Sensitivity Analysis

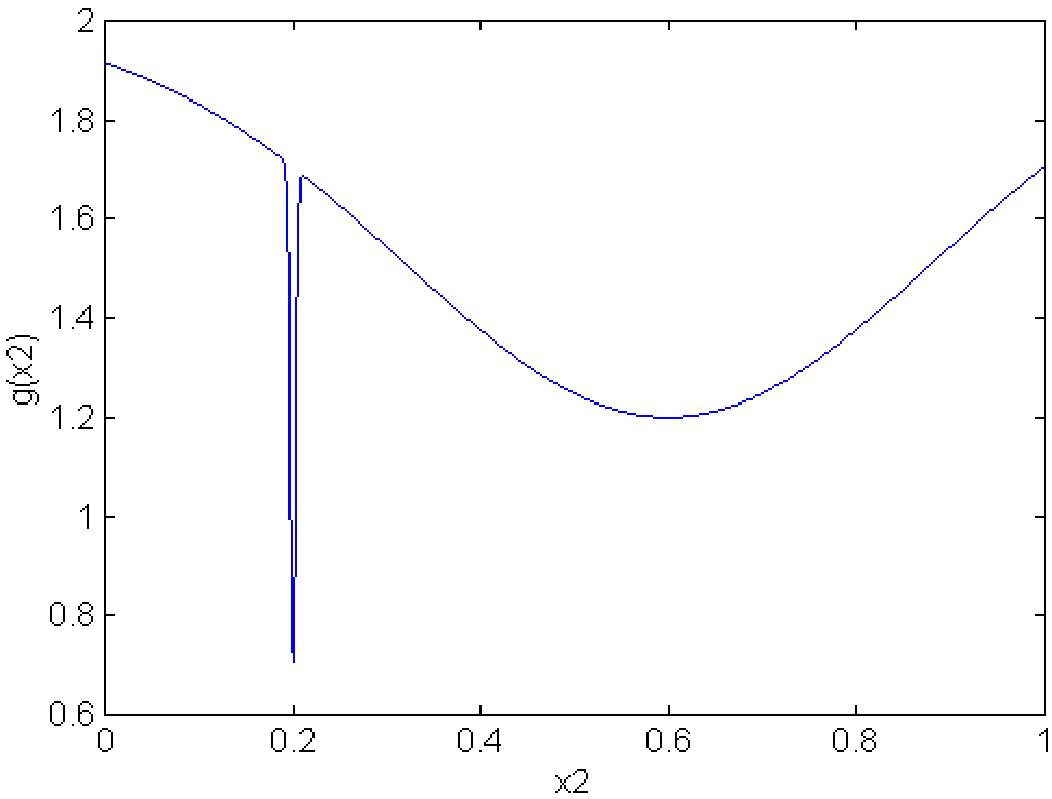
Design Optimization

TMKT48

LINKÖPINGS UNIVERSITET

Sensitivity Analysis

- We want to see how robust a suggested solution is
  - Choose a robust solution
- Identifies the variables that are not so important



Sensitivity Analysis

- We want to see how much each variable affects the objectives and constraints
  - Identifies the variables that are most important to optimize
  - Identifies the variables that are not so important

Local Sensitivity Analysis

Changes in design characteristics = Sensitivity matrix * Changes in design variables

$$
\begin{bmatrix} \Delta DC_1 \\ \Delta DC_2 \\ \vdots \\ \Delta DC_m \end{bmatrix} =
\begin{bmatrix}
\frac{\partial DC_1}{\partial x_1} & \frac{\partial DC_1}{\partial x_2} & \cdots & \frac{\partial DC_1}{\partial x_n} \\
\frac{\partial DC_2}{\partial x_1} & \frac{\partial DC_2}{\partial x_2} & \cdots & \frac{\partial DC_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial DC_m}{\partial x_1} & \cdots & \cdots & \frac{\partial DC_m}{\partial x_n}
\end{bmatrix}
\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}
$$

$$\Delta DC = S * \Delta x$$

Local Sensitivity Analysis

$$\Delta DC = S * \Delta x$$

- Analytical derivative

$$S_{ij} = \frac{\partial DC_i}{\partial x_j}$$

- Partial derivative

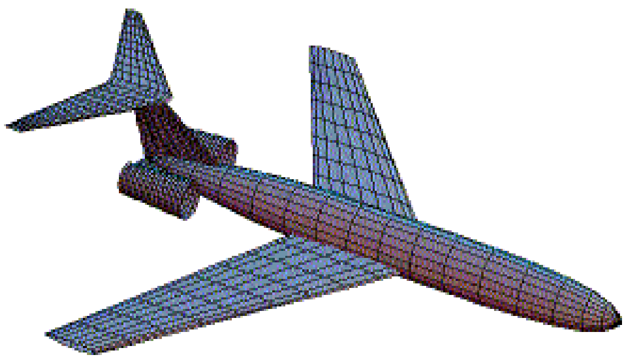
$$S_{ij} = \frac{DC_i(x + h) - DC_i(x)}{h}$$

Normalize the sensitivities

- To enable comparison of variables and DC's of different orders of magnitudes

$$S_{norm,ij} = S_{ij}\,\frac{x_{j0}}{DC_{i0}} = \frac{\partial DC_i}{\partial x_j}\frac{x_{j0}}{DC_{i0}}$$

Aircraft Sizing Example



Objective = Minimize Weight

$R > R_{\mathrm{ref}}$

$S_{\mathrm{Lo}} < S_{\mathrm{Lor}}$

$S_L < S_{\mathrm{Lr}}$

$W_1 > W_{1\,r}$

Sensitivity Matrix

	values	B	Cr	Ct	tc	T	W1	Wf
	values	2,16E+01	3,65E+00	1,55E+00	3,10E-02	1,73E+03	2,64E+04	2,22E+03
ObjF	-3,04E+04	-3,98E+05	4,76E+07	5,37E+06	-3,09E+08	5,80E+03	-2,05E+04	3,86E+04
R	1,00E+06	-2,44E+04	-1,16E+05	-1,14E+05	-1,50E+06	-4,10E-02	-4,00E+00	4,64E+02
SLo	1,00E+03	-1,13E+02	5,49E+01	-2,99E+01	8,87E+01	-1,46E+00	1,21E-01	1,21E-01
SL	4,83E+02	-5,07E-01	-1,71E+02	-1,46E+02	-5,70E+01	-1,17E-05	1,68E-02	1,68E-02
W0r	2,86E+04	6,66E-02	-2,67E+00	-3,48E-02	3,89E+01	-3,79E-05	4,70E-01	1,47E+00
Vs	3,57E+01	-8,25E-01	-3,40E+00	-3,43E+00	-1,57E-01	-3,46E-06	6,18E-04	6,14E-04
VLo	2,17E+01	-5,01E-01	-2,07E+00	-2,08E+00	-3,93E-02	-7,19E-07	3,77E-04	3,74E-04
VL	1,83E+01	-4,23E-01	-1,75E+00	-1,76E+00	-5,91E-02	-1,12E-06	3,17E-04	3,16E-04

- Useful for trade studies
- Overview is difficult due to large number of values and mix of units

Normalised sensitivity matrix

$$k_{ij} = \frac{\partial c_{s,i}}{\partial p_{s,j}} \frac{p_{s,j}}{c_{s,i}}$$

	Wing				Propulsion	Structure	fuel
	B	Cr	Ct	tc	T	W1	Wf
R	-0,53	-0,42	-0,18	-0,05	0	-0,11	1,03
SLo	-2,44	0,2	-0,05	0	-2,52	3,2	0,27
SL	-0,02	-1,29	-0,47	0	0	0,92	0,08
W0r	0	0	0	0	0	0,43	0,11
Vs	-0,5	-0,35	-0,15	0	0	0,46	0,04
VLo	-0,5	-0,35	-0,15	0	0	0,46	0,04
VL	-0,5	-0,35	-0,15	0	0	0,46	0,04

Normalised sensitivity matrix - Alt 2

$$k_{ij} = \frac{p_{s,j} \dfrac{\partial c_{s,i}}{\partial p_{s,j}}}{\sum_{l=1}^{N} \left| p_{s,l} \dfrac{\partial c_{s,i}}{\partial p_{s,l}} \right|}$$

The sum of the absolute numbers on each row is 1

	Wing				Propulsion	Structure	fuel
	B	Cr	Ct	tc	T	W1	Wf
R	-0,23	-0,18	-0,08	-0,02	0	-0,05	0,45
SLo	-0,28	0,02	-0,01	0	-0,29	0,37	0,03
SL	-0,01	-0,46	-0,17	0	0	0,33	0,03
W0r	0	0	0	0	0	0,79	0,21
Vs	-0,33	-0,23	-0,1	0	0	0,31	0,03
VLo	-0,33	-0,23	-0,1	0	0	0,31	0,03
VL	-0,33	-0,23	-0,1	0	0	0,31	0,03

Aggregation of the sensitivity matrix

	Wing				Propulsion	Structure	fuel
	B	Cr	Ct	tc	T	W1	Wf
R	-0,23	-0,18	-0,08	-0,02	0	-0,05	0,45
SLo	-0,28	0,02	-0,01	0	-0,29	0,37	0,03
SL	-0,01	-0,46	-0,17	0	0	0,33	0,03
W0r	0	0	0	0	0	0,79	0,21
Vs	-0,33	-0,23	-0,1	0	0	0,31	0,03
VLo	-0,33	-0,23	-0,1	0	0	0,31	0,03
VL	-0,33	-0,23	-0,1	0	0	0,31	0,03

		Propulsion	Structure	fuel
	Wing	T	W1	Wf
R	0,51	0	-0,05	0,45
SLo	0,31	-0,29	0,37	0,03
SL	0,64	0	0,33	0,03
W0r	0	0	0,79	0,21
Vs	0,66	0	0,31	0,03
VLo	0,66	0	0,31	0,03
VL	0,66	0	0,31	0,03

- An aggregated normalised sensitivity is the sum of absolute values of normalised sensitivities for a subsystem
- Overview is obtained through the introduction of hierarchy

Some remarks

- Can be seen as a linearization around a reference point
- Different points in the design space usually give different sensitivities

- Variables that the Design Characteristics are insensitive to might not need to be used as design variables in the optimization
  - We get a smaller design space =)

Exercise - Sensitivity analysis of El MC

$$
\begin{bmatrix} \mathrm{D}DC_1 \\ \mathrm{D}DC_2 \\ \vdots \\ \mathrm{D}DC_m \end{bmatrix}
=
\begin{bmatrix}
\frac{\partial DC_1}{\partial x_1} & \frac{\partial DC_1}{\partial x_2} & \cdots & \frac{\partial DC_1}{\partial x_n} \\
\frac{\partial DC_2}{\partial x_1} & \frac{\partial DC_2}{\partial x_2} & \cdots & \frac{\partial DC_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial DC_p}{\partial x_1} & \cdots & \cdots & \frac{\partial DC_p}{\partial x_n}
\end{bmatrix}
\begin{bmatrix} \mathrm{D}x_1 \\ \mathrm{D}x_2 \\ \vdots \\ \mathrm{D}x_n \end{bmatrix}
$$

Changes in design characteristics = Sensitivity matrix * Changes in design variables

Sensitivity analysis

- Start of in a good compromise point:
  u1=6.9, u2=3, v_shift=53.4
- Calculate the sensitivities for the characteristics acceleration, top speed and range
- Change the design variables:
  u1, u2 and v_shift as well as the parameters m, Cd, Ah_max, Umax
- Calculate the sensitivities for a 2% increase for the parameters

$$\frac{\partial C_i}{\partial x_i} \approx \frac{\Delta C_i}{\Delta x_i}$$

Sensitivity matrix

The script el_mc_sens.m helps you create the sensitivity matrix

$$\begin{bmatrix} \Delta DC_1 \\ \Delta DC_2 \\ \Delta DC_3 \end{bmatrix} = \begin{bmatrix} -1.5777 & 4.2180 & -0.0010 & -0.1359 & -4.3251 & 0.0000 & 0.2466 \\ -0.0091 & 7.4602 & 0.0004 & -0.1756 & -69.9901 & -0.0000 & 0.0015 \\ -14.7862 & 0.1061 & 0.0099 & -0.1625 & -15.4837 & 0.7365 & 0.0522 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_7 \end{bmatrix}$$

Normalized sensitivities

- Normalize the sensitivities according to the equation below.

$$S_{ij\text{norm}} = \frac{\partial DC_i}{\partial x_j} \frac{x_{0j}}{DC_{0i}}$$

$$S_{norm} = \frac{\Delta DC}{\Delta x} \frac{x_0}{DC_0}$$

Examine the normalized matrix

- Determine important parameters for the different characteristics. For example:
- Acceleration is mainly determined by the mass of the driver and the gear ratio. The energy in the battery (Ah_max) is of no importance.
- Top speed: .....
- Range: .....

Questions?