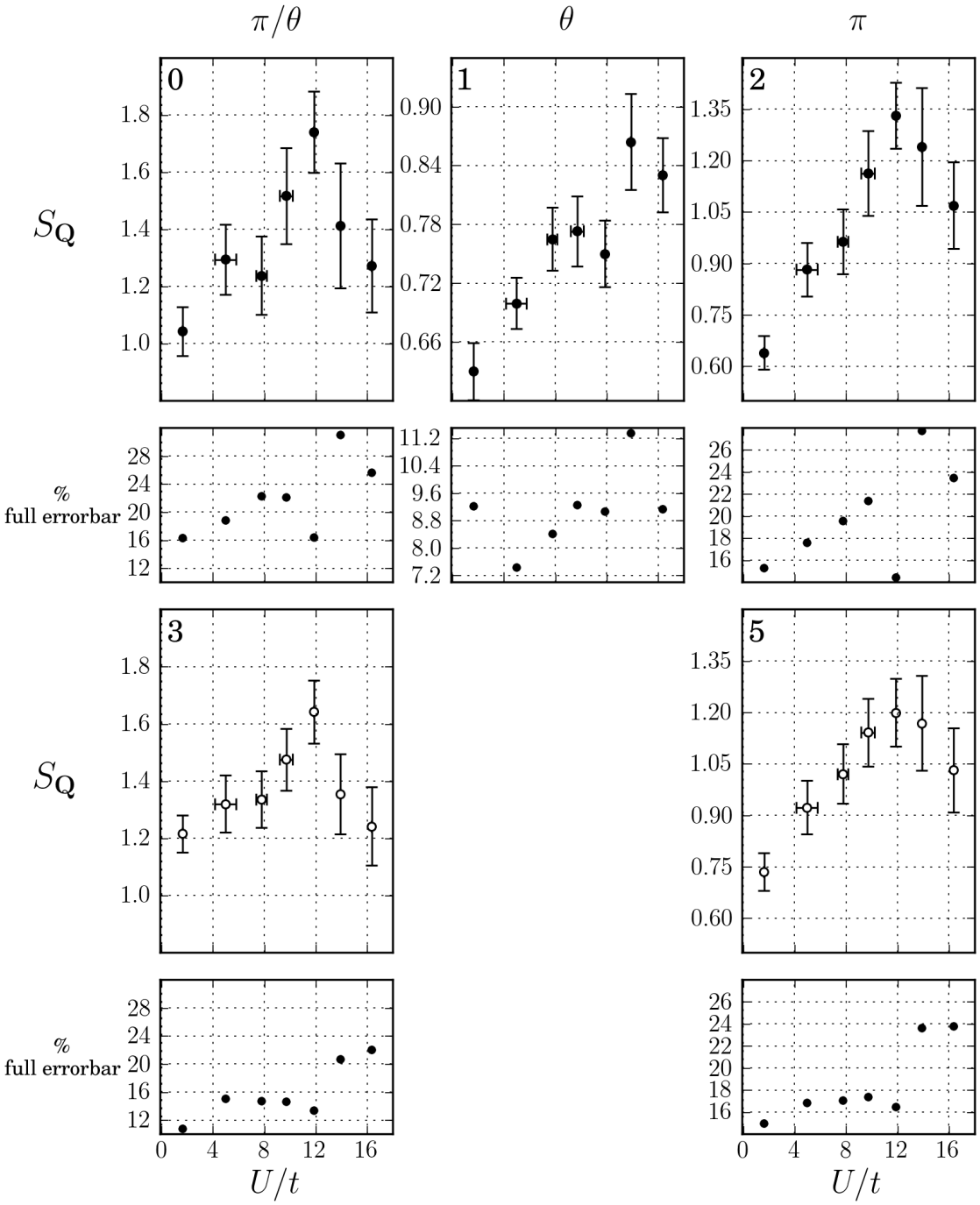

2.5 Comments

The first row and the third row of the plot shown above involve two different ways to get to the structure factor from our data. They are pretty close to each other within error bars but not fully consistent. We think that the most direct way to get to $S_{\boldsymbol{\pi}}$ is in the first row. The most direct way to get to the ratio $S_{\boldsymbol{\pi}}/S_{\boldsymbol{\theta}}$ is in the third row, since it uses the calibrated ratio $f_{(\boldsymbol{\pi}/\boldsymbol{\theta})_{\mathrm{MA}}}$ which only depends on the solid angle and responsivity of the cameras.

3 Sample inhomogeneity

The Quantum Monte Carlo (QMC) calculations done by the theorists in our collaboration are for systems with uniform filling, and at a fixed value of the interaction strength U/t. Our sample is far from these uniform conditions. Since it is a trapped finite sample, the filling (density) goes all the way down to zero. Typically, optical lattices are realized with laser beams that have large beam waists, so the parameters U and t have a negligible variation over the size of the sample. In our experiment we used a beam waist ($\approx 45\,\mu$m) which is comparable to the size of the sample ($1/e$ radius of $\approx 20\,\mu$m). For this reason we have a strong variation of U and t over the size of the sample. In our system, as you move from the center to the outside of the atom cloud the lattice gets shallower, which results in reduced U and increased t. The Hubbard interaction strength U/t thus suffers from both numerator and denominator which results in a significant variation of U/t over the cloud.

The motivation for reducing U and increasing t in length scales comparable with the sample is such that there may be entropy redistribution to the edges of the cloud. Near the edges the system has a larger entropy capacity due to the lower filling there. Additionally, since towards the edge the lattice is shallower, elastic collisions may lead to evaporation of the particles which may resulting in cooling of the system.

To incorporate the effects of inhomogeneity in the comparison between QMC and our measurements we will divide our sample into five bins, each with equal number of particles and then get the values of the filling n and of U/t for each of the bins. QMC calculations of the structure factor can be performed for each bin and then averaged before comparing with the experimental results.

To calculate the average value of n and U/t in each bin we need knowledge of the density profile of our cloud, as well as knowledge of our optical lattice potential. The potential can be calculated from the calibrated beam waists and power of our optical lattice beams.

For the density profile we can perform in-situ phase-contrast images of the atom cloud. Alternatively, with our known atom number and the calculated potential we can use a thermodynamic model of the system to calculate the expected density distribution. The thermodynamic model we use is the high temperature series expansion up to second order in T/t, which can be calculated readily and is accurate down to about $T/t \approx 1.8$. To avoid complications with the repeatability in the realization of a certain density distribution we choose this second alternative. We feed into the model the atom number measured in the experiment. As a sanity check we can plot a particular realization of the density distribution and compare it with the calculation from the thermodynamic model. We can check if the peak density, double occupancy, and cloud size roughly agree with the model.

In Fig. 1 we show a plot of the in-situ density distribution measured using phase-contrast imaging in a $5.5\,E_R$ lattice with $3.0\,E_R$ of green compensation. It can be seen that the distribution is not spherically symmetric and that the atoms are pushed to the left of the picture. The $1/e$ sizes are 15.1 μm and 23.8 μm. The number of atoms in the cloud is 360,000. The density at the peak is estimated from the number and the cloud sizes to be 6.53e12 cm^{-3} which corresponds to 0.98 atoms per site. The image shown was taken at a scattering length of 326 a_0.

If we put 360,000 atoms into the model we obtain the results shown in Fig. 2 for various values of the green compensation. The peak density and double occupancy depend sensitively on the green compensation. In order to calculate the density profiles that will be used to estimate the effect of inhomogeneity on the spin structure factor we have to pick a value for the green compensation. The value that we use in the experiment is $3\,E_R$, but our observations of the peak density and the double occupancy as a function of scattering length do not match the model at $3\,E_R$ compensation. Our measurement of the double occupancy in the $5.5\,E_R$ lattice is shown in Fig. 3 and it resembles the curve with a compensation of $2.6\,E_R$ in the model (the x axis in both plots is in different units but the points

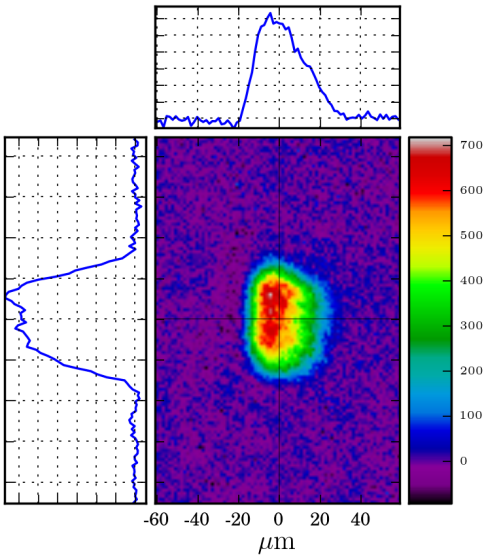

Figure 1: In-situ density distribution in a $5.5\,E_R$ lattice with $3\,E_R$ of green compensation.

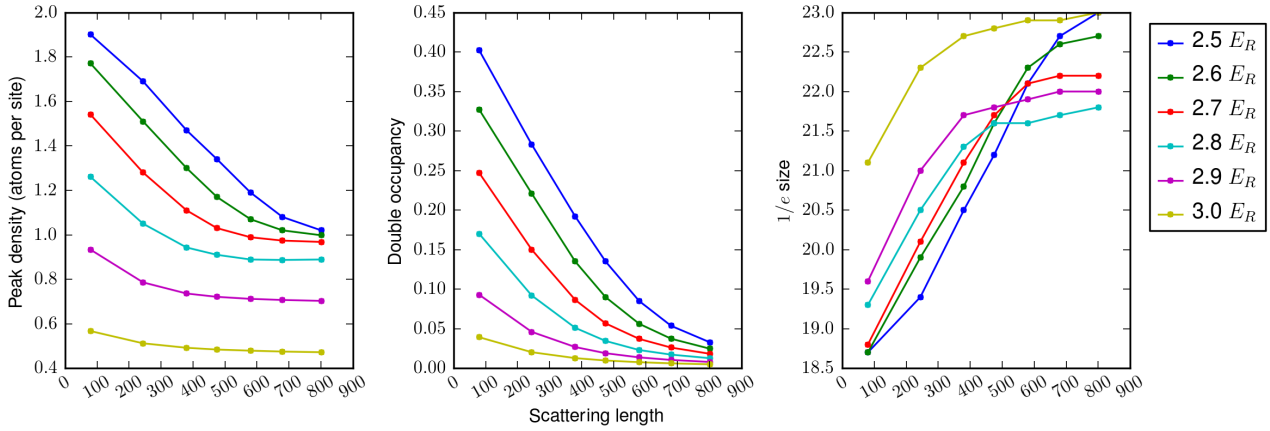

Figure 2: System parameters as a function of scattering length for various values of the green compensation. The model used is the high temperature series expansion.

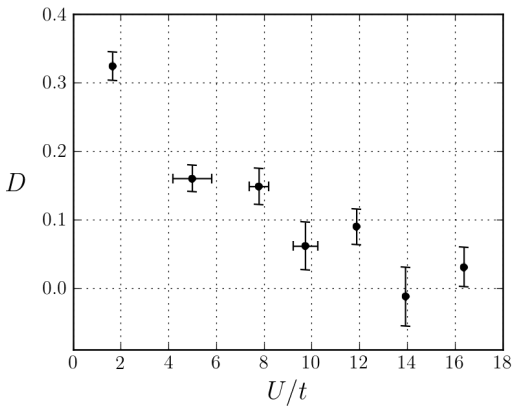

Figure 3: Double occupancy in the $5.5\,E_R$ lattice.

shown correspond between the two plots). Our measurement of the density at 326 a_0 is more consistent with a compensation of $2.8\,E_R$ in the model. The discrepancy between our measurements and the model is most likely due to calibration errors and our inexact knowledge of the potential, however since the temperature used for the model is $T/t = 1.8$ it is possible that the discrepancy is also due to a lower temperature in our system than the one this model can handle. In what follows we will use $2.7\,E_R$ in the model to split the difference between the peak density and double occupancy observation and stay somewhat consistent with our observations.

With all the parameters that go into the model in hand, we can go ahead and calculate density distributions. We then split those density distributions into bins with an equal number of atoms and calculate the average filling and U/t for each bin. An example of this calculation for a scattering length of 580 a_0, which corresponds to $[U/t]_0 = 11.9$ is shown in Fig. 4. $[U/t]_0$ stands for U/t at the center, which is the value that we have been using as the x axis in all our structure factor plots so far.

Below we proceed to tabulate the binned filling and U/t for the values of $[U/t]_0$ that correspond to the data that we have taken in the $5.5\,E_R$ lattice

[U/t]_0 = 1.636		
bin#	n	U/t
000	1.24	1.43
001	0.85	1.17
002	0.50	0.95
003	0.16	0.68
004	0.02	0.29

[U/t]_0 = 4.976		
bin#	n	U/t
000	1.04	4.26
001	0.71	3.41
002	0.43	2.75
003	0.14	1.92
004	0.02	0.78

[U/t]_0 = 7.771		
bin#	n	U/t
000	0.93	6.56
001	0.65	5.19
002	0.39	4.12
003	0.12	2.78
004	0.02	1.08

[U/t]_0 = 9.713		
bin#	n	U/t
000	0.89	8.19
001	0.63	6.41
002	0.37	5.01
003	0.11	3.36
004	0.01	1.30

[U/t]_0 = 11.86		
bin#	n	U/t
000	0.86	9.89
001	0.61	7.75
002	0.37	6.12
003	0.12	4.10
004	0.01	1.58

[U/t]_0 = 13.905		
bin#	n	U/t
000	0.85	11.60
001	0.59	8.98
002	0.36	7.07
003	0.12	4.76
004	0.01	1.78

[U/t]_0 = 16.359		
bin#	n	U/t
000	0.84	13.66
001	0.59	10.57
002	0.36	8.32
003	0.12	5.60
004	0.01	2.10

To make matters simpler, and since this treatment is only a crude estimation we create a single table averaging the seven tables above, such that we obtain values for the filling and percentage of $[U/t]_0$ for

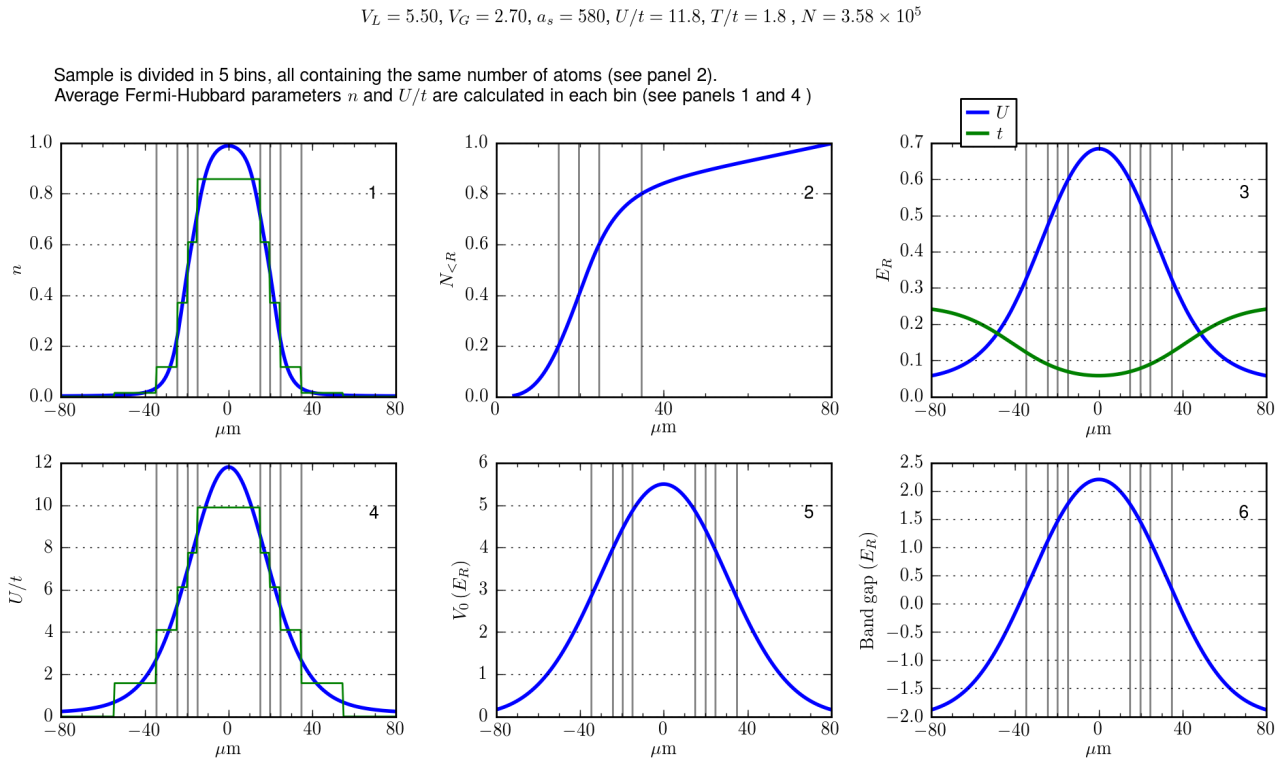

Figure 4: Binned filling and U/t are shown in the left column, panels 1 and 4. The average value in each bin is shown in green. The cumulative number up to a certain radius is shown in panel 2, this illustrates how all the bins have the same number of atoms. The radial dependence of other relevant parameters are shown in panels 3, 5, and 6.

a general case. This table is what we suggest should be used to estimate the effect of inhomogeneity of our sample on the structure factor. The column labeled n denotes the fillings for each of the five bins, and the last seven columns denote the value of U/t in each of the five bins for the seven values of $[U/t]_0$ that we have in our data.

bin#	n	%U/t	1.6	5.0	7.8	9.7	11.9	13.9	16.4
000	0.95	85	1.4	4.2	6.6	8.2	10.0	11.8	13.8
001	0.66	67	1.1	3.3	5.2	6.5	7.9	9.3	10.9
002	0.40	53	0.9	2.6	4.1	5.2	6.3	7.4	8.7
003	0.13	36	0.6	1.8	2.8	3.5	4.3	5.0	5.9
004	0.02	14	0.2	0.7	1.1	1.4	1.7	2.0	2.3

3.1 Raw data for S_π/S_θ and S_π

Below we show the raw data for the best estimate of S_π/S_θ which uses the simultaneous measurement of both cameras with a calibrated normalization. This is panel 3 on the figure with the data.

U/t	Spi/Sth	error
1.6	1.21	0.065
5.0	1.32	0.099
7.8	1.34	0.098
9.7	1.48	0.108
11.9	1.64	0.110
13.9	1.35	0.140
16.4	1.24	0.136

Below we show $S_{\boldsymbol{\pi}}$ determined from $S_{\boldsymbol{\pi}}/S_{\boldsymbol{\theta}}$ and $S_{\boldsymbol{\theta}}$, this is panel 5 on the figure with the data .

U/t	Spi	error
1.6	0.735	0.055
5.0	0.923	0.078
7.8	1.021	0.087
9.7	1.141	0.099
11.9	1.199	0.099
13.9	1.168	0.138
16.4	1.031	0.123

Below we show $S_{\boldsymbol{\pi}}$ determined using only the camera at $\boldsymbol{\pi}$, this is panel 2 on the figure with the data.

U/t	Spi	error
1.6	0.639	0.049
5.0	0.882	0.078
7.8	0.963	0.094
9.7	1.162	0.124
11.9	1.331	0.096
13.9	1.240	0.172
16.4	1.069	0.125