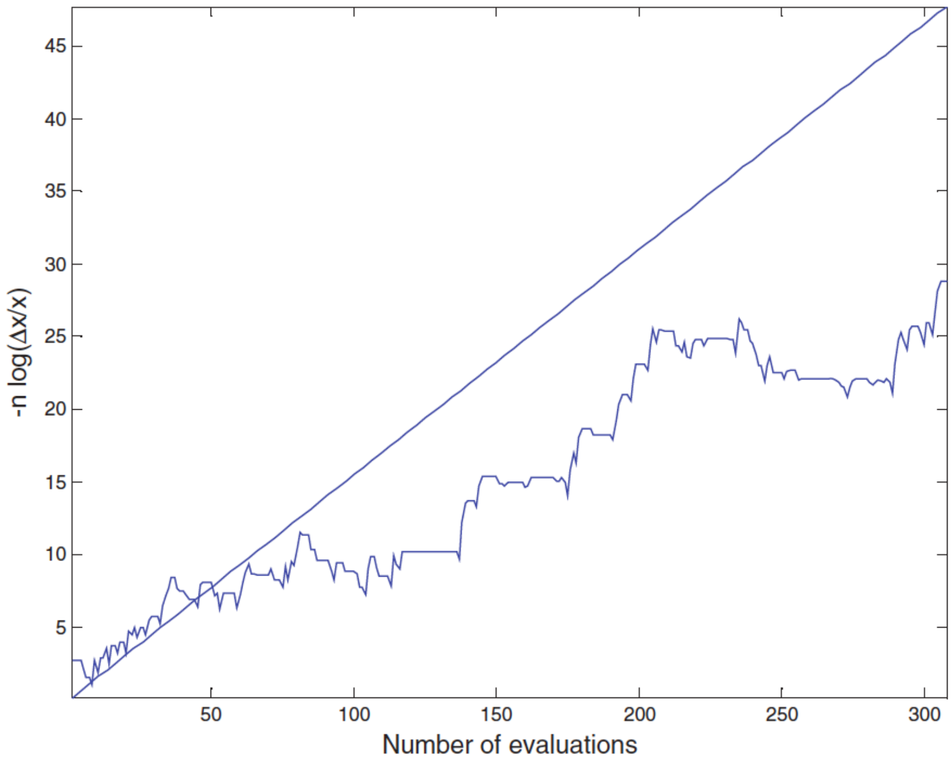
Example 3

- The speed is good in the beginning, then slow
- It seems easy to find a promising region, but difficult to find the optimum
- The optimization is probably OK



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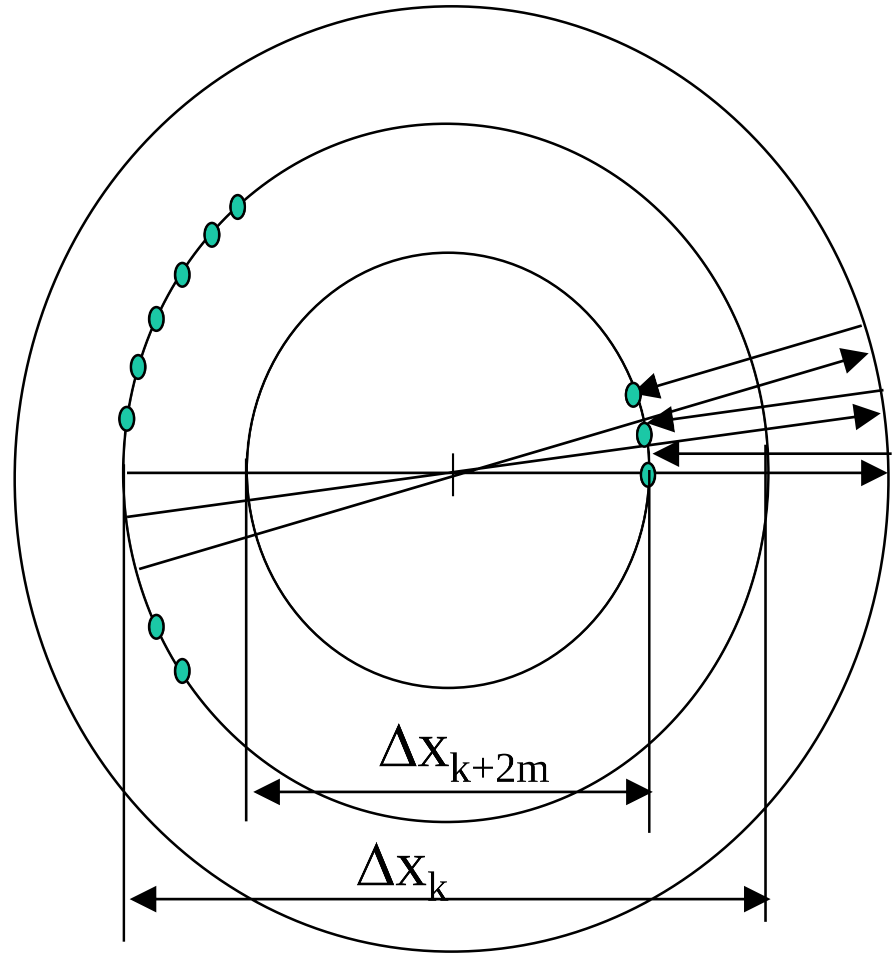
Theoretical Convergence Rate of Complex

- The average degree of contraction in each step

$$\frac{\Delta x_{k+1}}{\Delta x_k} = \left(\frac{\alpha}{2}\right)^{\frac{1}{2m}}$$

- We move each point on average two times
- There are m points

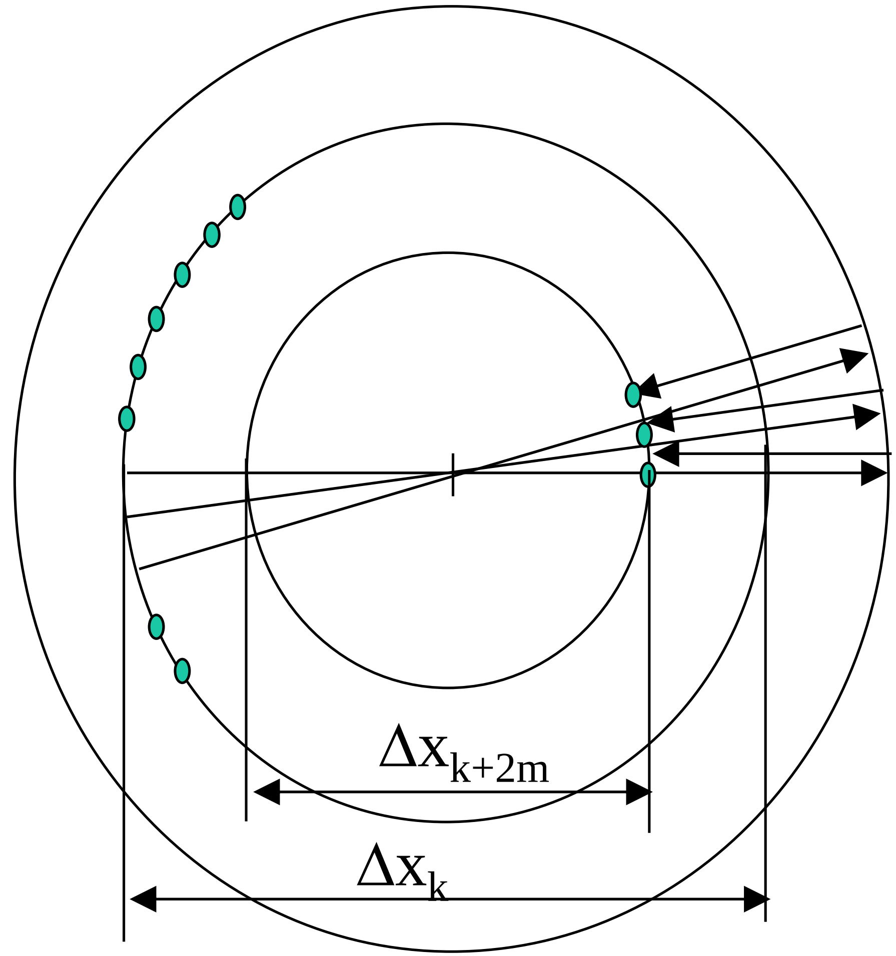


Theoretical Convergence Rate of Complex

- There are m points

$$\frac{\Delta x_{k+1}}{\Delta x_k} = \left(\frac{\alpha}{2}\right)^{\frac{1}{2m}}$$

- m=K*n
- n= number of variables
- K = 2 as standard setting



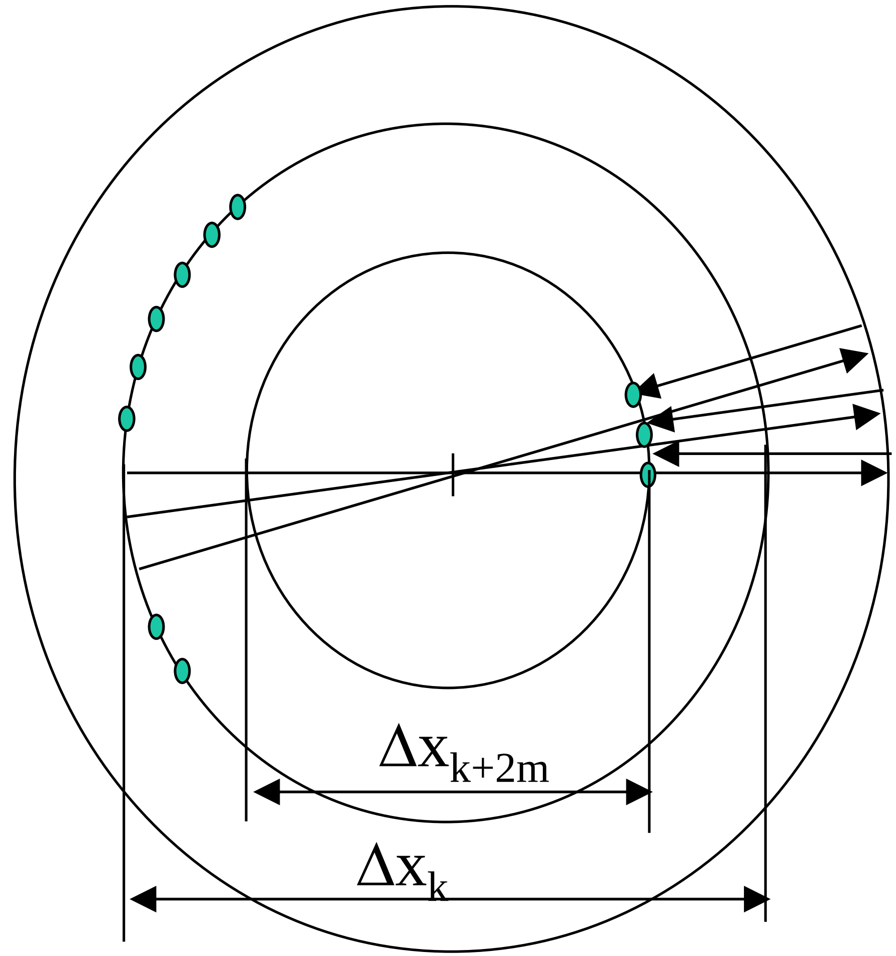
Theoretical Convergence Rate of Complex

- There are m points

$$\frac{Dx_{k+1}}{Dx_k} = \left(\frac{a}{2}\right)^{\frac{1}{2m}}$$

- m=K*n

$$\frac{\Delta x_{k+1}}{\Delta x_k} = \left(\frac{\alpha}{2}\right)^{\frac{1}{2\kappa n}}$$



Information Gain in the Complex Method

- Increase in Information = Reduced area in the design space where the optimum can be.
  - (Claude Shannon 1947)

- The increase in information in each step for Complex is: (Times n because there are n parameters that are gaining information)

$$\Delta I = -n \log_2 \frac{\Delta x_{k+1}}{\Delta x_k} = -n \log_2 \left(\frac{\alpha}{2} \right)^{\frac{1}{2\kappa n}} = -\log_2 \left(\frac{\alpha}{2} \right)^{\frac{1}{2\kappa}}$$

Complex Contraction/Convergence

- The increase in information I in each step

$$\Delta I = -\log_2 \left(\frac{\alpha}{2} \right)^{\frac{1}{2\kappa}}$$

- Example: α=1.3, K=2 yields

$$\Delta I = -\log_2 \left(\frac{1.3}{2} \right)^{\frac{1}{2*2}} = 0.155$$

- The amount of information gain in each step is ΔI=0.155

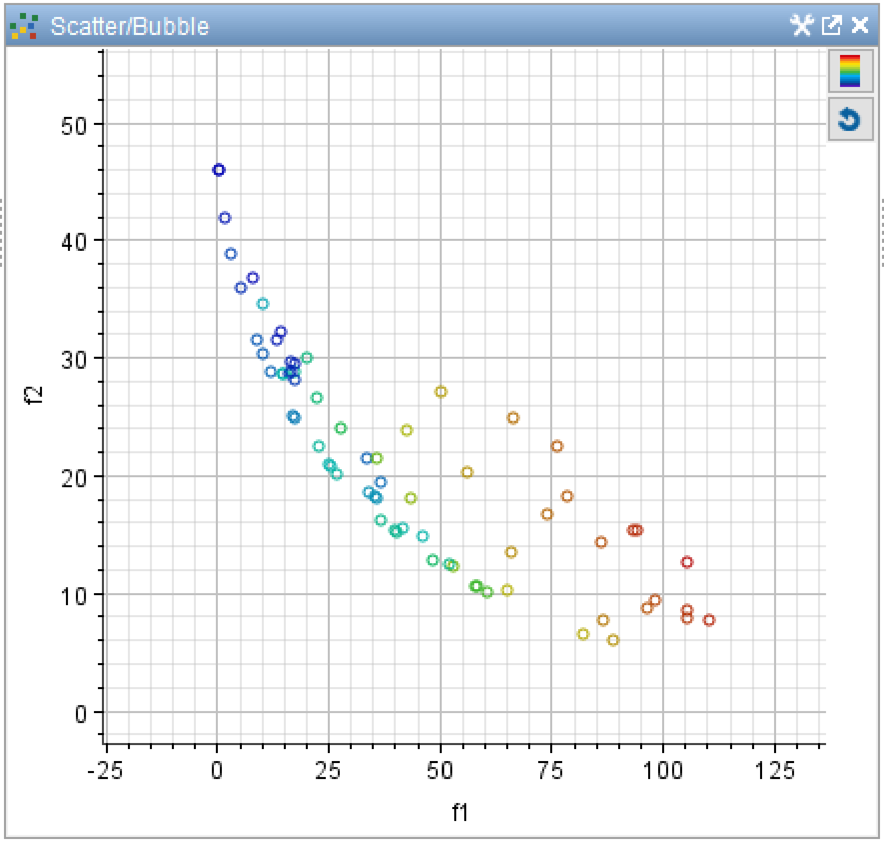
Choosing a Solution

Choosing a Solution

- Difficult to give specific instructions
  - Extremely problem/application dependent

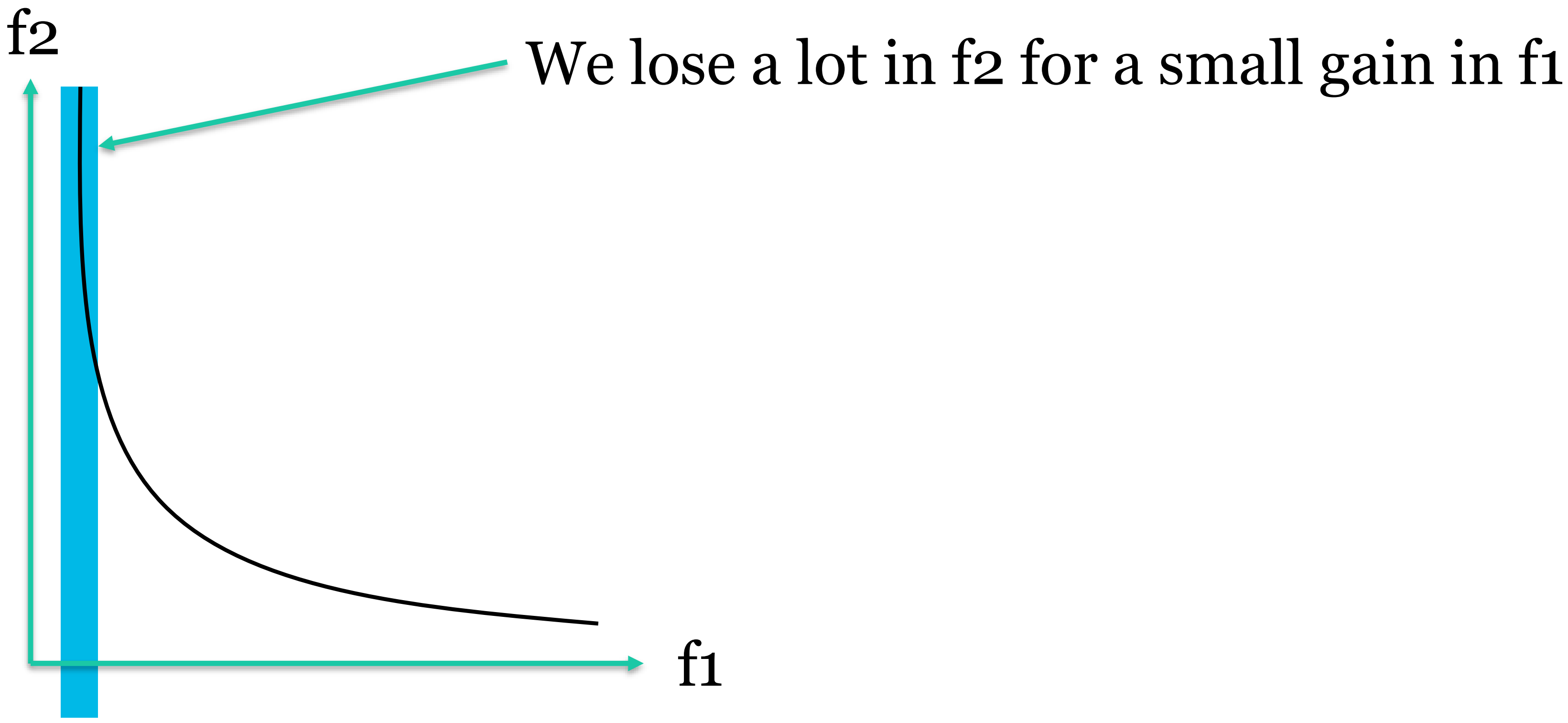
Choosing a Solution

- Possible to investigate graphically for two objectives.
- Difficult to display more than four identities in a graph
  - X1
  - X2
  - Color
  - Size



Choosing a Solution

- Avoid areas where you gain very little in f_i while f_j is drastically worsened

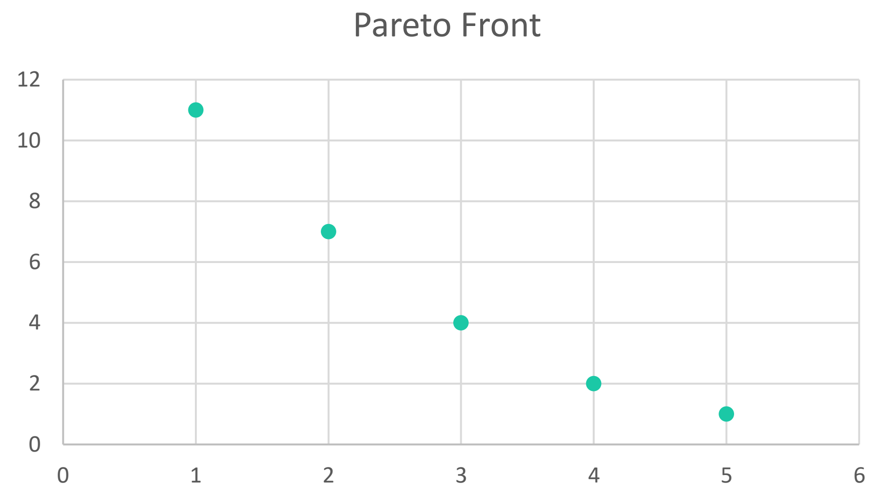


Numerical Methods for Choosing a Solution

- Similar to the MOO-lectures
  - Closest to utopian point
  - Fuzzy logic
  - Weighted sum

Example Pareto Front

f1	f2
1	11
2	7
3	4
4	2
5	1

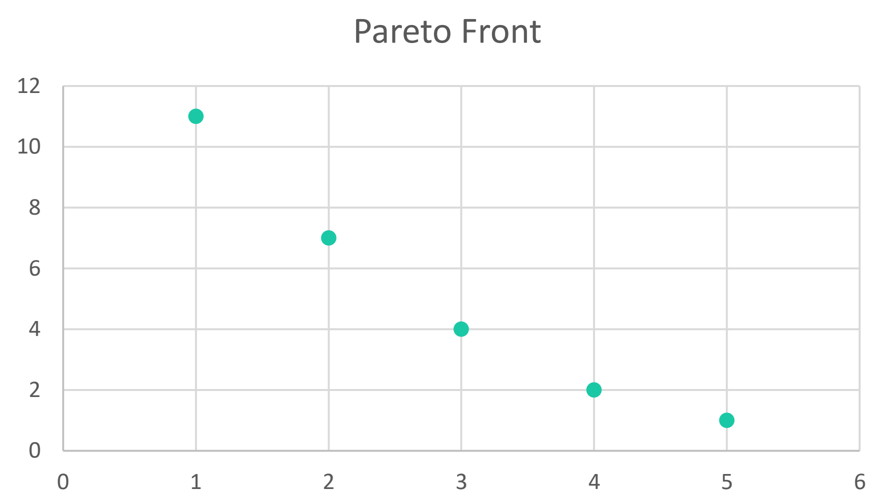


Numerical Methods for Choosing a Solution

$$F = \sum_{j=1}^{k} w_j \left(\frac{f_j}{f_{j0}}\right)^2$$

- Non-Linear Weighting
  - Normalized with $f_{jo}=f_j^*$
- The square penalizes the end solutions
  - 1^2=1
  - 0.5^2=0.25

f1	f2	Value
1	11	122
2	7	53
3	4	25
4	2	20
5	1	26



Numerical Methods for Choosing a Solution

- Wang, Z., & Rangaiah, G. P. (2017). Application and analysis of methods for selecting an optimal solution from the Pareto-optimal front obtained by multiobjective optimization. Industrial & Engineering Chemistry Research, 56(2), 560-574.

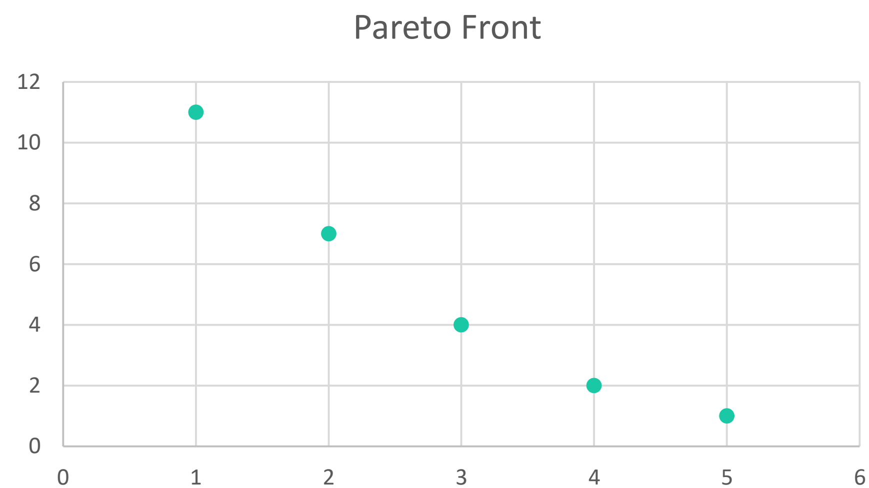
- They compared different numerical methods

- The methods were divided into three categories and the best in their study were…

Numerical Methods for Choosing a Solution

- Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)
  - Closest to Utopian point
  - Furthest from worst imaginable point

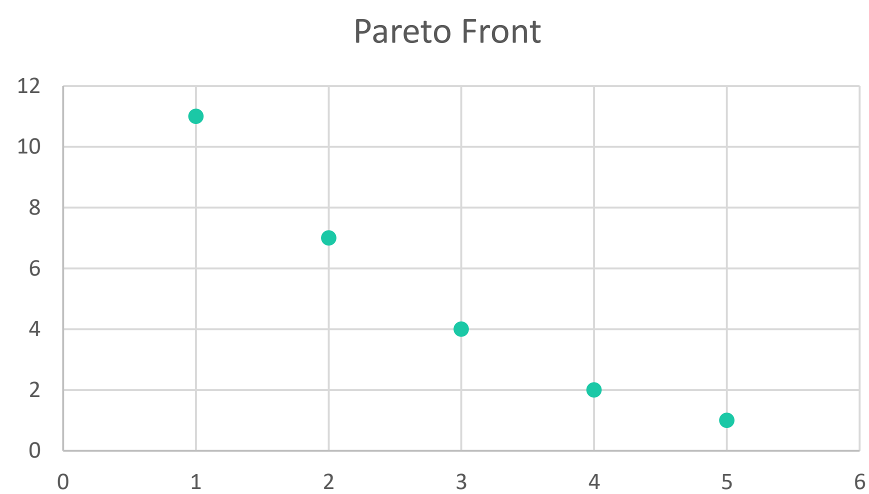
f1	f2	Value
1	11	0.427
2	7	0.522
3	4	0.624
4	2	0.618
5	1	0.573



Numerical Methods for Choosing a Solution

- Grey Relational Analysis (GRA)
  - Does not need any user input
  - The largest value is best

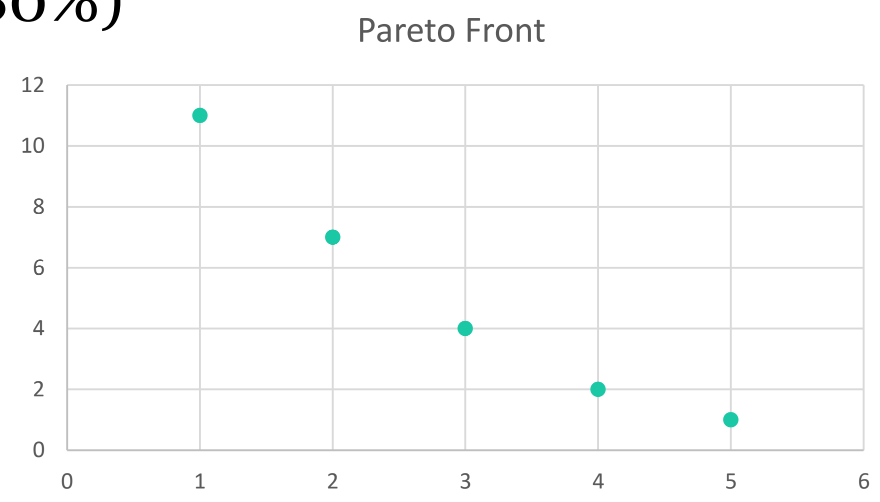
f1	f2	Value
1	11	0.3
2	7	0.285
3	4	0.287
4	2	0.296
5	1	0.3



Numerical Methods for Choosing a Solution

- Net Flow Method (NFM)
  - Three parameters needed
    - Indifference threshold (10% of objective range)
    - Preference threshold (20%)
    - Veto threshold (80%)

f1	f2	Value
1	11	0.64
2	7	0.25
3	4	0.29
4	2	-1.39
5	1	0.20



My Personal Way of Choosing

- See if I can find any obvious best point
- Remove points that have a really bad objective value
- Try to see if I can combine several objectives together so I only have 2-3 values to consider
  - > Easier to see graphically
- Pick 2-3 solutions that are good for different objectives
- Discuss the picked solutions with someone else
  - Project members
  - Managers

Questions?