Non-equilibrium many-body dynamics and heating of cold atoms in optical lattices

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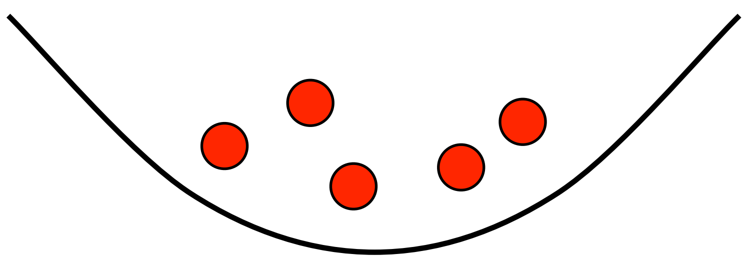
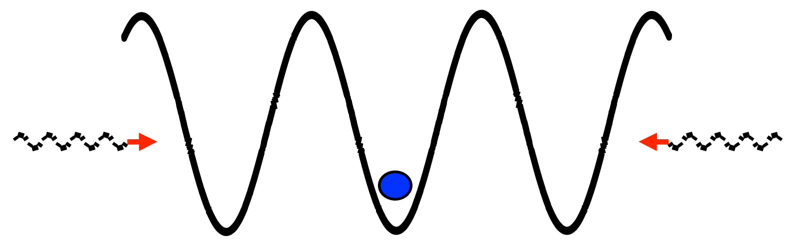
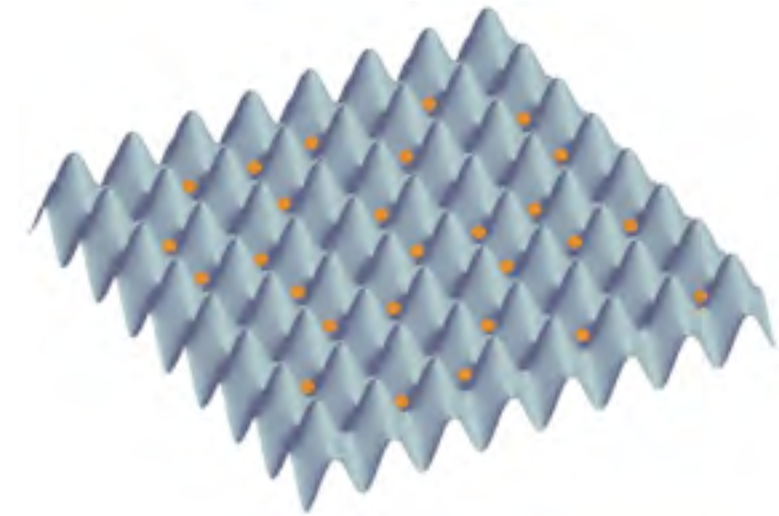
Atoms in 3D Optical Lattices:





AC-Stark Shift

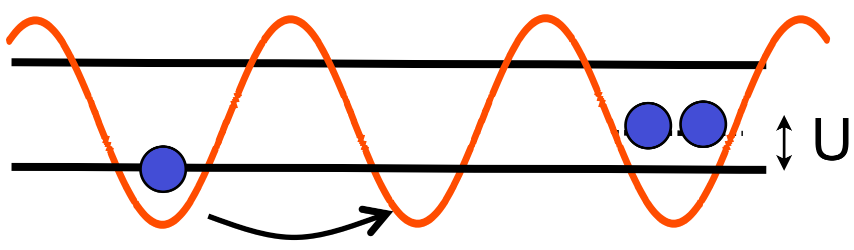


- BEC / degenerate Fermi gas

(Weakly interacting)

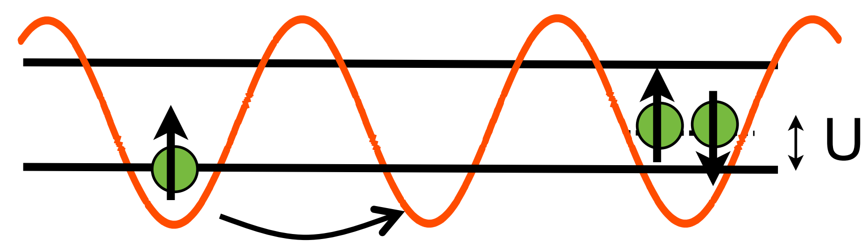
(Strongly interacting)

Bose-Hubbard: D. Jaksch et al. PRL ’98



$$H = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

(Fermi-) Hubbard:



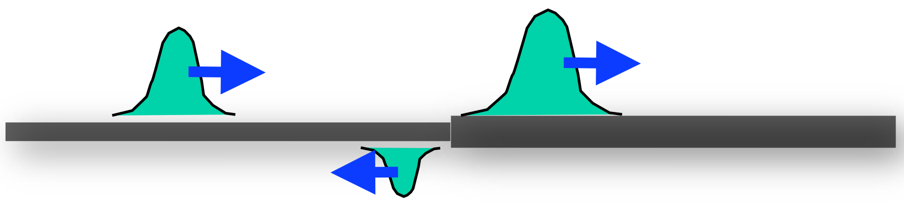
$$H = -J \sum_{\langle i,j \rangle,\sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger c_{i,\downarrow} c_{i,\uparrow}$$

- Realise strongly correlated lattice models
- Microscopic understanding
- Study thermodynamics / quantum phases

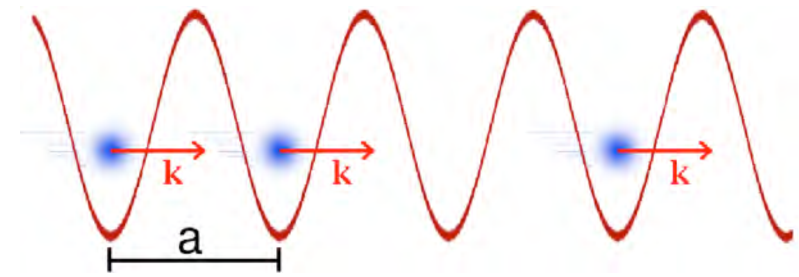
Experiments:
Munich, Zurich, NIST / JQI, MIT, Harvard, Innsbruck, Hamburg, Pisa, Florence, Oxford, Austin, Chicago, Penn State, Kyoto, Illanois, ...

Coherent non-equilibrium dynamics:

- Transport dynamics

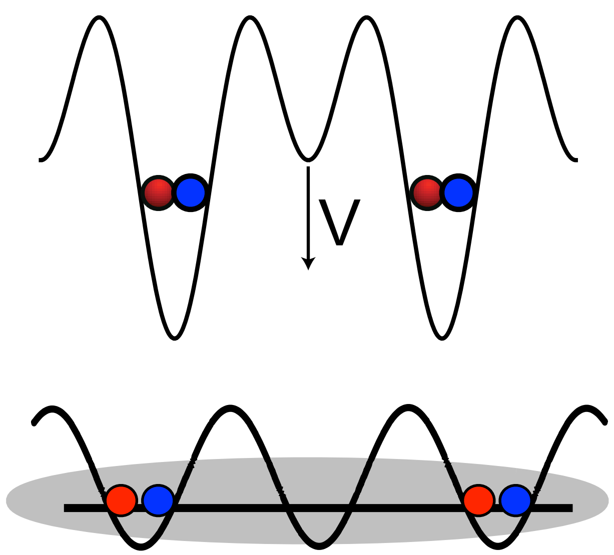


AJD, P. Zoller, and B. Trauzettel,
PRL **100**, 110404 (2008)



J. Schachenmayer, G. Pupillo, and AJD,
New J. Phys. **12**, 025014 (2010)

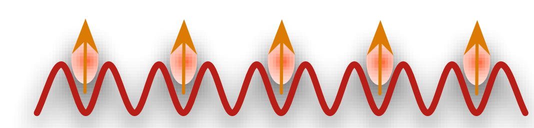
- Many-body State preparation (e.g., eta-pairs)



P. Rabl, AJD, P. O. Fedichev, J. I Cirac, and P. Zoller,
PRL **91**, 110403 (2003)

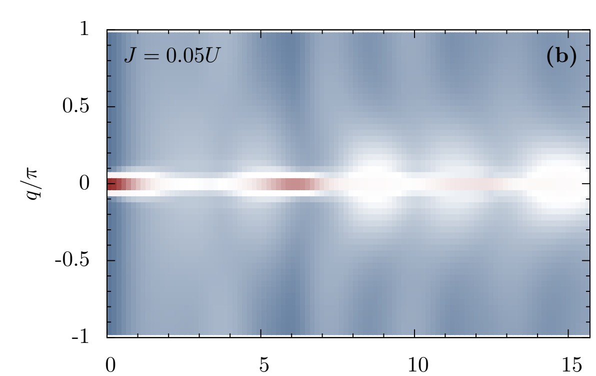
A. Kantian, AJD, and P. Zoller,
PRL **104**, 240406 (2010)

- Dynamical crystal formation
with Rydberg atoms / polar molecules



J. Schachenmayer, A. Micheli, I. Lesanovsky,
and AJD, New J. Phys. **12**, 103044 (2010)

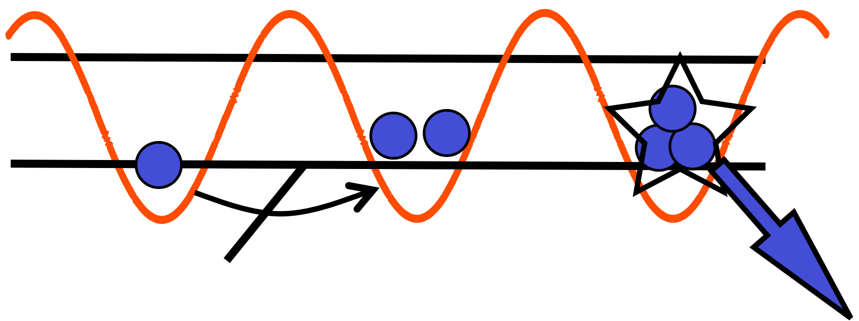
- Collapse/revival dynamics with fixed
atom number



J. Schachenmayer, AJD, and P. Zoller,
PRA in press, arXiv:1101.2385

Dissipative non-equilibrium dynamics:

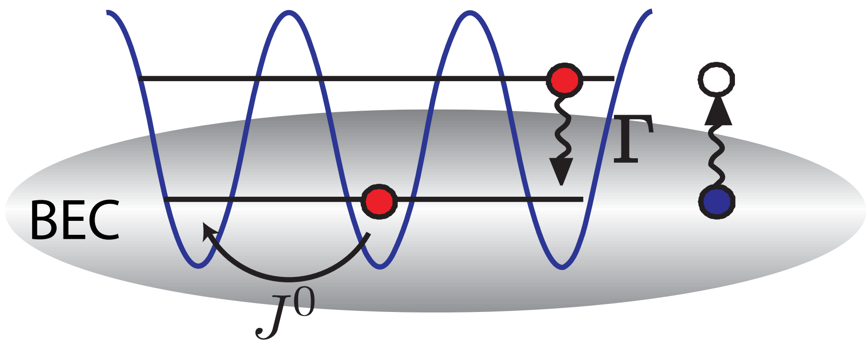
- Three-body loss



Dimer superfluid of Bosons:
AJD, J. Taylor, S. Diehl, M. Baranov, and P. Zoller
PRL **102**, 040402 (2009)
S. Diehl, M. Baranov, AJD, and P. Zoller
PRL **104**, 165301 (2010)

Colour superfluid in a 3-component Fermi gas:
A. Kantian, M. Dalmonte, S. Diehl, W. Hofstetter,
P. Zoller, and AJD, PRL **103**, 240401 (2009)

- State preparation by reservoir engineering

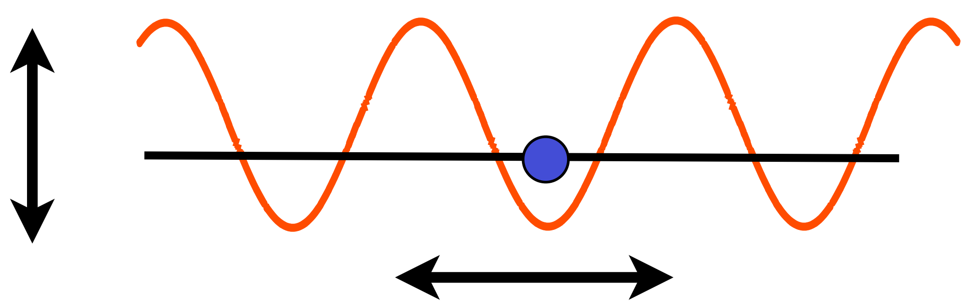


Dark State Cooling:
A. Griessner, AJD, S. R. Clark, D. Jaksch, P. Zoller,
PRL **97**, 220403 (2006)

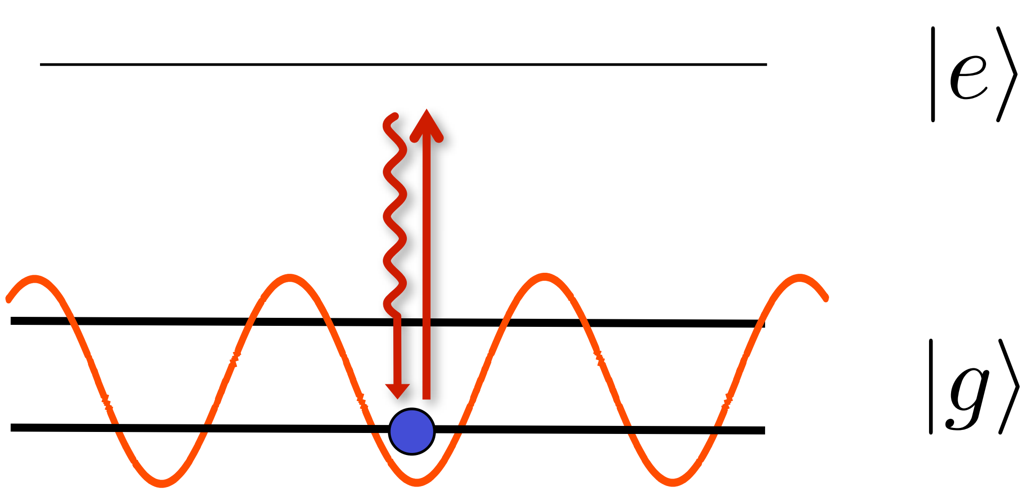
Dissipative preparation of a d-wave paired state:
S. Diehl, W. Yi, AJD, P. Zoller
PRL **105**, 227001 (2010)

Heating:

Via “classical” noise:
(e.g., lattice laser phase/amplitude noise)



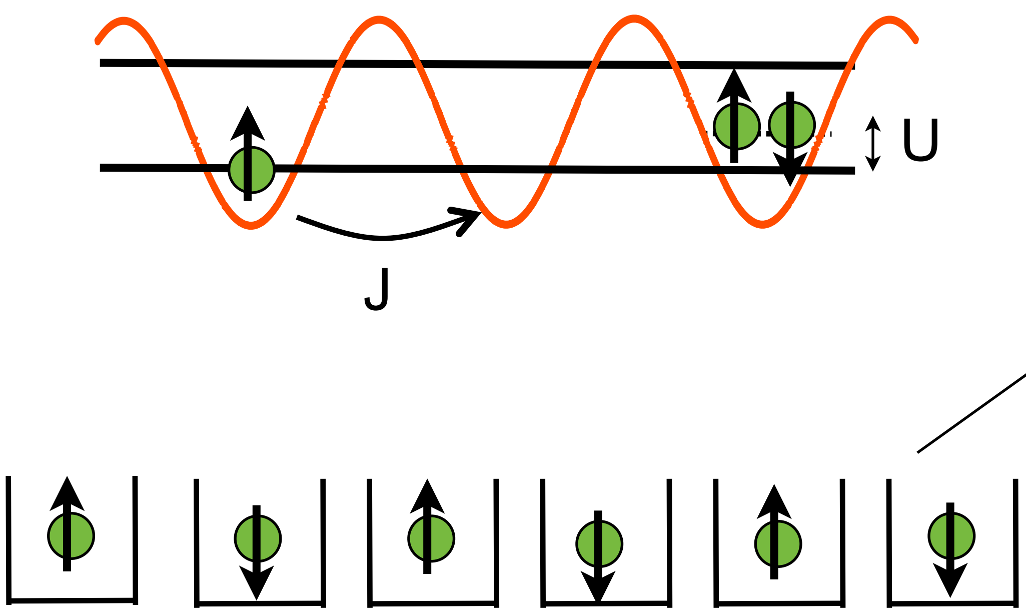
Via spontaneous emissions:

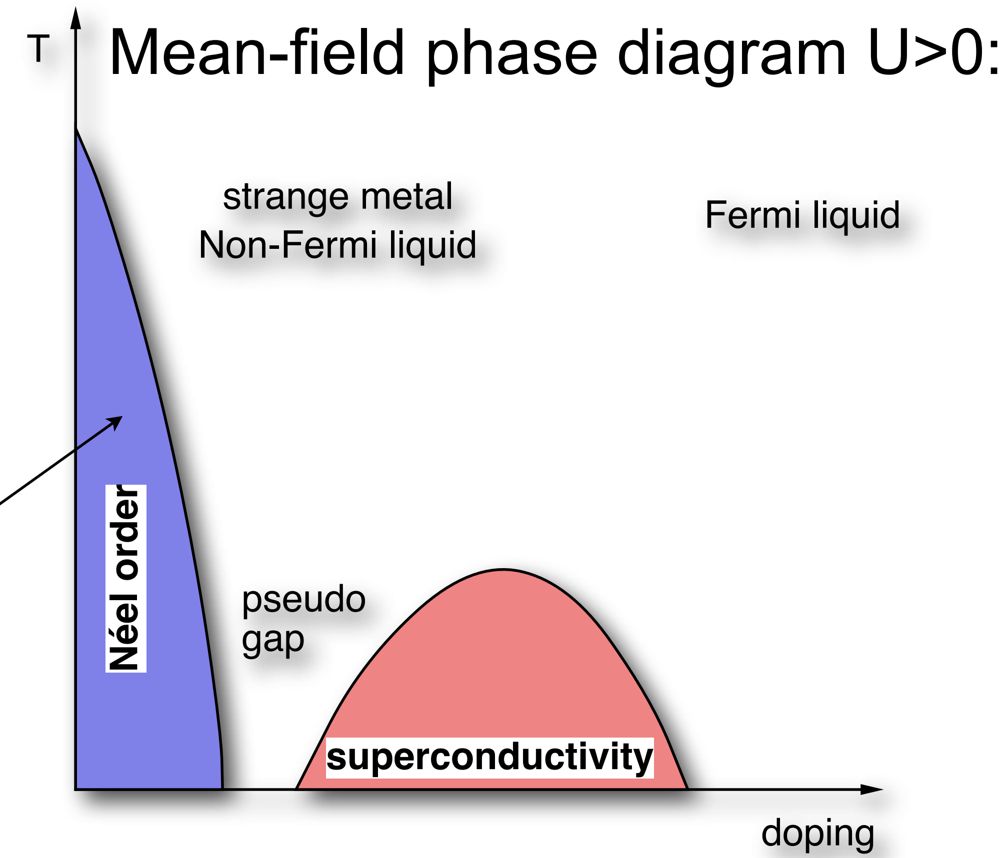


Quantum Simulation:

- Ground state properties / quantum phases of important models from solid state physics

  e.g., Potential connection to high Tc superconductivity:

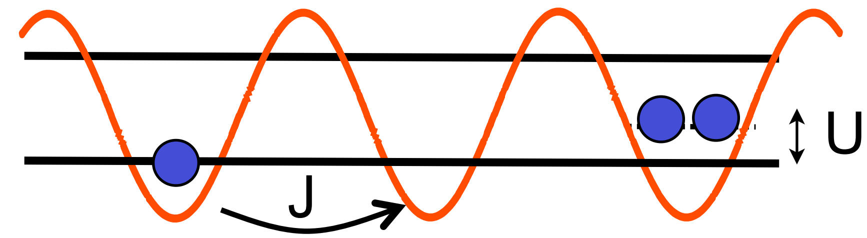




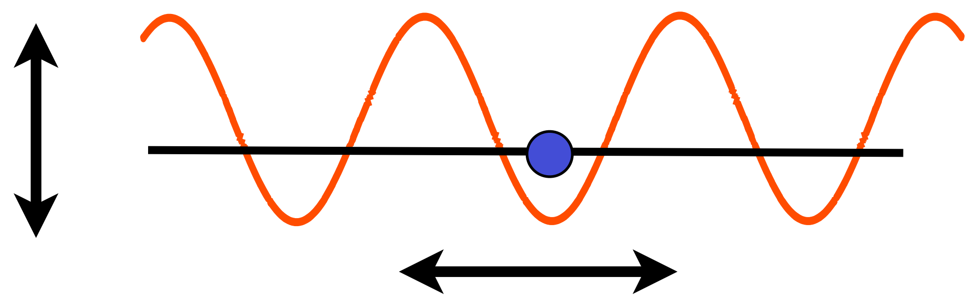
Key challenges:

- State preparation / cooling to low temperatures
- Characterisation and control of heating processes

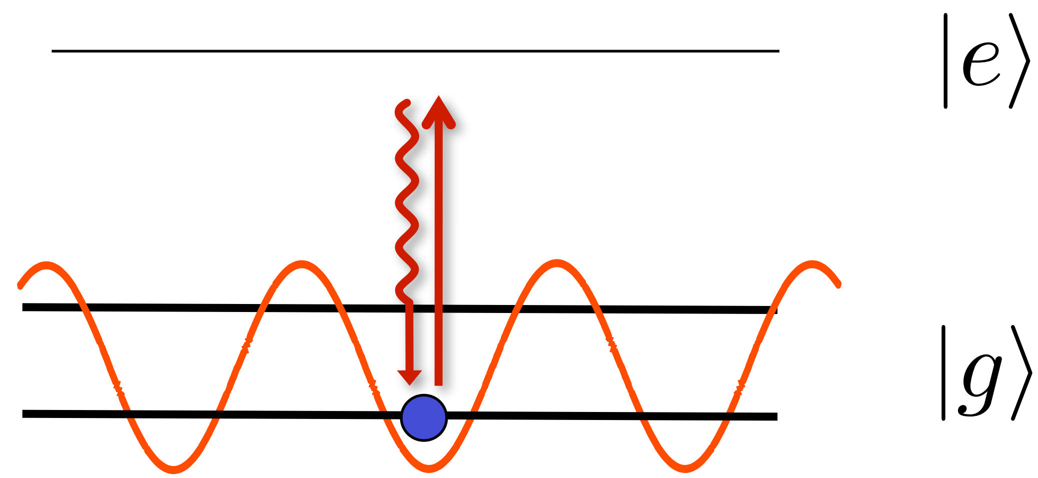
Heating in an optical lattice



Via “classical” noise:
(e.g., lattice laser phase/amplitude noise)



Via spontaneous emissions:

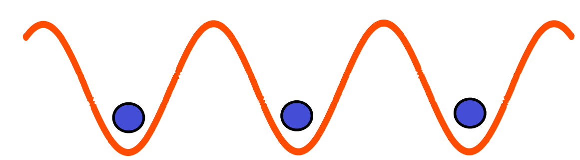


Also: collisional losses, background gas collisions....

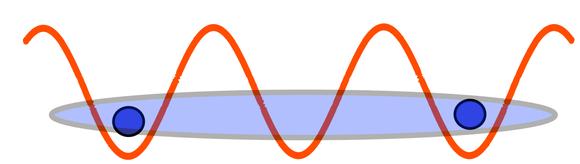
Questions:

- Interplay: **heating** with **many-body** physics
  Are some strongly correlated states more/less sensitive?

- How do we control / minimise heating in experiments?
  (e.g., choice of red vs. blue detuning for lattices)



Mott Insulator (U>>J)



Superfluid (J>>U)

Outline

Heating of many bosons due to spontaneous emissions

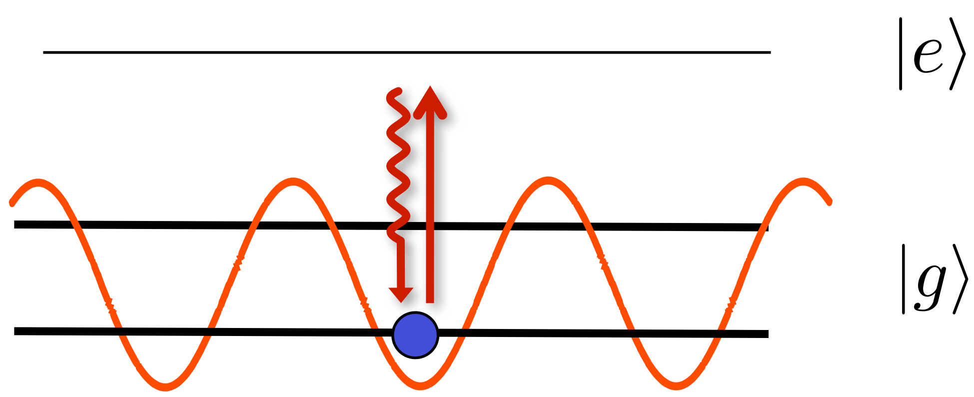
- Master equation
- Heating of single atoms
- Heating of many-body states
- Exact simulations in 1D

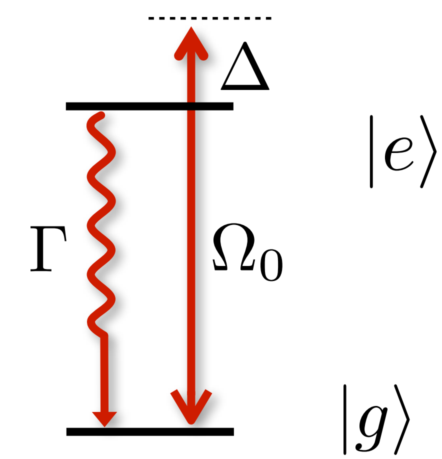
Work in progress

- Thermalisation of bosons in the presence of heating
- Light-assisted collisions
- Heating of many fermions via spontaneous emissions
- Amplitude and position fluctuations and heating

Heating of bosons via spontaneous emission

Theoretical Description (single atom)





- Spontaneous emissions from **two-level atoms**
- **Master equation** for spontaneous emissions in Born-Markov approximation

$$\dot{\rho} = -i\,[H,\rho] \;+\; \Gamma \int d^3\boldsymbol{u} N(\boldsymbol{u}) \left(C_{\boldsymbol{u}}\rho C_{\boldsymbol{u}}^{\dagger} - \frac{1}{2} C_{\boldsymbol{u}}^{\dagger} C_{\boldsymbol{u}}\rho - \frac{1}{2}\rho C_{\boldsymbol{u}}^{\dagger} C_{\boldsymbol{u}} \right)$$

$$C_{\boldsymbol{u}} = |g\rangle\langle e| e^{-ik_{eg}\boldsymbol{u}\cdot\hat{\boldsymbol{x}}}$$

atomic lowering operator

$$H = \frac{\hat{\boldsymbol{p}}^2}{2m} - \Delta|e\rangle\langle e| - \left(|g\rangle\langle e| \frac{\Omega(\hat{\boldsymbol{x}})}{2} + \text{h.c.} \right)$$

Kinetic energy

Detuning of excited state

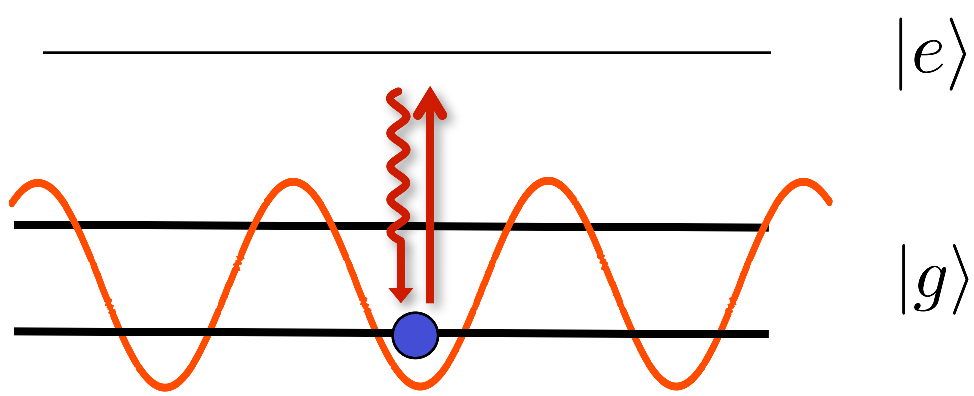
Coupling

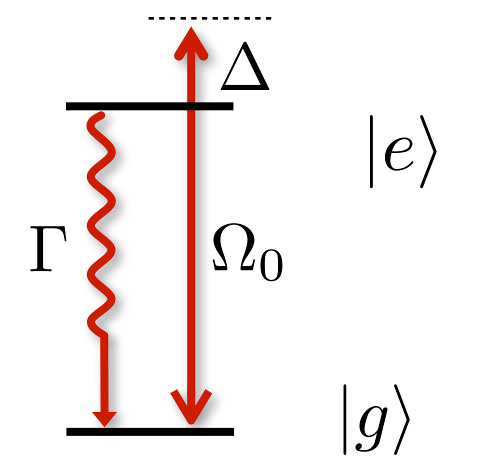
$$N(\boldsymbol{u}) = \frac{3}{8\pi}\left(1 - \left(\boldsymbol{u}\cdot\hat{\boldsymbol{d}}\right)^2\right)$$

Distribution of emitted photons

- Can be extended, e.g., with additional exit channels for internal state

Theoretical Description (single atom)





- Limit of **large detuning** - can adiabatically eliminate excited state $\Delta \gg \Gamma, \Omega_0$

$$\frac{d}{dt}\rho = -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^{\dagger}) + \mathcal{J}\rho$$

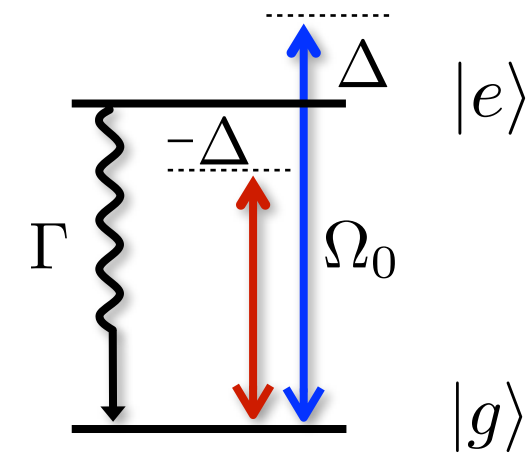
AC-Stark shift Decay

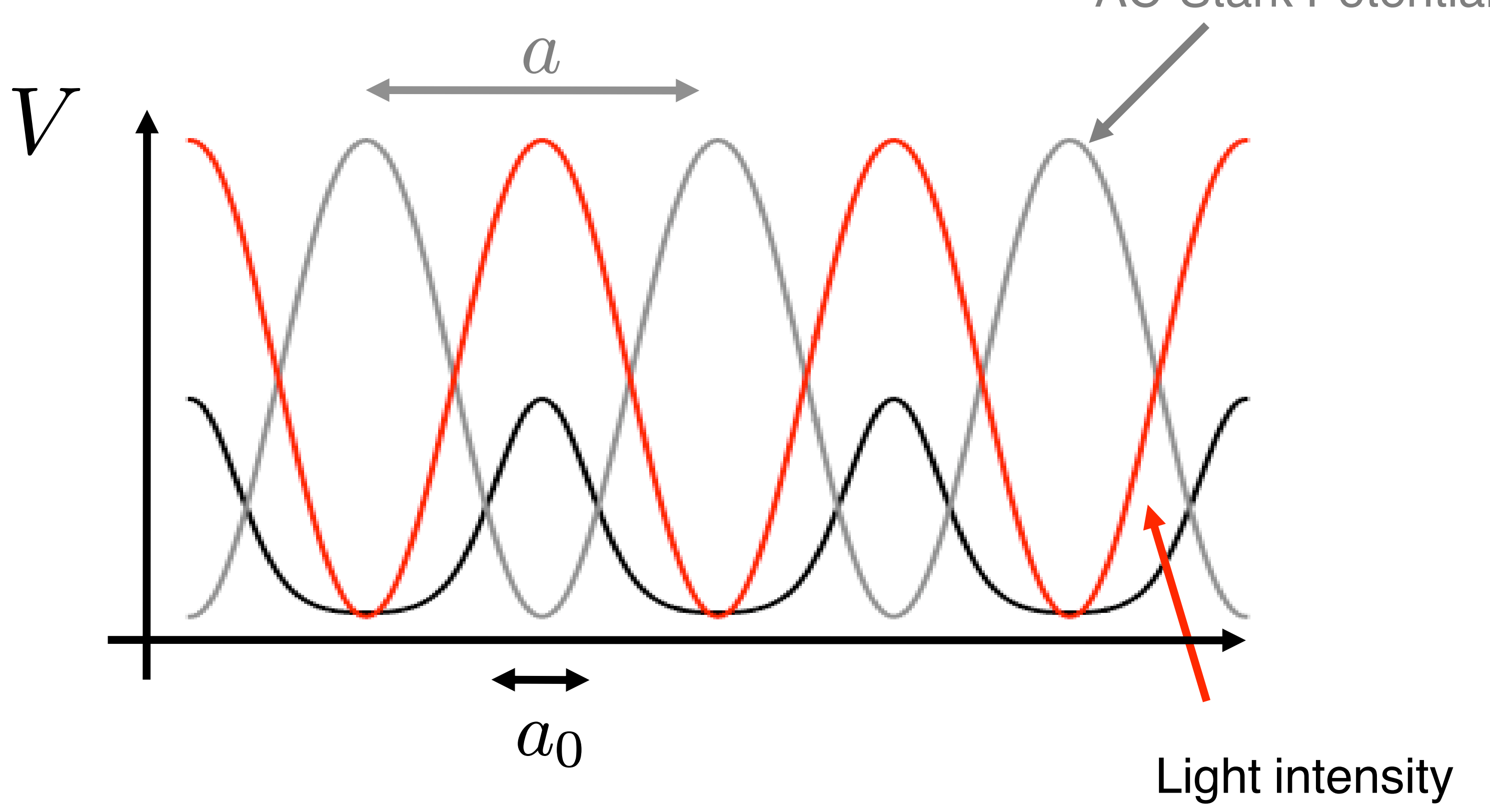
$$\begin{aligned} H_{\text{eff}} &= \frac{\hat{\boldsymbol{p}}^2}{2m} + \frac{|\Omega(\hat{\boldsymbol{x}})|^2}{4\Delta} - i\frac{1}{2}\frac{\Gamma|\Omega(\hat{\boldsymbol{x}})|^2}{4\Delta^2} \\ &\equiv \frac{\hat{\boldsymbol{p}}^2}{2m} + V_{\text{opt}}(\hat{\boldsymbol{x}}) - i\frac{\gamma(\boldsymbol{x})}{2}. \end{aligned}$$

$$\mathcal{J}\rho = \Gamma \int d^2\boldsymbol{u}\, N(\boldsymbol{u}) \left[e^{-ik_{eg}\boldsymbol{u}\cdot\hat{\boldsymbol{x}}}\frac{\Omega(\hat{\boldsymbol{x}})}{2\Delta}\right] \rho \left[e^{ik_{eg}\boldsymbol{u}\cdot\hat{\boldsymbol{x}}}\frac{\Omega^*(\hat{\boldsymbol{x}})}{2\Delta}\right]$$

Example: Red vs. Blue detuning, single atom

$$H = \frac{p^2}{2m} + \frac{|\Omega_0|^2}{4\Delta}$$





- Timescale for heating processes:

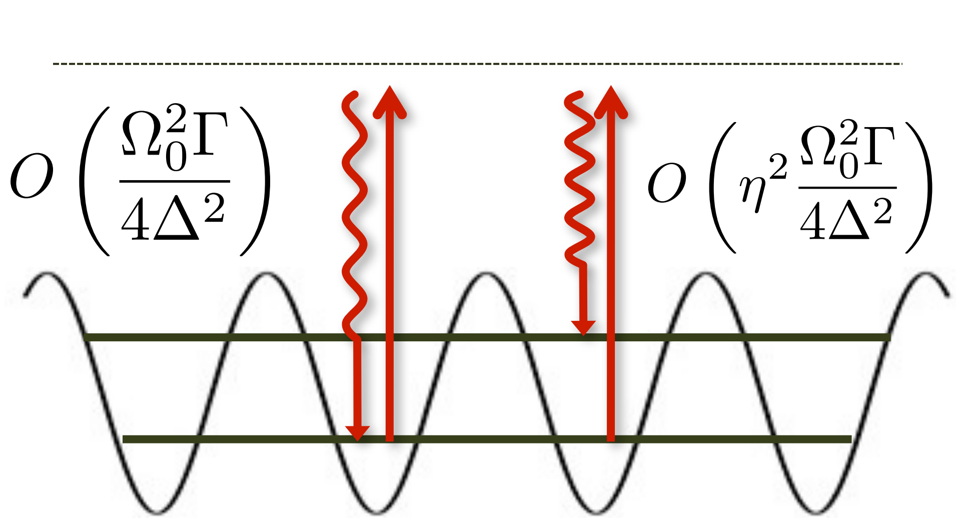
$$\Gamma_{\text{eff}} = \frac{\Omega_0^2}{4\Delta^2}\Gamma$$

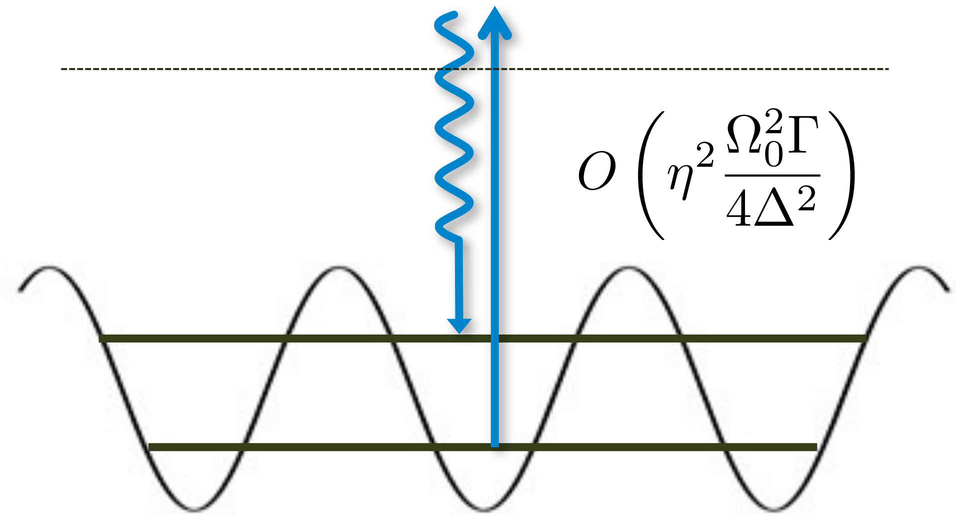
- Lamb-Dicke parameter:

$$\eta = \frac{\pi a_0}{a} = \left(\frac{1}{4V/E_R}\right)^{1/4}$$

J. P. Gordon and A. Ashkin, PRA **21,** 1606 (1980), F. Gerbier and Y. Castin, PRA **82,** 013615 (2010)

- Harmonic oscillator approximation, Lamb-Dicke limit: $\eta = \frac{\pi a_0}{a}, \quad kx = \eta(a + a^\dagger)$





$$|\langle f|\Omega(x)e^{ikx}|i\rangle|^2$$

initial state

final oscillator level

$$=|\langle f|\cos(kx)e^{ikx}|i\rangle|^2$$

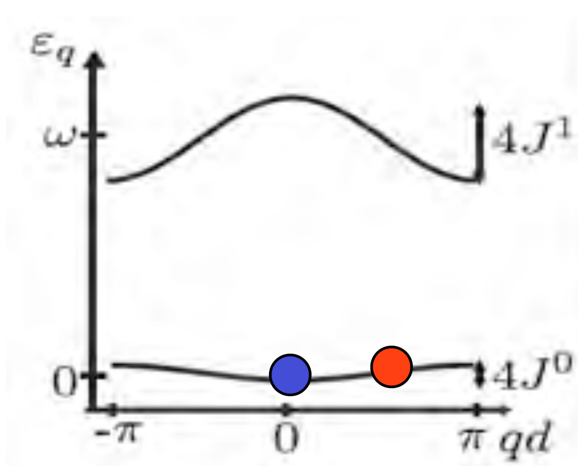
$$\approx|\langle f|(1-(kx)^2/2)(1+ikx)|i\rangle|^2$$

$$|\langle f|\Omega(x)e^{ikx}|i\rangle|^2$$

$$=|\langle f|\sin(kx)e^{ikx}|i\rangle|^2$$

$$\approx|\langle f|(x)(1+ikx)|i\rangle|^2$$

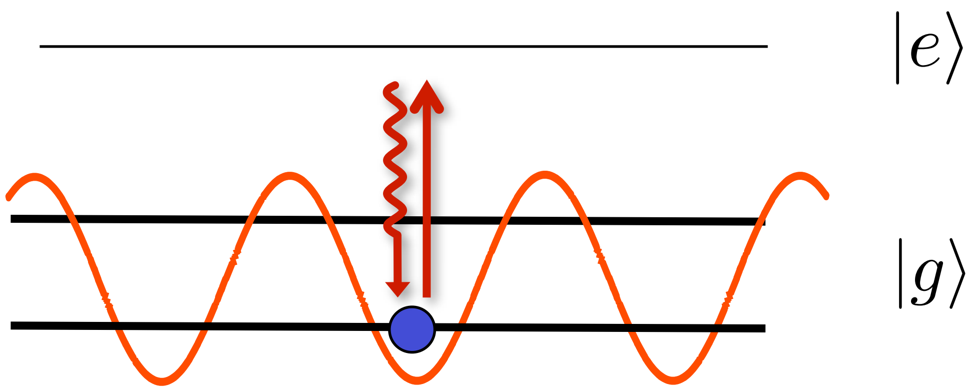
- General lattice: Lowest-band processes inelastic

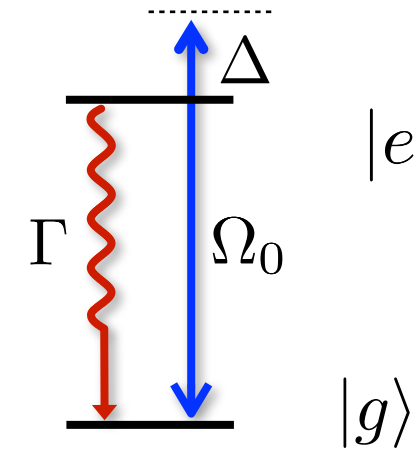


- Time-dependence of the mean energy: $\dot{E} = \frac{\Gamma|\Omega_0|^2}{4\Delta^2}E_R$

J. P. Gordon and A. Ashkin, PRA **21,** 1606 (1980), F. Gerbier and Y. Castin, PRA **82,** 013615 (2010)

Theoretical Description (many atoms)





- Spontaneous emissions from **two-level atoms**

- **Many-body** master equation for spontaneous emissions in Born-Markov approximation
- Limit of **large detuning** - eliminate excited state, obtain a description for motional states

$$\dot{\rho} = -i\left[\hat{H}, \rho\right] - \frac{1}{2}\frac{\Gamma}{4\Delta^2}\sum_{\mu}|\Omega_{0,\mu}|^2 \int d^3x \int d^3y F_{\mathbf{e}_\mu}(k(\mathbf{x}-\mathbf{y}))\epsilon_\mu(\mathbf{x})\epsilon_\mu(\mathbf{y}) \left[\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x}), \left[\hat{\psi}^\dagger(\mathbf{y})\hat{\psi}(\mathbf{y}), \rho\right]\right]$$

Many-body Hamiltonian (including optical potential)

multiple beams

bosonic field operator

$$F_{\mathbf{e}_\mu}(\xi) = \int d^2u \, \frac{3}{8\pi}\left(1 - (\mathbf{e}_\mu \cdot \mathbf{u})^2\right) e^{-i\mathbf{u}\cdot\xi}$$

- Can be extended, e.g., with additional exit channels for internal state

H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)

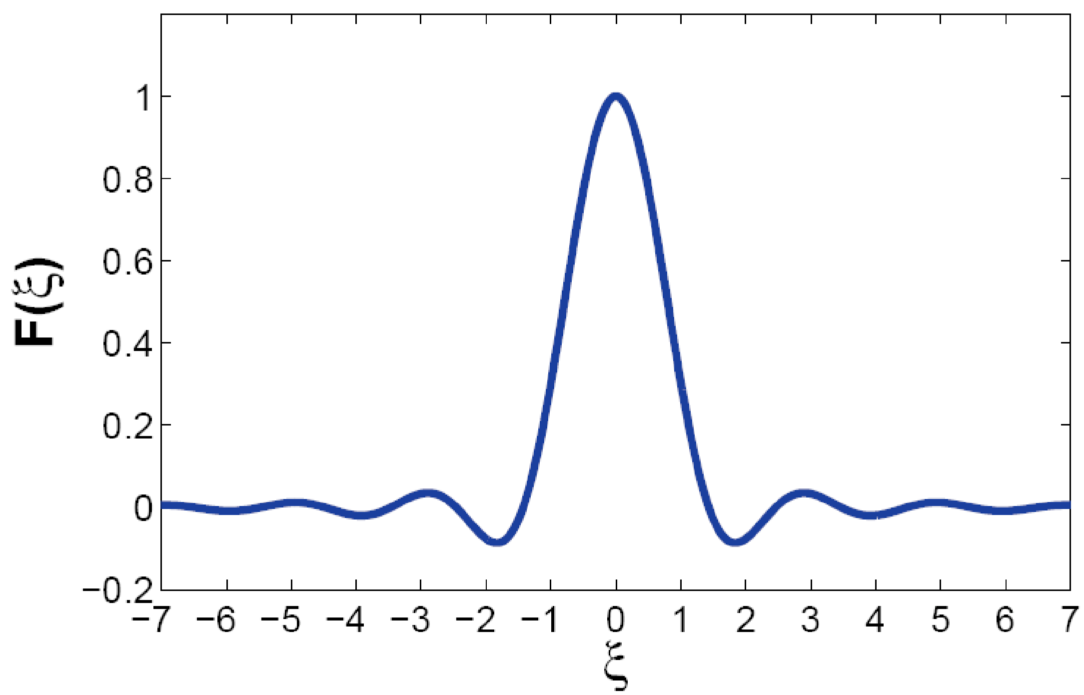
Localisation due to spontaneous emission

- Spontaneous emissions destroy long-range order by localising atoms

$$\dot{\rho} = -i\left[\hat{H},\rho\right] - \frac{1}{2}\frac{\Gamma}{4\Delta^2}\sum_{\mu}|\Omega_{0,\mu}|^2\int d^3x\int d^3y F_{\mathbf{e}_\mu}(k(\mathbf{x}-\mathbf{y}))\epsilon_\mu(\mathbf{x})\epsilon_\mu(\mathbf{y})\left[\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x}),\left[\hat{\psi}^\dagger(\mathbf{y})\hat{\psi}(\mathbf{y}),\rho\right]\right]$$

Many-body Hamiltonian
(including optical potential)

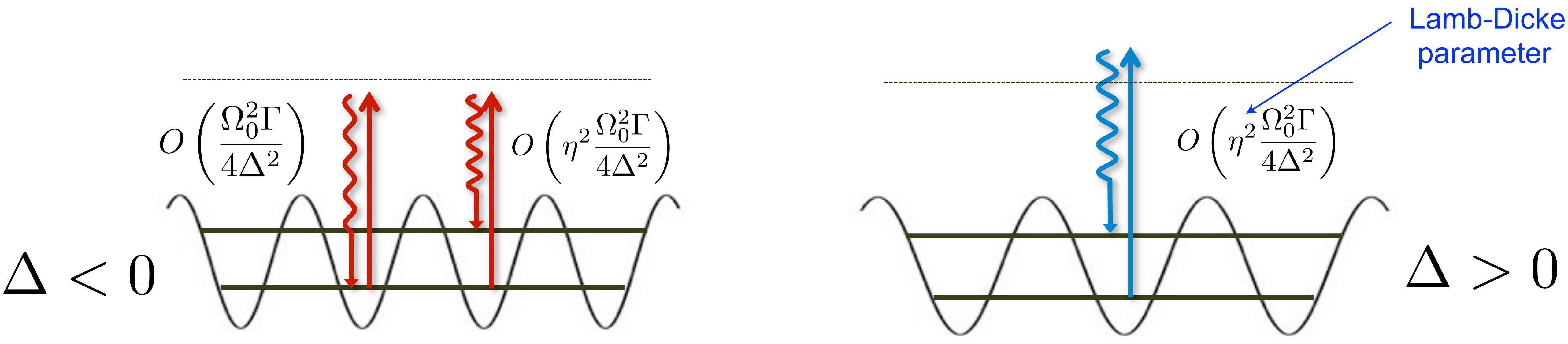
$$F_{\mathbf{e}_\mu}(\xi) = \int d^2u\,\frac{3}{8\pi}\left(1-(\mathbf{e}_\mu\cdot\mathbf{u})^2\right)e^{-i\mathbf{u}\cdot\xi}$$



H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)

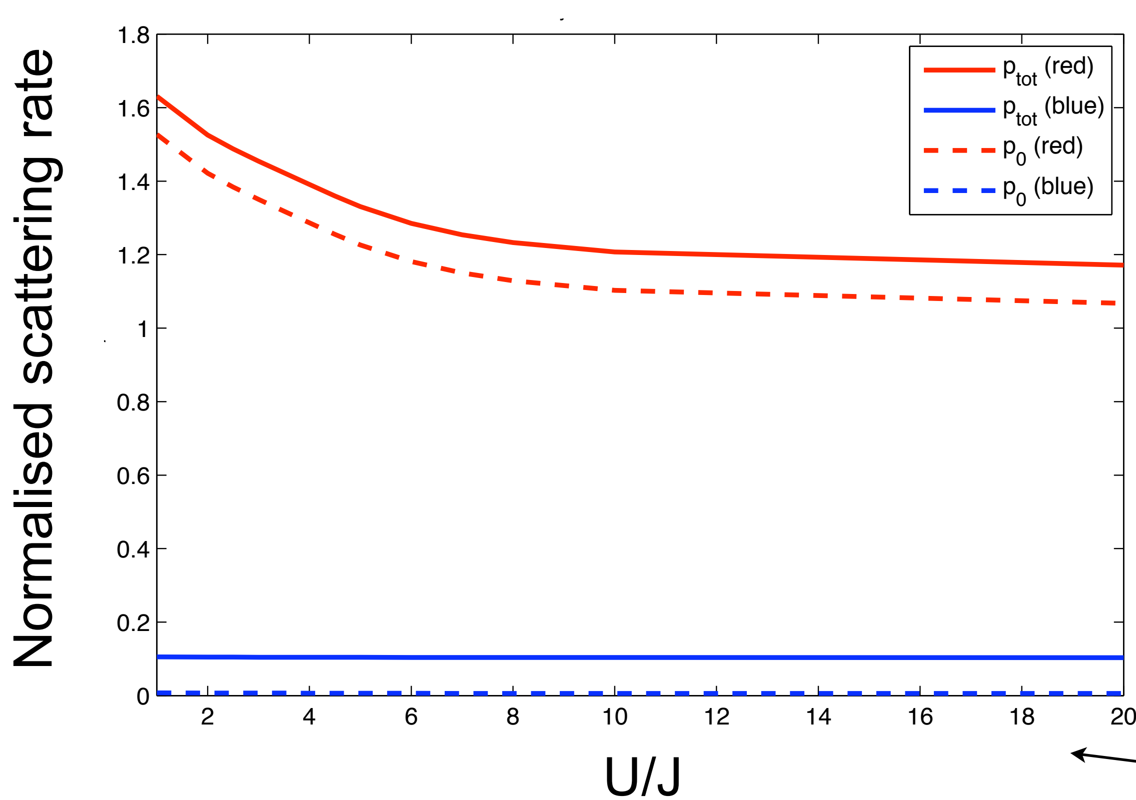
Example: Red vs. Blue detuning

H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)



- Red detuning gives rise to more spontaneous emission events in deep lattices
- Processes returning atoms to the lowest band are strongly suppressed for blue detuning

- Variation of total scattering rate with U/J (dashed lines:processes within lowest band)



- Total rate of energy increase is the same

$$\frac{d\langle \hat{H}\rangle}{dt} = \frac{\Gamma}{4\Delta^2}\sum_{\mu}|\Omega_{0,\mu}|^2 E_R N_{tot}$$

recoil energy

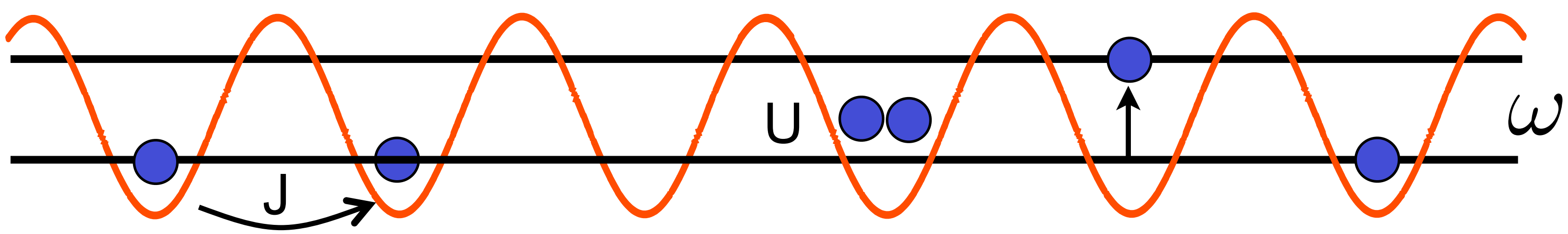
number of atoms

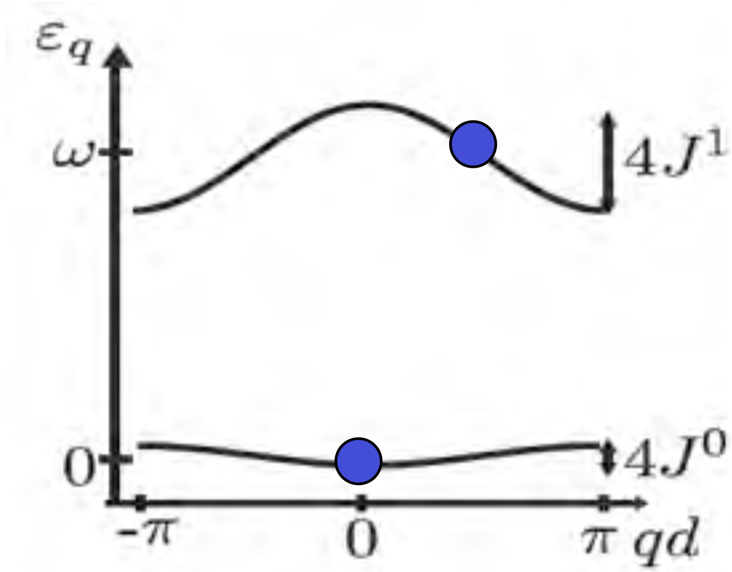
Estimates using many-body ground states from t-DMRG, typical parameters for Rb

Heating of many-body states

H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)

- Rate of energy increase is not a complete characterisation, as the system will **not thermalise on experimental timescales**



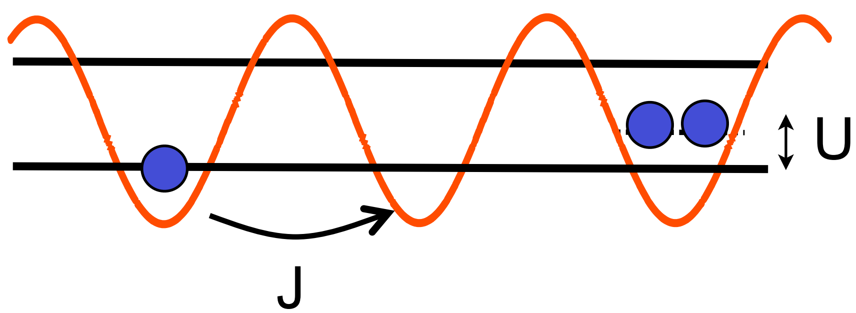


- This is particularly true of **atoms excited to higher bands**, where

$$\omega \gg U, J$$

[possibility of thermalisation in the lowest band, see later, and also c.f. “thermal” momentum distributions in S. Trotzky et al., Nature Phys. **6**, 998 (2010)]

- To understand change in the many-body state we should examine the **characteristic correlation functions** and see how order is destroyed by heating

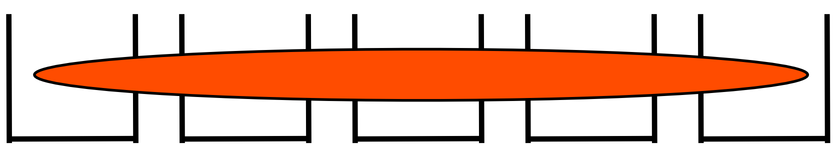


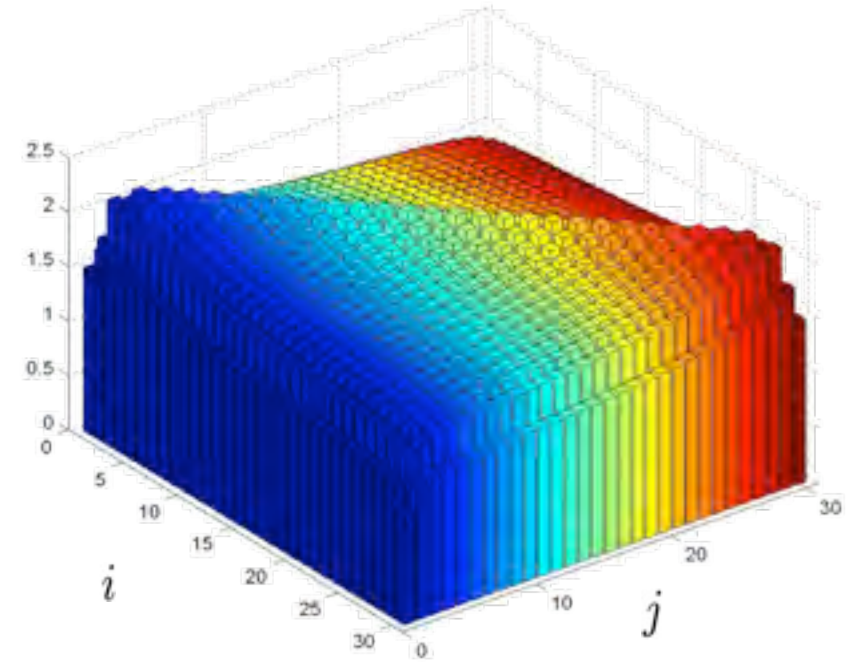
$$\hat{H} = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

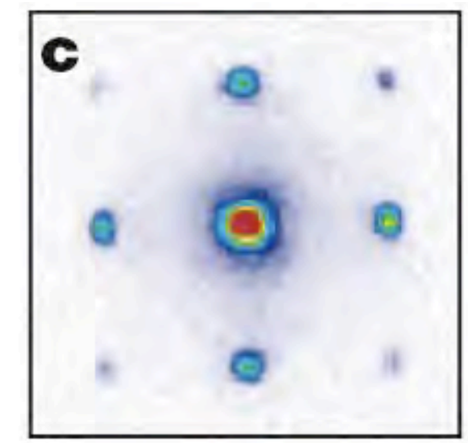
$$\langle b_i^\dagger b_j \rangle$$

Momentum Distribution

- **Superfluid** J>>U

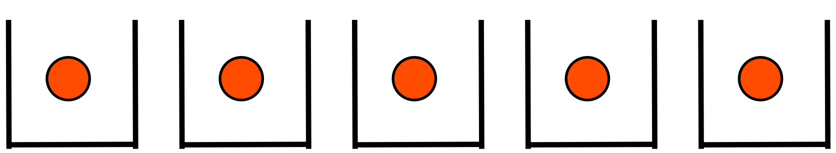


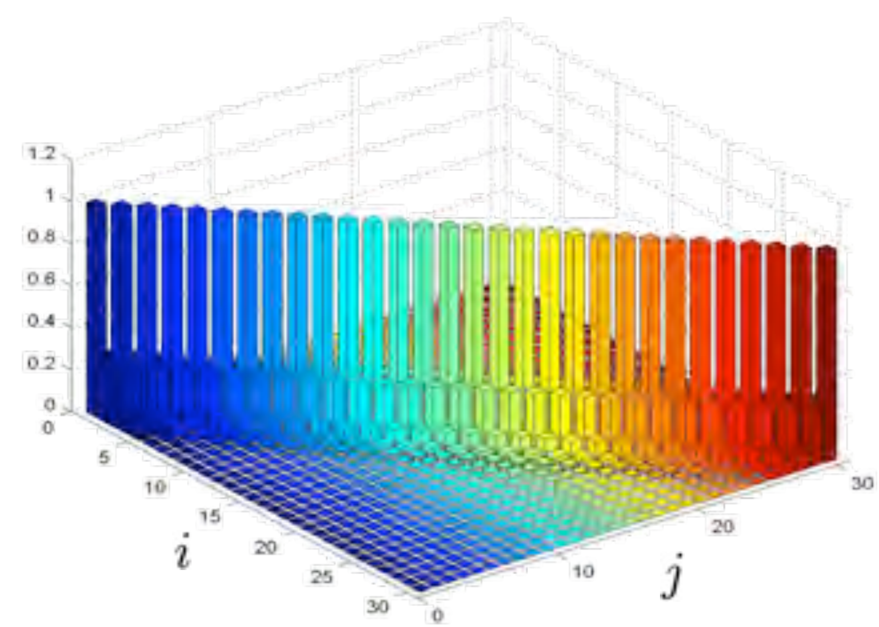


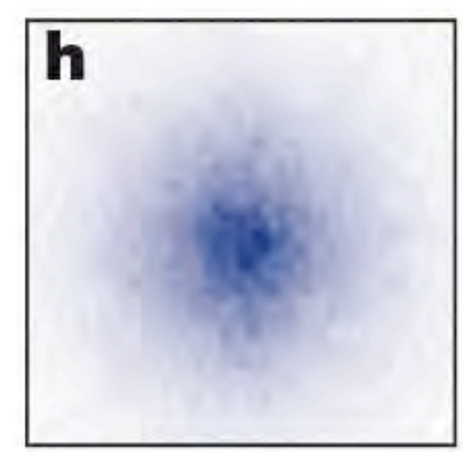


delocalised atoms: BEC

- **Mott Insulator** J<<U







"atoms lower their energy by minimising the number of atoms per site

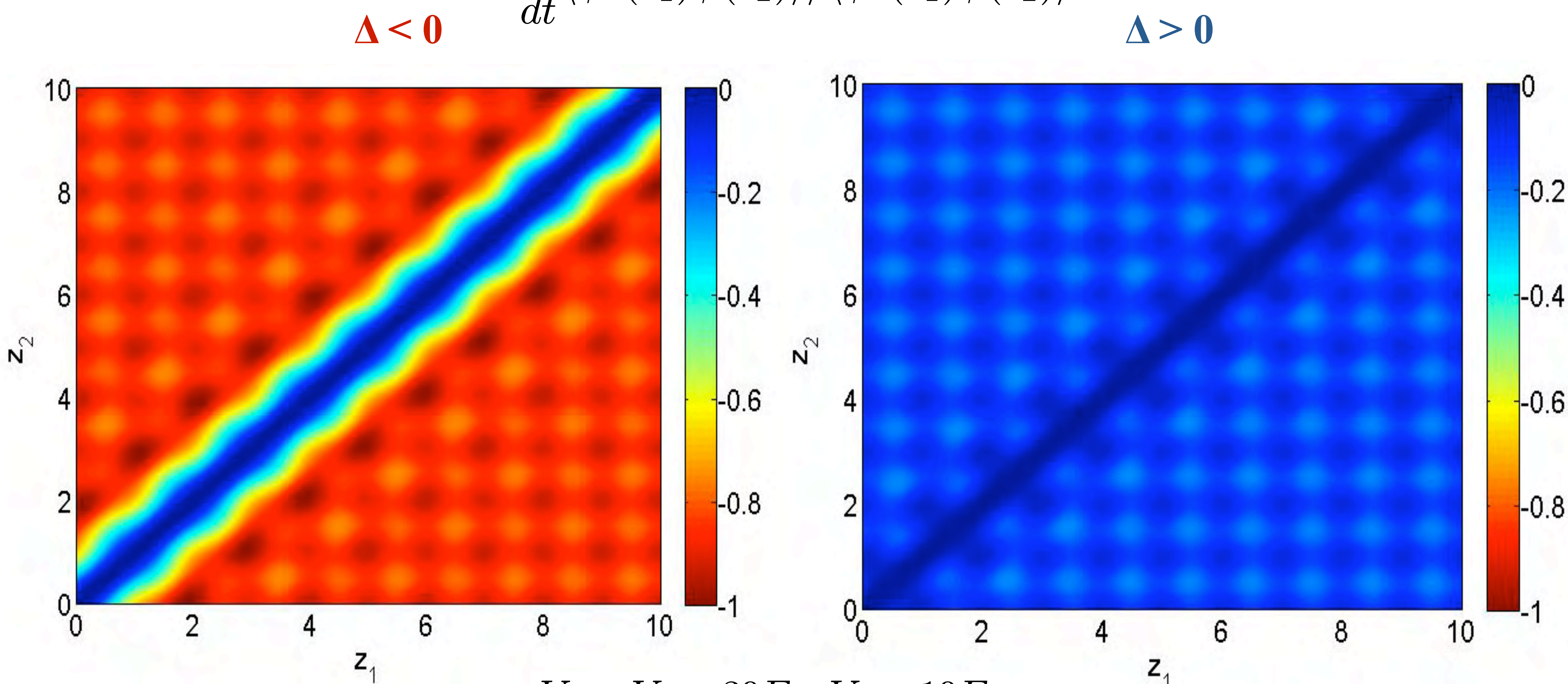
(Greiner et al., Nature '02)

Decay of characteristic correlations

H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)

- 1st order perturbation theory calculation (in $\Gamma_{\rm eff}/J$), neglecting interactions after a spontaneous emission event:

$$\frac{d}{dt}\langle\hat{\psi}^\dagger(z_1)\hat{\psi}(z_2)\rangle/\langle\hat{\psi}^\dagger(z_1)\hat{\psi}(z_2)\rangle$$



$$V_x = V_y = 30E_R; V_z = 10E_r$$

- Red-detuned lattice: More scattering events, more rapid localisation
- Interaction dependence: Superfluid order strongly affected, Mott Insulator relatively robust

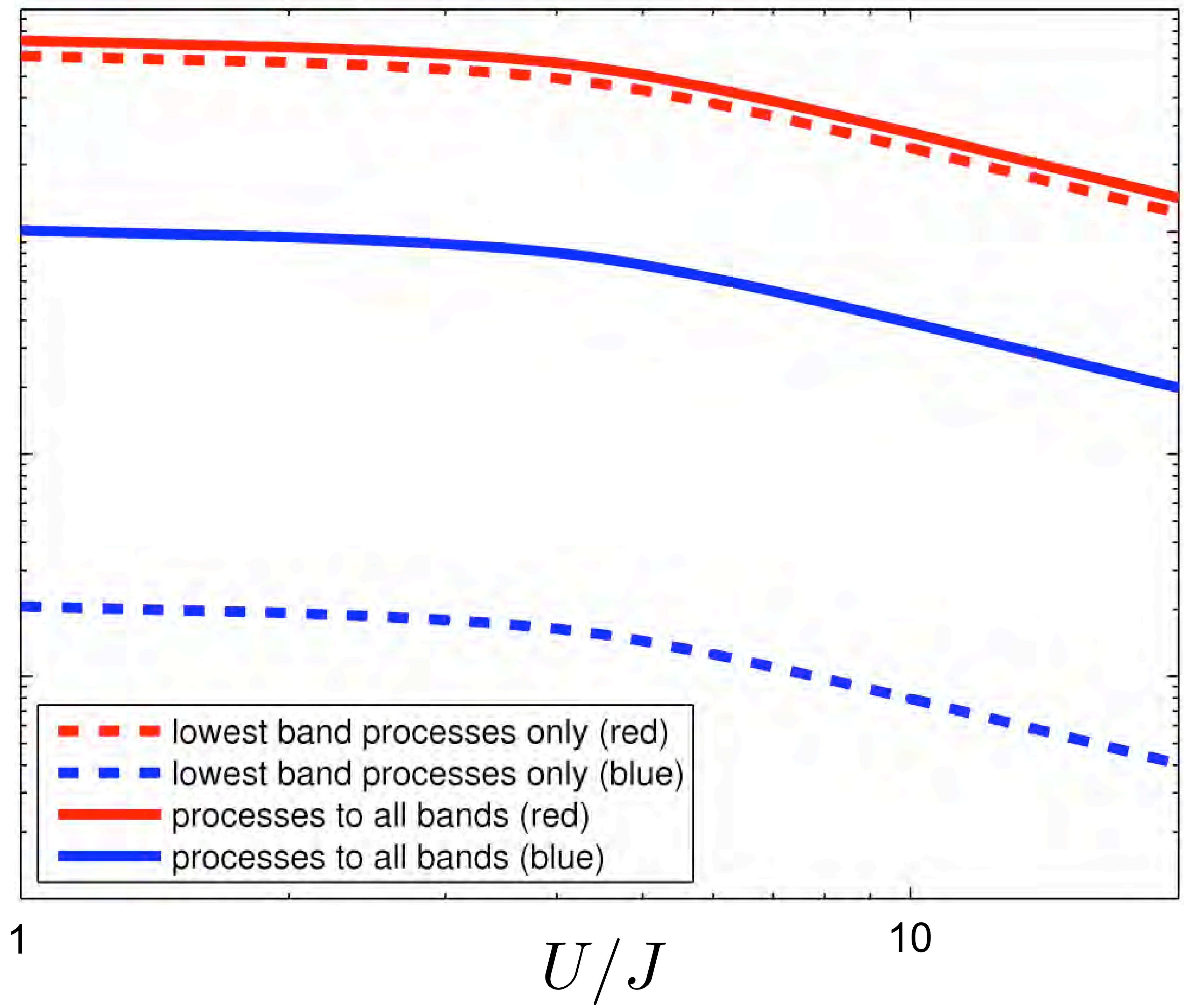
Decay of characteristic correlations

H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)

- Interaction dependence: Superfluid order strongly affected, Mott Insulator relatively robust
- 1st order perturbation theory calculation (in Γ_{eff}/J), neglecting scattering after loss:

$$\frac{d}{dt}\langle b^\dagger_{0,i} b^\dagger_{0,i+1}\rangle$$

(normalised to total decay rate, logarithmic scale)



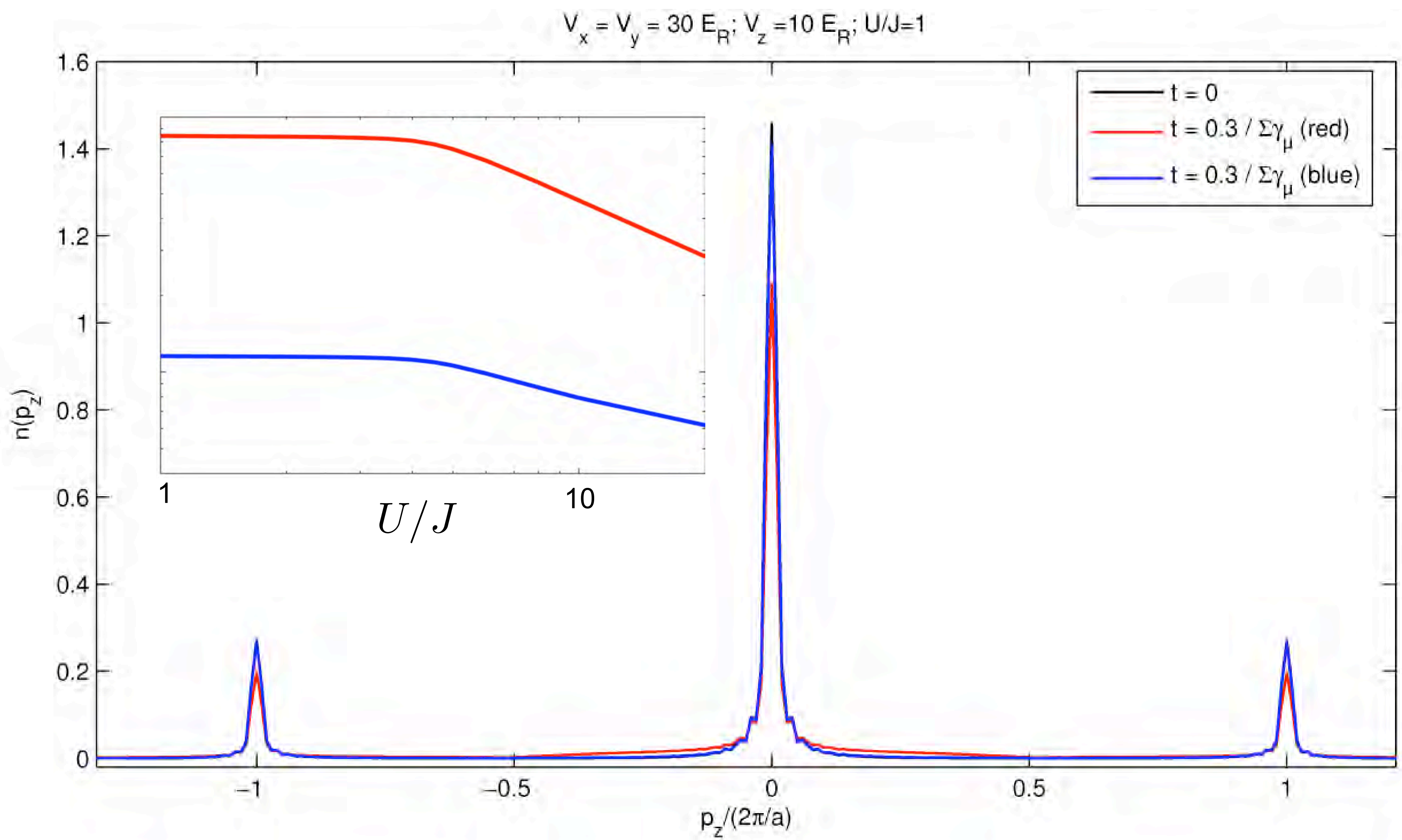
Computed using ground states in a quasi-1D lattice from t-DMRG calculations.

$V_x = V_y = 30\ E_R$; $V_z = 10\ E_R$; $(J_z = 0.0192\ E_R)$

Momentum Distributions

H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)

- We compute the corresponding decay rates of the characteristic peaks in the momentum distributions from a superfluid state (inset: decay rate, central peak)

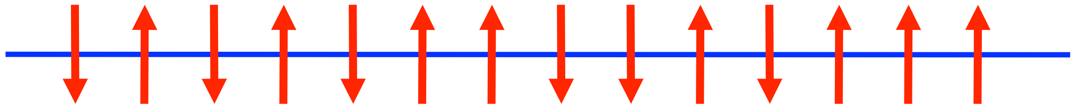


Time-dependent dynamics in 1D: t-DMRG + Quantum Trajectories

A. J. Daley et al., Phys. Rev. Lett **102**, 040402 (2009).

t-DMRG Algorithm

- Integration of many-body Schrödinger eq. in 1D on reduced Hilbert space.



- Works in 1D, near equilibrium
- Compute ground states / time evolution
- Direct determination of dynamics for typical experimental parameters

Quantum Trajectories

$$\dot{\rho} = -i[H,\rho] - \frac{\Gamma}{2}\sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger\right]$$

- Developed to treat master equations (H. Carmichael; K. Mølmer, Y. Castin, J. Dalibard; C. Gardiner & P. Zoller)
- Evolve stochastic trajectories (states)

$$H_{\text{eff}} = H - i\frac{\Gamma}{2}\sum_m c_m^\dagger c_m$$

- Quantum Jumps

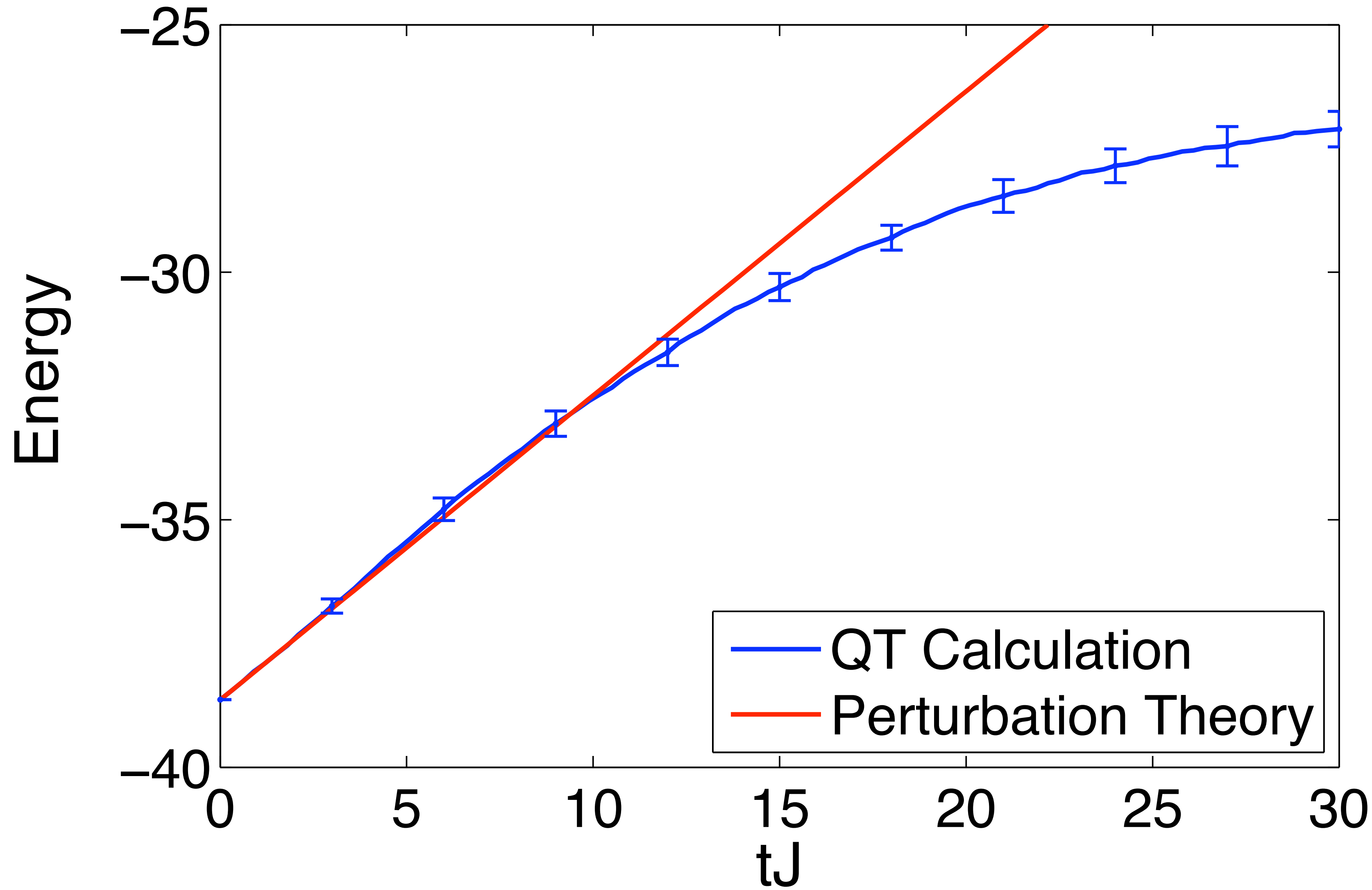
$$|\psi\rangle = \frac{c_m|\psi\rangle}{||c_m|\psi\rangle||}$$

  - Norm decays below random threshold
  - Jump operator chosen randomly

- Expectation values by stochastic average.

Full time-dependent calculation for heating (here: lowest band)

- Time-dependent DMRG + Quantum trajectories calculation



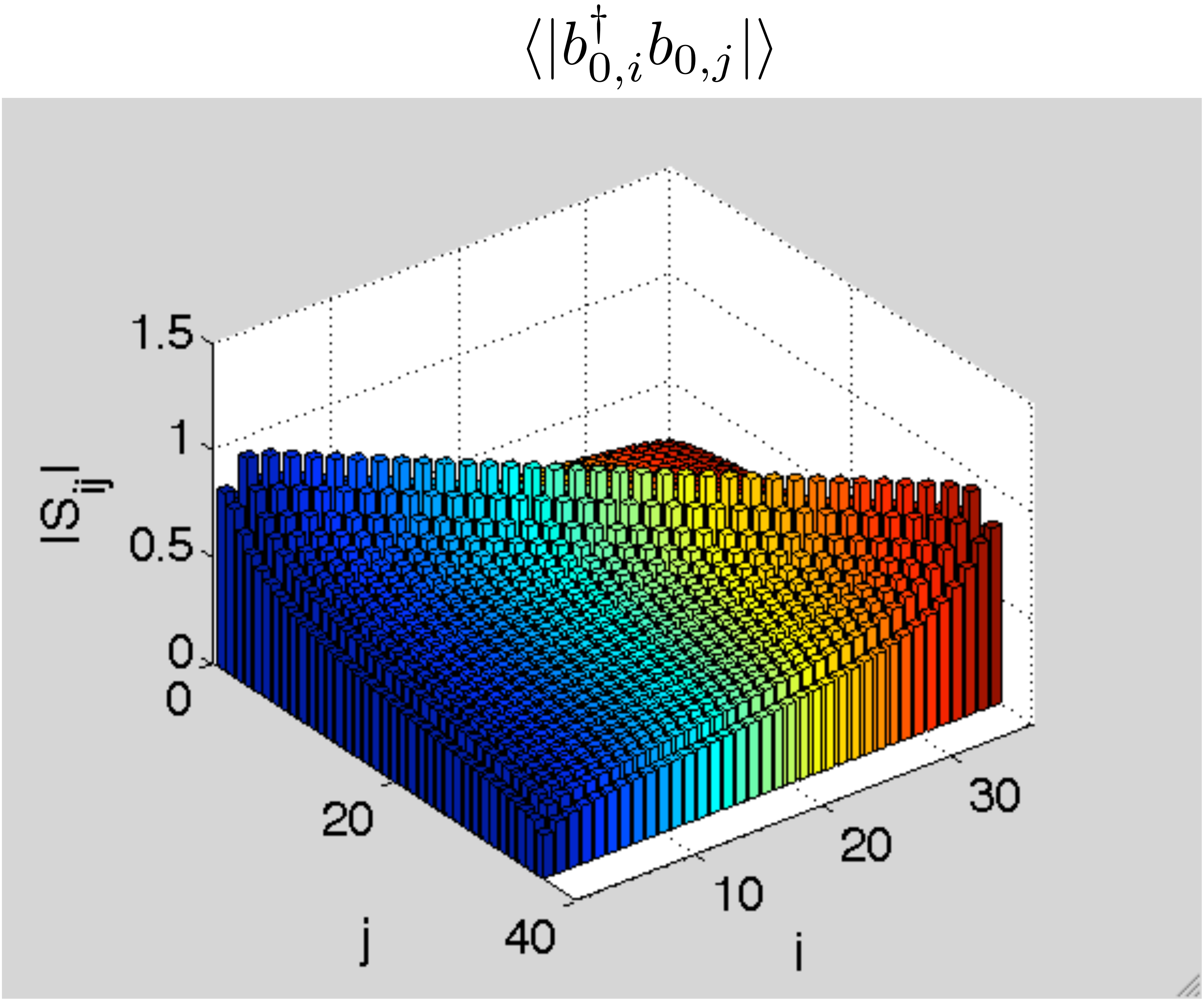
36 sites, 36 particles
U/J=3
Γ = 0.01J
1000 Trajectories

Here: we use master equation in lowest order approximation, neglecting off-diagonal terms

$$\dot{\rho} = -i\left[\hat{H}, \rho\right] + \sum_{\boldsymbol{n},\mu} \gamma_{\boldsymbol{n},\mu} \left(b^\dagger_{\boldsymbol{n},\boldsymbol{i}} b_{0,\boldsymbol{i}} \rho b^\dagger_{0,\boldsymbol{i}} b_{\boldsymbol{n},\boldsymbol{i}} - \frac{1}{2} b^\dagger_{0,\boldsymbol{i}} b_{\boldsymbol{n},\boldsymbol{i}} b^\dagger_{\boldsymbol{n},\boldsymbol{i}} b_{0,\boldsymbol{i}} \rho - \rho \frac{1}{2} b^\dagger_{0,\boldsymbol{i}} b_{\boldsymbol{n},\boldsymbol{i}} b^\dagger_{\boldsymbol{n},\boldsymbol{i}} b_{0,\boldsymbol{i}} \right)$$

band site

Decay of correlations during heating

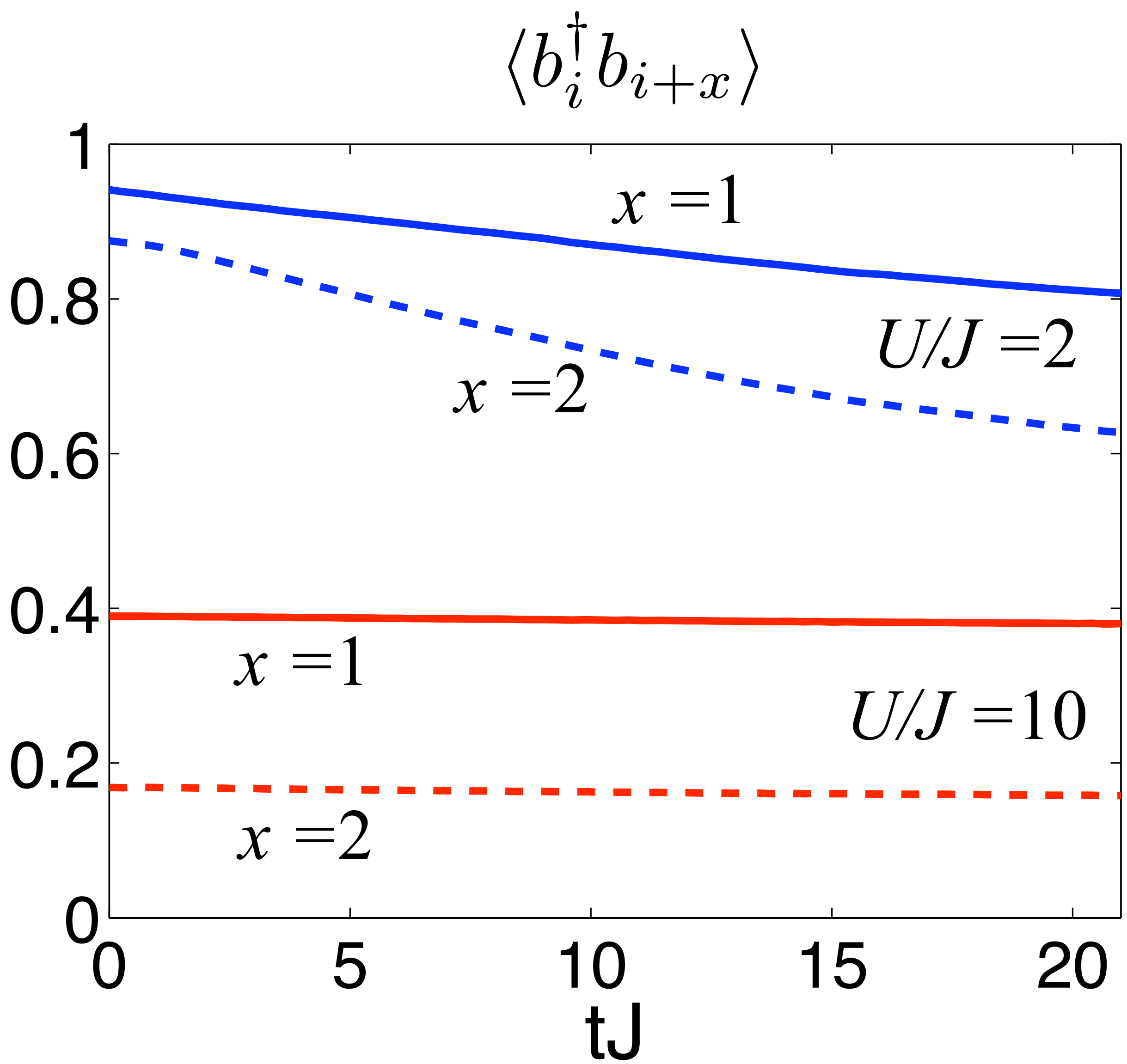


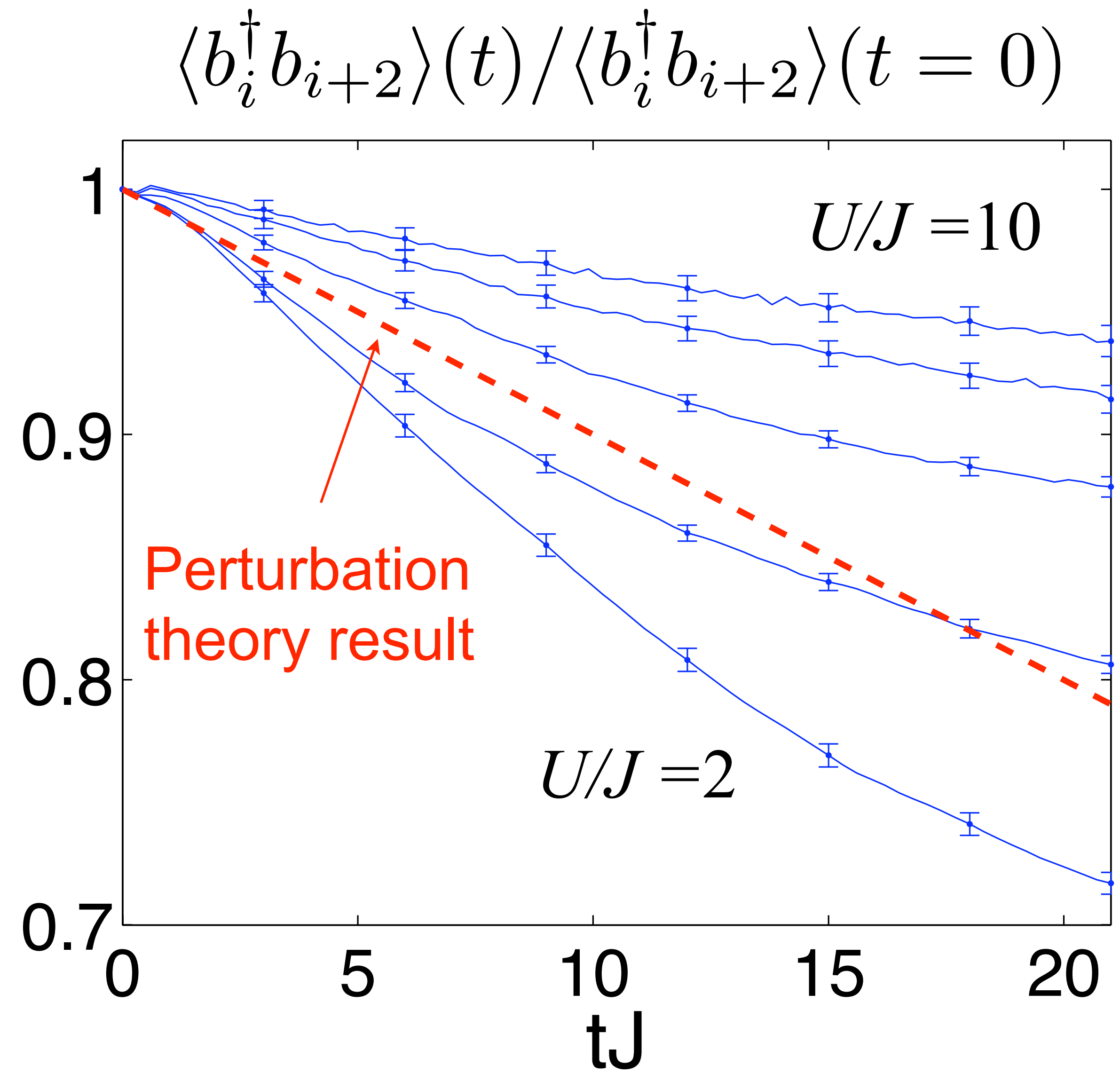
36 sites, 36 particles, U/J=3, Γ = 0.01J
1000 Trajectories, total time tJ=30

Decay of correlations during heating

H. Pichler, A. J. Daley, P. Zoller, PRA **82**, 063605 (2010)

- Time-dependent DMRG + Quantum trajectories calculation:
  Dynamics beyond the perturbation theory result, including (partial) thermalisation
- (Partial) thermalisation leads to more rapid decay of correlations in SF
- Mott Insultator is even more robust





36 sites, 36 particles
U/J=3, Γ = 0.01J
1000 Trajectories

Work in progress...

- Thermalisation of Bosons
- Light-assisted collisions
- Heating of Fermions via spontaneous emissions
- Amplitude and position fluctuations and heating

Heating due to spontaneous emissions - further work in progress.....

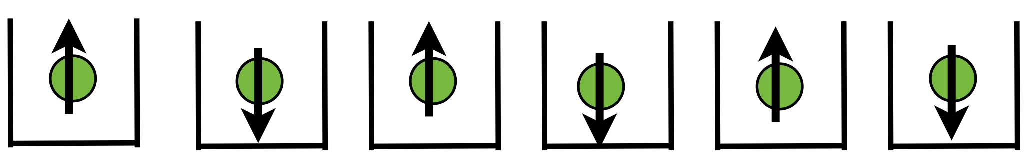
Light-assisted collisions for Bosons

- Trade-off between heating due to spontaneous emissions, and light assisted collisions in blue-detuned lattices.

Heating of Fermions via spontaneous emissions

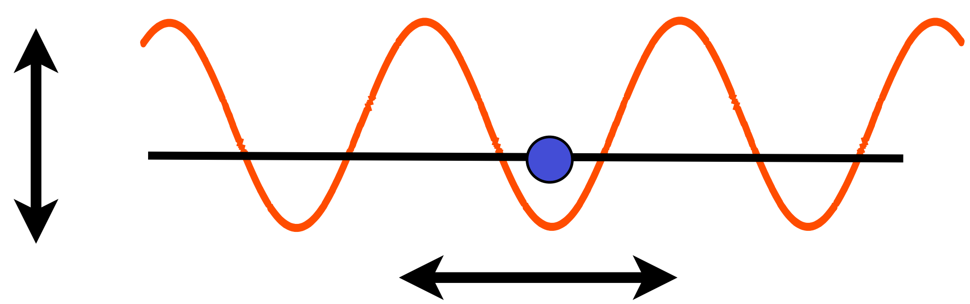
- Additional physics of internal states, spin flips / spin coherence in spontaneous emissions (coherence of nuclear spins in Li / Sr / Yb can be preserved if laser is far-detuned)

- Robustness of Mott insulator is probably good news for quantum magnetism (e.g., AF phase), *provided spontaneous emissions do not affect spin coherence*

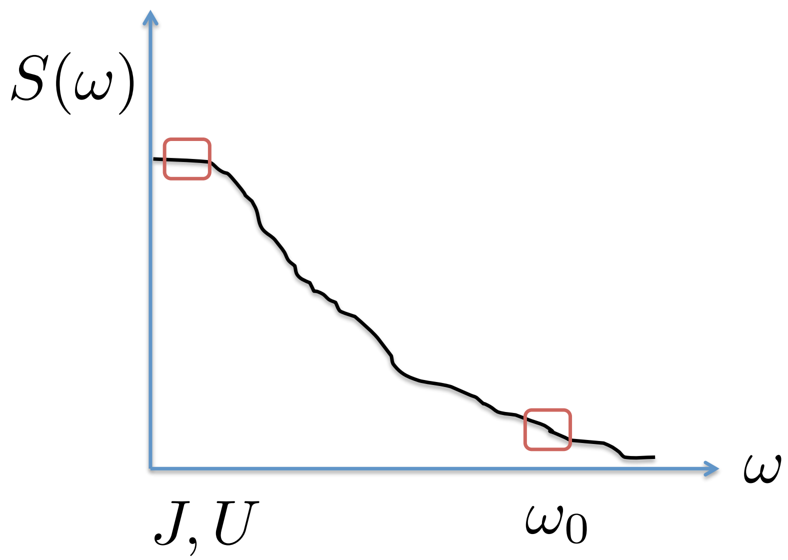


- Example Questions:
  - What happens in the weakly-interacting limit?
  - What happens away from half-filling?
  - What happens when spin coherence is not perfectly maintained?

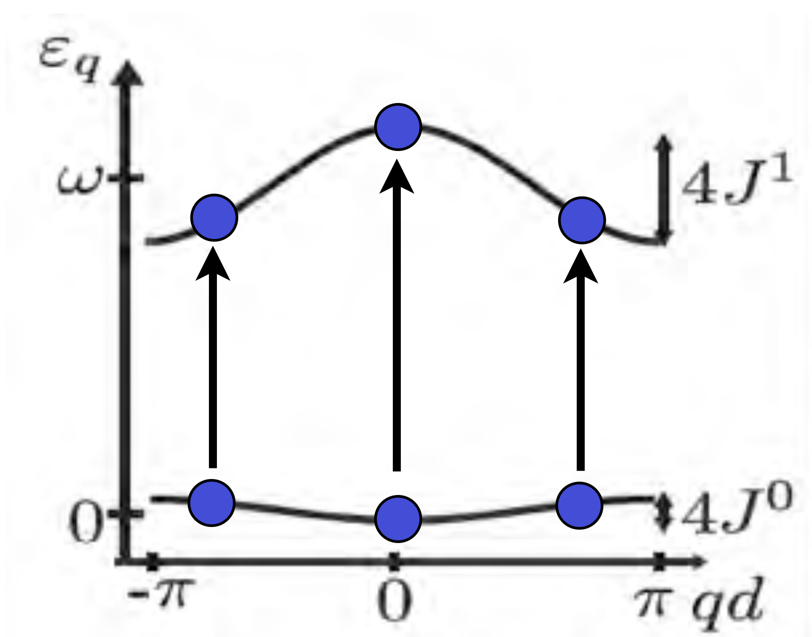
Heating due to classical noise on the lattice potential



- Amplitude noise from intensity fluctuations
- Position noise, e.g., from jittering optics



- Form of the noise spectrum dependent on experiment.
- Transitions/heating determined by noise spectrum around J, U (intraband) and the bandgap (interband)



- Spatially uniform noise - single particle transitions do not change the quasimomentum
- Heating of the system due to interband transitions relies on thermalisation between bands
- In progress: Stochastic differential equation + t-DMRG analysis

H. Pichler, C. Spee, J. Schachenmayer, P. Zoller, A. J. Daley, work in progress

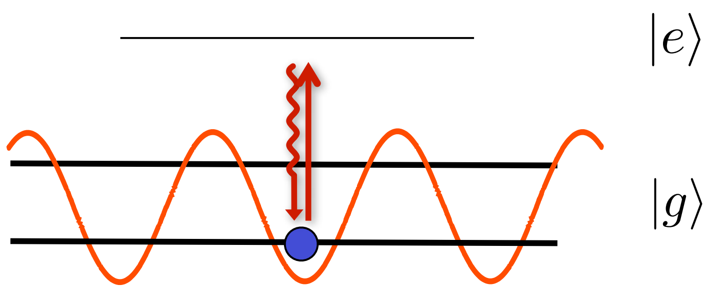
Summary / Outlook

Heating of many-body states

- Depends strongly on the **many-body state**
- Intrinsically **non-equilibrium system**
- **Mean energy is not sufficient** to characterise change in many-body properties

Heating of bosons due to spontaneous emisisons

- Breakdown of long-range order in SF due to localisation
- Stronger in red-detuned lattices (more scattering events)
- MI affected only by (rare) transitions to higher bands



Outlook:

- Thermalisation in the presence of heating (?)
- Potential for trade-off with light-assisted collisions in blue-detuned lattice
- Investigation of heating due to amplitude/phase fluctuations on lattice
- Heating of Fermions