1 Dipole potentials for lithium

The depth (in μK) for a 1064 nm beam of power P (in mW) and beam waist w (in μm) is given by

$$U = u\frac{P}{w^2} \tag{1}$$

where

$$u = 38.709 \tag{2}$$

The recoil energy for a 1064 nm photon is E_R =1.4 μK.

2 Calibration of lattice depth

To calibrate the lattice depth we perform lattice phase modulation spectroscopy. We do so by modulating the frequency of the lattice AOM. This has the effect of shaking the lattice wells back and forth. The lattice depth is given by

$$V_0 = \frac{4u}{E_R}\frac{\sqrt{P_\mathrm{I}P_\mathrm{R}}}{w_\mathrm{I}w_\mathrm{R}} \tag{3}$$

where the subscripts I, R stand for input and retro respectively. We define the retro factor r as $P_\mathrm{R} = rP_\mathrm{I}$, so we have

$$V_0 = \frac{4uP_\mathrm{I}}{E_R}\frac{\sqrt{r}}{w_\mathrm{I}w_\mathrm{R}} \tag{4}$$

3 Measurement of radial frequency in lattice configuration

Lithium mass: The radial frequency is determined by the potential energy profile and the mass of lithium. Below we devise a convenient unit for the lithium mass:

$$m = 6\,\mathrm{AMU} = \frac{6h}{0.4\,\mu\mathrm{m}^2} = \frac{6\times 48\,\mu\mathrm{K/MHz}}{0.4\,\mu\mathrm{m}^2} = 7.2\text{e-}4\,\frac{\mu\mathrm{K}}{\mu\mathrm{m}^2\ \mathrm{kHz}} \tag{5}$$

With the rotators in lattice configuration we can make a radial frequency measurement. The square of this radial frequency is given by

$$\nu_{Lr}^2 = \frac{u}{m\pi^2}\left(\frac{P_\mathrm{R}}{w_\mathrm{R}^4} + \frac{\sqrt{P_\mathrm{R}P_\mathrm{I}}}{w_\mathrm{I}w_\mathrm{R}^3} + \frac{P_\mathrm{I}}{w_\mathrm{I}^4} + \frac{\sqrt{P_\mathrm{R}P_\mathrm{I}}}{w_\mathrm{R}w_\mathrm{I}^3}\right) \tag{6}$$

Using the retro factor, we have

$$\nu_{Lr}^2 = \frac{uP_\mathrm{I}}{m\pi^2}\left(\frac{r}{w_\mathrm{R}^4} + \frac{\sqrt{r}}{w_\mathrm{I}w_\mathrm{R}^3} + \frac{1}{w_\mathrm{I}^4} + \frac{\sqrt{r}}{w_\mathrm{R}w_\mathrm{I}^3}\right) \tag{7}$$

4 Measurement of radial frequency in dimple configuration

With the rotators in dimple configuration we can make a radial frequency measurement. The square of this radial frequency is given by

$$\nu_{Dr}^2 = \frac{uP_\mathrm{I}}{m\pi^2}\left(\frac{1}{w_\mathrm{I}^4} + \frac{r}{w_\mathrm{R}^4}\right) \tag{8}$$

5 Combining all the above

Using the results from the sections above we have

$$\begin{aligned}
\frac{V_0}{P_{\mathrm{I}}}\frac{E_R}{4u} &= \frac{\sqrt{r}}{w_{\mathrm{I}}w_{\mathrm{R}}} \\
\frac{\nu_{Lr}^2}{P_{\mathrm{I}}}\frac{m\pi^2}{u} &= \left(\frac{r}{w_{\mathrm{R}}^4} + \frac{\sqrt{r}}{w_{\mathrm{I}}w_{\mathrm{R}}^3} + \frac{1}{w_{\mathrm{I}}^4} + \frac{\sqrt{r}}{w_{\mathrm{R}}w_{\mathrm{I}}^3}\right) \\
\frac{\nu_{Dr}^2}{P_{\mathrm{I}}}\frac{m\pi^2}{u} &= \left(\frac{1}{w_{\mathrm{I}}^4} + \frac{r}{w_{\mathrm{R}}^4}\right)
\end{aligned} \tag{9}$$

The numerical factors that show up are

$$\begin{aligned}
\frac{E_R}{4u} &= 9.04\text{e-}3 \equiv x_1 \\
\frac{m\pi^2}{u} &= 1.84\text{e-}4 \equiv x_2
\end{aligned} \tag{10}$$

In the experiment we measure the lattice depth, the trap frequencies, and the beam power, so the equations above are written as

$$\begin{aligned}
\frac{P_{\mathrm{I}}}{V_0}\left[\frac{\mathrm{mW}}{E_R}\right] &= x_1\frac{w_{\mathrm{I}}w_{\mathrm{R}}}{\sqrt{r}} \\
\frac{P_{\mathrm{I}}}{\nu_{Lr}^2}\left[\frac{\mathrm{mW}}{\mathrm{kHz}^2}\right] &= \frac{x_2}{\left(\frac{r}{w_{\mathrm{R}}^4} + \frac{\sqrt{r}}{w_{\mathrm{I}}w_{\mathrm{R}}^3} + \frac{1}{w_{\mathrm{I}}^4} + \frac{\sqrt{r}}{w_{\mathrm{R}}w_{\mathrm{I}}^3}\right)} \\
\frac{P_{\mathrm{I}}}{\nu_{Dr}^2}\left[\frac{\mathrm{mW}}{\mathrm{kHz}^2}\right] &= \frac{x_2}{\left(\frac{1}{w_{\mathrm{I}}^4} + \frac{r}{w_{\mathrm{R}}^4}\right)}
\end{aligned} \tag{11}$$