The Complex Method

Design Optimization
TMKT48

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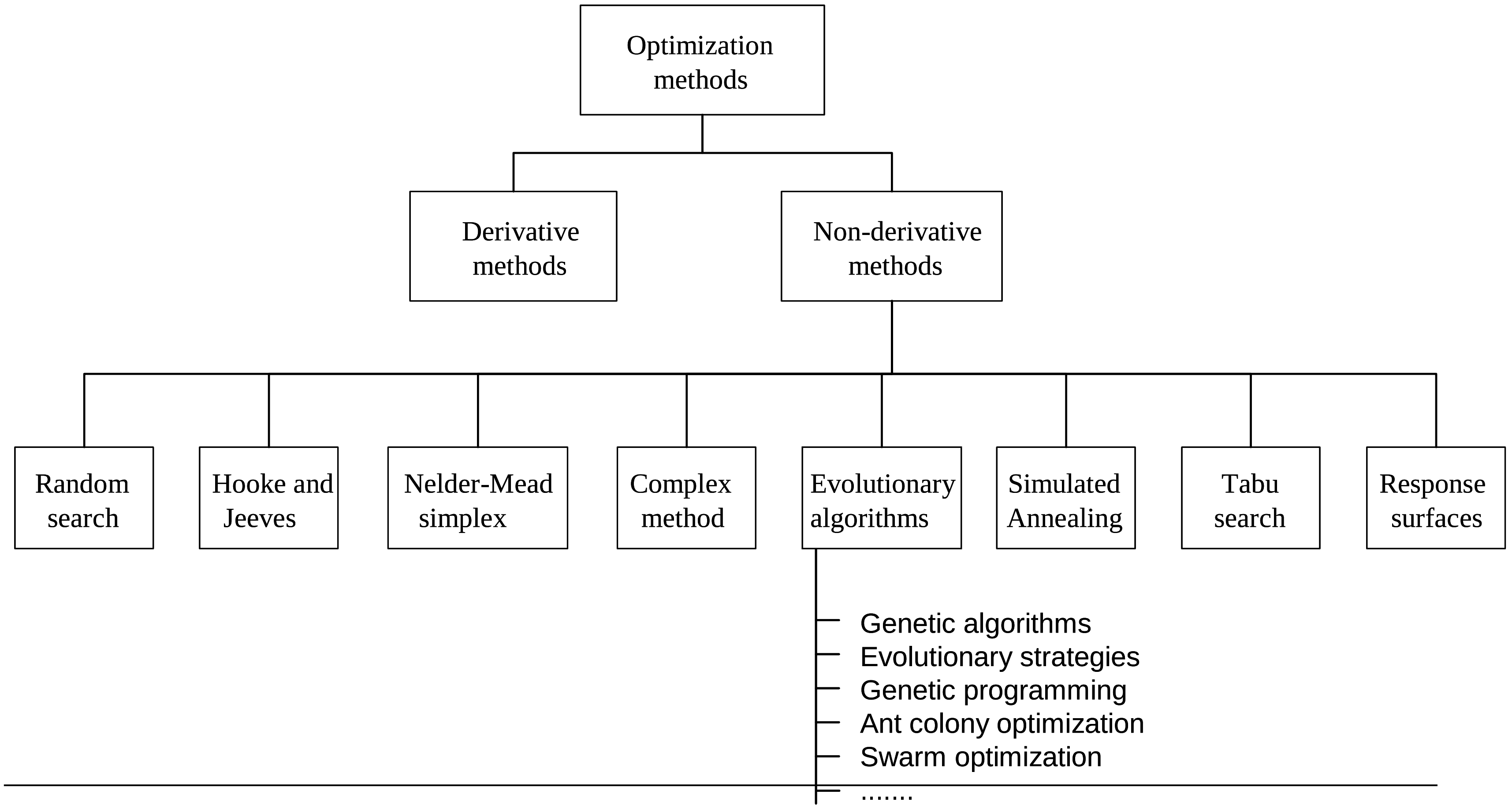
Non-derivative methods



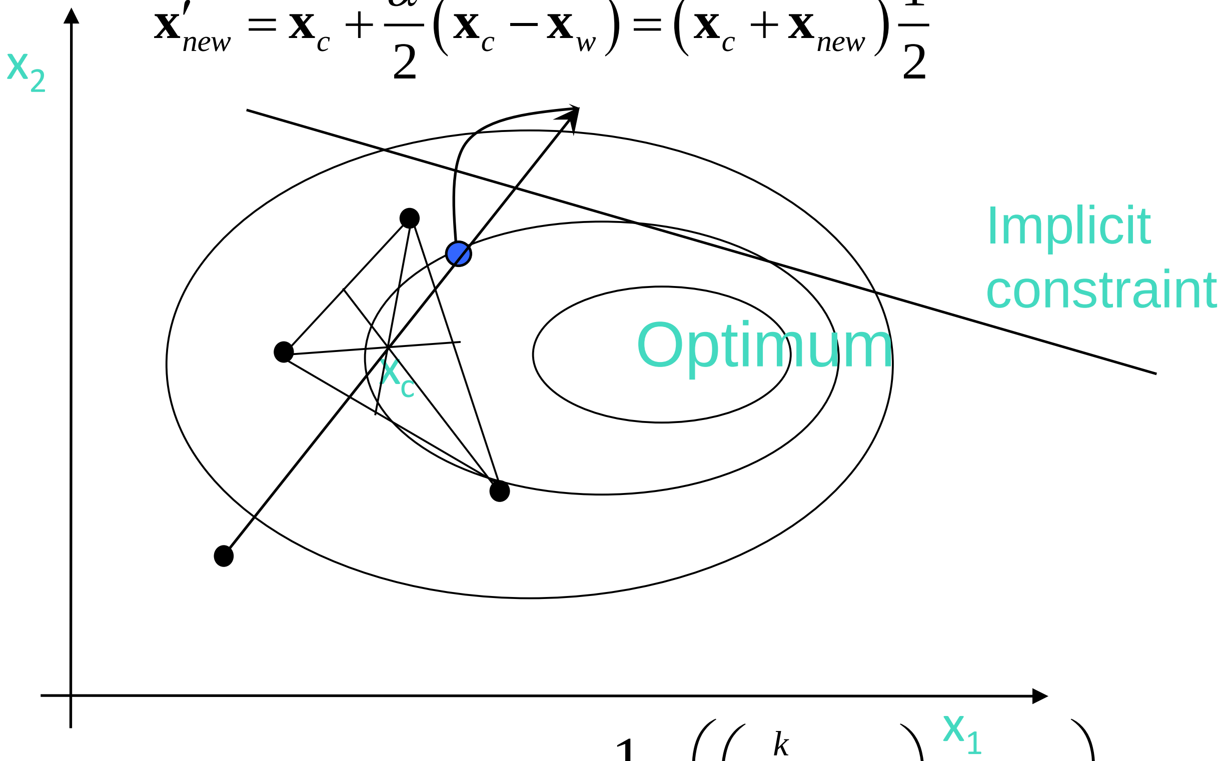
The Complex method: History

- It was developed in the mid 60’s by Box
- The Complex method is based on the Nelder-Mead Simplex method.
- The Complex methods extends the Simplex method so it works on constrained optimization problems.
- Both are iterative search processes.
- The simplex uses $k=n+1$ points whereas the Complex method uses $k \geq n+1$ points.
- Complex is easy applicable to a wide range of engineering problems.
- The Complex method has been modified and improved by Krus et al.

The Complex Method

$$\mathbf{x}_{new} = \mathbf{x}_c + \alpha(\mathbf{x}_c - \mathbf{x}_w)$$

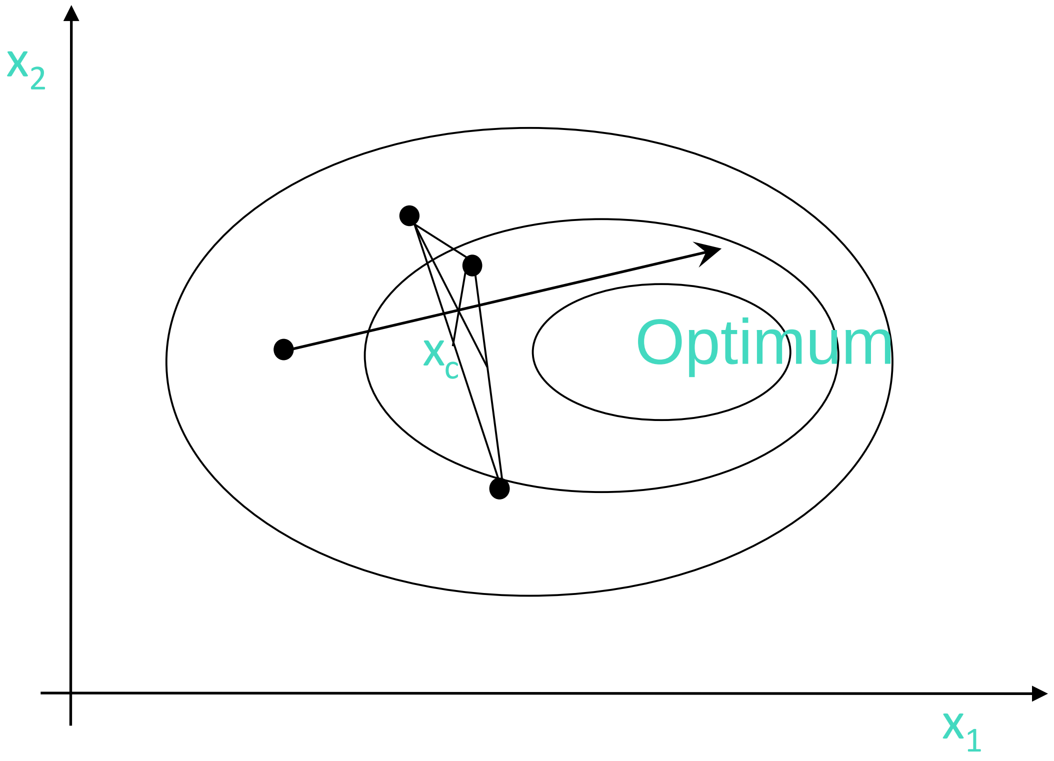
$$\mathbf{x}'_{new} = \mathbf{x}_c + \frac{\alpha}{2}(\mathbf{x}_c - \mathbf{x}_w) = (\mathbf{x}_c + \mathbf{x}_{new})\frac{1}{2}$$



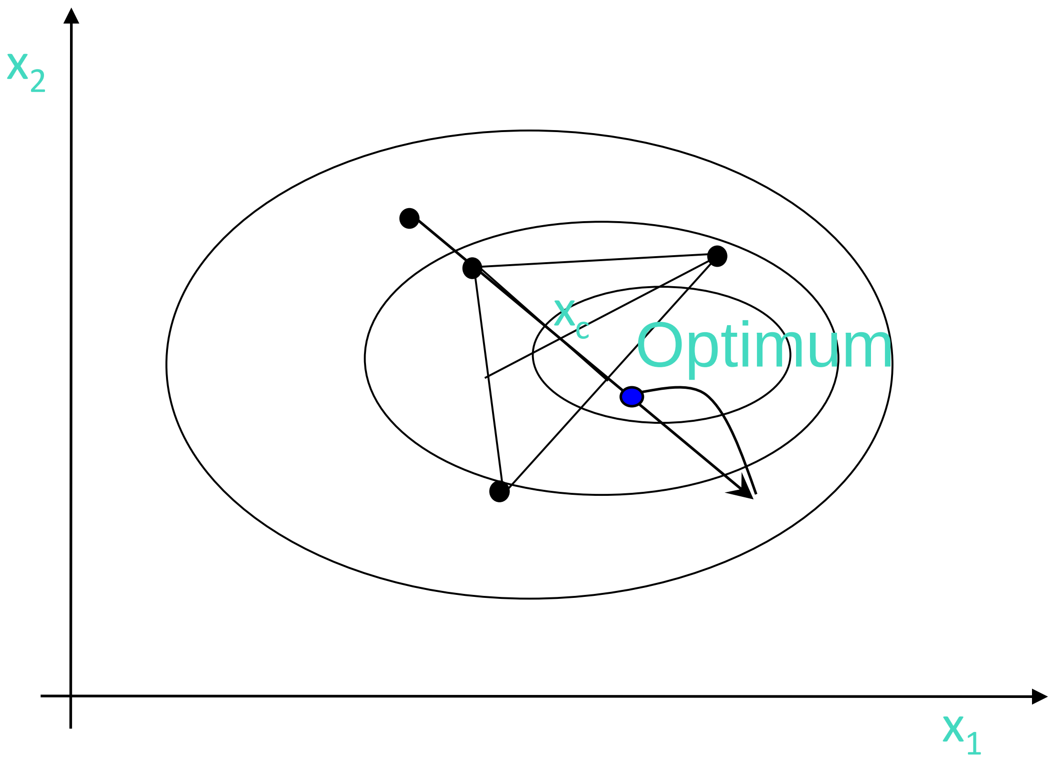
$$x_{c,j} = \frac{1}{k-1}\left(\left(\sum_{i=1}^{k} x_{i,j}\right) - x_{w,j}\right), j = 1 \ldots n$$

…

The Complex Method



The Complex method



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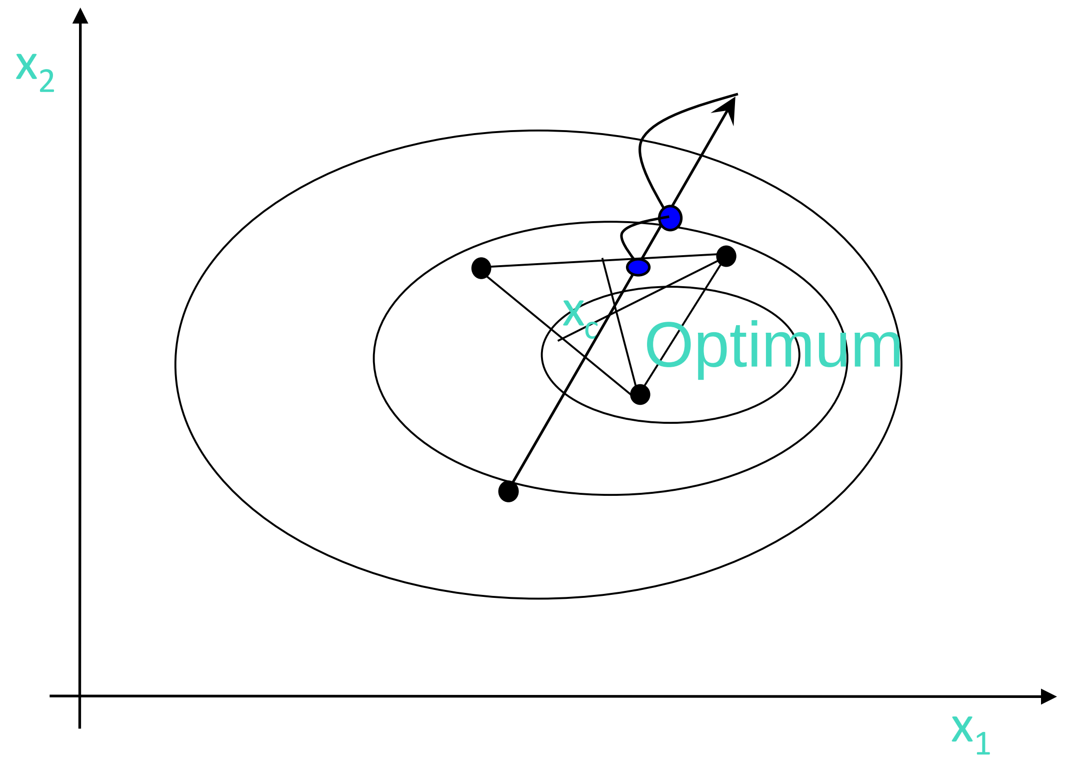

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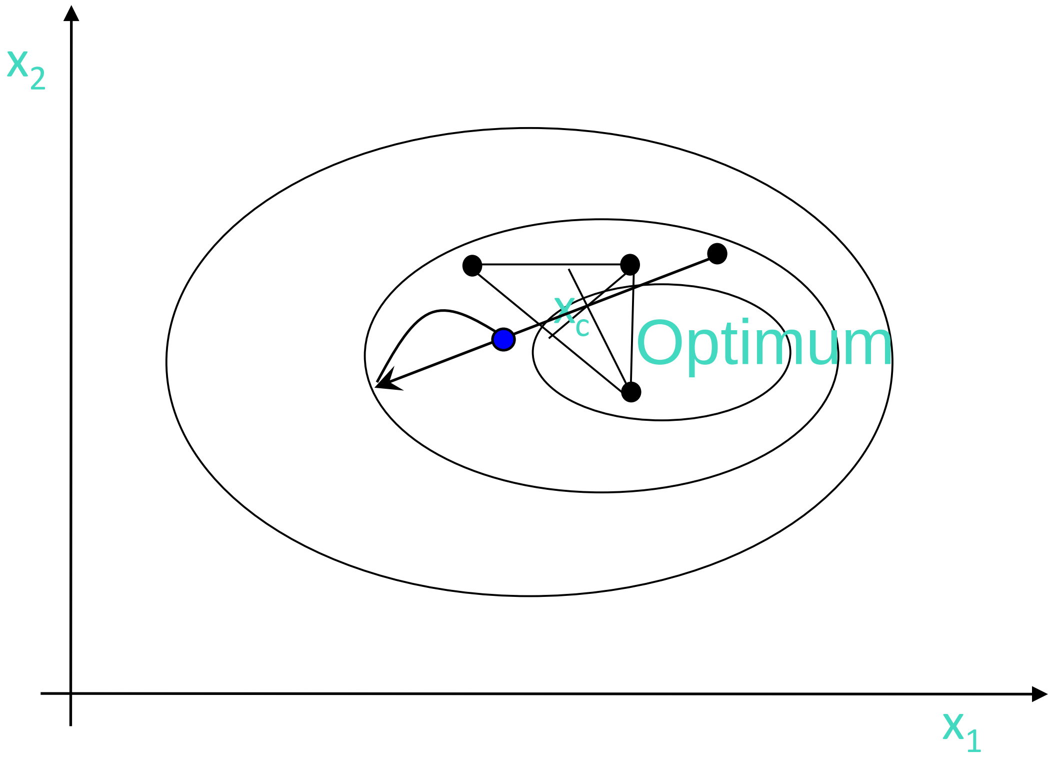
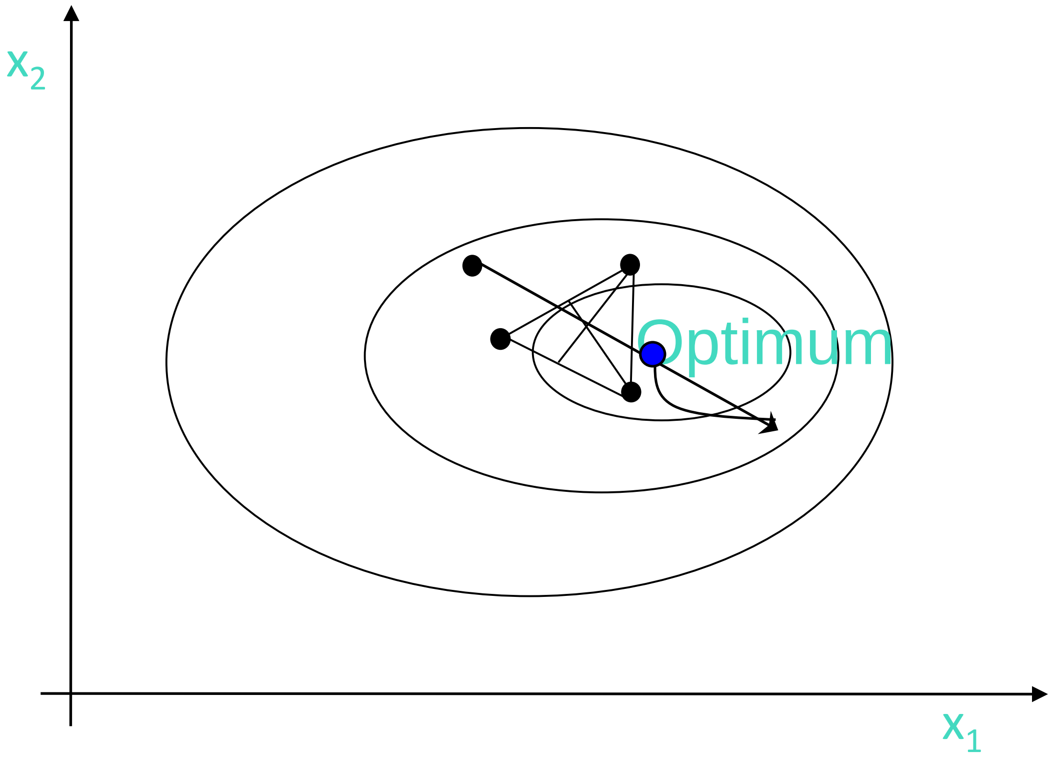


The Complex Method



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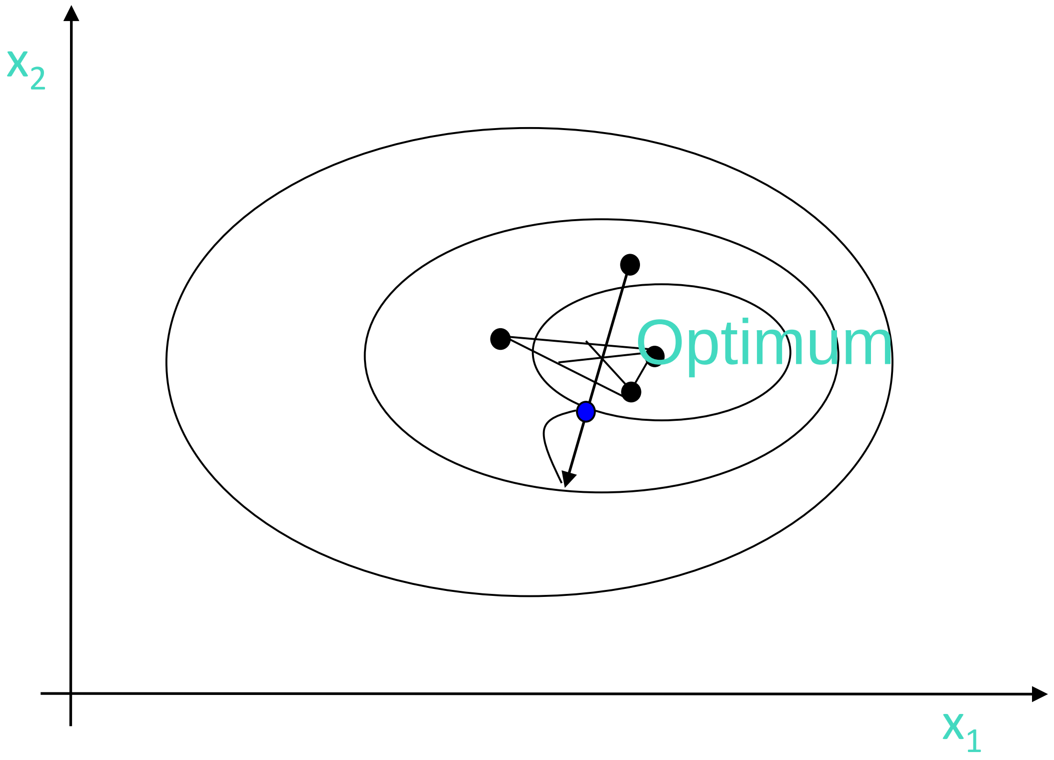

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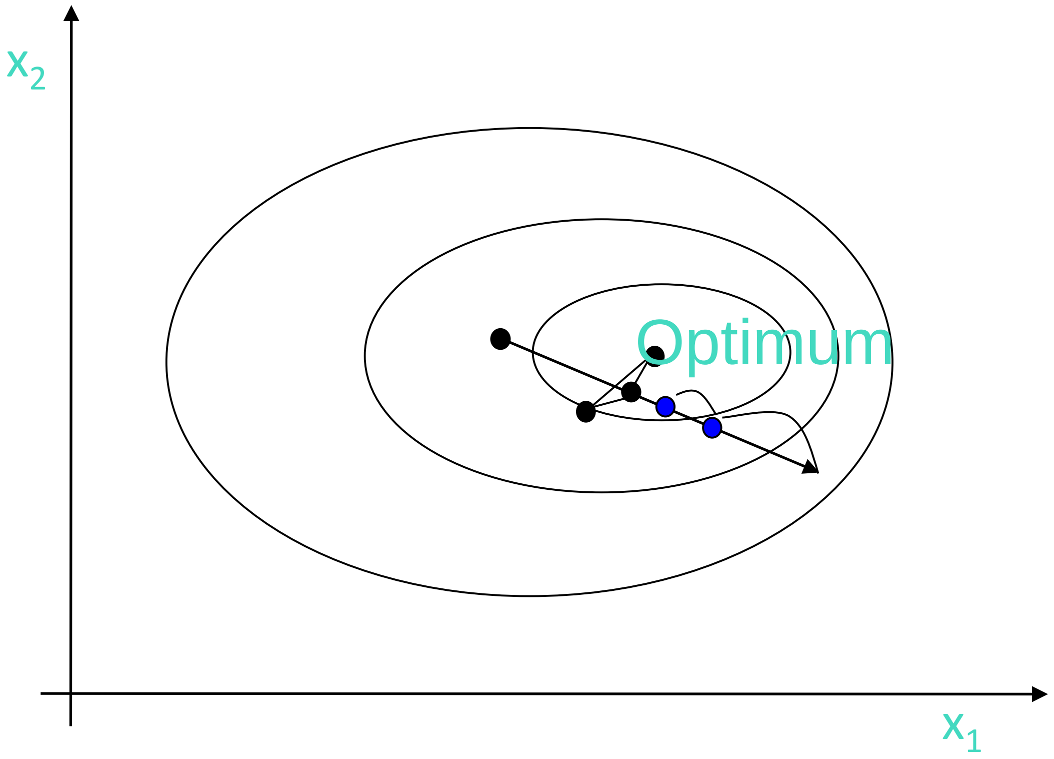

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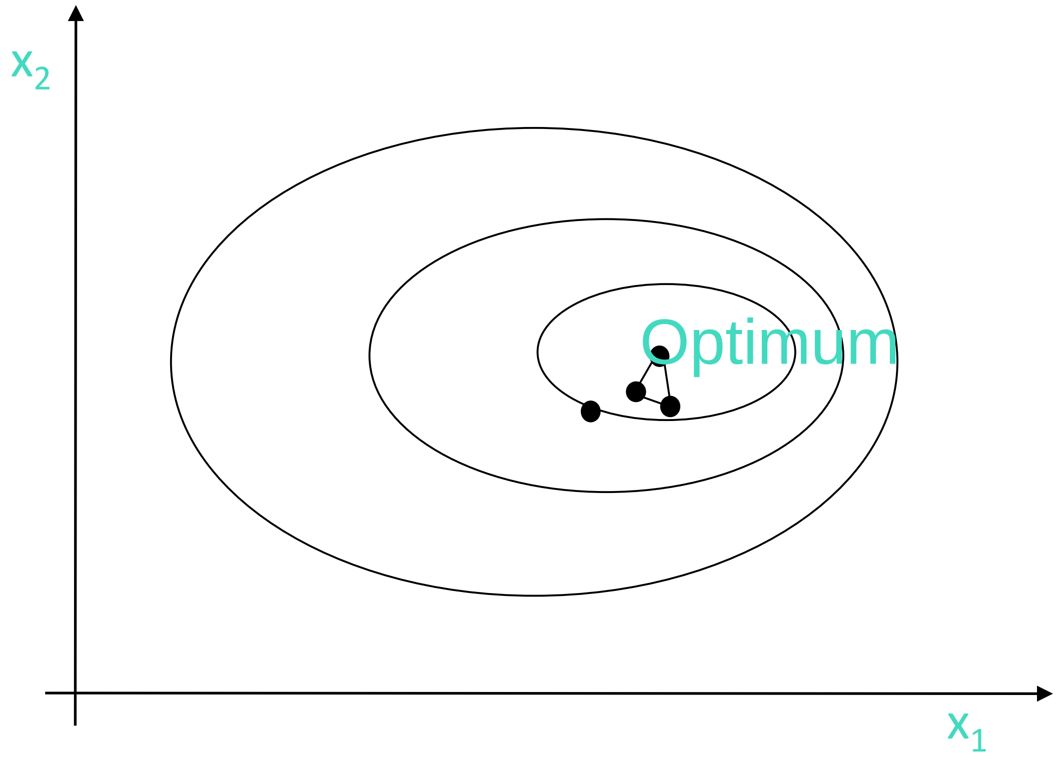

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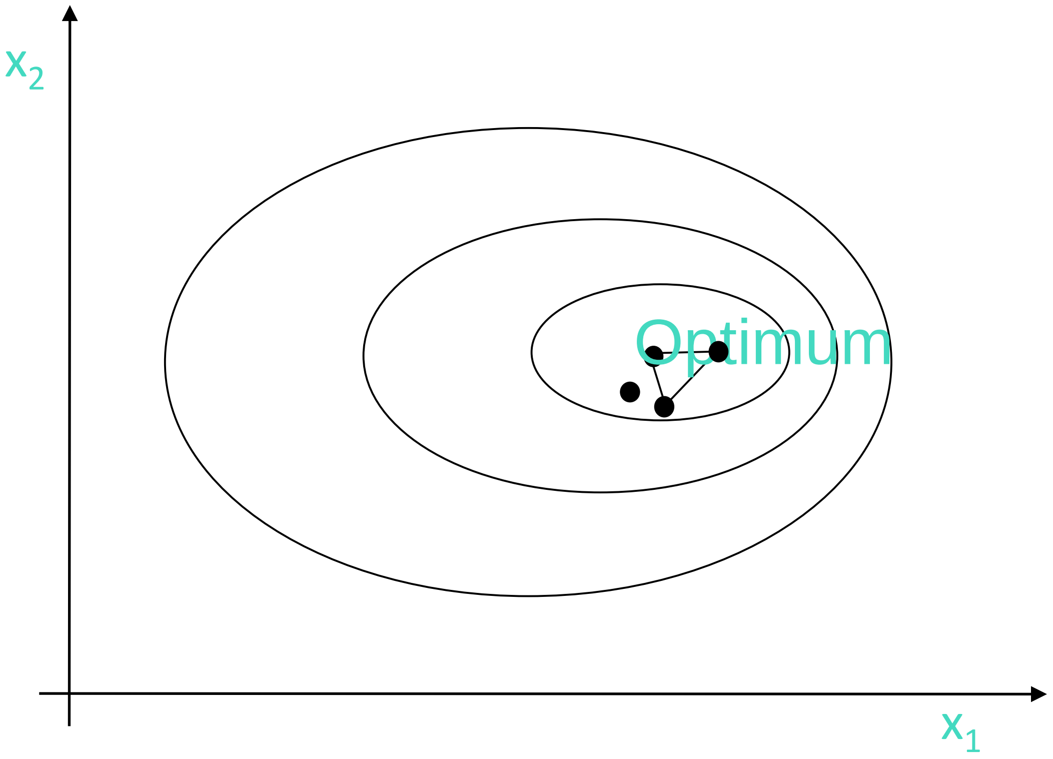

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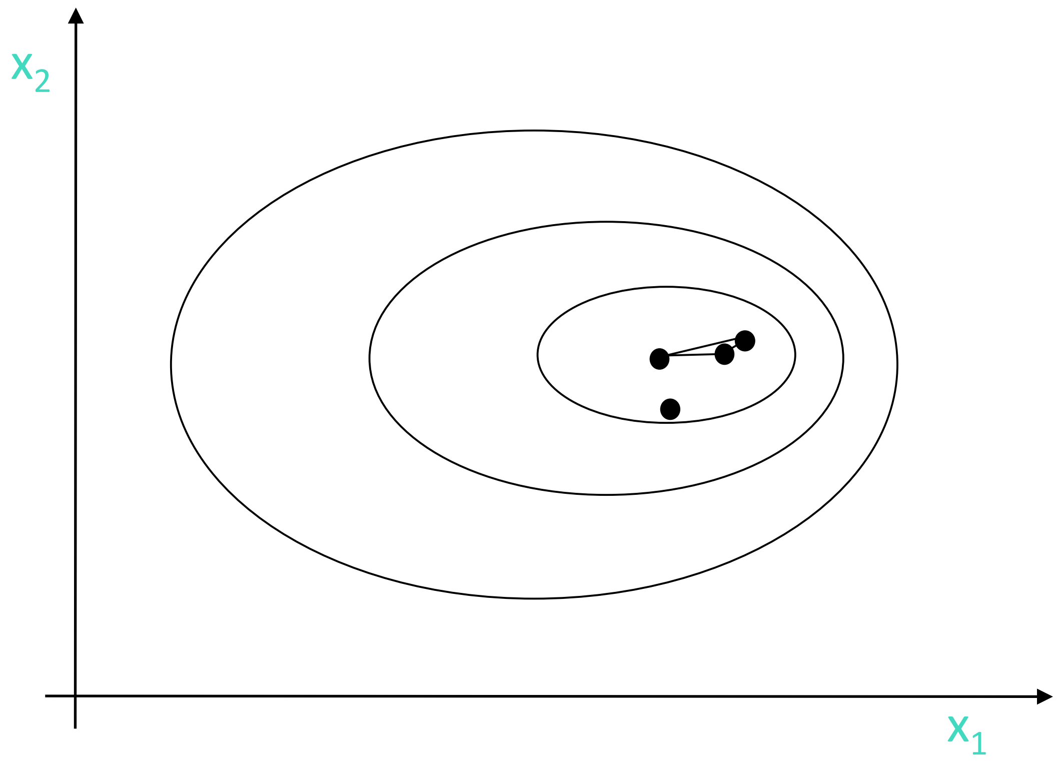
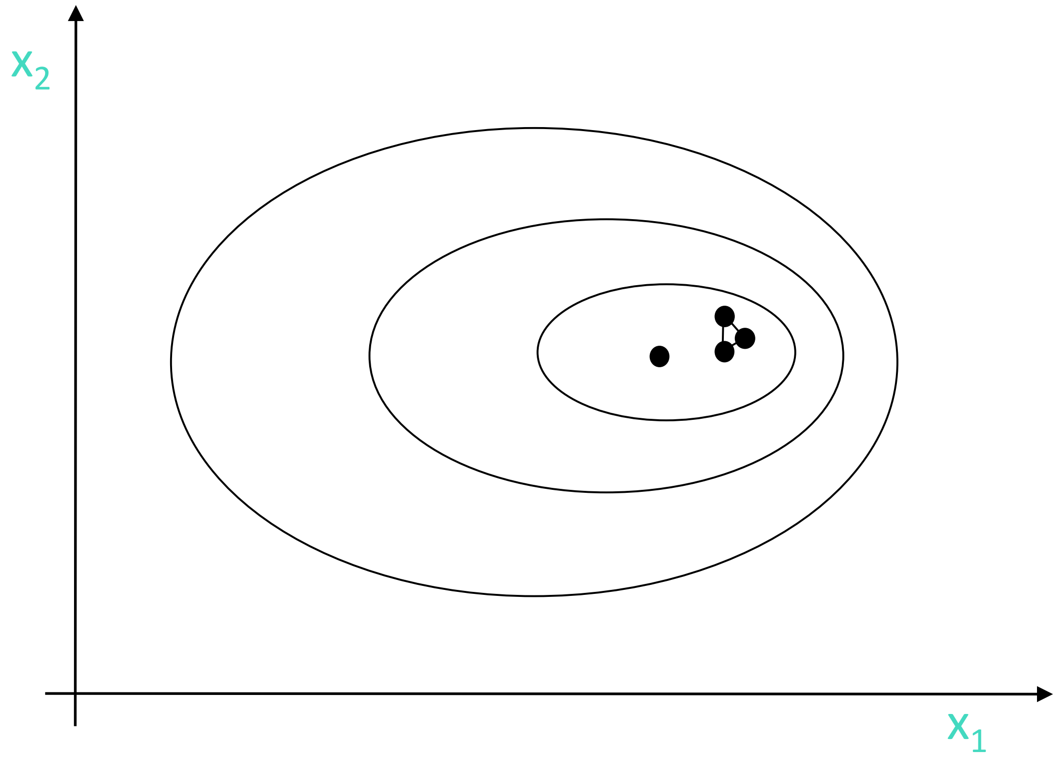


The Complex method



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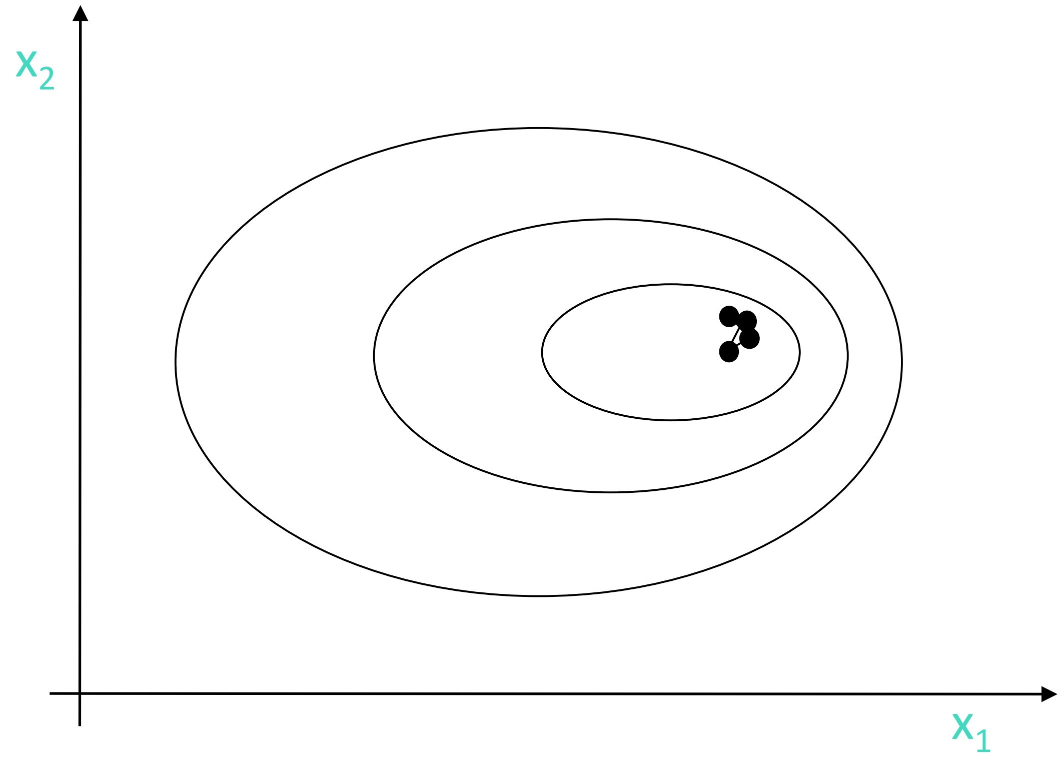
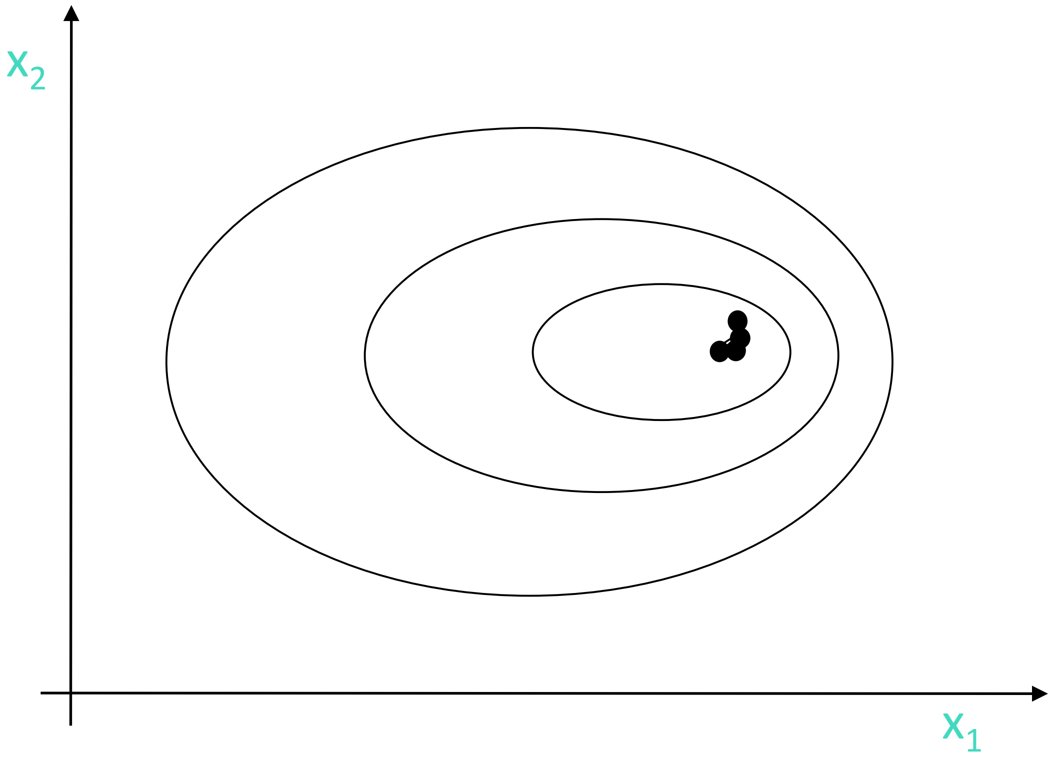


The Complex method



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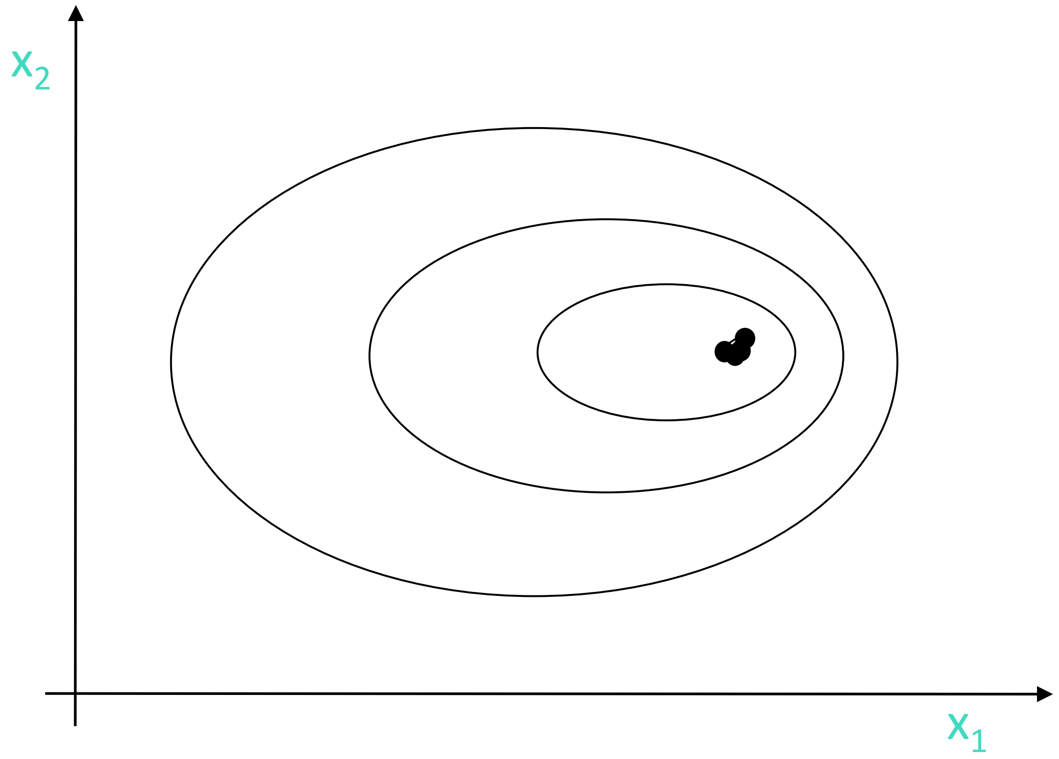

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Question?

- Will the Complex algorithm return the same optimum every time if you try to optimize the same problem with the same algorithm settings?

- We can only be sure about it if the problem is uni-modal since the starting points are randomized

Termination Criteria

Convergence in function values

$$\max_{i=1,\ldots,k}\left(f\left(\mathbf{x}_i\right)\right)-\min_{i=1,\ldots,k}\left(f\left(\mathbf{x}_i\right)\right)\leq\varepsilon_f$$

Convergence in optimization variables

$$\max_{j=1,\ldots,n}\left(\max_{i=1,\ldots,k}\left(x_{ij}\right)-\min_{i=1,\ldots,k}\left(x_{ij}\right)\right)\leq\varepsilon_v$$

Max number of evaluations

number of evaluations > MaxNoEvals

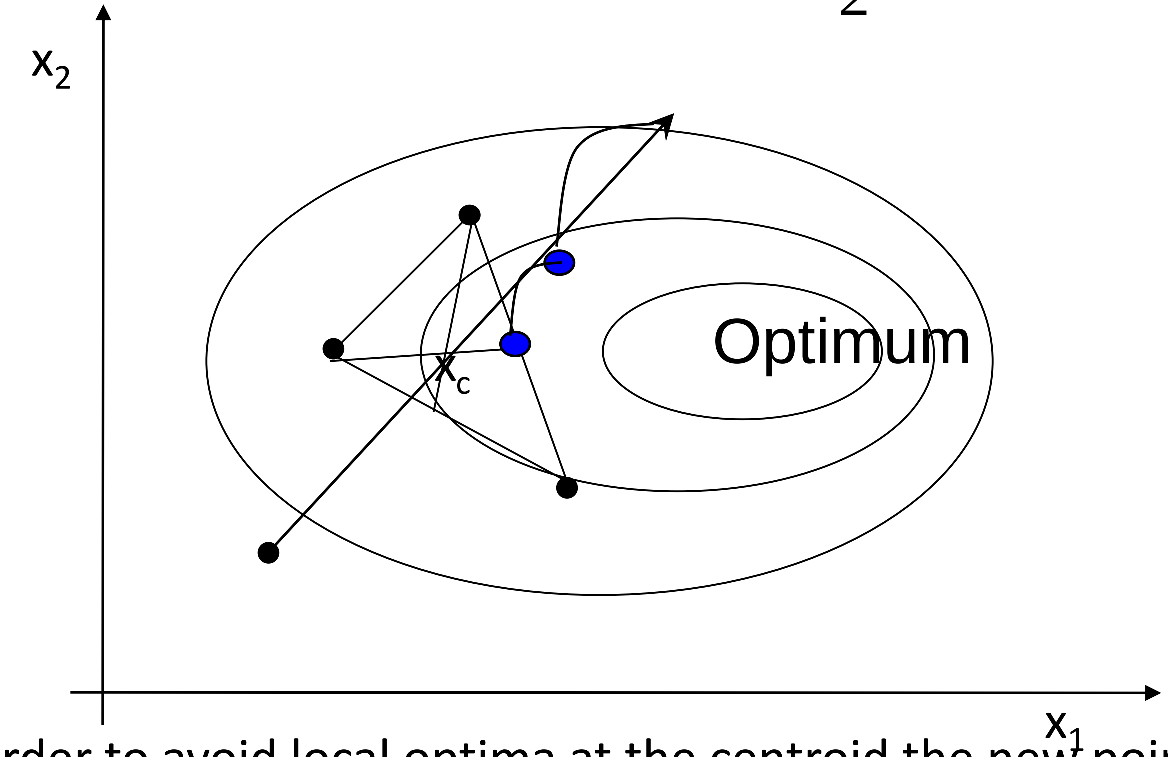
Question?

- Why might we want to normalize the values when we calculate the spread?

- The design variables might have different order of magnitude, for example length (meters) and stress (pascal)
- It is nice to have the spread expressed as a percentage

Improved Complex

$$\mathbf{x}'_{new} = \left(\left((1-a)\mathbf{x}_c + a\mathbf{x}_{best} \right) + \mathbf{x}_{new} \right) \frac{1}{2} \qquad a = 1 - e^{-\frac{iter}{4}}$$

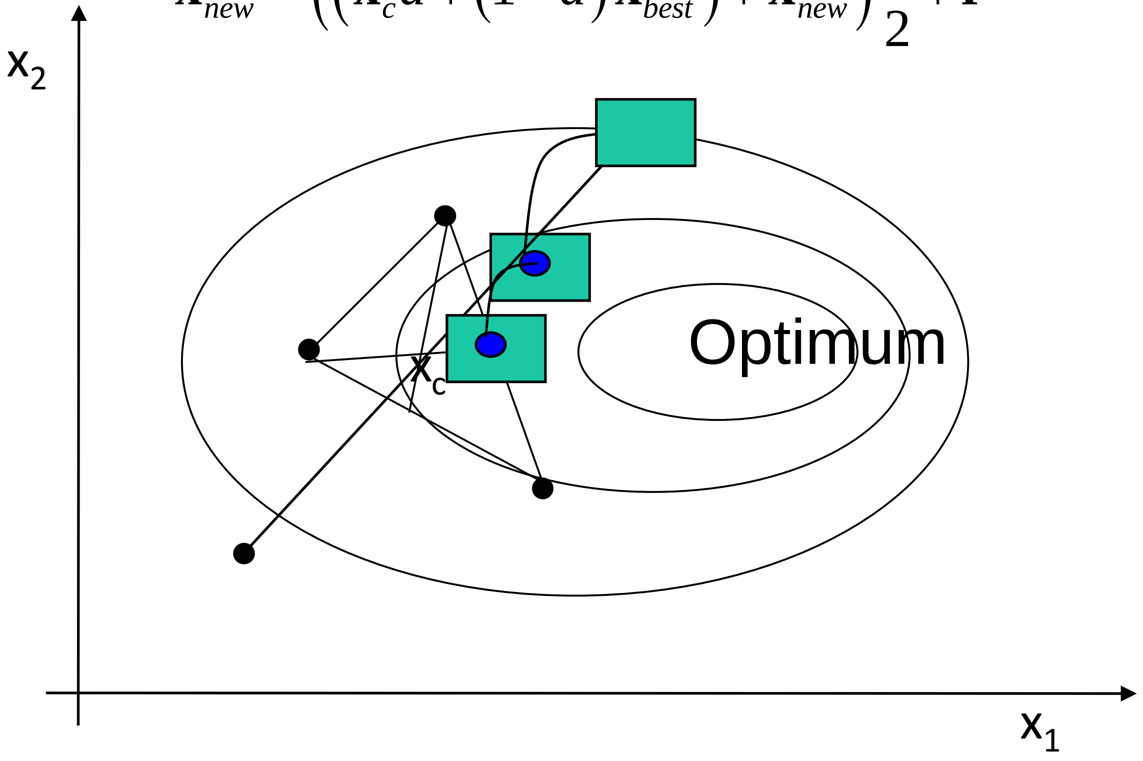


In order to avoid local optima at the centroid the new point could gradually be moved towards the best point in the complex.

Complex R

$$\mathbf{x}_{new} = \mathbf{x}_c + \alpha(\mathbf{x}_c - \mathbf{x}_w) + \mathbf{r}$$

$$\mathbf{x}'_{new} = \left(\left(\mathbf{x}_c a + (1-a)\mathbf{x}_{best}\right) + \mathbf{x}_{new}\right)\frac{1}{2} + \mathbf{r}$$



To further increase the robustness of the algorithm some random noise could be added to each new point.

Complex RF

- In order to track dynamic object functions it is necessary to introduce a forgetting factor
- This is done by decreasing the function value each time it is not re-evaluated
- This is also an advantage in “tricky” non-dynamic object functions such as discrete functions with plateaus.

Procedure of the original Complex method

```
Generate starting points
Calculate objective function
Identify the worst point
While stop criteria is not met
    Calculate centroid
    Reflect worst point through centroid
    Calculate objective function for the new point
    Identify the worst point
    While the new point is the worst
        Move the new point towards the centroid
        Calculate the objective function
    end while
    Identify the worst point in the new complex
    Check stop criteria
end while
Output the optimal point
```

Procedure of the improved Complex method

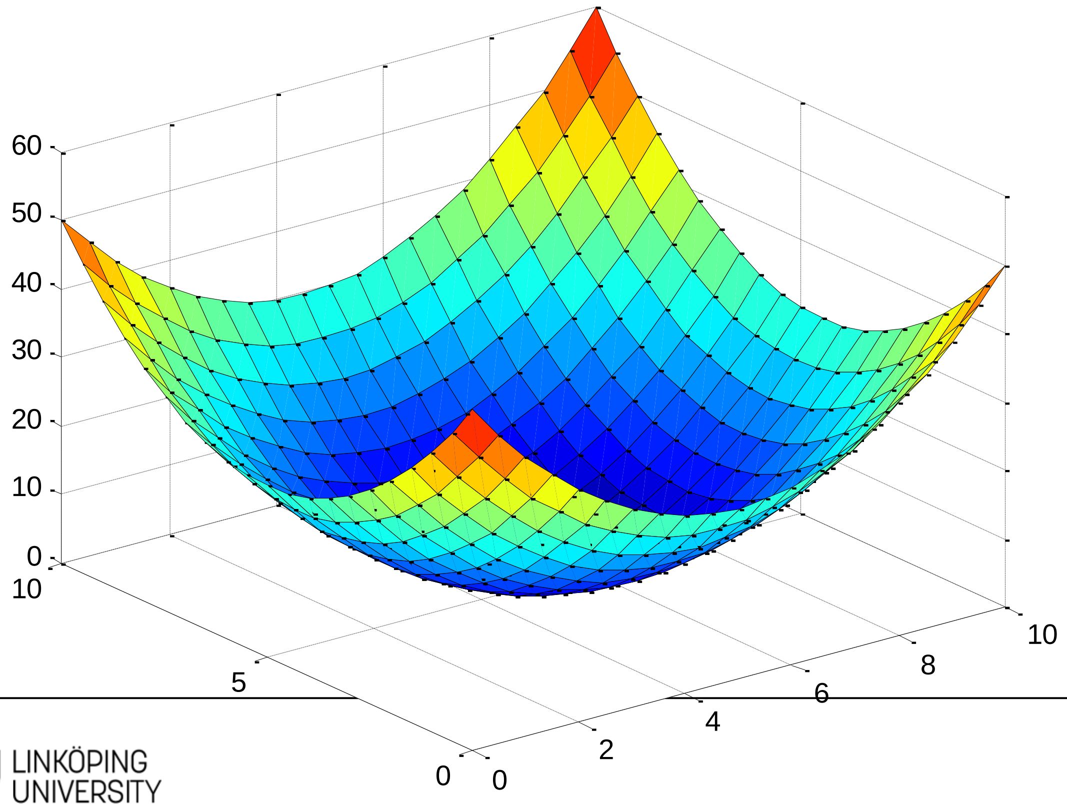
```
Generate starting points
Calculate objective function
Identify the worst point
While stop criteria is not meet
    Make all points worse (Forgetting)
    Calculate centroid
    Reflect worst point through centroid (add noise R)
    Calculate objective function for the new point
    Identify the worst point
    While the new point is the worst
        Move the new point towards the centroid
        Calculate the objective function
    end while
    Identify the worst point in the new complex
    Check stop criteria
end while
Output the optimal point
```

Add R + move towards best

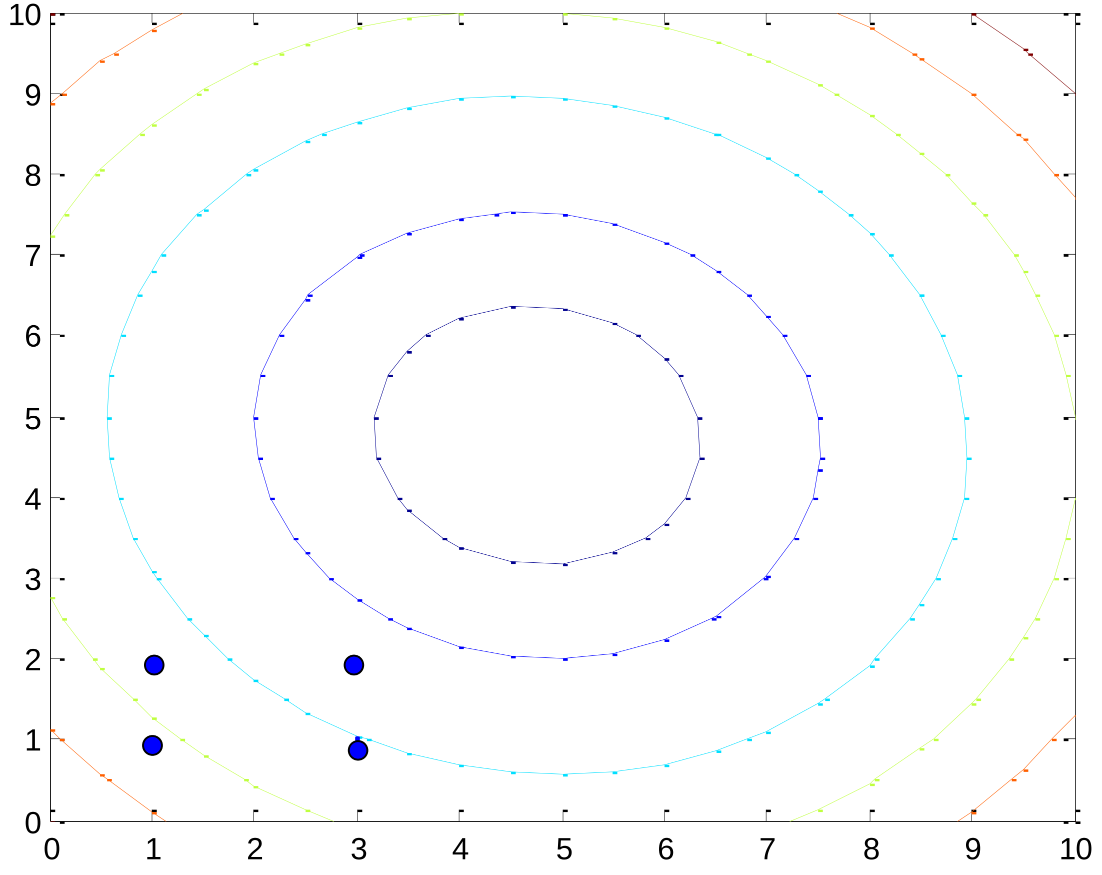
Test Example: Test 1

$$f = (x_1-5)^2 + (x_2-5)^2 + 0.1 \cdot x_1 \cdot x_2$$

$$0 < x_1 < 10 \,,\quad 0 < x_2 < 10$$



Procedure of the Complex method



Starting points

$\mathbf{x}_1$=[1, 1], f = 32.1

$\mathbf{x}_2$=[1, 2], f = 25.2

$\mathbf{x}_3$=[3, 1] , f = 20.3

$\mathbf{x}_4$=[3, 2] , f = 13.6

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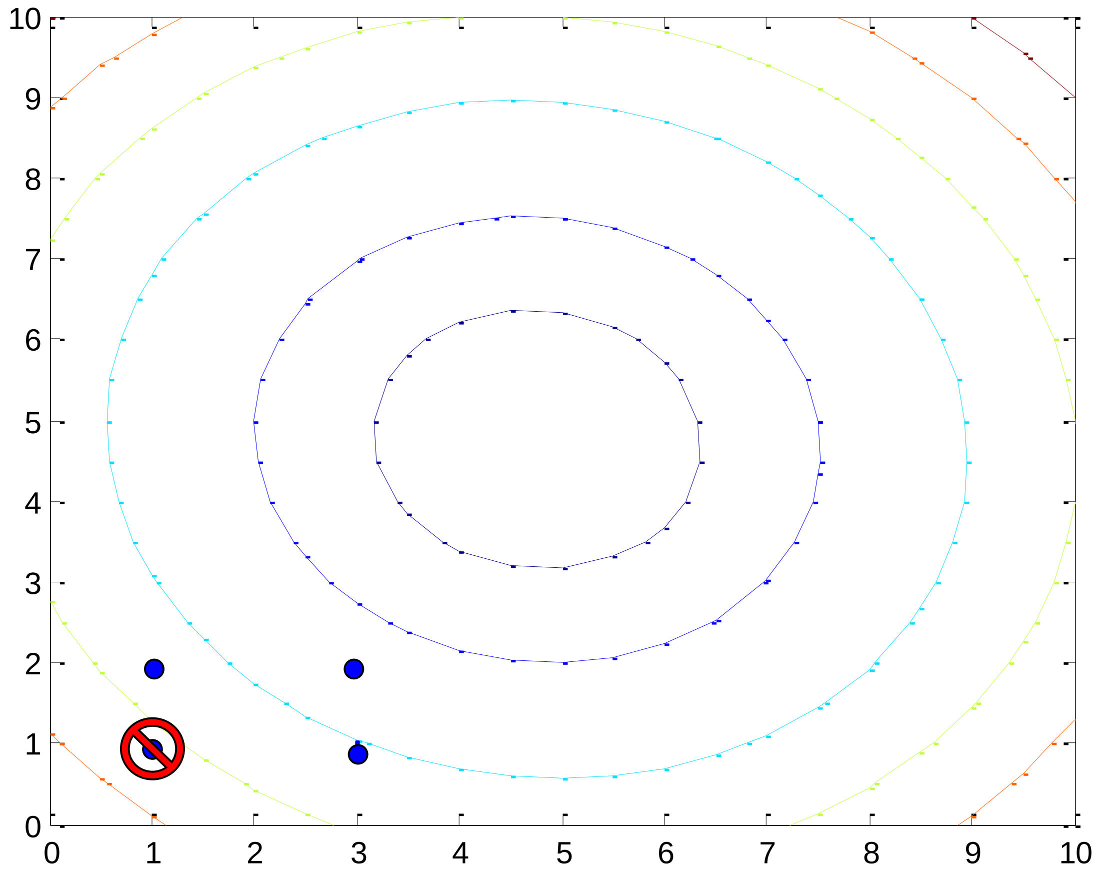
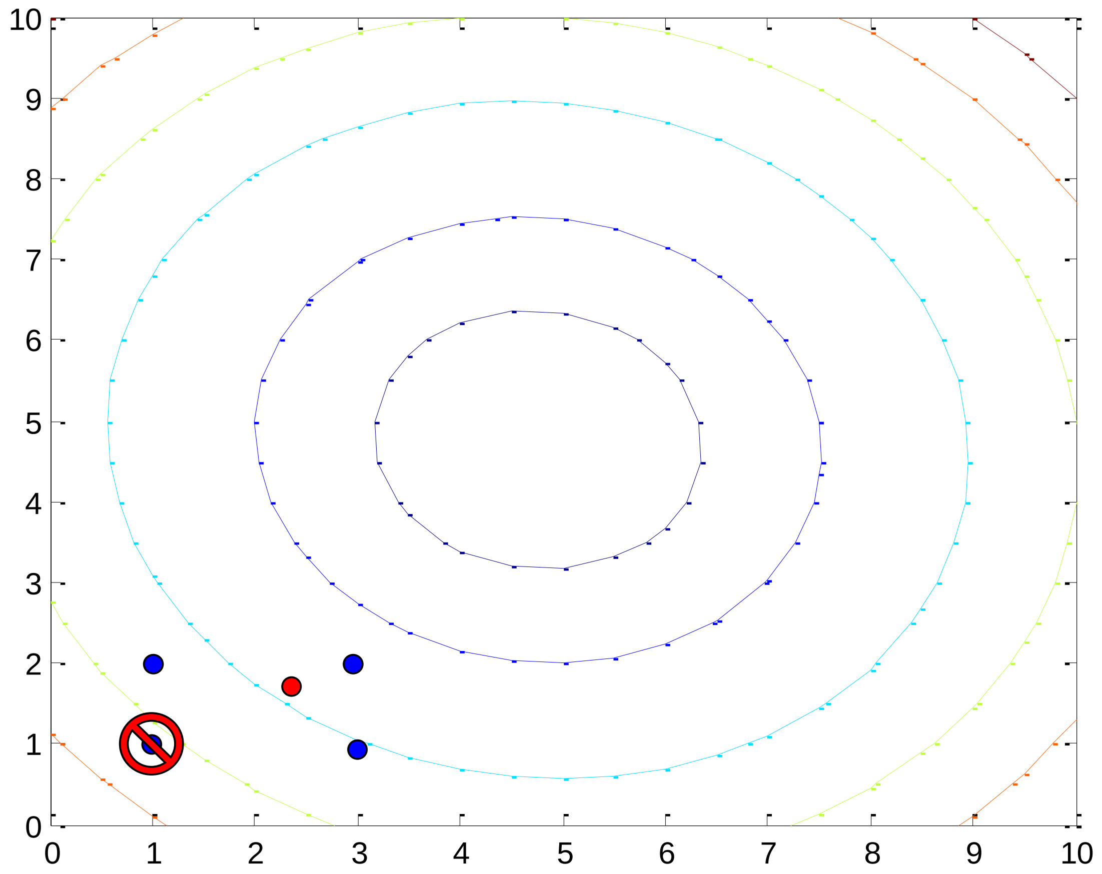
Procedure of the Complex method



Identify the worst point

$\mathbf{x}_1$ is worst.

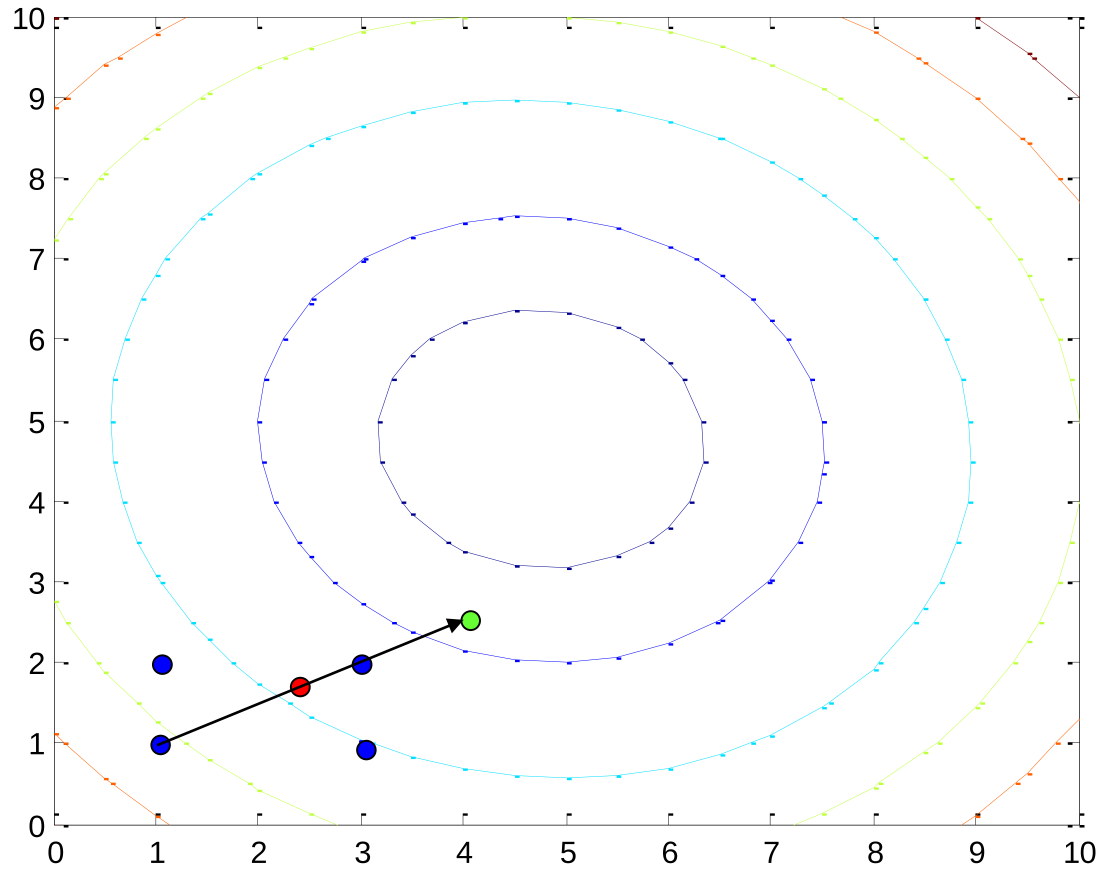
Procedure of the Complex method



Calculate the centroid of the other three points

$\mathbf{x}_c = [2.33, 1.67]$

Procedure of the Complex method



Calculate the new point

$\mathbf{x}_{new} = [4.07, 2.53]$

$f_{new} = 8.0$

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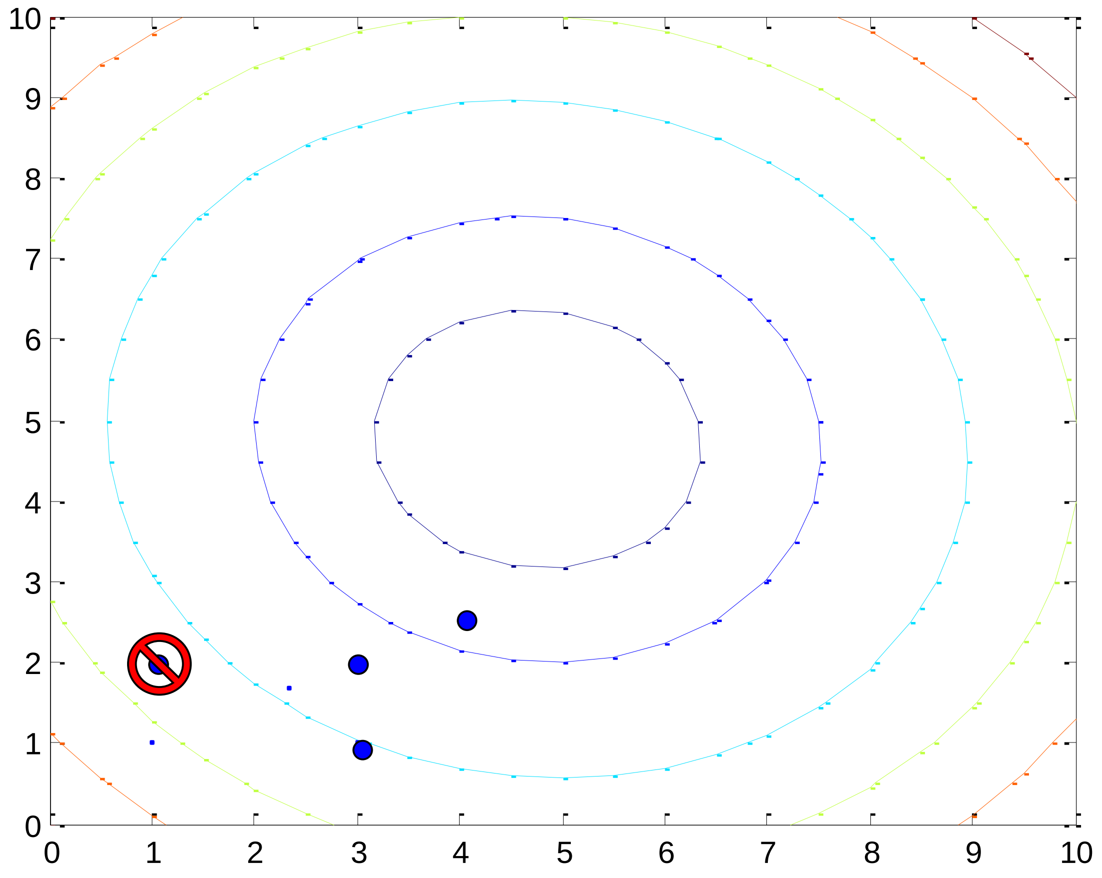
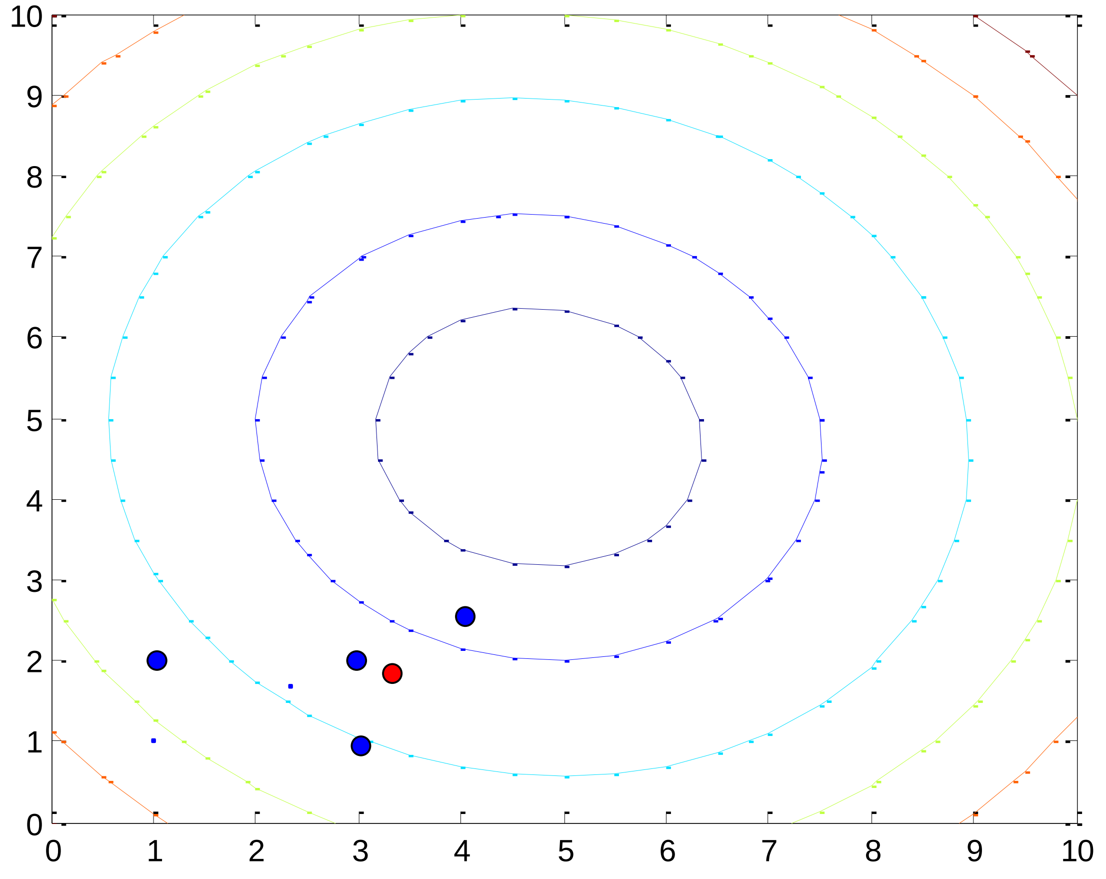
Procedure of the Complex method



Identify the worst point

x = [1, 2] is the worst

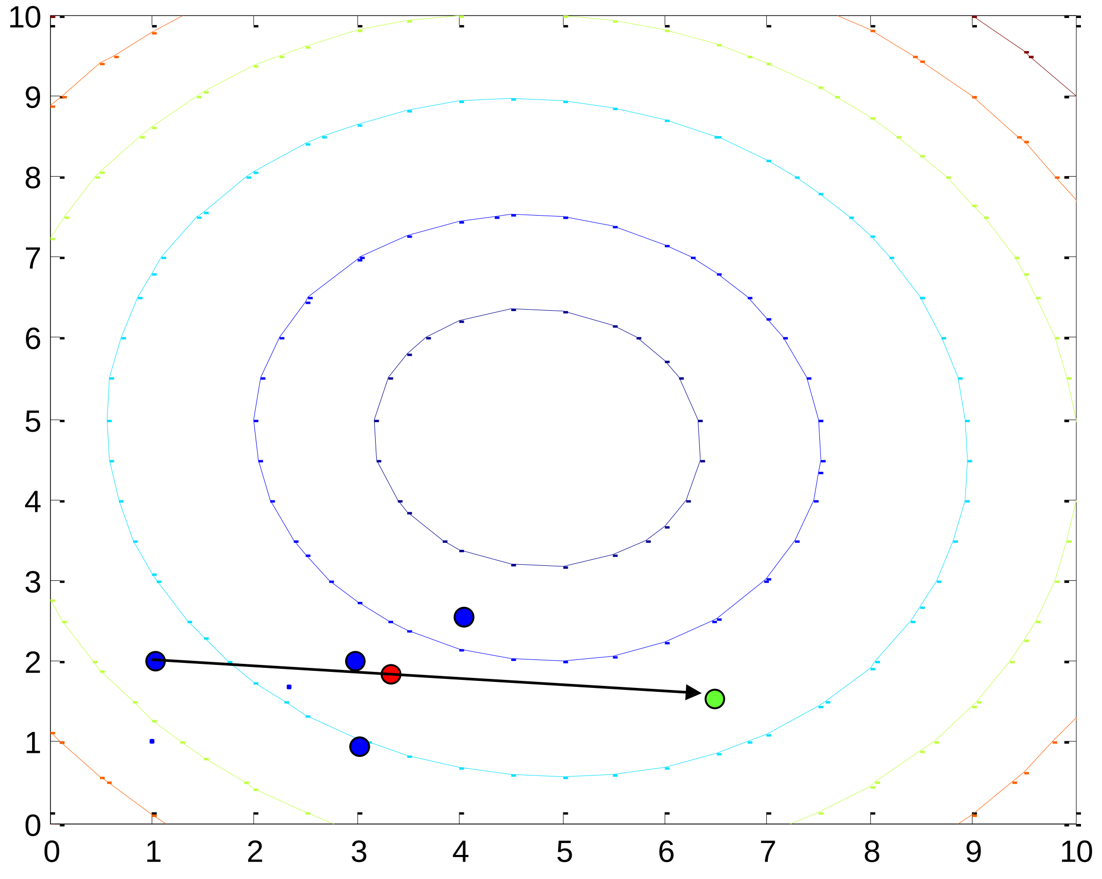
Procedure of the Complex method



Calculate the centroid

$\mathbf{x}_c = [3.36, 1.84]$

Procedure of the Complex method

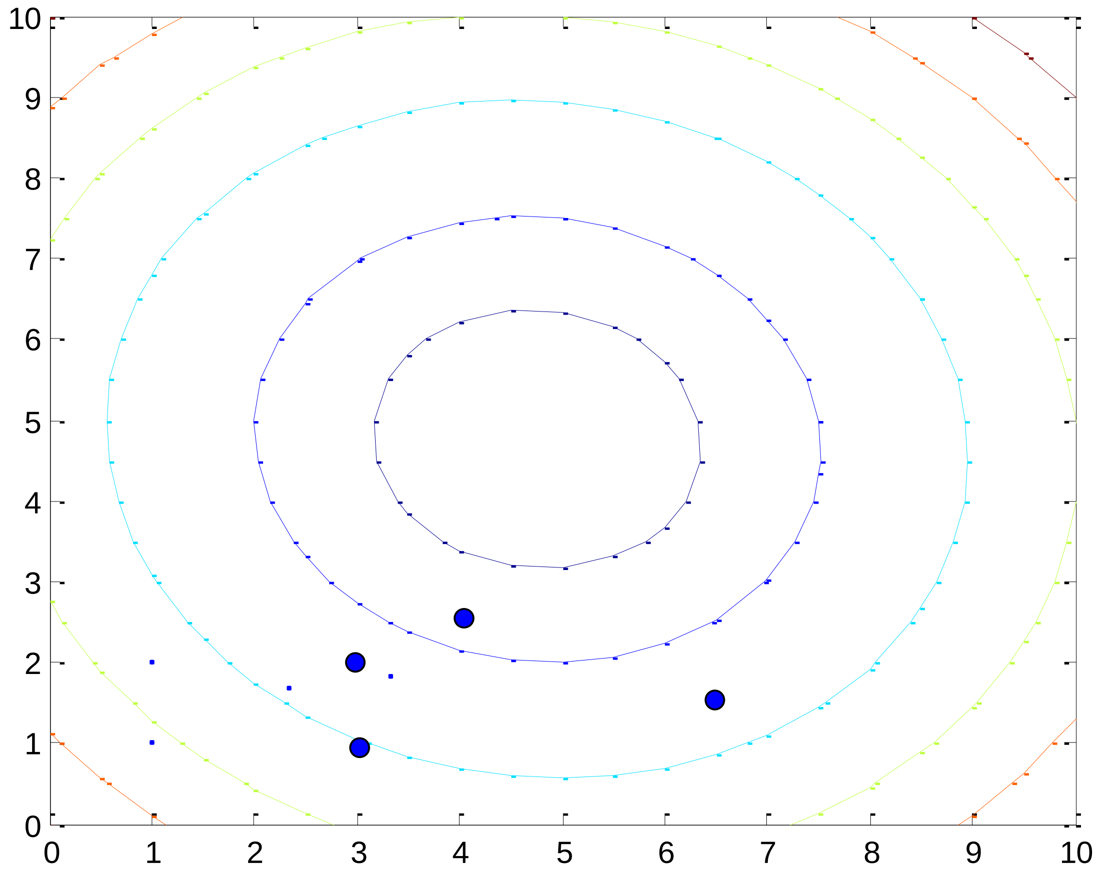


Calculate the new point

$\mathbf{x}_{new}$ = [6.42, 1.63]

f_{new}= 14.41

Procedure of the Complex method



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MATLAB Commands

```
>>x_low=[0 0];
>>x_up=[10 10];

>>[x,f,x_hist,f_hist]=complexrf('test1',x_low,x_up);

>>plot(x_hist(:,1))
>>figure
>>plot(x_hist(:,2))
>>figure
>>plot(f_hist(:,1))
>>cmplx_mov(x_hist,4,1,2)
```

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Questions?