1 Dipole potential and scattering rate

The dipole potential [1] produced by far-detuned light of frequency ω_L on a two-level atom is given by

$$U_{\text{dip}}(\boldsymbol{r}) = -\frac{3\pi c^2}{2\omega_0^3}\Gamma\left(\frac{1}{\omega_0-\omega_L}+\frac{1}{\omega_0+\omega_L}\right)I(\boldsymbol{r}) \tag{1}$$

The associated photon scattering rate is

$$\Gamma_{\text{sc}} = \frac{3\pi c^2}{2\hbar\omega_0^3}\Gamma^2\left(\frac{\omega_L}{\omega_0}\right)^3\left(\frac{1}{\omega_0-\omega_L}+\frac{1}{\omega_0+\omega_L}\right)^2 I(\boldsymbol{r}) \tag{2}$$

The beams that we use in our lattice have a Gaussian cross section such that the intensity (of a single beam) is given by

$$I(\boldsymbol{r}) = \frac{2P}{\pi w^2}\exp\left[-2\frac{r_\perp^2}{w^2}\right] \tag{3}$$

where $r_\perp$ is the perpendicular distance from the beam axis and we neglect the small variation of the intensity along the beam axis.

We will calculate the heating rate due to spontaneous emission at the center of the potential, where the intensity of all the beams is the largest. We define the constnats u_L and h_L according to

$$\begin{aligned} k_{\text{B}}u_L &= \frac{3\pi c^2}{2\omega_0^3}\Gamma\left(\frac{1}{\omega_0-\omega_L}+\frac{1}{\omega_0+\omega_L}\right) \\ h_L &= \frac{3\pi c^2}{2\hbar\omega_0^3}\Gamma^2\left(\frac{\omega_L}{\omega_0}\right)^3\left(\frac{1}{\omega_0-\omega_L}+\frac{1}{\omega_0+\omega_L}\right)^2 \end{aligned} \tag{4}$$

Which allows writing the dipole potential and the scattering rate simply as

$$\begin{aligned} U_0/k_{\text{B}} &= u_L I_0 \\ \Gamma_{\text{sc}0} &= h_L I_0 \end{aligned} \tag{5}$$

In our experiments we use lasers at 1064 nm and 532 nm, the values of u_L and h_L for this two wavelengths are given in Table 1 towards the end of this document.

2 Compensated lattice potential: definitions and simplified expressions

An optical lattice potential results due to the stationary interference pattern of two or more laser beams. The most common configuration, and the one relevant to our work, consists on a laser beam that is retroreflected upon itself, which produces a standing wave with nodes separated by half the wavelength of the light. Ideally the retroreflected path would have the same power and beam waist (at the position of the atoms) as the input path, however in practice this is not the case. The light for the lattice is brought to the apparatus using an optical fiber; the retro beam alignment process consists of maximizing the light that goes back trough this fiber. The alignment process guarantees that (within ∼5%) the input and retro beam waists will be the same. In this treatment we will initially consider different input and retro waists, but in the end we will set them to be equal. On the other hand, the retro reflected power can be significantly lower due to losses on the retro path, so we do consider a different power for the input and retro beams throughout.

We define a factor, R, which characterizes the lossses of the retro path, such that the power of the retro beam is $P_{\rm r} = P_{\rm i}R$, where r,i stand for retro and input respectively. Below we tabulate the beam waists and retro factors for the three axes of our simple cubic lattice potential:

	LATTICE	BEAM 1	BEAM 2	BEAM 3
w	beam waist	48.3	45.9	41.3
R	retro factor	0.93	0.77	0.68

The calibration method used to obtain these values will be presented in a later section.

In our setup we have the possibility of changing the polarization of the retro beam. We use a liquid crystal retarder, which allows us to accurately set the polarization of the retro beam parallel or perpendicular to that of the input beam. In between these two values we can continuously set the fraction of retro power that has the same polarization of the input beam, and this allows us to smoothly change the potential from a dimple (minimize retro power at input polarization) to a lattice (maximize retro power at input polarization). We will use the letter α to refer to the fraction of power in the retro beam that has the same polairzation as the input beam, and thus can interfere with it to form the lattice.

2.1 Electric field

The electric field on the axis of our lattice (input propagating along $+z$) can be determined as a sum of the input and retro electric fields. We independently treat the interfering ($\|$) and non-interfering ($\perp$) fields.

$$\sqrt{\frac{2}{\epsilon_0 c}} E_{1D\|}(x,y,z) = A e^{ikz} + B e^{-ikz+\phi_{\|}(\alpha)}$$
$$\sqrt{\frac{2}{\epsilon_0 c}} E_{1D\perp}(x,y,z) = C e^{-ikz+\phi_{\perp}(\alpha)}$$

where

$$\begin{aligned} A &= \sqrt{\frac{2P_{\rm i}}{\pi w_{\rm i}^2}} \exp\left[-\frac{x^2+y^2}{w_{\rm i}^2}\right] \\ B &= \sqrt{\frac{2P_{\rm i}R\alpha}{\pi w_{\rm r}^2}} \exp\left[-\frac{x^2+y^2}{w_{\rm r}^2}\right] \\ C &= \sqrt{\frac{2P_{\rm i}R(1-\alpha)}{\pi w_{\rm r}^2}} \exp\left[-\frac{x^2+y^2}{w_{\rm r}^2}\right] \end{aligned} \tag{6}$$

Notice that the field can have phase shifts that depend on α. We have not characterized those phase shift for our retarder setup. We will focus only on situations where $\alpha = 0$ or 1, so that we can neglect them.

2.2 Lattice depth

To obtain the lattice depth we need to set $\alpha = 1$ and look at the oscillatory part of the interfering intensity $(\epsilon_0 c/2)|E_{1D\|}|^2$. The lattice depth will have a gaussian transverse profile as a function of x, y.

To get the lattice depth at the axis of the beam we set $x = 0, y = 0$ and obtain

$$(\epsilon_0 c/2)|E_{1D\|}|^2 = I_{1D\|}(z) = \frac{8}{\pi}\sqrt{\frac{P_{\mathrm{i}}^2 R}{w_{\mathrm{in}}^2 w_{\mathrm{r}}^2}}\cos^2(kz) - \frac{4}{\pi}\sqrt{\frac{P_{\mathrm{i}}^2 R}{w_{\mathrm{in}}^2 w_{\mathrm{r}}^2}} + \frac{2P_{\mathrm{i}}}{\pi w_{\mathrm{in}}^2} + \frac{2P_{\mathrm{i}}R}{\pi w_{\mathrm{r}}^2} \tag{7}$$

The lattice depth is the dipole potential produced by the oscillatory part of $I_{1D\|}$

$$s = \frac{V_{\mathrm{latt}}}{E_{R,\mathrm{ir}}} = \frac{k_{\mathrm{B}}|u_{\mathrm{ir}}|}{E_{R,\mathrm{ir}}}\frac{8}{\pi}\frac{P_{\mathrm{i}}\sqrt{R}}{w_{\mathrm{in}}w_{\mathrm{r}}} \tag{8}$$

Assuming equal beam waists for input and retro this becomes

$$s = \frac{k_{\mathrm{B}}|u_{\mathrm{ir}}|}{E_{R,\mathrm{ir}}}\frac{8}{\pi}\frac{P_{\mathrm{i}}\sqrt{R}}{w^2} \tag{9}$$

2.3 Lattice and dimple radial frequencies

To find the lattice and dimple radial frequencies we have to look at the full intensity

$$I_{1D}(r) = (\epsilon_0 c/2)(|E_{1D\|}|^2 + |E_{1D\perp}|^2)$$

and set $z = 0$ and $x^2 + y^2 = r^2$.

2.3.1 Lattice $\alpha = 1$

$$I_{1D}(r) = \frac{4}{\pi}\frac{P_{\mathrm{i}}\sqrt{R}}{w_{\mathrm{in}}w_{\mathrm{r}}}\exp\left[-\frac{r^2}{w_{\mathrm{in}}^2} - \frac{r^2}{w_{\mathrm{r}}^2}\right] + \frac{2P_{\mathrm{i}}}{\pi w_{\mathrm{in}}^2}\exp\left[-2\frac{r^2}{w_{\mathrm{in}}^2}\right] + \frac{2P_{\mathrm{i}}R}{\pi w_{\mathrm{r}}^2}\exp\left[-2\frac{r^2}{w_{\mathrm{r}}^2}\right] \tag{10}$$

Expanding the exponentials around $r = 0$ gives

$$I_{1D}(r) \approx I_{1D}(0) - \frac{1}{2}\left[\frac{8}{\pi}\frac{P_{\mathrm{i}}\sqrt{R}}{w_{\mathrm{in}}w_{\mathrm{r}}}\left(\frac{1}{w_{\mathrm{in}}^2} + \frac{1}{w_{\mathrm{r}}^2}\right) + \frac{2P_{\mathrm{i}}}{\pi w_{\mathrm{in}}^2}\frac{4}{w_{\mathrm{in}}^2} + \frac{2P_{\mathrm{i}}R}{\pi w_{\mathrm{r}}^2}\frac{4}{w_{\mathrm{r}}^2}\right] r^2 \tag{11}$$

Recall that $U(\boldsymbol{r}) = u_L k_{\mathrm{B}} I(\boldsymbol{r})$. The radial frequency is

$$\nu_{\mathrm{latt}}^2 = \frac{|u_{\mathrm{ir}}|k_{\mathrm{B}}}{m\pi^2}P_{\mathrm{i}}\left[\frac{2}{\pi}\frac{\sqrt{R}}{w_{\mathrm{in}}w_{\mathrm{r}}}\left(\frac{1}{w_{\mathrm{in}}^2} + \frac{1}{w_{\mathrm{r}}^2}\right) + \frac{2}{\pi w_{\mathrm{in}}^4} + \frac{2R}{\pi w_{\mathrm{r}}^4}\right] \tag{12}$$

which for equal beam waists becomes

$$\nu_{\mathrm{latt}}^2 = \frac{|u_{\mathrm{ir}}|k_{\mathrm{B}}}{m\pi^2}P_{\mathrm{i}}\left[\frac{2 + 4\sqrt{R} + 2R}{\pi w^4}\right] \tag{13}$$

2.3.2 Dimple $\alpha = 0$

$$I_{1D}(r) = \frac{2P_{\mathrm{i}}}{\pi w_{\mathrm{in}}^2}\exp\left[-2\frac{r^2}{w_{\mathrm{in}}^2}\right] + \frac{2P_{\mathrm{i}}R}{\pi w_{\mathrm{r}}^2}\exp\left[-2\frac{r^2}{w_{\mathrm{r}}^2}\right] \tag{14}$$

$$I_{1D}(r) \approx I_{1D}(0) - \frac{1}{2}\left[\frac{2P_{\rm i}}{\pi w_{\rm in}^2}\frac{4}{w_{\rm in}^2} + \frac{2P_{\rm i}R}{\pi w_{\rm r}^2}\frac{4}{w_{\rm r}^2}\right] r^2 \tag{15}$$

$$\nu_{\rm dimp}^2 = \frac{|u_{\rm ir}|k_{\rm B}}{m\pi^2}P_{\rm i}\left[\frac{2}{\pi w_{\rm in}^4} + \frac{2R}{\pi w_{\rm r}^4}\right] \tag{16}$$

For equal beam waists

$$\nu_{\rm dimp}^2 = \frac{|u_{\rm ir}|k_{\rm B}}{m\pi^2}P_{\rm i}\left[\frac{2+2R}{\pi w^4}\right] \tag{17}$$

Lithium mass. For the mass of lithium which appears in the expressions for the radial frequency we use the convenient expresssion

$$\frac{m}{k_{\rm B}} = 6\,\frac{\rm AMU}{k_{\rm B}} = 6\frac{h/k_{\rm B}}{0.4\,\mu{\rm m}^2\,{\rm MHz}} = \frac{6(48\,\mu{\rm K/MHz})}{0.4\mu{\rm m}^2\,{\rm MHz}} = 7.2\text{e-}4\frac{\mu{\rm K}}{\mu{\rm m}^2\,{\rm kHz}^2} \tag{18}$$

A numerical value for $\frac{|u_L|k_{\rm B}}{m\pi^2}$ is listed in Table 1.

2.4 Compensation beams

Overlapped on each of our lattice axes we have a repulsive compensation beam at a wavelength of 532 nm. This is a single gaussian beam with a potential given by

$$U_{c1D}(x,y) = u_{\rm gr}k_{\rm B}\frac{2P_{\rm gr}}{\pi w_{\rm gr}^2}\exp\left[-2\frac{r^2}{w_{\rm gr}^2}\right] \tag{19}$$

and radial frequency

$$\nu_{\rm gr}^2 = \frac{|u_{\rm gr}|k_{\rm B}}{m\pi^2}\frac{2P_{\rm gr}}{\pi w_{\rm gr}^4} \tag{20}$$

The beam waist for each of the three beams is shown below

	COMPENSATION	BEAM 1	BEAM 2	BEAM 3
w	beam waist	42.9	41.4	40.4

3 Calibration of the compensated lattice

This section is not written yet.

4 Fermi temperature for a compensated dimple

In our experiment we specify the powers of the IR beams by the lattice depth that they would generate if α were equal to 1, that is

$$P_{\rm i} = s\frac{E_{R,{\rm ir}}}{k_{\rm B}|u_{\rm ir}|}\frac{\pi w_{\rm ir}^2}{8\sqrt{R}} \tag{21}$$

$$\nu_{\rm ir,dimp}^2 = s\frac{E_{R,{\rm ir}}}{m\pi^2}\frac{1+R}{4w_{\rm ir}^2\sqrt{R}} \tag{22}$$

$$\nu_{\mathrm{ir,latt}}^2 = s\frac{E_{R,\mathrm{ir}}}{m\pi^2}\frac{1+R+\sqrt{R}}{2w_{\mathrm{ir}}^2\sqrt{R}} \tag{23}$$

A value for $\frac{E_{R,\mathrm{ir}}}{m\pi^2}$ can be found in Table. 1.

For the green beams we specify the depth of the potential produced by each beam in units of the IR recoil, we call this quantity g.

$$P_{\mathrm{gr}} = g\frac{E_{R,\mathrm{ir}}}{k_{\mathrm{B}}|u_{\mathrm{gr}}|}\frac{\pi w_{\mathrm{gr}}^2}{2} \tag{24}$$

$$\nu_{\mathrm{gr}}^2 = g\frac{E_{R,\mathrm{ir}}}{m\pi^2}\frac{1}{w_{\mathrm{gr}}^2} \tag{25}$$

Along each axis we have that the radial frequency of the compensated dimple potential is

$$\begin{aligned}\nu_{\mathrm{comp}}^2 &= \nu_{\mathrm{ir}}^2 - \nu_{\mathrm{gr}}^2 \\ &= \frac{E_{R,L}}{m\pi^2}(s\varphi_{\mathrm{dimp}} - g\varphi_{\mathrm{gr}})\end{aligned} \tag{26}$$

where we have defined

$$\varphi_{\mathrm{latt}} = \frac{1+R+\sqrt{R}}{2w_{\mathrm{ir}}^2\sqrt{R}} \qquad \varphi_{\mathrm{dimp}} = \frac{1+R}{4w_{\mathrm{ir}}^2\sqrt{R}} \qquad \varphi_{\mathrm{gr}} = \frac{1}{w_{\mathrm{gr}}^2} \tag{27}$$

For each of the three axes we have

	AXIS 1	AXIS 2	AXIS 3	units
φ_{dimp}	2.14e-4	2.39e-4	2.99e-4	$\mu\mathrm{m}^{-2}$
φ_{latt}	6.433e-4	7.160e-4	8.903e-4	$\mu\mathrm{m}^{-2}$
φ_{gr}	5.434e-4	5.834 e-4	6.127 e-4	$\mu\mathrm{m}^{-2}$

In order to produce spherically symmetric samples we set s for all three beams to the same (such that if α were equal to 1 the lattice depths would be the same along all three directions) and we adjust the green powers of beams 1 and 2 to match the compensated radial frequency of beam 3. To calculate the Fermi temperature we will set the radial frequencies for beams 1 and 2 equal to that of beam 3, which is given by

$$\nu_{\mathrm{comp},3}^2 = \frac{E_{R,\mathrm{ir}}}{m\pi^2}(s\varphi_{\mathrm{dimp},3} - g\varphi_{\mathrm{gr},3}) \tag{28}$$

When the dimple beams in all three axis are turned on at the same time, the squares of the trap frequencies add up. Beams 1, 2, 3 propagate along x, y, and z respectively. And so we have

$$\begin{aligned}\nu_{\mathrm{comp},x}^2 &= \nu_{\mathrm{comp},2}^2 + \nu_{\mathrm{comp},3}^2 \\ \nu_{\mathrm{comp},y}^2 &= \nu_{\mathrm{comp},3}^2 + \nu_{\mathrm{comp},1}^2 \\ \nu_{\mathrm{comp},z}^2 &= \nu_{\mathrm{comp},1}^2 + \nu_{\mathrm{comp},2}^2\end{aligned} \tag{29}$$

The Fermi temperature for a spin mixture of N total atoms is

$$k_{\mathrm{B}}T_F = h(3N)^{1/3}\left[\prod_i \nu_{x_i}\right]^{1/3} \tag{30}$$

$$
\begin{aligned}
T_F &= \frac{h}{k_\mathrm{B}}(3N)^{1/3}\left(2\nu_{\mathrm{comp},3}\right) \\
&= \frac{h}{k_\mathrm{B}}2(3N)^{1/3}\left(\frac{E_{R,L}}{m\pi^2}\right)^{1/2}\left(s\varphi_{\mathrm{dimp},3} - g\varphi_{\mathrm{gr},3}\right)^{1/2} \\
&= \left[48\text{e-}3\,\mu\mathrm{K\,kHz}^{-1}\right]2(3N)^{1/3}\left[14.1\,\mu\mathrm{m\,kHz}\right]\left(2.99\,s - 6.13\,g\right)^{1/2}\left[0.01\,\mu\mathrm{m}^{-1}\right] \\
&= \left[13.5\,\mathrm{nK}\right]\times(3N)^{1/3}\sqrt{2.99\,s - 6.13\,g}
\end{aligned}
\tag{31}
$$

5 Heating

A detailed treatment of heating as a diffusion of momentum has been carried out by Gordon and Ashkin [2] and also more recently in [3, 4, 5]. These references stay within the rotating wave approximation, so here we have adapted their formulas to include the counter-rotating term as well as a factor of $(\omega_L/\omega_0)^3$ that corrects the decay rate for the different density of states at ω_L. From [2] the rate of momentum diffusion in the low saturation regime is given by

$$
D_p = \frac{\hbar^2 k^2}{2}\frac{3\pi c^2}{2\hbar\omega_0^3}\frac{\Gamma^2}{\Delta^2}I(\boldsymbol{r})\left(1 + \frac{1}{k^2}\left|\frac{\nabla(\langle g|\boldsymbol{d}|e\rangle\cdot\boldsymbol{E}(\boldsymbol{r}))}{\langle g|\boldsymbol{d}|e\rangle\cdot\boldsymbol{E}(\boldsymbol{r})}\right|^2\right)
\tag{32}
$$

where $\boldsymbol{E}(\boldsymbol{r})e^{-i\omega t}$ is the complex classical field and $I(\boldsymbol{r})$ is the intensity. If the polarization of the field is $\boldsymbol{\varepsilon}$ such that $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{\varepsilon}E(\boldsymbol{r})$, we have

$$
\begin{aligned}
D_p &= \frac{\hbar^2 k^2}{2}\frac{3\pi c^2}{2\hbar\omega_0^3}\frac{\Gamma^2}{\Delta^2}I(\boldsymbol{r})\left(1 + \frac{1}{k^2}\left|\frac{\langle g|\boldsymbol{d}\cdot\boldsymbol{\varepsilon}|e\rangle\nabla E(\boldsymbol{r})}{\langle g|\boldsymbol{d}\cdot\boldsymbol{\varepsilon}|e\rangle E(\boldsymbol{r})}\right|^2\right) \\
&= \frac{\hbar^2 k^2}{2}\frac{3\pi c^2}{2\hbar\omega_0^3}\frac{\Gamma^2}{\Delta^2}I(\boldsymbol{r})\left(1 + \frac{1}{k^2}\left|\frac{\nabla E(\boldsymbol{r})}{E(\boldsymbol{r})}\right|^2\right)
\end{aligned}
\tag{33}
$$

At this point we generalize the result of Gordon and Ashkin to far detuned light by doing the replacement

$$
\frac{1}{\Delta^2} \to \left(\frac{\omega_L}{\omega_0}\right)^3\left(\frac{1}{\omega_0 - \omega_L} + \frac{1}{\omega_0 + \omega_L}\right)^2
\tag{34}
$$

which results in

$$
D_p = \frac{\hbar^2 k^2}{2}\frac{3\pi c^2}{2\hbar\omega_0^3}\Gamma^2\left(\frac{\omega_L}{\omega_0}\right)^3\left(\frac{1}{\omega_0 - \omega_L} + \frac{1}{\omega_0 + \omega_L}\right)^2 I(\boldsymbol{r})\left(1 + \frac{1}{k^2}\left|\frac{\nabla E(\boldsymbol{r})}{E(\boldsymbol{r})}\right|^2\right)
\tag{35}
$$

The momentum diffusion term is defined as

$$
D_p = \frac{1}{2}\left(\langle p^2\rangle - \langle\boldsymbol{p}\rangle\cdot\langle\boldsymbol{p}\rangle\right)
\tag{36}
$$

so for an atom that starts at rest the energy deposited per unit time is $\dot{E} = D_p/m$.

For a plane wave propagating along x, $E(\boldsymbol{r})$ as defined above is $E_0 e^{ikx}$, so this gives a heating rate

$$
\begin{aligned}
\dot{E} &= E_{R,L}\frac{3\pi c^2}{2\hbar\omega_0^3}\Gamma^2\left(\frac{\omega_L}{\omega_0}\right)^3\left(\frac{1}{\omega_0-\omega_L}+\frac{1}{\omega_0+\omega_L}\right)^2 2I_0 \\
&= 2E_{R,L}\Gamma_{sc,0} \\
&= 2E_{R,L}\frac{h_L}{k_{\mathrm{B}}|u_L|}U_0
\end{aligned}
\tag{37}
$$

For a standing wave, such as that produced in a optical lattice $E(\boldsymbol{r}) = E_0\cos(kx)$ and $I(\boldsymbol{r}) = I_{\max}\cos^2(kx)$ which results in a heating rate

$$
\begin{aligned}
\dot{E} &= E_{R,L}\frac{3\pi c^2}{2\hbar\omega_0^3}\Gamma^2\left(\frac{\omega_L}{\omega_0}\right)^3\left(\frac{1}{\omega_0-\omega_L}+\frac{1}{\omega_0+\omega_L}\right)^2 I_{\max}\cos^2(kx)\left(1+\frac{\sin^2(kx)}{\cos^2(kx)}\right) \\
&= E_{R,L}\Gamma_{sc,\max}
\end{aligned}
\tag{38}
$$

We see that the heating rate is independent of x and so it is the same for a red or blue detuned lattice.

6 Heating due to optical lattice beams

As we saw in §2, our lattice potential suffers from losses on the retro path. The intensity for one of the axis with the light propagating along the x direction (neglecting the transverse profile $\exp[-2r_\perp^2/w^2]$) is given by

$$
I_{1D}(x) = \frac{2P_{\mathrm{i}}}{\pi w^2}\left(4\sqrt{R}\cos^2(kz) - 2\sqrt{R} + R + 1\right)
\tag{39}
$$

The corresponding electric field is

$$
\begin{aligned}
E_{1D}(x) &\propto \sqrt{\frac{2P_{\mathrm{i}}}{\pi w^2}}e^{ikx} + \sqrt{\frac{2P_{\mathrm{i}}R}{\pi w^2}}e^{-ikx} \\
&\propto \sqrt{\frac{2P_{\mathrm{i}}}{\pi w^2}}2\sqrt{R}\cos(kx) + \sqrt{\frac{2P_{\mathrm{i}}}{\pi w^2}}(1-\sqrt{R})e^{ikx}
\end{aligned}
\tag{40}
$$

such that

$$
\begin{aligned}
\left|\frac{\nabla E(\boldsymbol{r})}{E(\boldsymbol{r})}\right|^2 &= \frac{\left|-2k\sqrt{R}\sin(kx) + ik(1-\sqrt{R})e^{ikx}\right|^2}{\left|2\sqrt{R}\cos(kx) + (1-\sqrt{R})e^{ikx}\right|^2} \\
&= k^2\frac{4R\sin^2(kx) + (1-\sqrt{R})^2 + 2\sqrt{R}\sin(kx)(1-\sqrt{R})(2\sin(kx))}{4R\cos^2(kx) + (1-\sqrt{R})^2 + 2\sqrt{R}\cos(kx)(1-\sqrt{R})(2\cos(kx))} \\
&= k^2\frac{4\sqrt{R}\sin^2(kx) + (1-\sqrt{R})^2}{4\sqrt{R}\cos^2(kx) + -2\sqrt{R} + R + 1}
\end{aligned}
\tag{41}
$$

The spatially dependent factor that appears in Gordon and Ashkin's formula is

$$
\begin{aligned}
I(\boldsymbol{r})\left(1+\frac{1}{k^2}\left|\frac{\nabla E(\boldsymbol{r})}{E(\boldsymbol{r})}\right|^2\right) &= \frac{2P_\mathrm{i}}{\pi w^2}\left(4\sqrt{R}\cos^2(kz)-2\sqrt{R}+R+1+4\sqrt{R}\sin^2(kx)+(1-\sqrt{R})^2\right) \\
&= \frac{2P_\mathrm{i}}{\pi w^2}2(1+R)
\end{aligned}
\tag{42}
$$

The heating rate for the 1D lattice is then

$$
\begin{aligned}
\dot{E}_{1D} &= E_{R,L}\frac{3\pi c^2}{2\hbar\omega_0^3}\Gamma^2\left(\frac{\omega_L}{\omega_0}\right)^3\left(\frac{1}{\omega_L-\omega_0}+\frac{1}{\omega_0+\omega_L}\right)^2\frac{2P_\mathrm{i}}{\pi w^2}2(1+R) \\
&= E_{R,L}h_L\frac{2P_\mathrm{i}}{\pi w^2}2(1+R)
\end{aligned}
\tag{43}
$$

where h_L is defined in Eq. 5, and its value for 1064 nm and 532 nm appears in Table 1.

The lattice depth in our retroreflected setup is given by the $\cos^2$ modulation of I_{1D}. This can be read off of Eq. 39:

$$
I_\mathrm{lattice} = \frac{8P_\mathrm{i}}{\pi w^2}\sqrt{R} \tag{44}
$$

Using the factor u_λ defined above, this produces a lattice depth

$$
V_\mathrm{lattice} = k_\mathrm{B}|u_\lambda|\frac{8P_\mathrm{i}\sqrt{R}}{\pi w^2} \tag{45}
$$

The lattice depth in units of E_R is usually represented by $s = V_\mathrm{lattice}/E_R$.

$$
s = \frac{k_\mathrm{B}|u_\lambda|}{E_{R,\lambda}}\frac{8P_\mathrm{i}\sqrt{R}}{\pi w^2} \tag{46}
$$

This result can be incorporated into the heating rate obtained above

$$
\boxed{\dot{E}_{1D} = sE_{R,L}\frac{h_L E_{R,L}}{k_\mathrm{B}|u_\lambda|}\frac{(1+R)}{2\sqrt{R}}} \tag{47}
$$

Numerical values for $\frac{h_L E_{R,L}}{k_\mathrm{B}|u_\lambda|}$ are given in Table 1. We can also write down the expression for it:

$$
\frac{h_L E_{R,L}}{k_\mathrm{B}|u_\lambda|} = E_{R,L}\frac{\Gamma}{\hbar}\left(\frac{\omega_L}{\omega_0}\right)^3\left|\frac{1}{\omega_0-\omega_L}+\frac{1}{\omega_0+\omega_L}\right| \approx \frac{E_{R,L}}{\hbar|\Delta|}\Gamma \tag{48}
$$

where on the far right we have neglected the counter-rotating term and the different density of states, as it is customarily done in the rotating wave approximation (RWA). Within RWA the heating rate is

$$
\dot{E}_{1D} = V_\mathrm{latt}\frac{E_{R,L}}{\hbar|\Delta|}\Gamma\frac{(1+R)}{2\sqrt{R}} \tag{49}
$$

7 Heating due to green compensation beams

Our green compensation beams are simply gaussian beams for which the heating rate was derived in Eq. 37. Since we use this beams to compensate a lattice formed by the 1064 nm IR beams we specify

	1064 nm	532 nm	units
u_λ	-60.81	+61.99	$\mu\mathrm{K}\dfrac{\mu\mathrm{m}^2}{\mathrm{mW}}$
h_λ	0.0876	0.728	$\mathrm{s}^{-1}\dfrac{\mu\mathrm{m}^2}{\mathrm{mW}}$
$\dfrac{E_{R,\lambda}}{k_\mathrm{B}}$	1.41	5.64	μK
$\dfrac{h_\lambda E_{R,\lambda}}{k_\mathrm{B}\lvert u_\lambda\rvert}$	2.03e-3	6.62e-2	s^{-1}
$\dfrac{\lvert u_\lambda\rvert k_\mathrm{B}}{m\pi^2}$	8.55e3	8.72e3	$\frac{\mu\mathrm{m}^4\,\mathrm{kHz}^2}{\mathrm{mW}}$
$\dfrac{E_{R,L}}{m\pi^2}$	198.4	793.68	$\mu\mathrm{m}^2\,\mathrm{kHz}^2$

Table 1: Helpful constants to calculate dipole potentials, trapping frequencies and heating rates.

the repulsive compensation at the center in units of the IR recoil, $E_{R,\mathrm{ir}}$:

$$g = \frac{U_{0,\mathrm{g}}}{E_{R,\mathrm{ir}}} \tag{50}$$

The heating rate due to each of the compenstion beams is then

$$\boxed{\dot{E}_g = 2E_{R,\mathrm{ir}}\frac{h_g E_{R,\mathrm{g}}}{k_\mathrm{B}|u_g|}g} \tag{51}$$

8 Total heating rate in the compensated lattice

We can then add up the contributions of the IR and green to obtain

$$\begin{aligned}\frac{\dot{E}_{1D,\mathrm{tot}}}{E_{R,\mathrm{ir}}} &= \frac{h_{ir}E_{R,\mathrm{ir}}}{k_\mathrm{B}|u_{ir}|}\left(s\frac{1+R}{2\sqrt{R}} + 2g\,\frac{h_g E_{R,\mathrm{g}}}{k_\mathrm{B}|u_g|}\Big/\frac{h_{ir}E_{R,\mathrm{ir}}}{k_\mathrm{B}|u_{ir}|}\right)\\ &= \frac{h_{ir}E_{R,\mathrm{ir}}}{k_\mathrm{B}|u_{ir}|}\left(s\frac{1+R}{2\sqrt{R}} + \kappa g\right)\end{aligned} \tag{52}$$

where we have defined $\kappa = \frac{h_g E_{R,\mathrm{g}}}{k_\mathrm{B}|u_g|}\Big/\frac{h_{ir}E_{R,\mathrm{ir}}}{k_\mathrm{B}|u_{ir}|}$ (For reference $\kappa = 65.22$). Including the contribution from the three orthogonal axes that form our lattice we obtain

$$\frac{\dot{E}_\mathrm{tot}}{E_{R,\mathrm{ir}}} = \frac{h_{ir}E_{R,\mathrm{ir}}}{k_\mathrm{B}|u_{ir}|}\sum_{i=1,2,3}\left(s_i\frac{1+R_i}{2\sqrt{R_i}} + \kappa g_i\right) \tag{53}$$

If we assume that the energy increase due to heating is redistributed equally in all three dimensions we have for the rate of increase in temperature

$$\boxed{\dot{T} = \frac{\dot{E}_\mathrm{tot}}{3k_\mathrm{B}} = \frac{E_{R,\mathrm{ir}}}{3k_\mathrm{B}}\frac{h_{ir}E_{R,\mathrm{ir}}}{k_\mathrm{B}|u_{ir}|}\sum_{i=1,2,3}\left(s_i\frac{1+R_i}{2\sqrt{R_i}} + \kappa g_i\right)} \tag{54}$$

Since $\overline{R} = 0.8$ and we typically use the same depth for all ir beams and nearly the same depth for all green beams this can be approximated by

$$\begin{aligned} \dot{T} &\approx \frac{E_{R,\mathrm{ir}}}{k_\mathrm{B}} \frac{h_{ir} E_{R,\mathrm{ir}}}{k_\mathrm{B}|u_{ir}|} (s + \kappa g) \\ &\approx 2.9(s + 65g)\,\mathrm{nK/s} \end{aligned} \tag{55}$$

where, once again, s is the lattice depth in IR recoils, and g is the compensation, also in IR recoils. It becomes obvious that our heating rate is completely dominated by the compensating beams. Typical values are $s = 7.0$ and $g = 2.9$.

Note that the large difference in the heating rate between green and IR can be mostly atrributed to the density of states factor and the recoil energy. The density of states factor is $(\omega_L/\omega_0)^3 = (\lambda_0/\lambda)^3$, which is $\sim$2 for green and $\sim$0.25 for IR, giving a factor of 8. The recoil scales as λ^{-2} which gives another factor of 4. This accounts for a factor of 32, the remaining factor of 2 is due to the different heating rates for a traveling wave and a standing wave.

9 Total heating rate in the compensated lattice

In dimple configuration ($\alpha = 1$), and using Eq. 46 to relate s and the input power P_i, the 1D heating rate is

$$\dot{E} = E_{R,\mathrm{ir}} = \frac{h_\mathrm{ir} E_{R,\mathrm{ir}}}{k_\mathrm{B} u_\mathrm{ir}} \frac{s}{\sqrt{R}} \tag{56}$$

Note that

$$\frac{1}{\sqrt{R}} \approx \frac{1 + R}{2\sqrt{R}}$$

for the values of R that we deal with. This means that Eq. 55 is still valid for the heating rate in dimple configuration.

References

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