# Lecture 2: Ultracold fermions

Fermions in optical lattices. Fermi Hubbard model.
Current state of experiments

Lattice modulation experiments

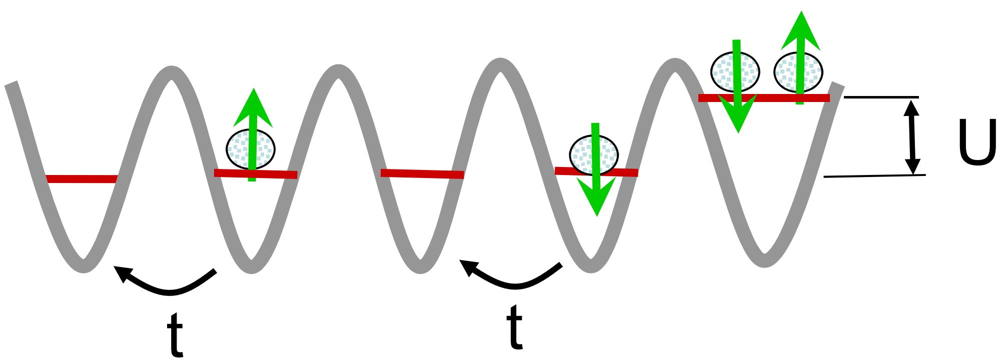
Doublon lifetimes

Stoner instability

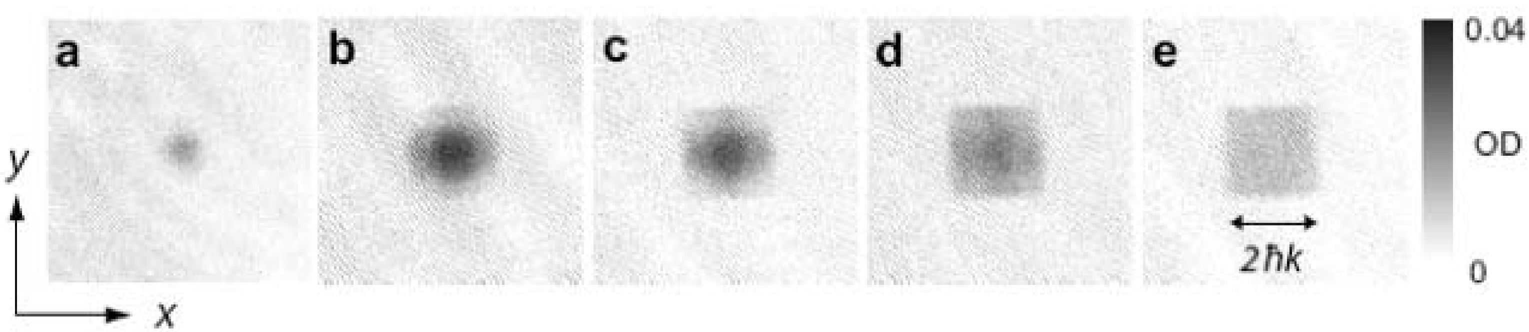
# Ultracold fermions in optical lattices

# Fermionic atoms in optical lattices

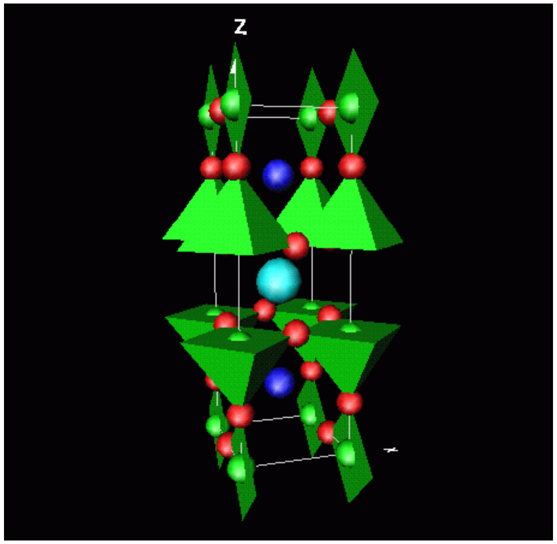
$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i}$$



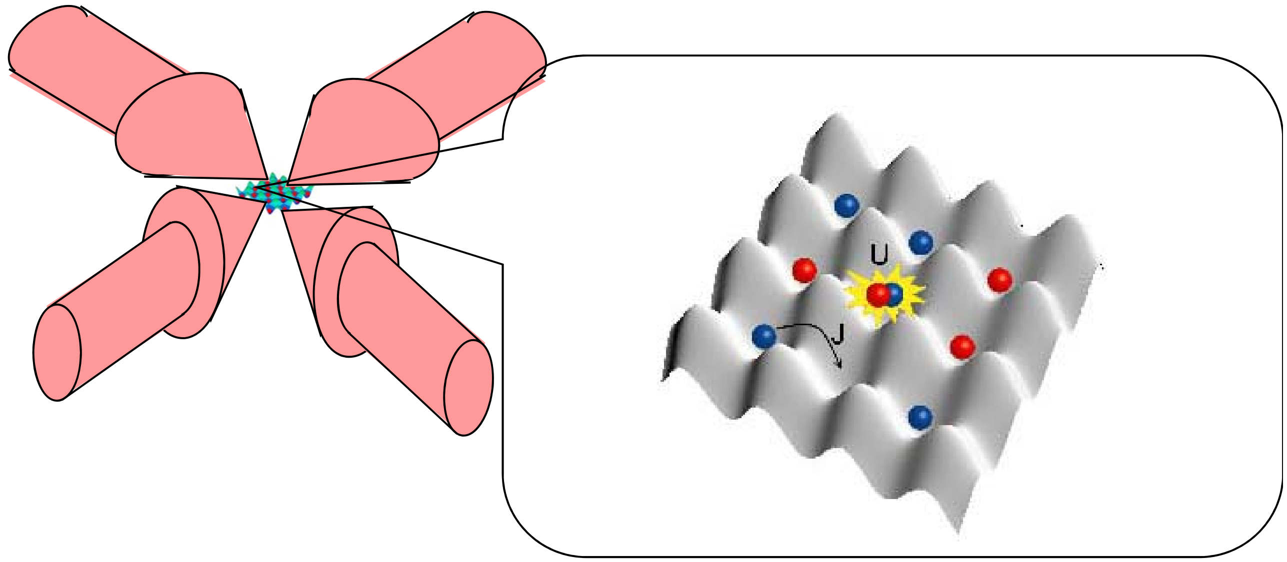

Experiments with fermions in optical lattice, Kohl et al., PRL 2005



# Quantum simulations with ultracold atoms




$YBa_2Cu_3O_7$




Atoms in optical lattice

Antiferromagnetic and superconducting Tc of the order of 100 K

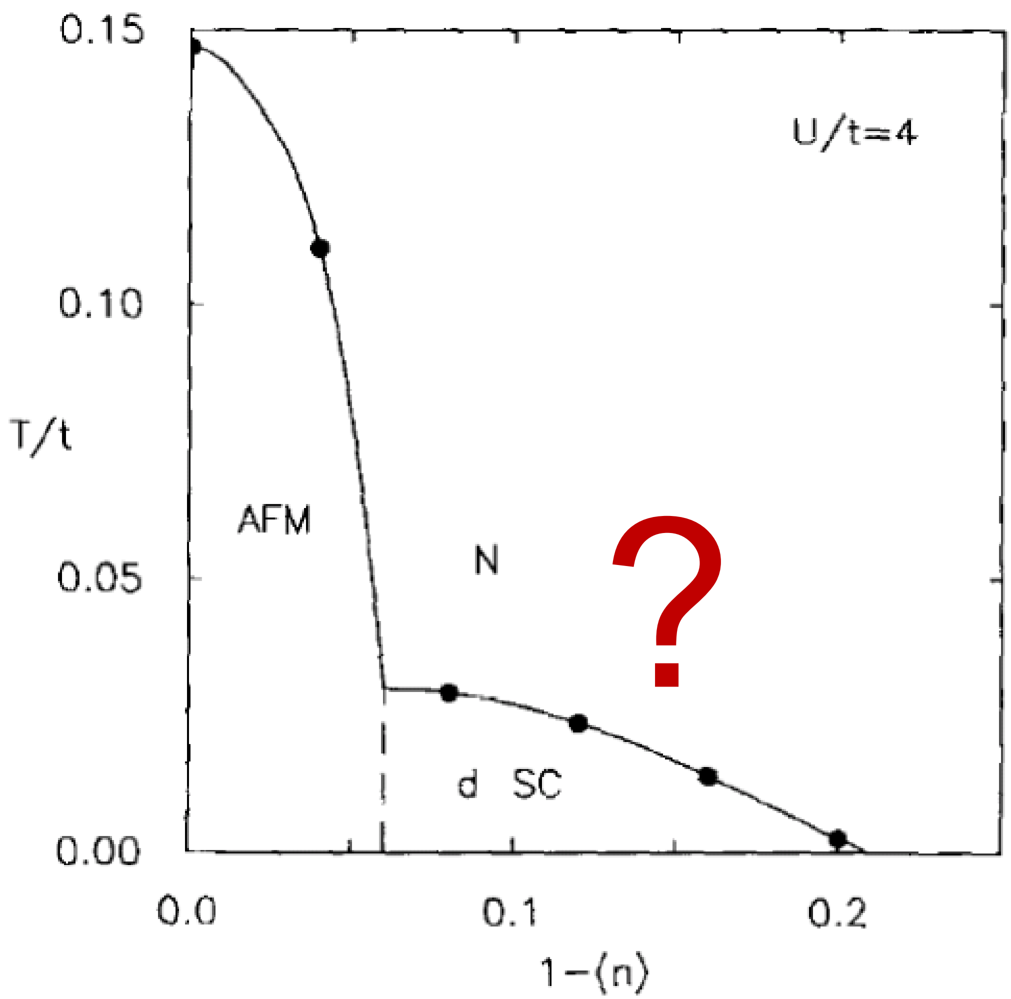
Antiferromagnetism and pairing at sub-micro Kelvin temperatures

## Same microscopic model

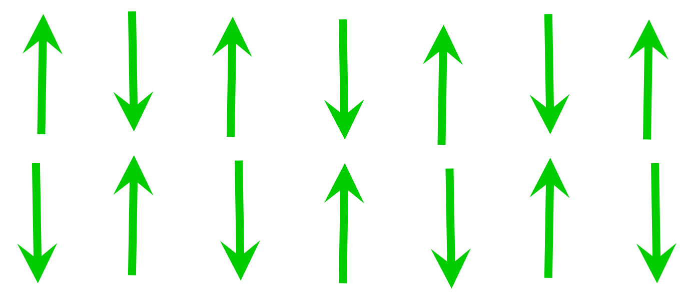
$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i}$$

# Positive U Hubbard model

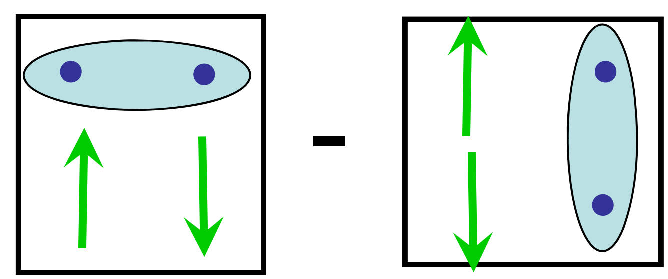
**Possible phase diagram. Scalapino, Phys. Rep. 250:329 (1995)**



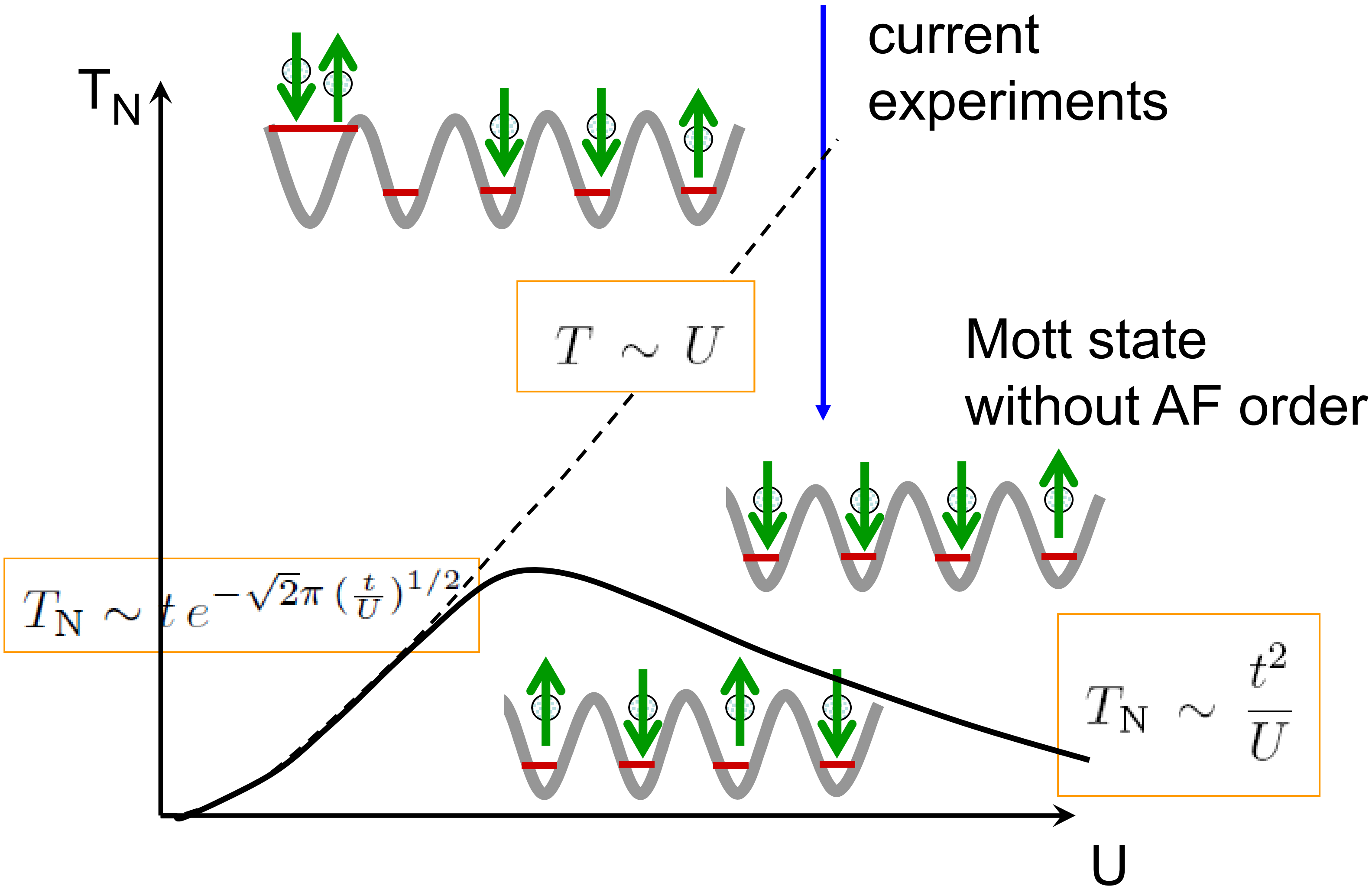

**Antiferromagnetic insulator**



**D-wave superconductor**

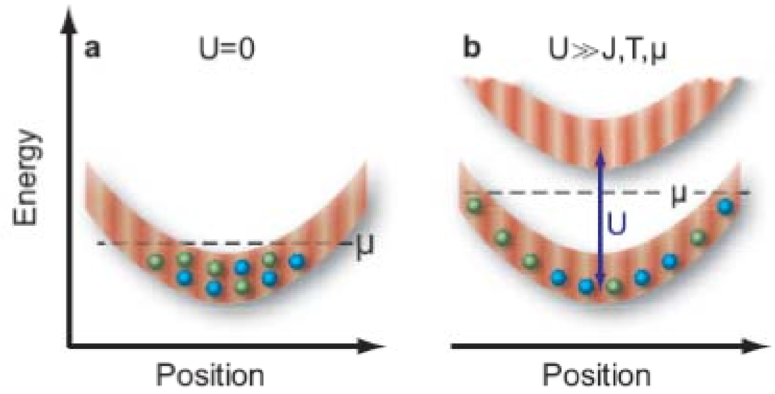


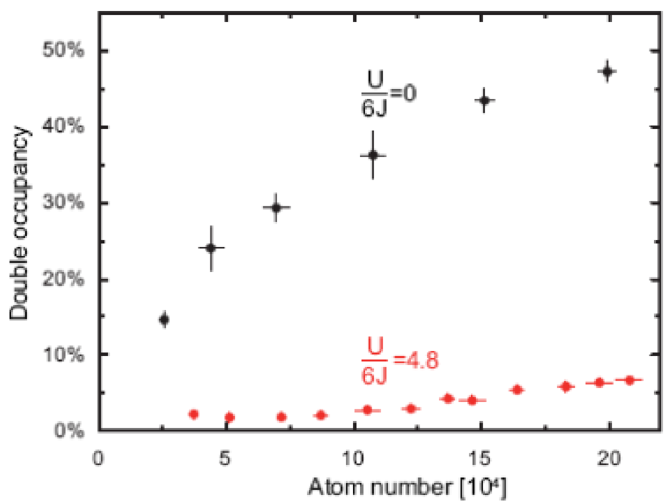
# Repulsive Hubbard model at half-filling



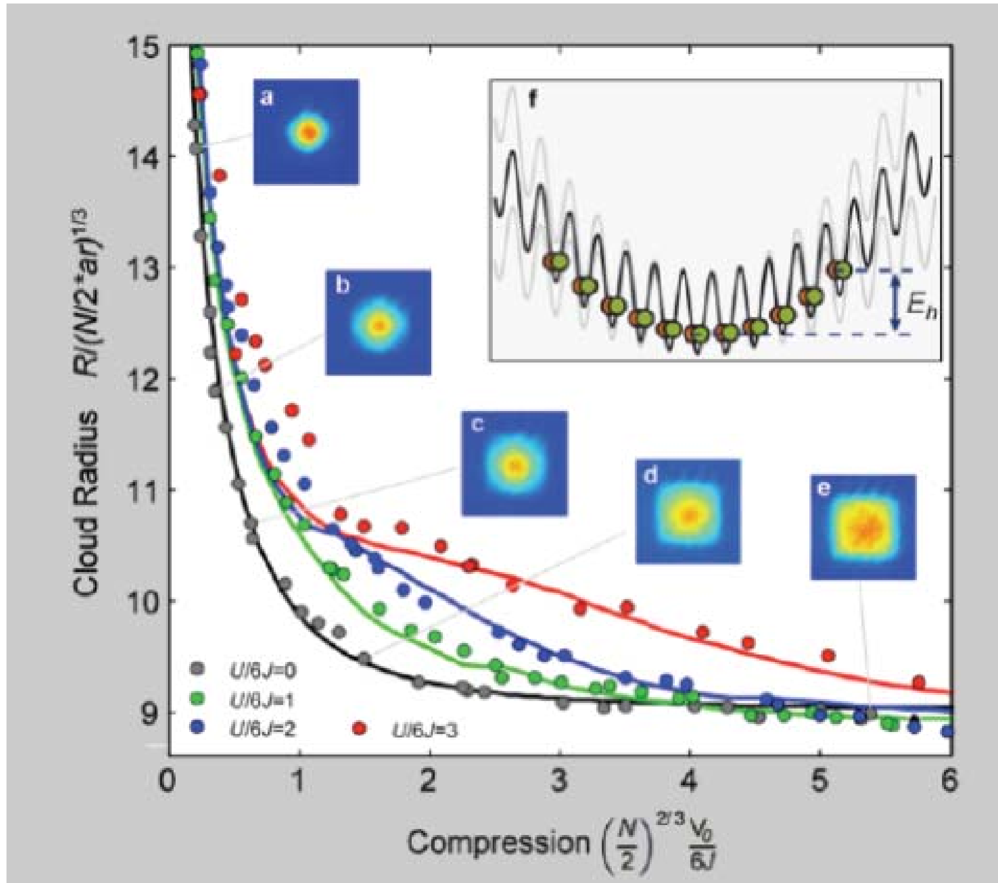
# Signatures of incompressible Mott state of fermions in optical lattice

Suppression of double occupancies
R. Joerdens et al., Nature (2008)

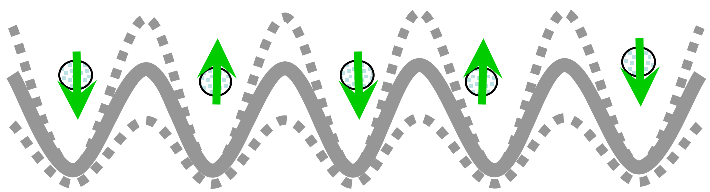



Compressibility measurements
U. Schneider et al., Science (2008)



# Lattice modulation experiments with fermions in optical lattice.



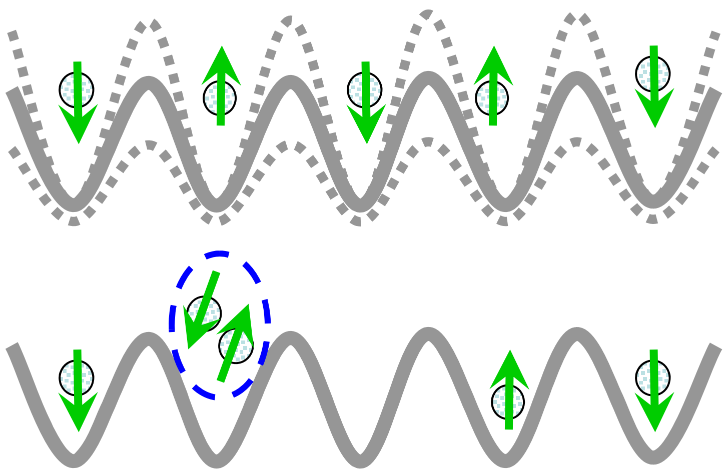
## Probing the Mott state of fermions

Sensarma, Pekker, Lukin, Demler, PRL (2009)

Related theory work: Kollath et al., PRL (2006)
Huber, Ruegg, PRB (2009)
Orso, Iucci, et al., PRA (2009)

# Lattice modulation experiments

## Probing dynamics of the Hubbard model



Modulate lattice potential $V_0$

Measure number of doubly occupied sites

$$t \sim \exp(-\sqrt{V_0/E_R}) \qquad U \sim \left(\frac{V_0}{E_R}\right)^{3/4}$$

Main effect of shaking: modulation of tunneling

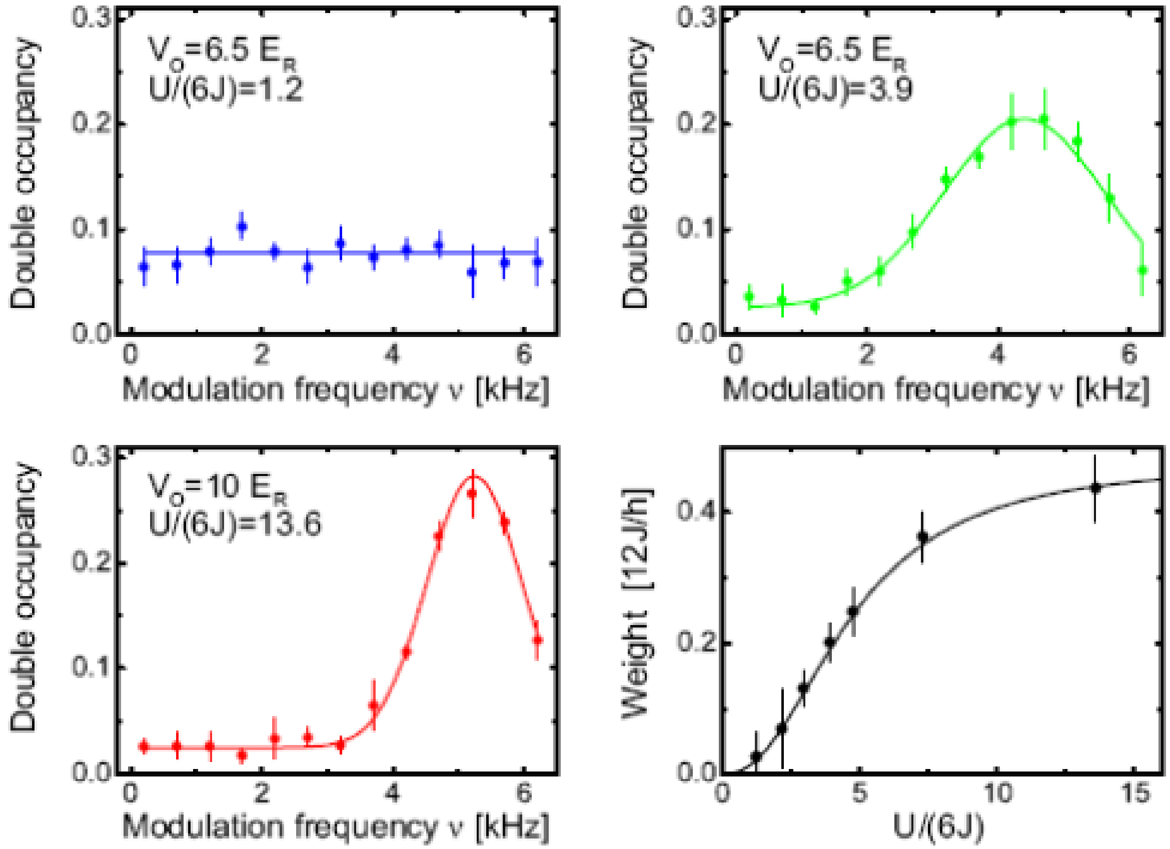
$$\mathcal{H}_{\text{pert}}(\tau) = \lambda t \cos\omega\tau \sum_{\langle ij\rangle\sigma} c^\dagger_{i\sigma} c_{j\sigma}$$

Doubly occupied sites created when frequency $\omega$ matches Hubbard $U$

# Lattice modulation experiments

## Probing dynamics of the Hubbard model

R. Joerdens et al., Nature 455:204 (2008)



# Mott state

Regime of strong interactions *U>>t.*

Mott gap for the charge forms at $T \sim U$

Antiferromagnetic ordering at $T_N \sim J = \frac{4t^2}{U}$

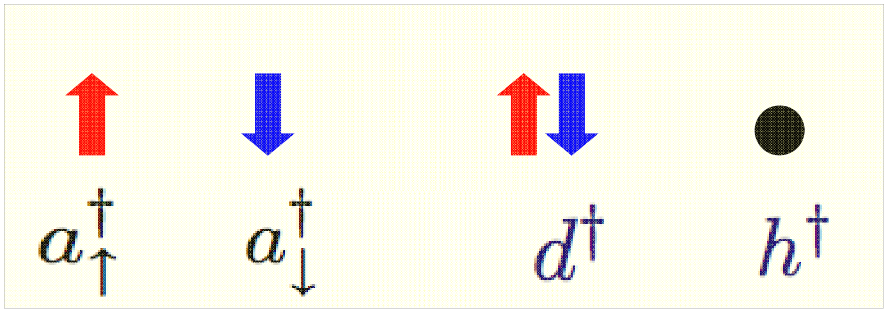
"High" temperature regime $T_N << T << U$

All spin configurations are equally likely.
Can neglect spin dynamics.

"Low" temperature regime $T \leq T_N$

Spins are antiferromagnetically ordered
or have strong correlations

# Schwinger bosons and Slave Fermions




Bosons Fermions

$$c_{i\sigma}^\dagger = a_{i\sigma}^\dagger h_i + \sigma a_{i-\sigma} d_i^\dagger$$

Constraint :

$$a_{i\sigma}^\dagger a_{i\sigma} + d_i^\dagger d_i + h_i^\dagger h_i = 1$$

Singlet Creation $$A_{ij}^\dagger = a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger - a_{i\downarrow}^\dagger a_{j\uparrow}^\dagger$$

Boson Hopping $$F_{ij}^\dagger = a_{i\uparrow}^\dagger a_{j\uparrow} + a_{i\downarrow}^\dagger a_{j\downarrow}$$

# Schwinger bosons and slave fermions

Fermion hopping

$$c_{i\uparrow}^\dagger c_{j\uparrow} + c_{i\downarrow}^\dagger c_{j\downarrow} + \text{h.c.} = (d_i^\dagger d_j - h_i^\dagger h_j) F_{ij} + d_i^\dagger h_j^\dagger A_{ij} + \text{h.c.}$$

Propagation of holes and doublons is coupled to spin excitations.
Neglect spontaneous doublon production and relaxation.

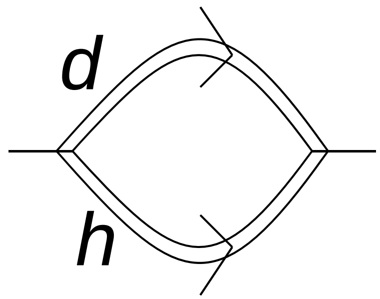
Doublon production due to lattice modulation perturbation

$$\mathcal{H}(\tau) = \lambda t \sin \omega\tau \sum_{\langle ij \rangle} (d_i^\dagger h_j^\dagger A_{ij} + \text{h.c.})$$

Second order perturbation theory. Number of doublons

$$N_d(\tau) = t^2 \lambda^2 \int_0^\tau dt' \int_0^\tau dt'' \sin[\omega t'] \sin[\omega t'']$$
$$\sum_{\langle ij \rangle \langle lm \rangle} \langle A_{ij}^\dagger(t') d_i(t') h_j(t') h_m^\dagger(t'') d_l^\dagger(t'') A_{lm}(t'') \rangle$$

# “Low” Temperature $T << T_N$



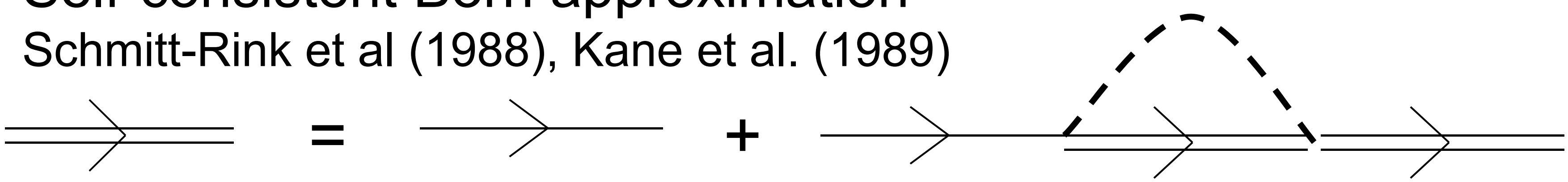

Schwinger bosons Bose condensed

Propagation of holes and doublons strongly affected by interaction with spin waves

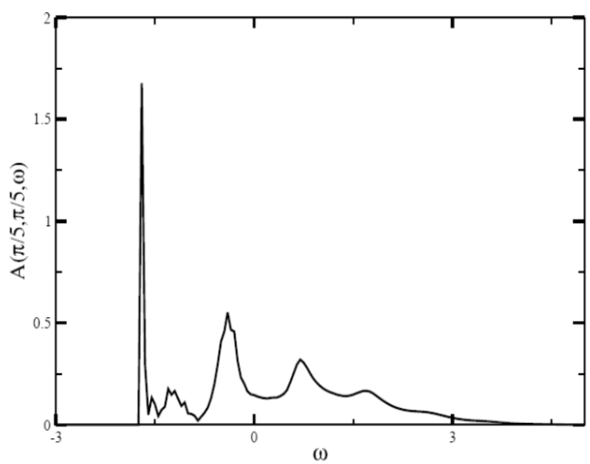
Assume independent propagation of hole and doublon (neglect vertex corrections)

## Self-consistent Born approximation

Schmitt-Rink et al (1988), Kane et al. (1989)




## Spectral function for hole or doublon



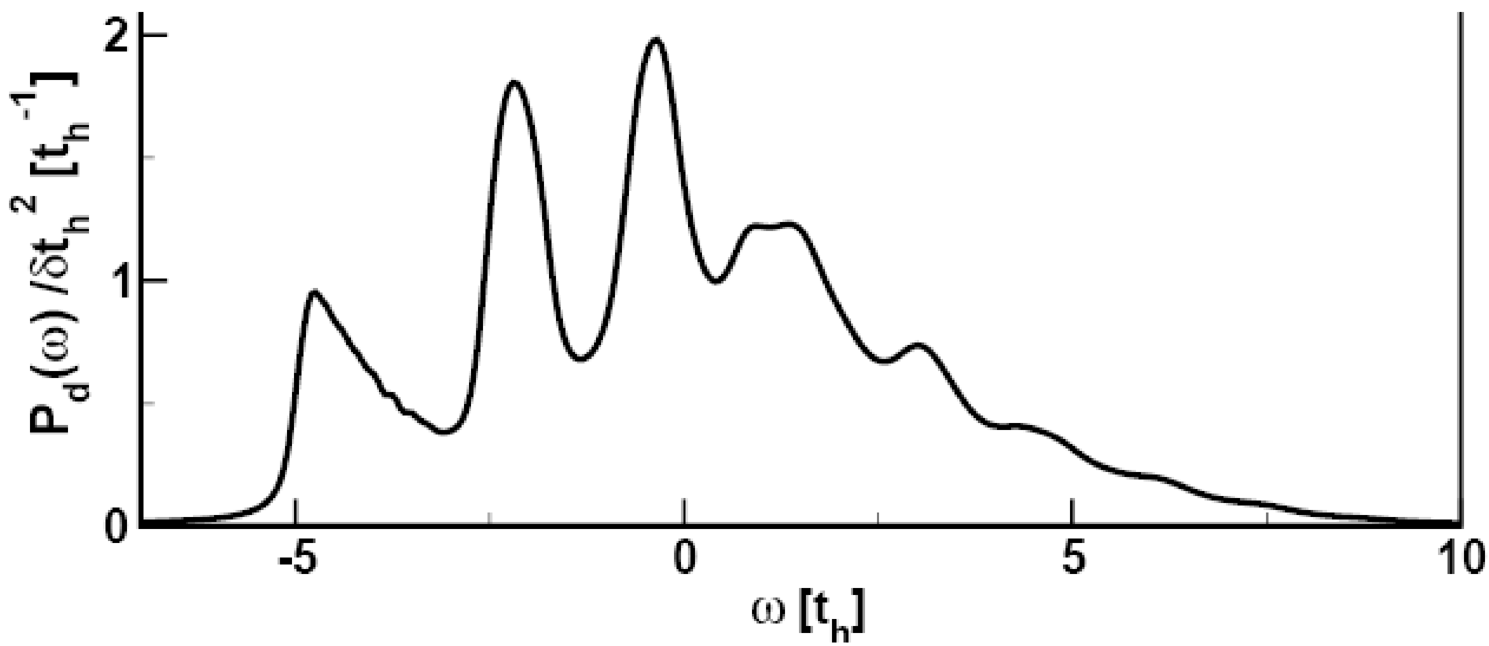

Sharp coherent part:
dispersion set by $t^2/U$, weight by $t/U$

Incoherent part:
dispersion $4\,t\ \times$ dimension

Oscillations reflect shake-off processes of spin waves

# "Low" Temperature $T << T_N$

Rate of doublon production

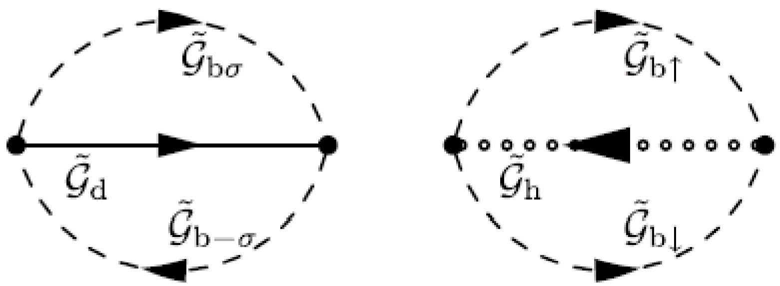



- Sharp absorption edge due to coherent quasiparticles
- Broad continuum due to incoherent part
- Spin wave shake-off peaks

# "High" Temperature

$$T_N << T << U$$

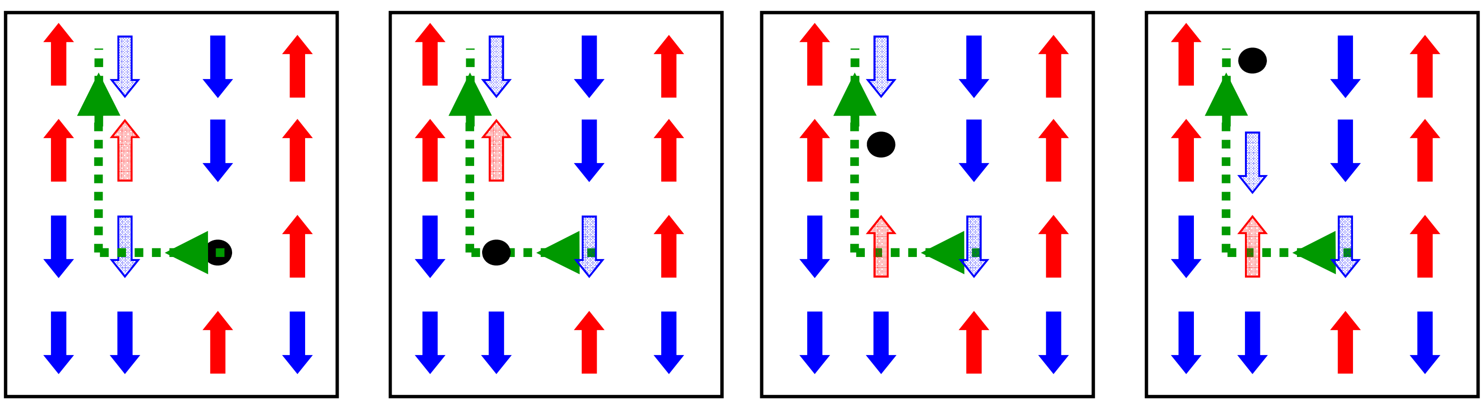
Calculate self-energy of doublons and holes interacting with incoherent spin excitations (Schwinger bosons) in the non-crossing approximation




Sensarma et al., PRL (2009)
Tokuno et al. arXIv:1106.1333

## Equivalent to Retraceable Path Approximation

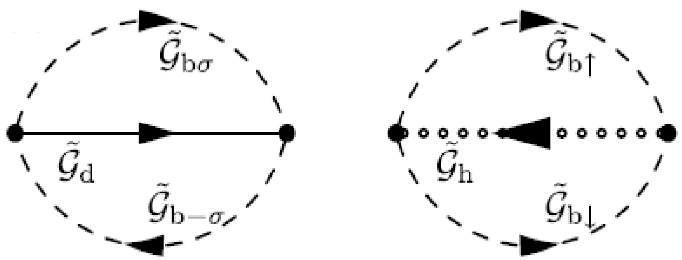
***Brinkmann & Rice, 1970***



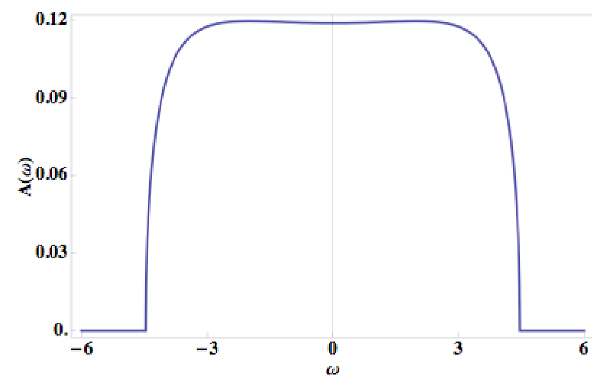
In calculating spectral function consider paths with no closed loops

# "High" Temperature

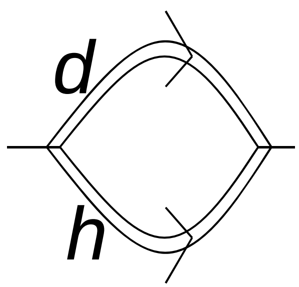
$$T_N << T << U$$

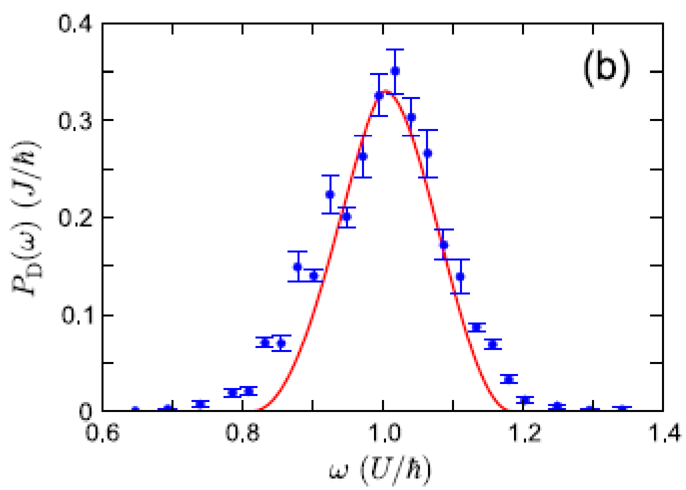



Spectral Fn. of single hole




# Doublon production rate



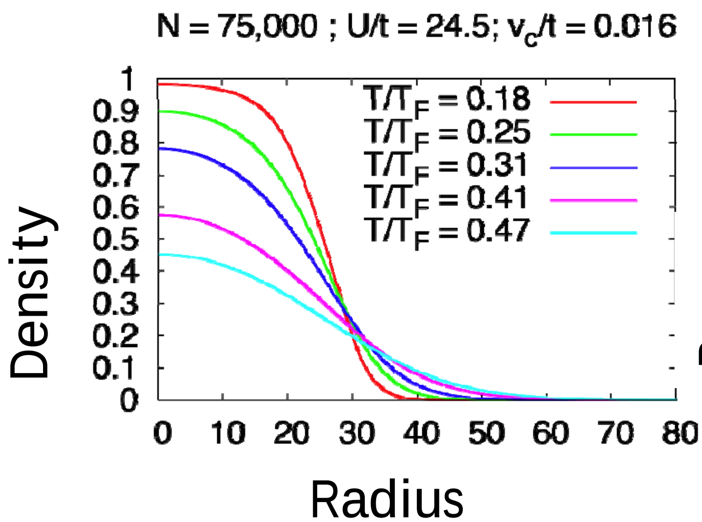



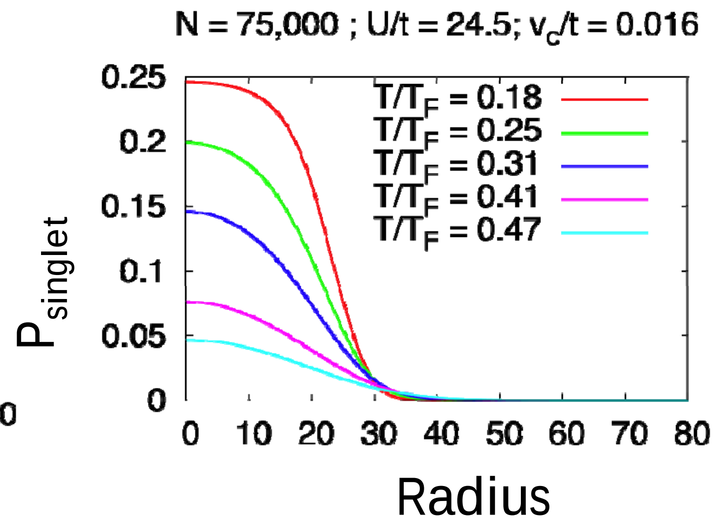

Experiment:
R. Joerdens et al.,
Nature 455:204 (2008)

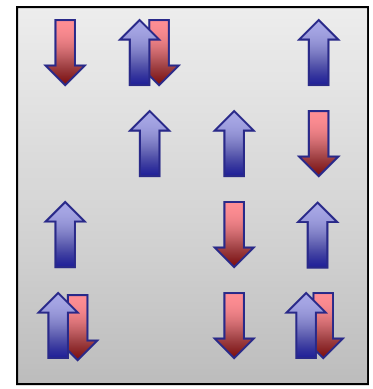
Theory:
Sensarma et al., PRL (2009)
Tokuno et al. arXIv:1106.1333

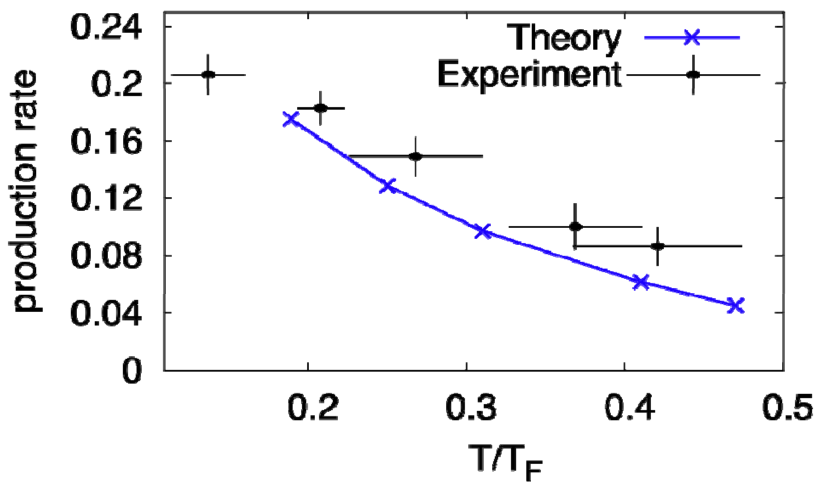
# Temperature dependence

Reduced probability to find a singlet on neighboring sites



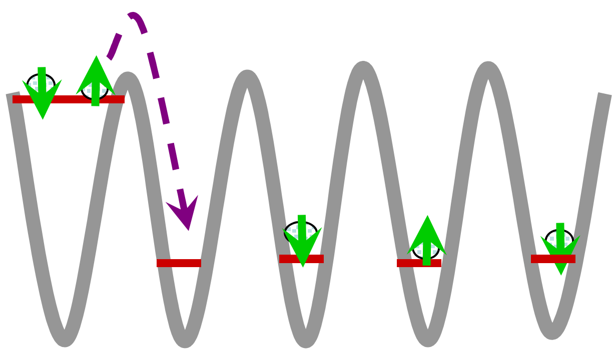






D. Pekker et al., upublished

# Fermions in optical lattice.
# Decay of repulsively bound pairs



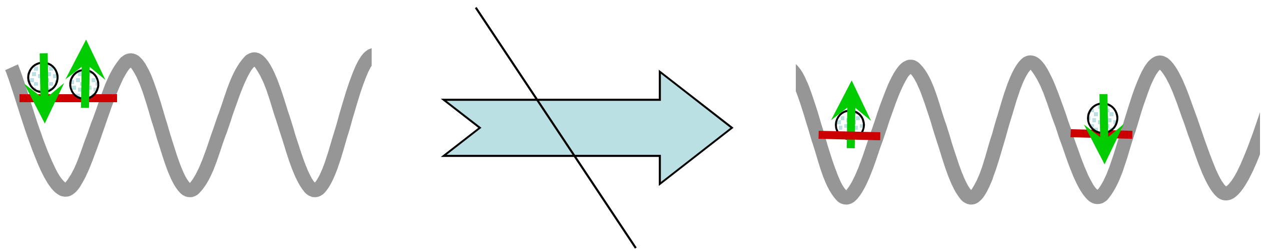
Experiments: ETH group
Theory: Sensarma, Pekker, et. al.

Ref: N. Strohmaier et al., PRL 2010

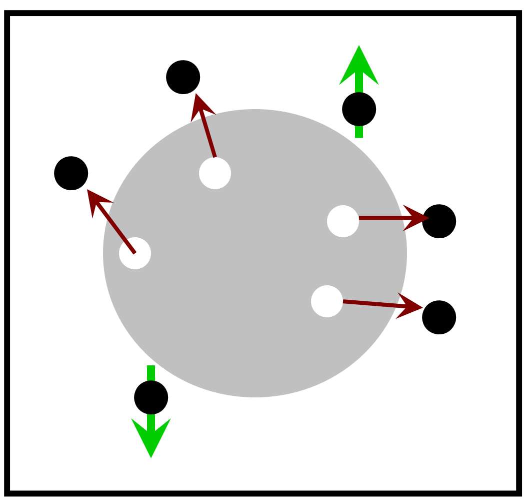
# Fermions in optical lattice.
# Decay of repulsively bound pairs

## Doublons – repulsively bound pairs

What is their lifetime?



Direct decay is not allowed by energy conservation

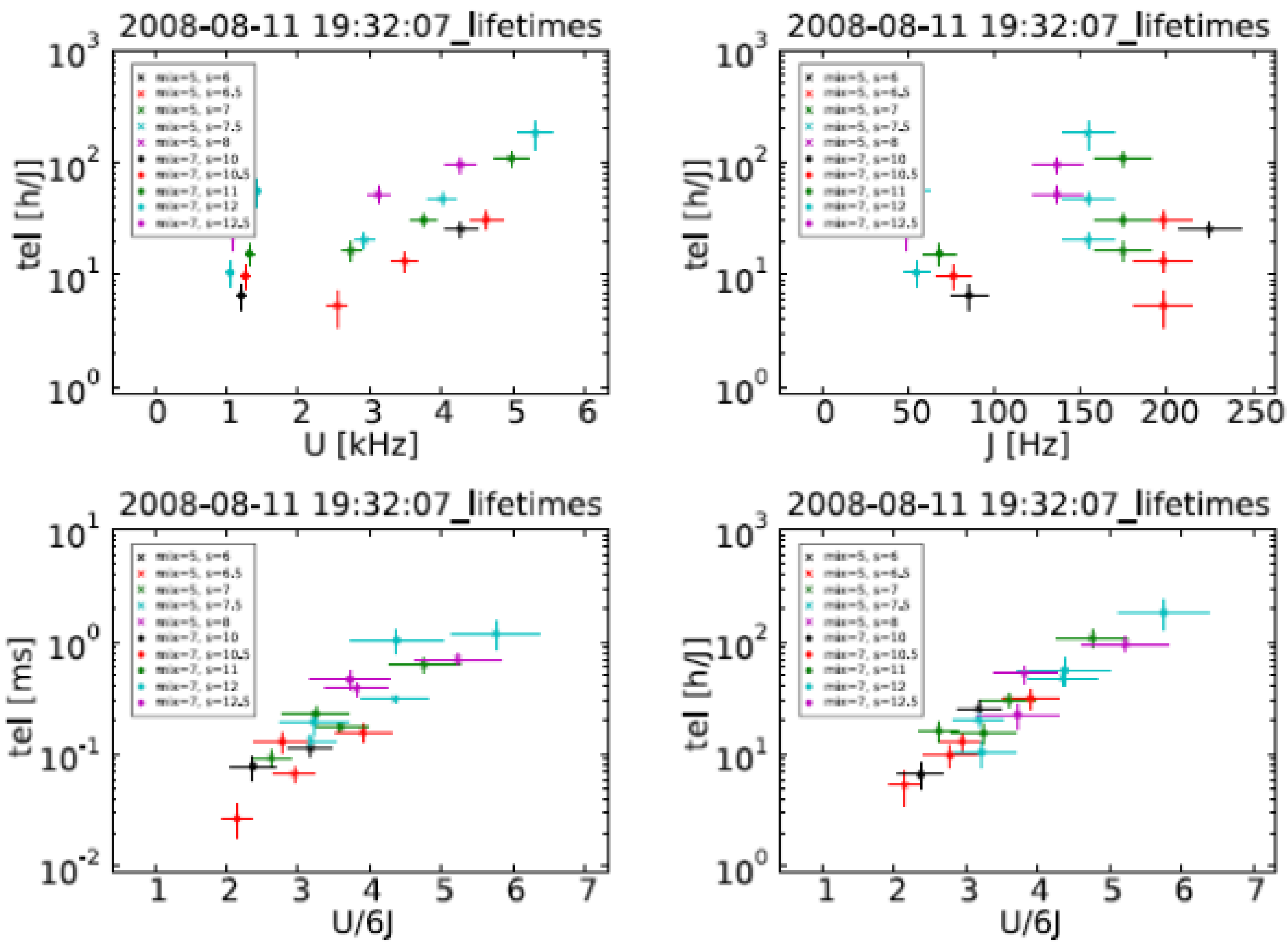


Excess energy U should be converted to kinetic energy of single atoms

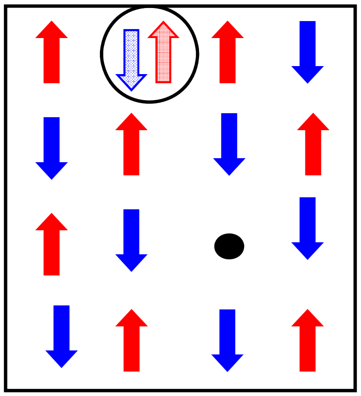
Decay of doublon into a pair of quasiparticles requires creation of many particle-hole pairs

# Fermions in optical lattice. Decay of repulsively bound pairs

Experiments: N. Strohmaier et. al.



# Relaxation of doublon- hole pairs in the Mott state



Energy U needs to be absorbed by spin excitations

❖Energy carried by

spin excitations

$\sim J = 4t^2/U$

❖ Relaxation requires

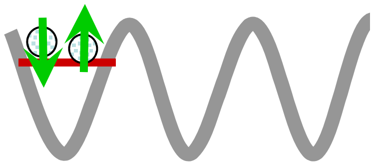
creation of $\sim U^2/t^2$

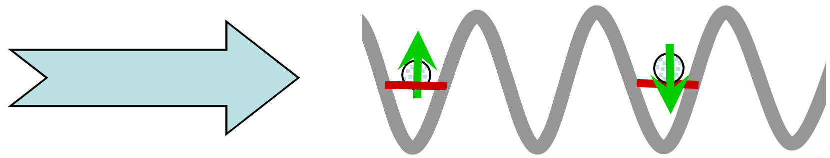
spin excitations

Relaxation rate $W \sim t(t/U)^{U^2/t^2}$ Sensarma et al., PRL 2011

Very slow, not relevant for ETH experiments

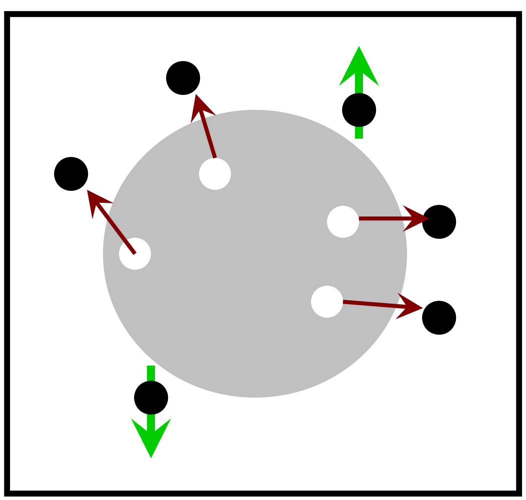
# Doublon decay in a compressible state



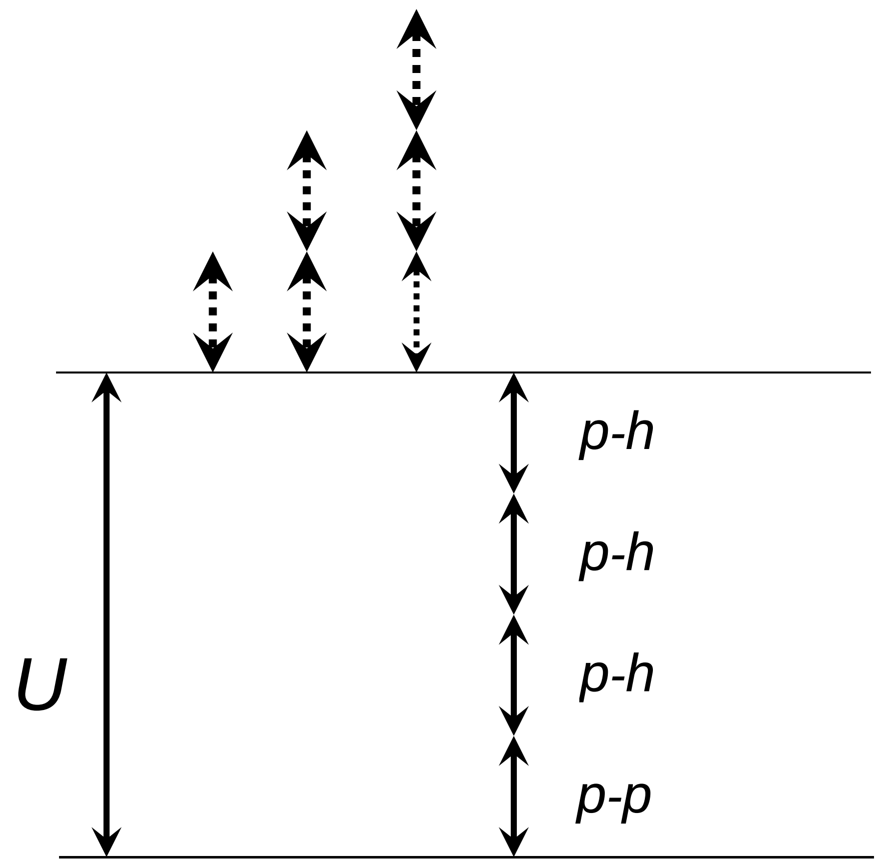


Excess energy U is converted to kinetic energy of single atoms

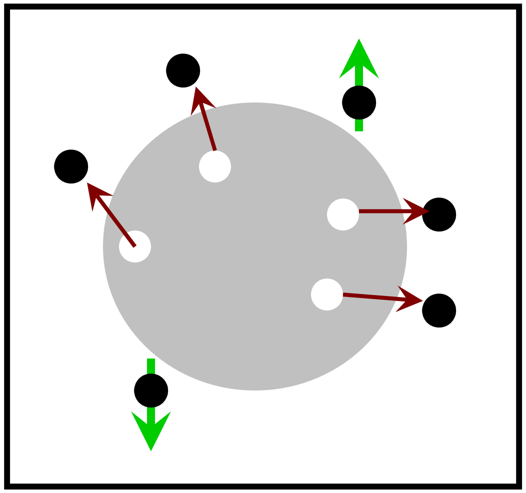
## Compressible state: Fermi liquid description



Doublon can decay into a pair of quasiparticles with many particle-hole pairs



# Doublon decay in a compressible state

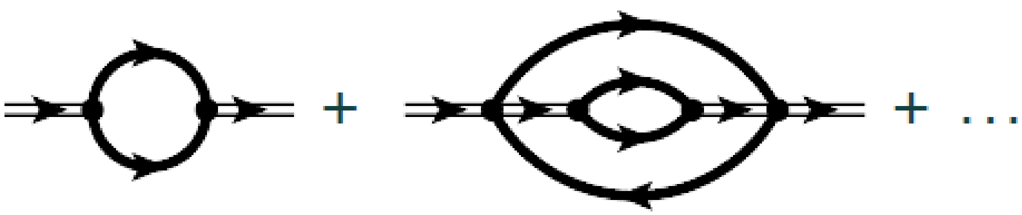


Perturbation theory to order n=U/6t

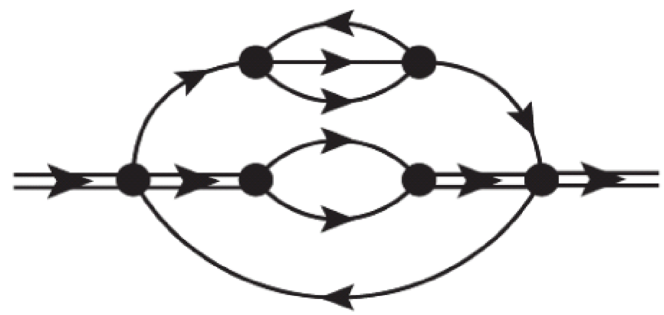
Decay probability

$$P \sim \left(\frac{t}{U}\right)^{\mathrm{const}\cdot\frac{U}{6t}} \sim e^{-\,\mathrm{const}\cdot\frac{U}{6t}\cdot\log(\frac{U}{t})}$$

**Doublon Propagator**

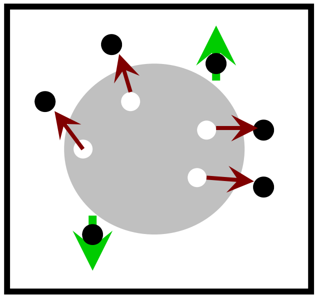


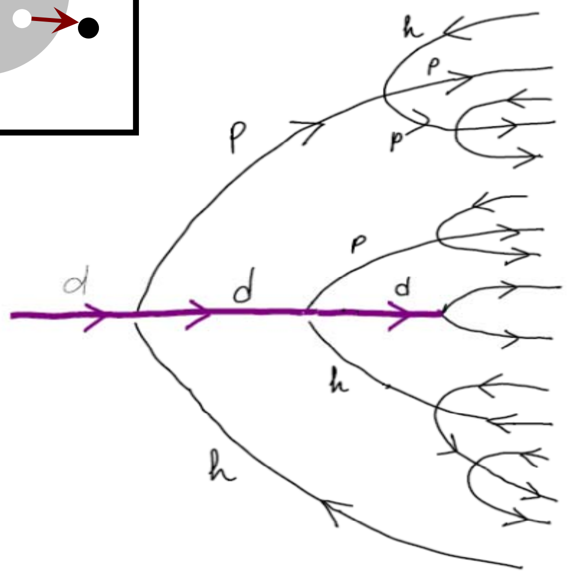
**Interacting "Single" Particles**

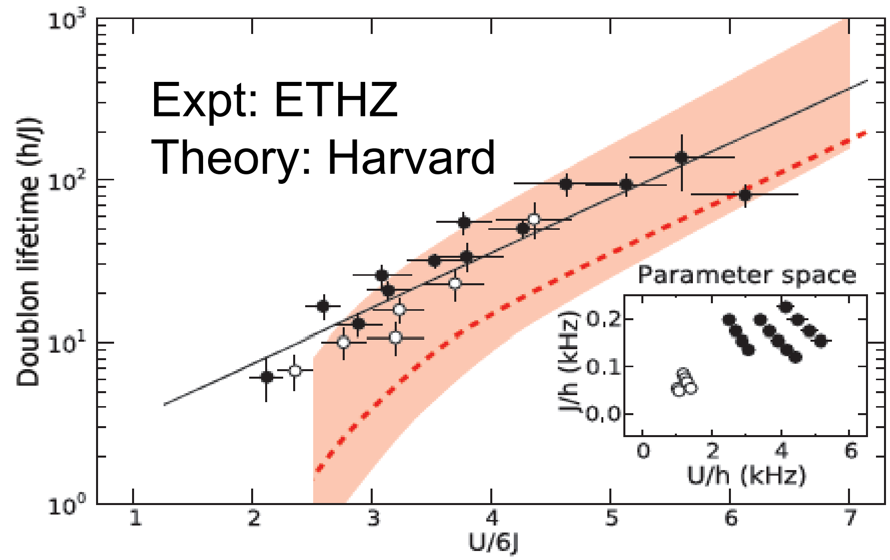


# Doublon decay in a compressible state

N. Strohmaier et al., PRL 2010






To calculate the rate: consider processes which maximize the number of particle-hole excitations

# Why understanding doublon decay rate is interesting

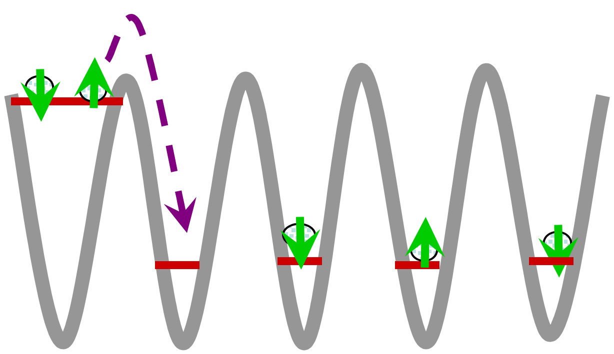
Important for adiabatic preparation of strongly correlated systems in optical lattices

Response functions of strongly correlated systems at high frequencies. Important for numerical analysis.

Prototype of decay processes with emission of many interacting particles.
Example: resonance in nuclear physics: (i.e. delta-isobar)

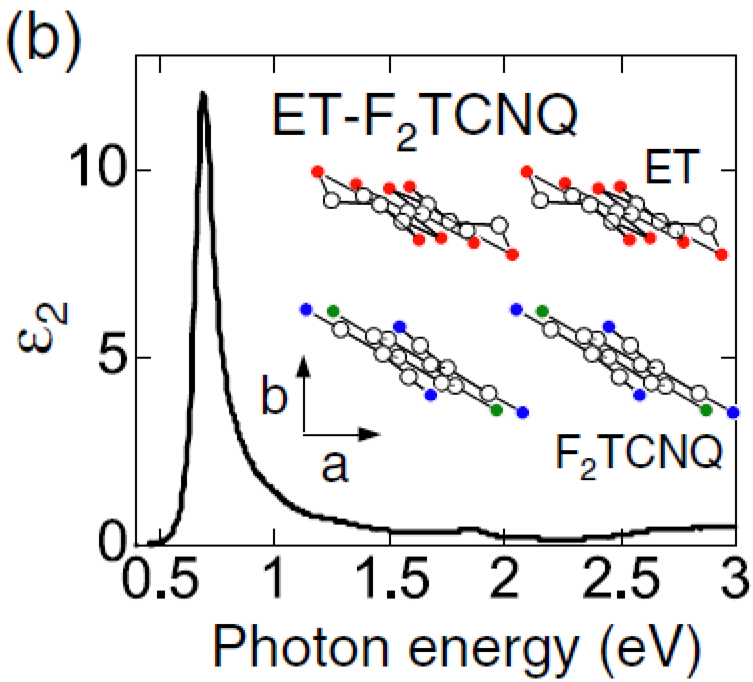
Analogy to pump and probe experiments in condensed matter systems

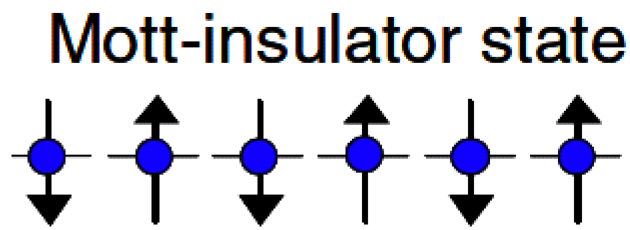
# Doublon relaxation in organic Mott insulators ET-$F_2$TCNQ



# One dimensional Mott insulator ET-$F_2$TCNQ

bis(ethylenedithio)tetrathiafulvalene difluorotetracyanoquinodimethane



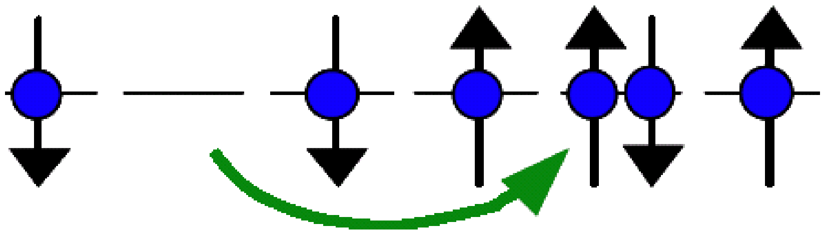

$t$=0.1 eV

$U$=0.7 eV

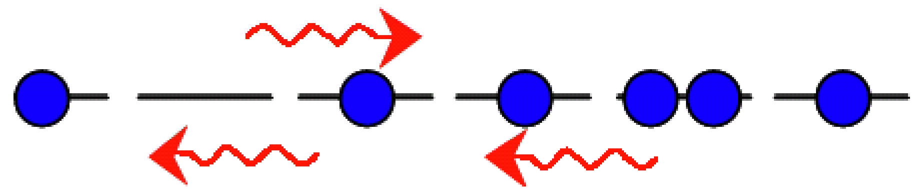
# Photoinduced metallic state

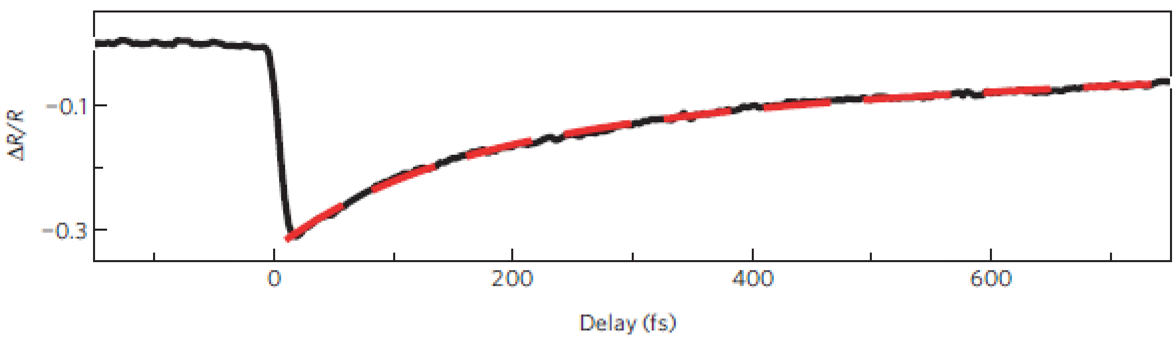
H. Okamoto et al., PRL 98:37401 (2007)
S. Wall et al. Nature Physics 7:114 (2011)

## Photoexcitations



## Conducting state by photo-doping






Surprisingly long relaxation time 840 fs

h/t = 40 fs

# Photoinduced metallic state

H. Okamoto et al., PRL 98:37401 (2007)
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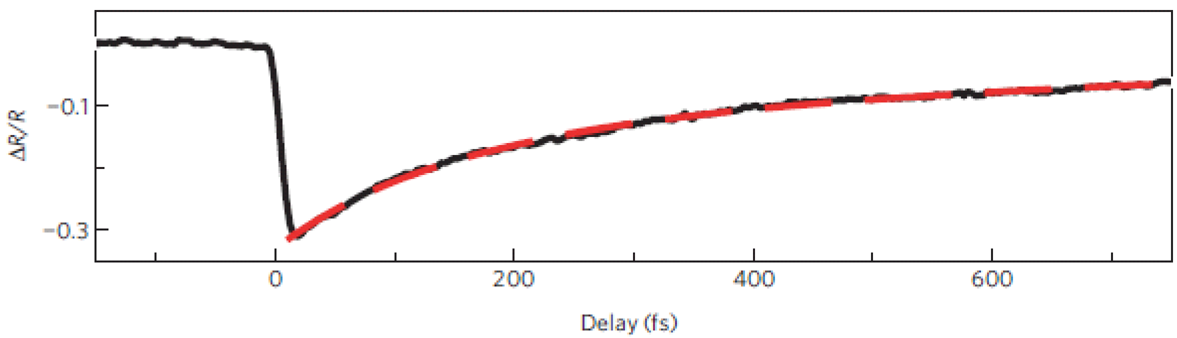
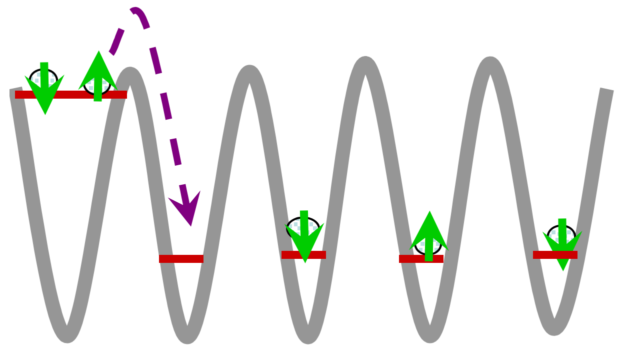


$$\tau \sim \frac{2}{\mathrm{t}}\ e^{1.6\,\frac{U}{\mathrm{w}}}$$

t=0.1 eV
w=4t=0.4eV
U=0.7 eV

$\sim$1400 fs

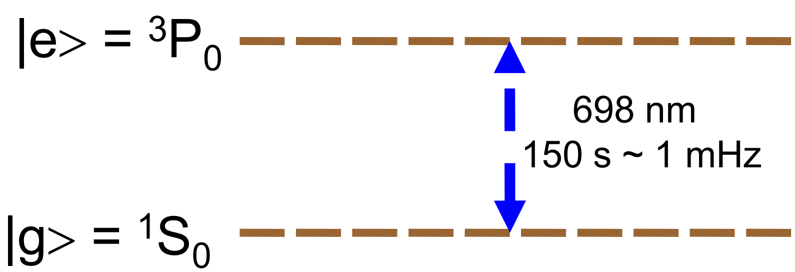
comparable to experimentally measured 840 ms

# Exploring beyond simple Hubbard model with ultracold fermions

# SU(N) Hubbard model with Ultracold Alkaline-Earth Atoms

Theory: A. Gorshkov, et al., Nature Physics 2010

Ex: ${}^{87}$Sr (I = 9/2)




Experiments:
realization of SU(6) fermions
Takahashi et al. PRL (2010)
Also J. Ye et al., Science (2011)

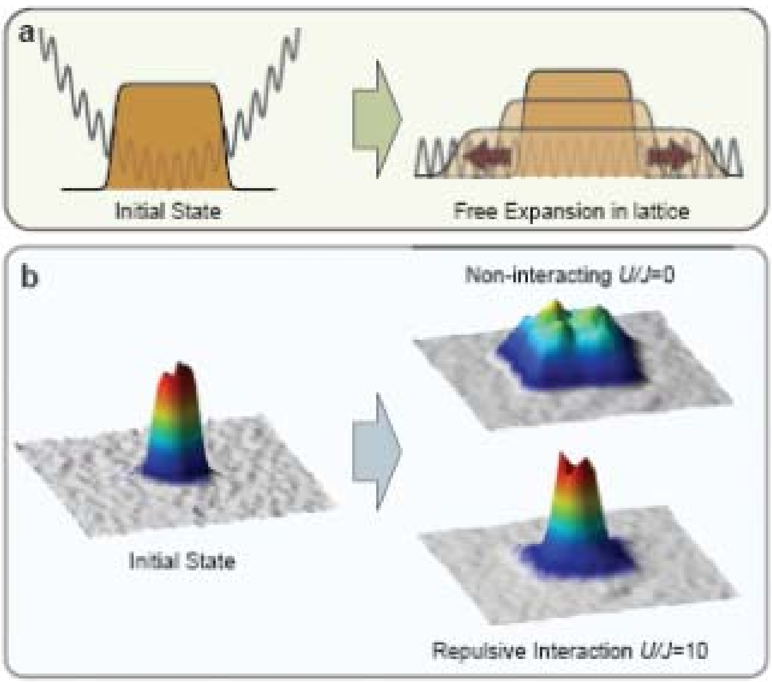
Nuclear spin decoupled from electrons SU(N=2I+1) symmetry
→ SU(N) Hubbard models ⇒ valence-bond-solid & spin-liquid phases

- orbital degree of freedom ⇒ spin-orbital physics

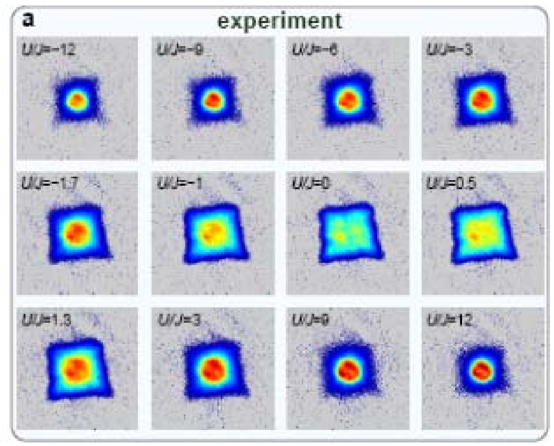
→ Kugel-Khomskii model [transition metal oxides with perovskite structure]
→ SU(N) Kondo lattice model [for N=2, colossal magnetoresistance in manganese oxides and heavy fermion materials]

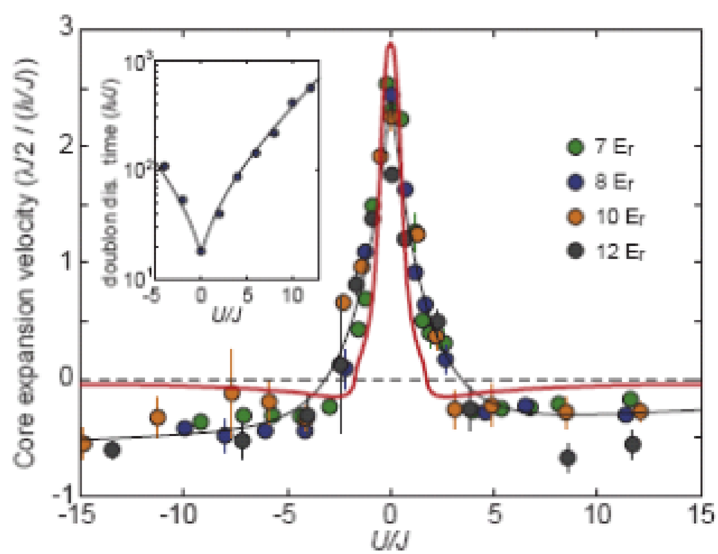
# Nonequilibrium dynamics: expansion of interacting fermions in optical lattice

U. Schneider et al., Nature Physics 2012




New dynamical symmetry: identical slowdown of expansion for attractive and repulsive interactions



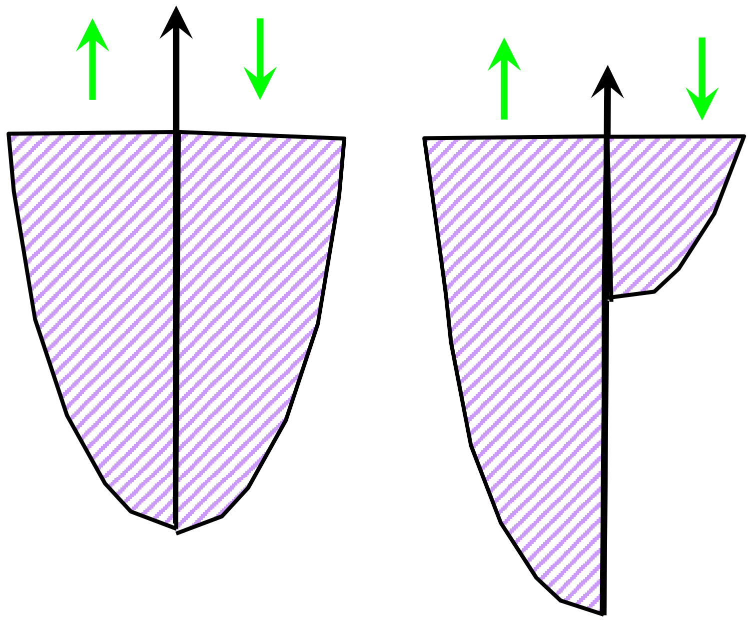
# Competition between pairing and ferromagnetic instabilities in ultracold Fermi gases near Feshbach resonances

Phys. Rev. Lett. 2010

D. Pekker, M. Babadi, R. Sensarma, N. Zinner,
L. Pollet, M. Zwierlein, E. Demler

Motivated by experiments of G.-B. Jo et al., Science (2009)

# Stoner model of ferromagnetism



Spontaneous spin polarization decreases interaction energy but increases kinetic energy of electrons

Mean-field criterion

$$U\,N(0) = 1$$

*U* – interaction strength

*N(0)* – density of states at Fermi level

Kanamori's counter-argument: renormalization of U.

$$U_{\rm eff} = \frac{U}{1 + U\chi_0} \sim \frac{U}{1+\frac{U}{E_F}} < E_F \quad \text{then} \quad U_{\rm eff}\,N(0) < 1$$

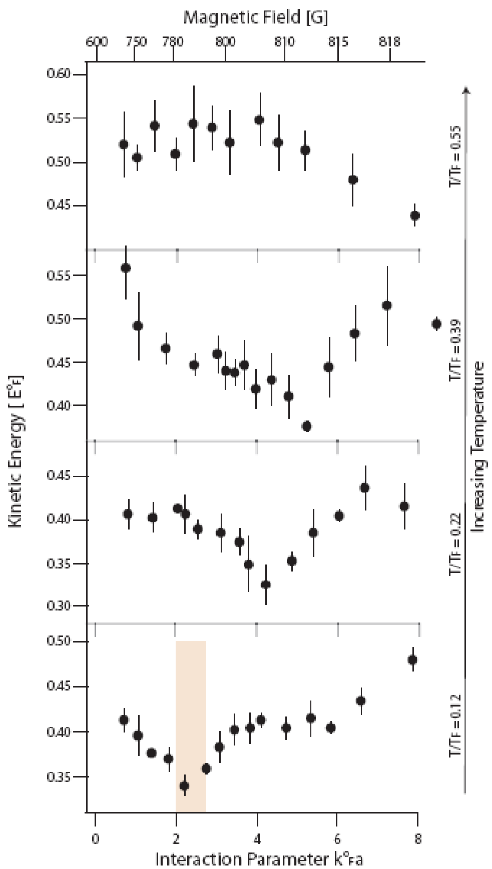
Theoretical proposals for observing Stoner instability with cold gases:

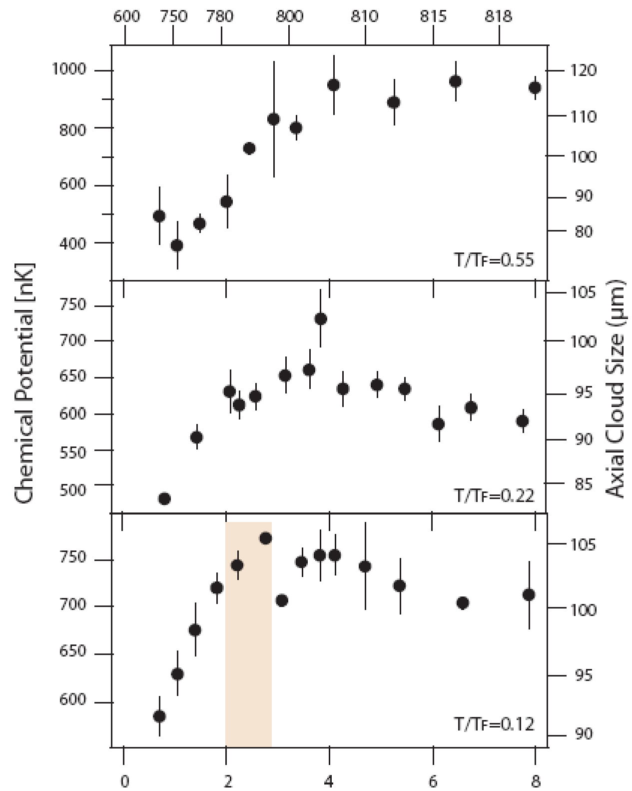
Salasnich et. al. (2000); Sogo, Yabu (2002); Duine, MacDonald (2005); Conduit, Simons (2009); LeBlanck et al. (2009); …

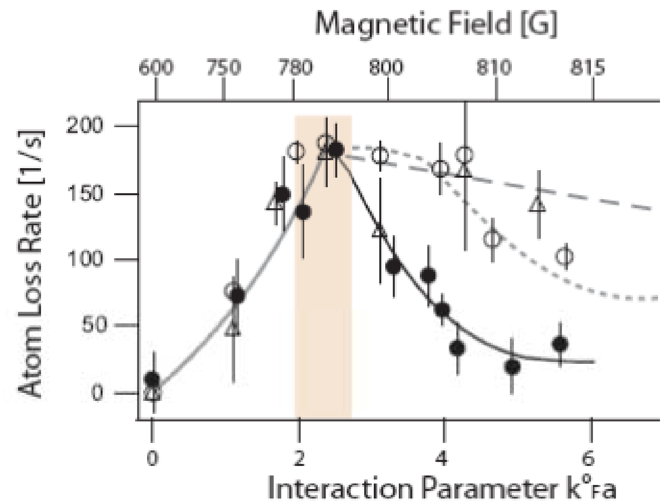
Recent work on hard sphere potentials: Pilati et al. (2010); Chang et al. (2010)

# Observation of itinerant ferromagnetism in a strongly interacting Fermi gas of ultracold atoms

Gyu-Boong Jo,[1*] Ye-Ryoung Lee,[1] Jae-Hoon Choi,[1] Caleb A. Christensen,[1] Tony H. Kim,[1] Joseph H. Thywissen,[2] David E. Pritchard[1], and Wolfgang Ketterle,[1]




Experiments were done dynamically.
What are implications of dynamics?
Why spin domains could not be observed?

Earlier work by C. Salomon et al., 2003

Is it sufficient to consider effective model with repulsive interactions when analyzing experiments?

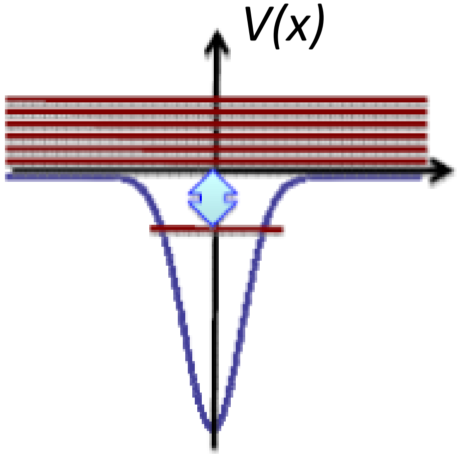
Feshbach physics beyond effective repulsive interaction

# Feshbach resonance

Interactions between atoms are intrinsically attractive
Effective repulsion appears due to low energy bound states

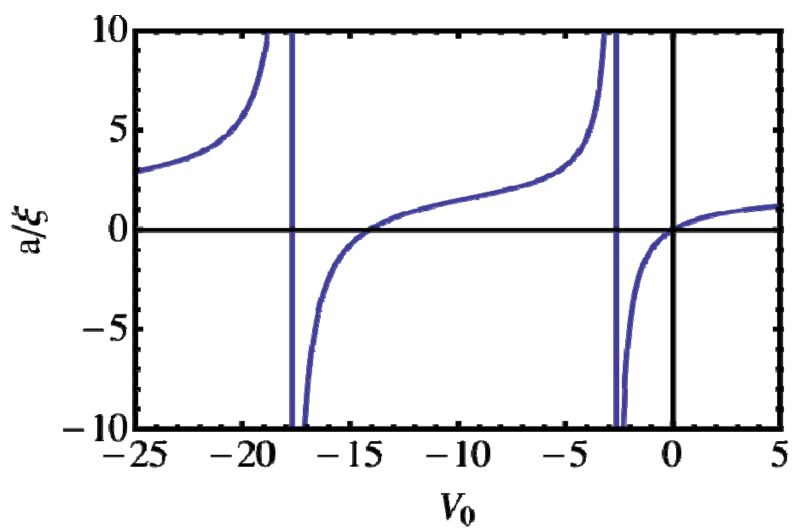
Example:

$$H = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + V_0 \xi^{-2} e^{-(\hat{x}_1 - \hat{x}_2)^2/2\xi^2}$$



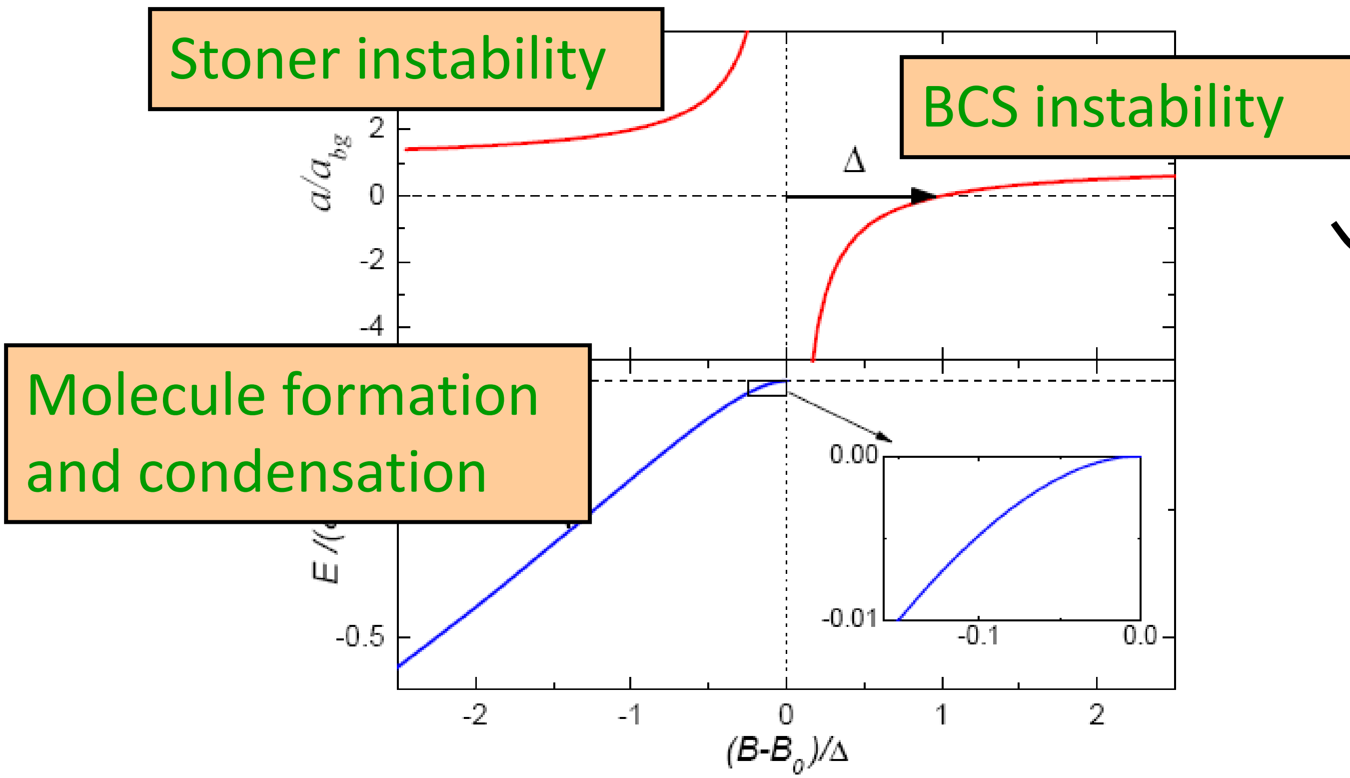

$V_0$ tunable by the magnetic field
Can tune through bound state

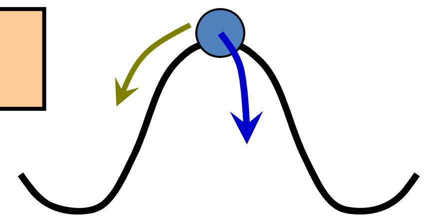
scattering length



# Feshbach resonance

## Two particle bound state formed in vacuum

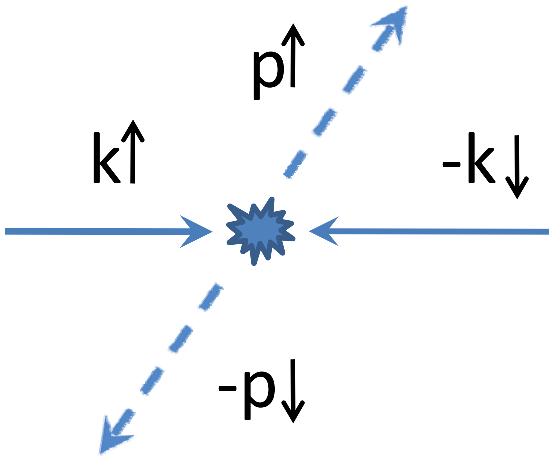


This talk: Prepare Fermi state of weakly interacting atoms.
Quench to the BEC side of Feshbach resonance.
System unstable to both molecule formation
and Stoner ferromagnetism. Which instability dominates ?

# Pair formation

# Two-particle scattering in vacuum




$$|\,\Psi\,\rangle = \left( c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} + \sum_{p} \psi_p c^{\dagger}_{p\uparrow} c^{\dagger}_{-p\downarrow} \right) |\,0\,\rangle$$

Microscopic Hamiltonian

$$\mathcal{H} = \sum_{p\sigma} \epsilon_p c^{\dagger}_{p\sigma} c_{p\sigma} + \frac{1}{V} \sum_{p,p'} V(p-p') c^{\dagger}_{p\uparrow} c^{\dagger}_{-p\downarrow} c_{-p'\downarrow} c_{p'\uparrow}$$

Schrödinger equation $\quad E\,|\,\Psi\,\rangle \;=\; \mathcal{H}\,|\,\Psi\,\rangle$

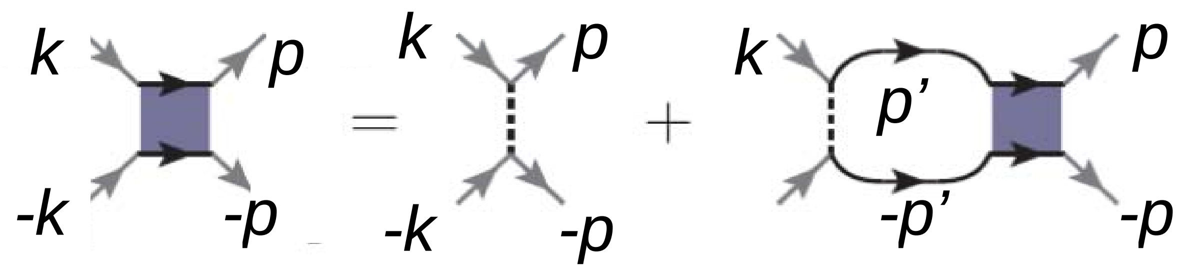
$$E\psi_p = 2\epsilon_p \psi_p + V(k-p) + \frac{1}{V} \sum_{p'} V(p-p')\psi_{p'}$$

# T-matrix

$$T_E(p,k) = (E - 2\epsilon_p)\psi_p$$

Lippman-Schwinger equation

$$\boxed{T_E(p,k) = V(p-k) + \frac{1}{V}\sum_{p'} V(p-p')\frac{T_E(p',k)}{E - 2\epsilon_{p'} + i0}}$$




On-shell T-matrix. Universal low energy expression

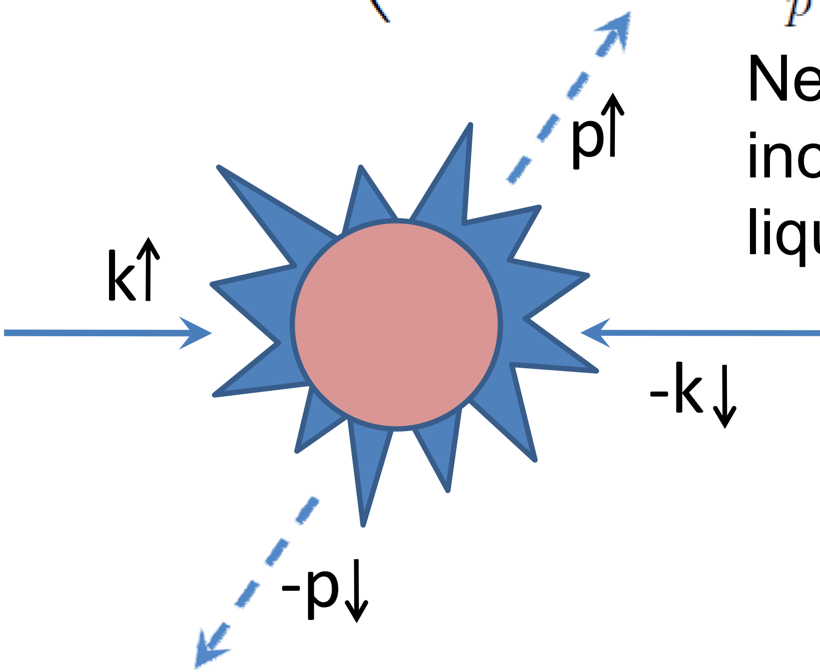
$$T_E = -\frac{m}{4\pi}\left(-\frac{1}{a} + r_e m E - i\sqrt{mE}\right)^{-1}$$

For positive scattering length bound state at $E = -\frac{1}{ma^2}$ appears as a pole in the T-matrix

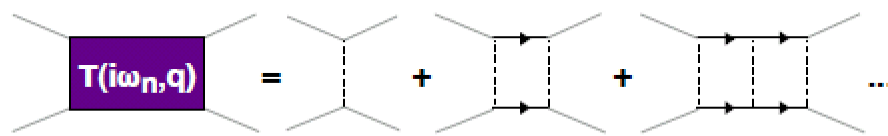
# Cooperon

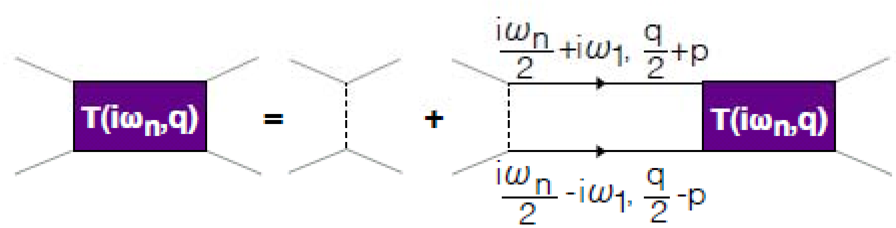
## Two particle scattering in the presence of a Fermi sea

$$|\,\Psi\,\rangle = \left( c^{\dagger}_{k\uparrow}\, c^{\dagger}_{-k\downarrow} + \sum_{p} \psi_p\, c^{\dagger}_{p\uparrow}\, c^{\dagger}_{-p\downarrow} \right) |\,\mathrm{FL}\,\rangle = \mathrm{O}^{\dagger}\,|\,\mathrm{FL}\,\rangle$$




Need to make sure that we do not include interaction effects on the Fermi liquid state in scattered state energy

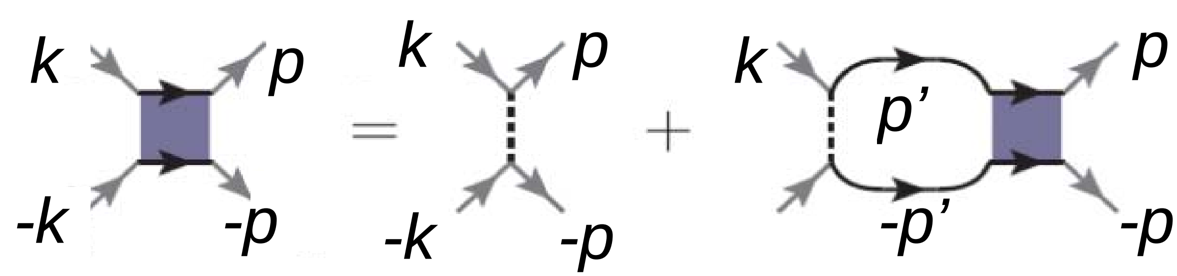


# Cooperon vs T-matrix

$$T_E(p,k) = V(p-k) + \frac{1}{V}\sum_{p'} V(p-p')\frac{T_E(p',k)}{E-2\epsilon_{p'}+i0}$$

$$C_E(p,k) = V(p-k) + \frac{1}{V}\sum_{p'} V(p-p')\frac{C_E(p',k)\,(1-2n_{p'})}{(E-2(\epsilon_{p'}-\mu)+i0)}$$




$$\boxed{C_E^{-1} = T_{E+2\mu}^{-1} + \int \frac{d^3 p}{(2\pi)^3}\frac{2\,n_p}{(E-2(\epsilon_p-\mu))}}$$

# Cooper channel response function

## Linear response theory

$$\mathcal{H} = \mathcal{H}_0 + \frac{1}{V}\sum_k \left( h_\omega^\Delta \, e^{-i\omega t} c_{k\uparrow}^\dagger \, c_{-k\downarrow}^\dagger + \text{c.c.} \right)$$

Induced pairing field $\qquad \Delta_\omega = \frac{1}{V}\sum_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle_t \; e^{i\omega t}$

Response function $\qquad \chi_\omega^\Delta = \frac{\Delta_\omega}{h_\omega^\Delta} = \frac{1}{V^2}\sum_{k,p} C_\omega(k,p) f_k f_p$

Poles of the Cooper channel response function are given by $C_\omega$

# Cooper channel response function

Linear response theory $\Delta_{q,\omega} = \chi^{\Delta}_{q,\omega}\, h^{\Delta}_{q,\omega}$

Poles of the response function, $\left(\chi^{\Delta}_{q,\,\omega_q}\right)^{-1} = 0$, describe collective modes

Time dependent dynamics $\Delta_q\,(t) \sim e^{\,i\,\omega_q\,t}$

When the mode frequency has negative imaginary part, the system is unstable

$$\omega_q = -i\Gamma_q$$

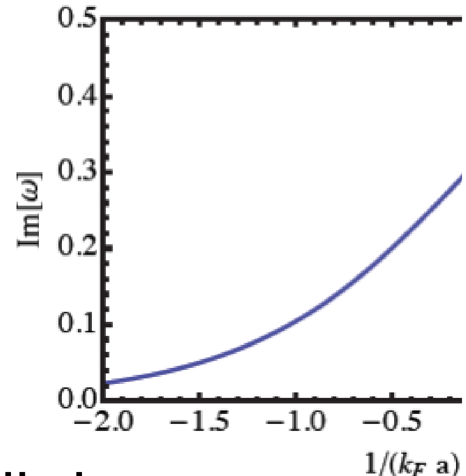
$$\Delta_q\,(t) \sim e^{\Gamma_q\,t}$$

# Pairing instability regularized

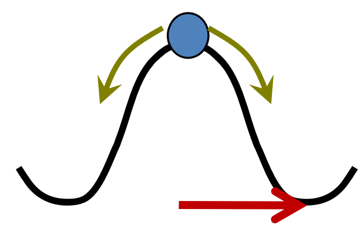
$$T^{-1}_{E+2E_F-q^2/m} + \int \frac{d^3k}{(2\pi)^3} \frac{n(\frac{q}{2}+k)+n(\frac{q}{2}-k)}{(E+2E_F-\epsilon_{\frac{q}{2}+k}-\epsilon_{\frac{q}{2}-k})} = 0$$

$$T_E = \frac{m}{4\pi}\left(\frac{1}{a}+i\sqrt{mE}\right)^{-1}$$

BCS side $\quad \Gamma \approx \frac{8}{e^2} E_F\, e^{-\pi/2\, k_F\, a}$




Instability rate coincides with the equilibrium gap
(Abrikosov, Gorkov, Dzyaloshinski)

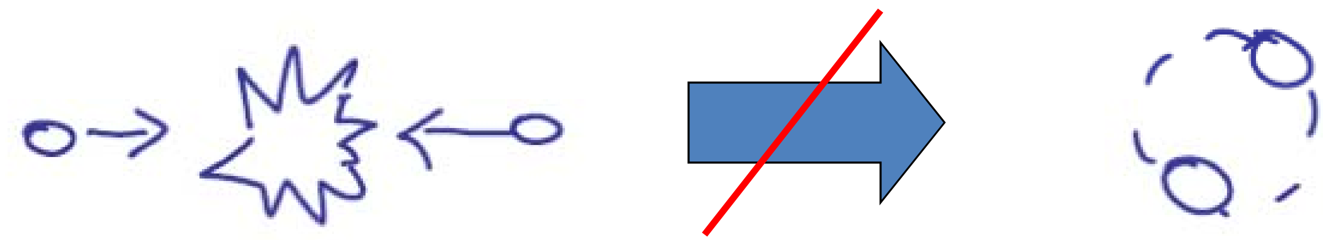


Instability to pairing even on the BEC side

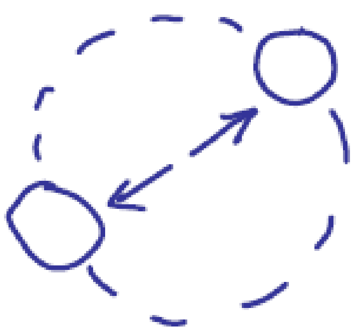
Related work: Lamacraft, Marchetti, 2008

# Pairing instability

Intuition: two body collisions do not lead to molecule formation on the BEC side of Feshbach resonance. Energy and momentum conservation laws can not be satisfied.

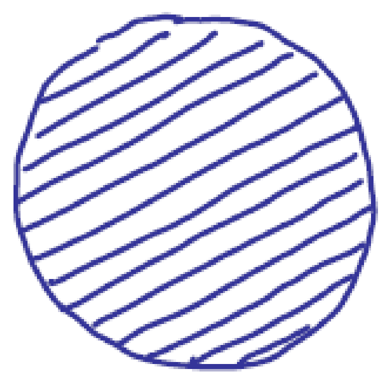


This argument applies in vacuum. Fermi sea prevents formation of real Feshbach molecules by Pauli blocking.



Molecule

$|k| \leq 1/a_{\rm s}$



Fermi sea

$|k| \leq k_{\rm F}$

# Pairing instability

Time dependent variational wavefunction

$$|\,\Psi(t)\,\rangle \;=\; \prod_k \left(\, u_k\,(t) \;+\; v_k\,(t)\, c^\dagger_{k\uparrow}\, c^\dagger_{-k\downarrow}\,\right) |\, 0\,\rangle$$

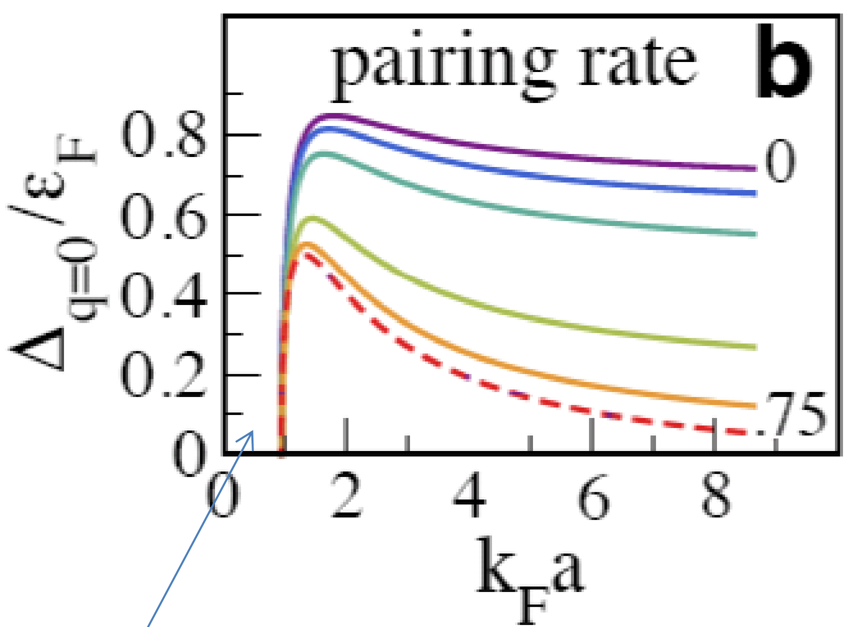
Time dependence of $u_k(t)$ and $v_k(t)$ due to $\triangle_{BCS}(t)$

For small $\triangle_{BCS}(t)$:

$$\frac{d}{dt} \log\, \Delta_{\rm BCS} \;=\; \Delta$$

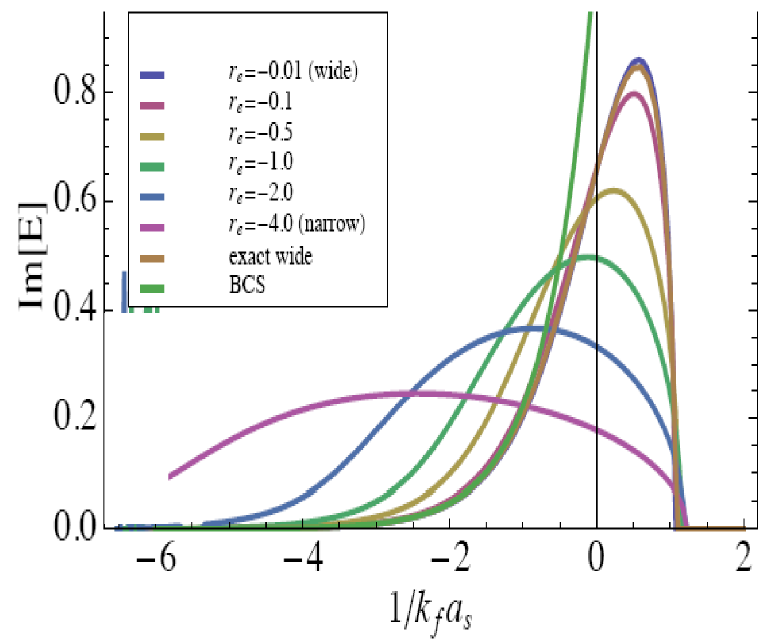
# Pairing instability

Effects of finite temperature




Three body recombination as in Shlyapnikov et al., 1996; Petrov, 2003; Esry 2005

From wide to narrow resonances




Observed in recent experiments by Grimm's group, arXiv:1112.0020

# Magnetic instability

# Stoner instability. Naïve theory

$$\mathcal{H}_0 = \sum_{p\sigma} (\, \epsilon_p - \mu \,)\, c^\dagger_{p\sigma}\, c_{p\sigma} + U \int d^3 r \, n_\uparrow(r)\, n_\downarrow(r)$$

$$U = 4\pi a_s / m$$

Linear response theory $\mathcal{H} = \mathcal{H}_0 - (\, h^\alpha_{q\,\omega}\, e^{-i\,\omega\, t}\, S^\alpha_q + \text{c.c.}\,)$

$$S^\alpha_q = \frac{1}{2V} \sum_{p\,\sigma\,\sigma'} c^\dagger_{p+q\,\sigma} \sigma^\alpha_{\sigma\,\sigma'}\, c_{p\,\sigma'}$$

Spin response function $\langle\, S^\alpha_{q\,\omega} \,\rangle = \chi^{\mathrm{S}}_{q\,\omega}\, h^\alpha_{q\,\omega}$

Spin collective modes are given by the poles of response function

$$(\, \chi^{\mathrm{S}}_{q\,\omega_q} \,)^{-1} = 0$$

Negative imaginary frequencies correspond to magnetic instability

# RPA analysis for Stoner instability

$$\mathcal{H}_{\text{eff}} = \sum \epsilon_p \, c^\dagger_{p\sigma} \, c_{p\sigma} - U \left( \langle S^z_{-q}(t) \rangle \, S^z_q + \langle S^z_q(t) \rangle \, S^z_{-q} \right) - \left( h^\alpha_{q\,\omega} \, e^{-i\,\omega\,t} \, S^\alpha_{-q} + \text{c.c.} \right)$$

Self-consistent equation on response function

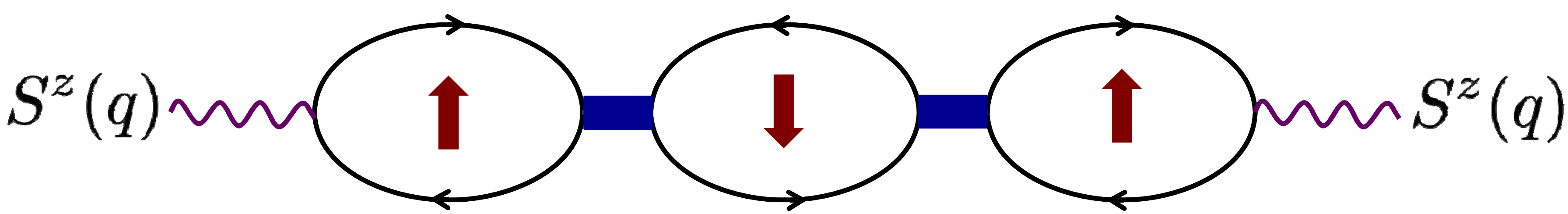
$$\langle S^z_{q\,\omega} \rangle = \chi^0_{q\,\omega} \left( h^\alpha_{q\,\omega} + U \langle S^z_{q\,\omega} \rangle \right)$$

Spin susceptibility for non-interacting gas

$$\chi^0_{q\,\omega} = \int \frac{d^3 p}{(2\,\pi)^3} \, \frac{n_{p+q} - n_p}{\left( \omega - (\epsilon_{p+q} - \epsilon_p) \right)}$$

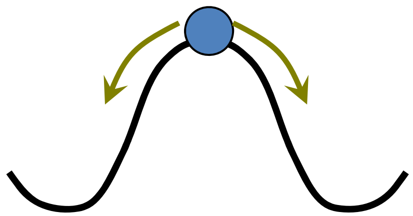
RPA expression for the spin response function

$$\boxed{\chi_{q\,\omega} = \frac{\chi^0_{q\,\omega}}{1 - U \, \chi^0_{q\,\omega}}}$$



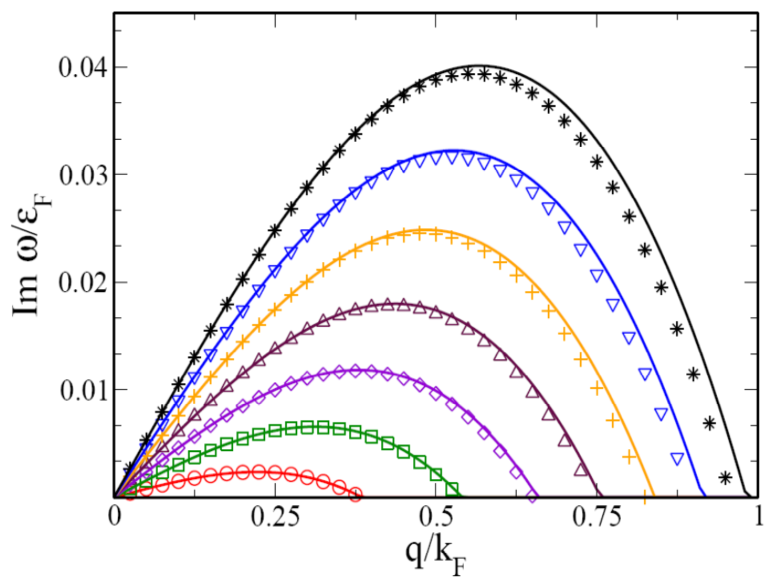
# Quench dynamics across Stoner instability

Stoner criterion $U_c = N(0)^{-1}$



For $U>U_c$ unstable collective modes

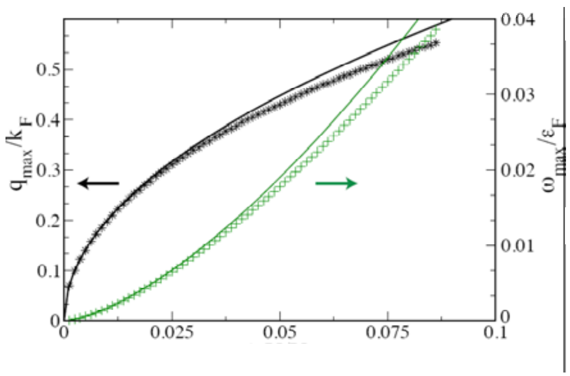
$$\omega_q = -i\Gamma_q \qquad S_q^z(t) \sim e^{-i\omega_q t} \sim e^{\Gamma_q t}$$



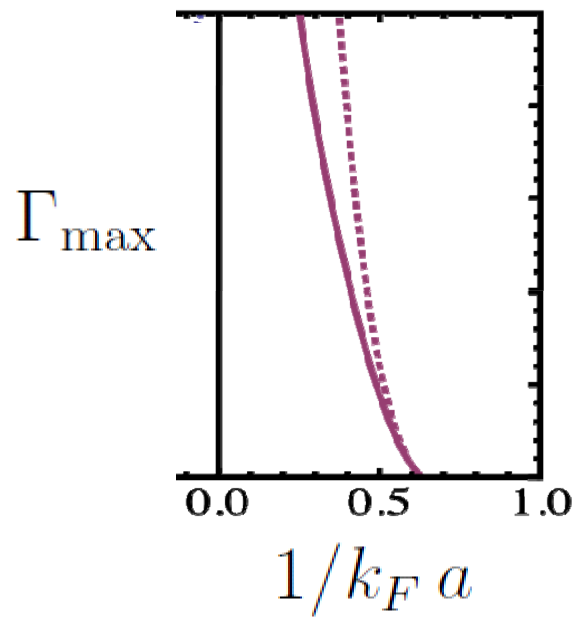

Unstable modes determine characteristic lengthscale of magnetic domains

# Stoner quench dynamics in D=3

## Scaling near transition




$$u = \frac{U}{U_c} - 1$$




Growth rate of magnetic domains

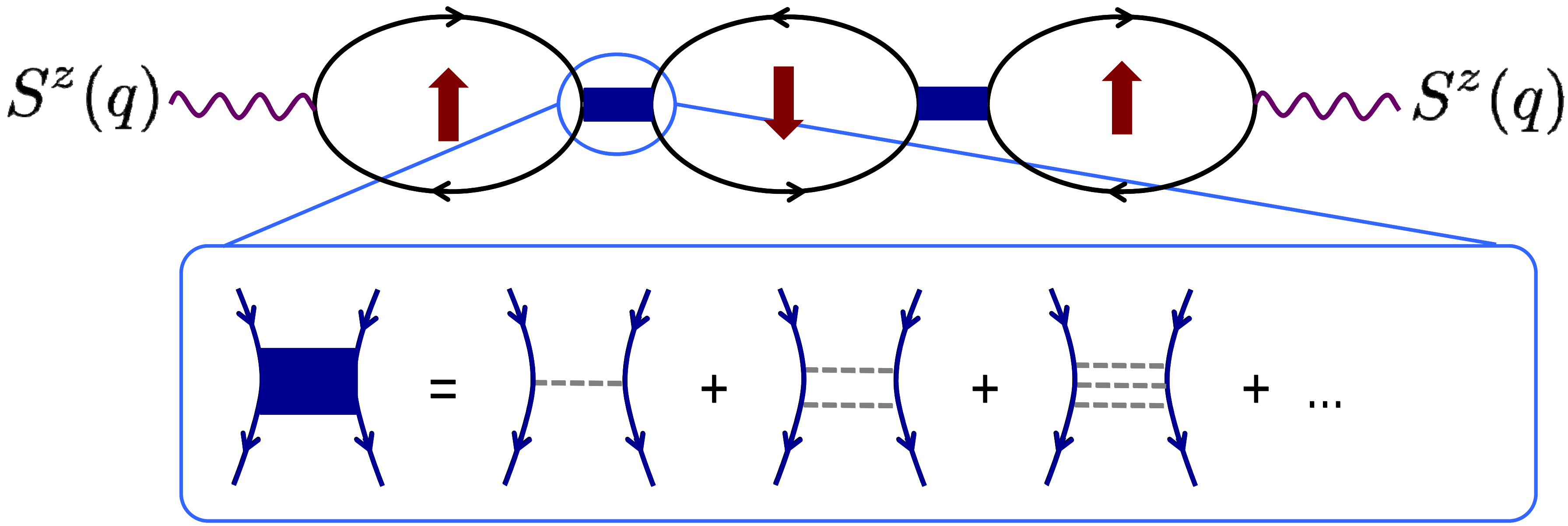
$$\Gamma_q \sim E_F\, u^{3/2}$$

Domain size

$$\xi \sim \lambda_F\, u^{-1/2}$$

Unphysical divergence
of the instability rate at unitarity

# Stoner instability

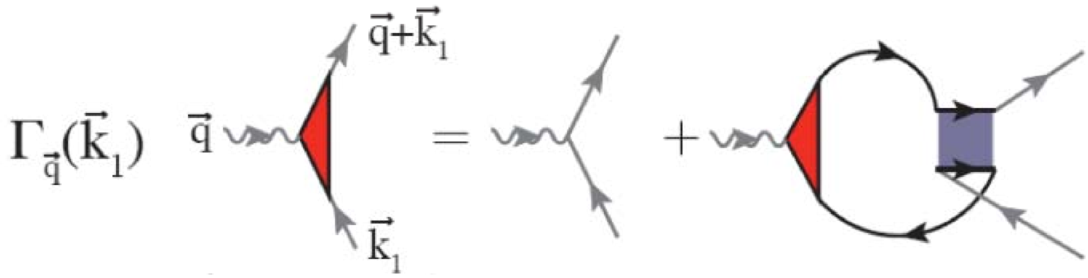



Stoner instability is determined by two particle scattering amplitude

Divergence in the scattering amplitude arises from bound state formation. Bound state is strongly affected by the Fermi sea.

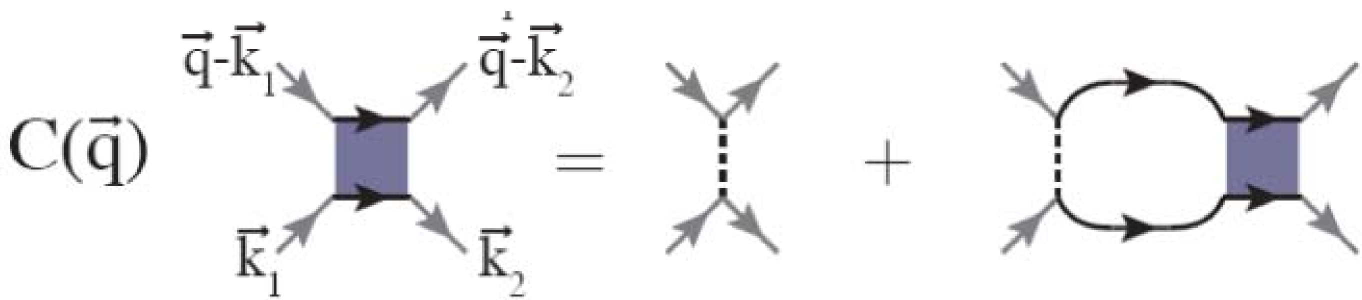
# Stoner instability

RPA spin susceptibility



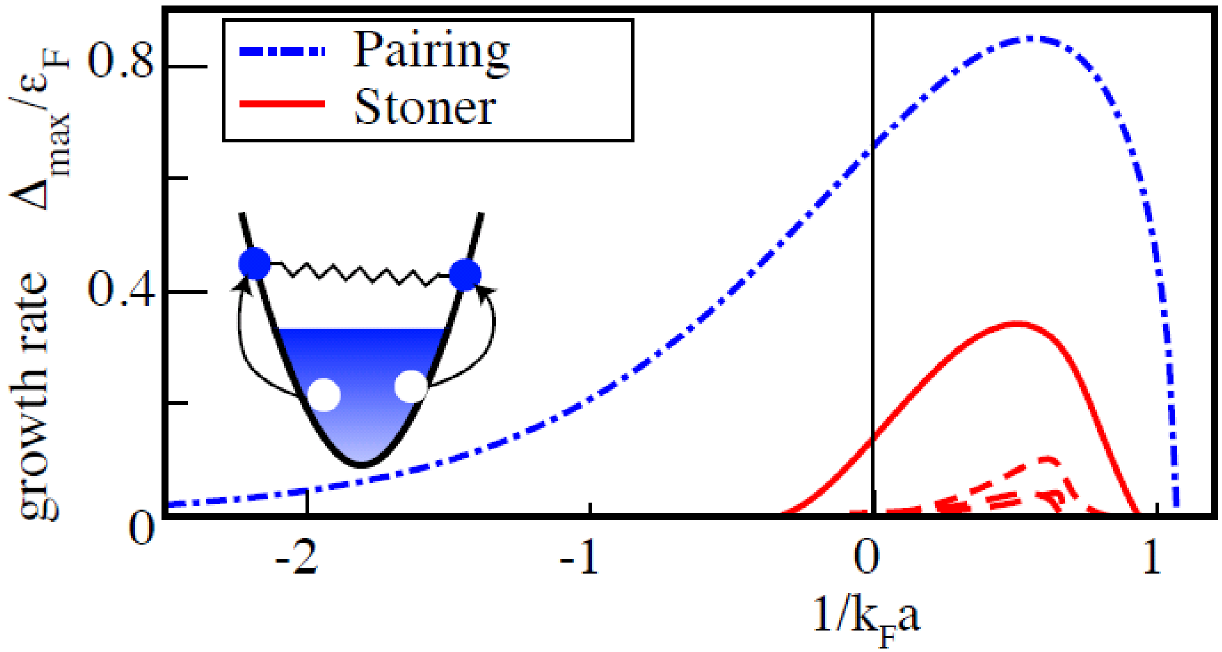

$$\Gamma_{\mathbf{q},\omega}(\hat{\mathbf{k}}_1)=1+\int\frac{d\hat{\mathbf{k}}_2}{4\pi}\Gamma_{\mathbf{q},\omega}(\hat{\mathbf{k}}_2)C(\hat{\mathbf{k}}_1+\hat{\mathbf{k}}_2,\omega)I_{\mathbf{q},\omega}(\hat{\mathbf{k}}_2)$$

Interaction = Cooperon




$$C^{-1}(E,\mathbf{q})=\tau^{-1}\left(E+2\epsilon_f-\mathbf{q}^2/4m\right)+\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{n^F(\frac{\mathbf{q}}{2}+\mathbf{k})+n^F(\frac{\mathbf{q}}{2}-\mathbf{k})}{E-\epsilon_{\frac{\mathbf{q}}{2}+\mathbf{k}}-\epsilon_{\frac{\mathbf{q}}{2}-\mathbf{k}}}$$

# Stoner instability



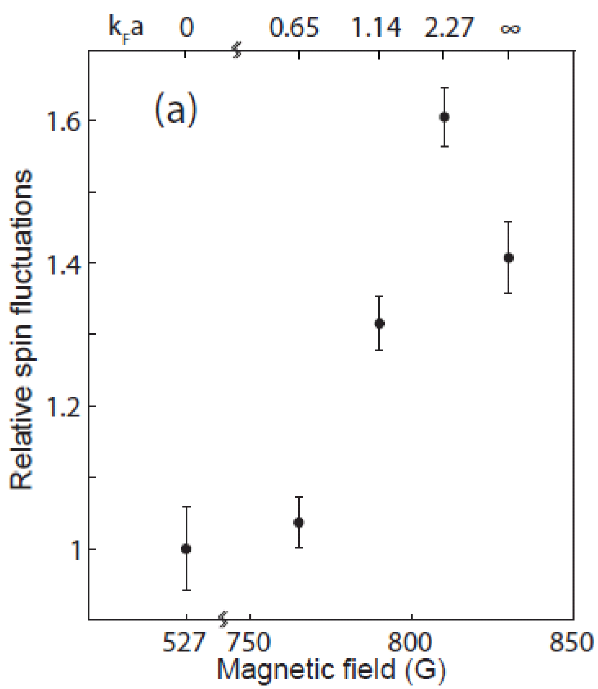

Pairing instability always dominates over pairing

If ferromagnetic domains form, they form at large q

# Tests of the Stoner magnetism

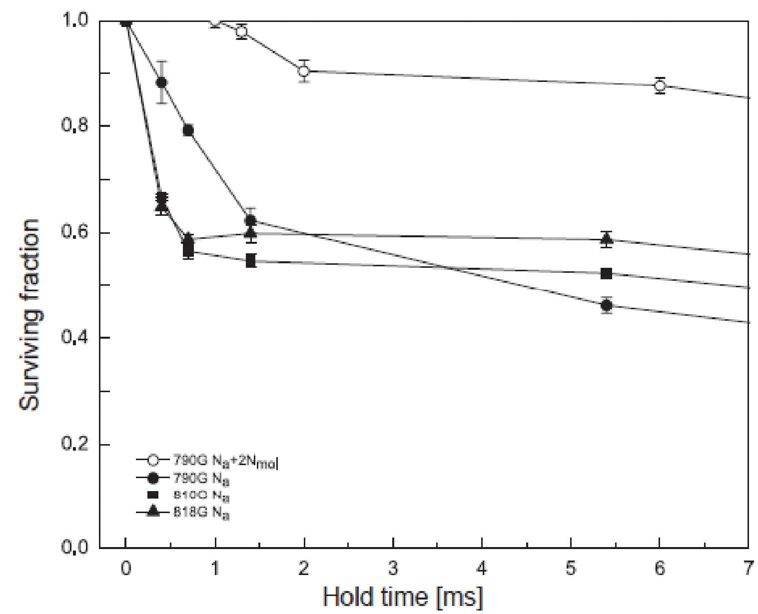
C. Sanner et al., arXiv:1108.2017

Spin fluctuations relative to noninteracting fermions




Only short range correlations and no domain formation

Extremely fast molecule formation rate at short times




Atoms loss rate is 13% of $E_F$ on resonance (averaged over trap)

# Lecture 2: Ultracold fermions

Fermions in optical lattices. Fermi Hubbard model.
Current state of experiments

Lattice modulation experiments

Doublon lifetimes

Stoner instability

# Future directions in ultracold atoms

## Nonequilibrium quantum many-body dynamics

Long intrinsic time scales

- Interaction energy and bandwidth ~ 1kHz
- System parameters can be changed over this time scale

Decoupling from external environment

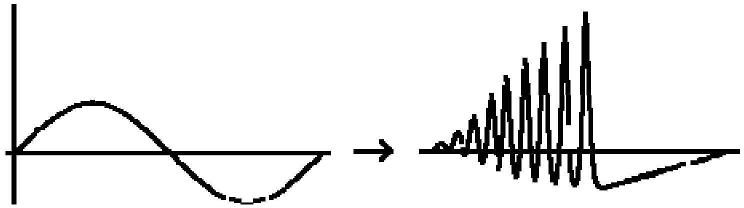
- Long coherence times

Can achieve highly non equilibrium quantum many-body states

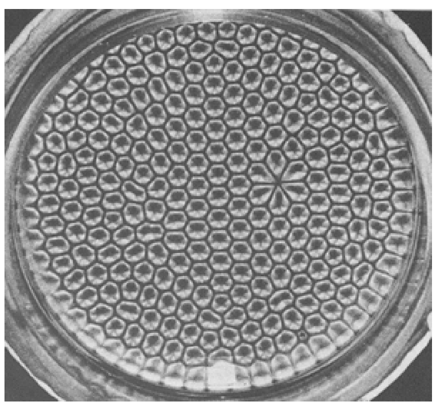
$H_i \rightarrow H_f$ $\qquad\qquad$ $|\,\Psi(t)\,\rangle = e^{-iH_f t}\,|\,\Psi_i\,\rangle$

## Emergent phenomena in dynamics of classical systems

Solitons in nonlinear wave propagation

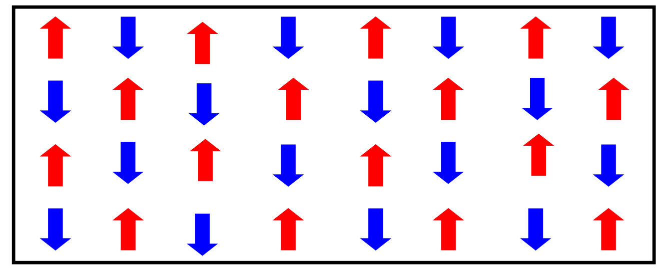


Bernard cells in the presence of T gradient

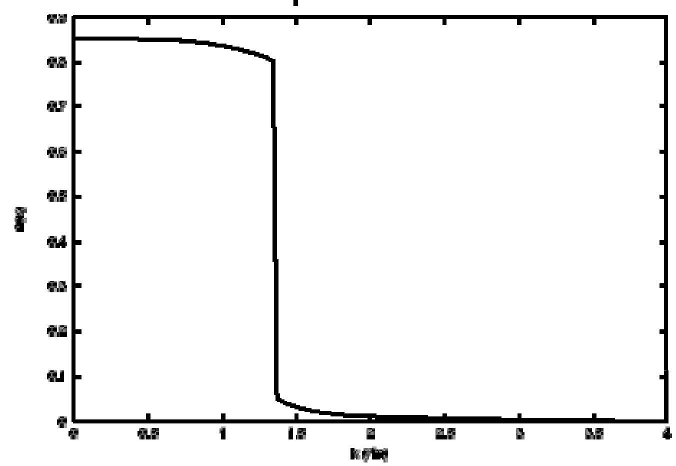


## Universality in quantum many-body systems in equilibrium

Broken symmetries

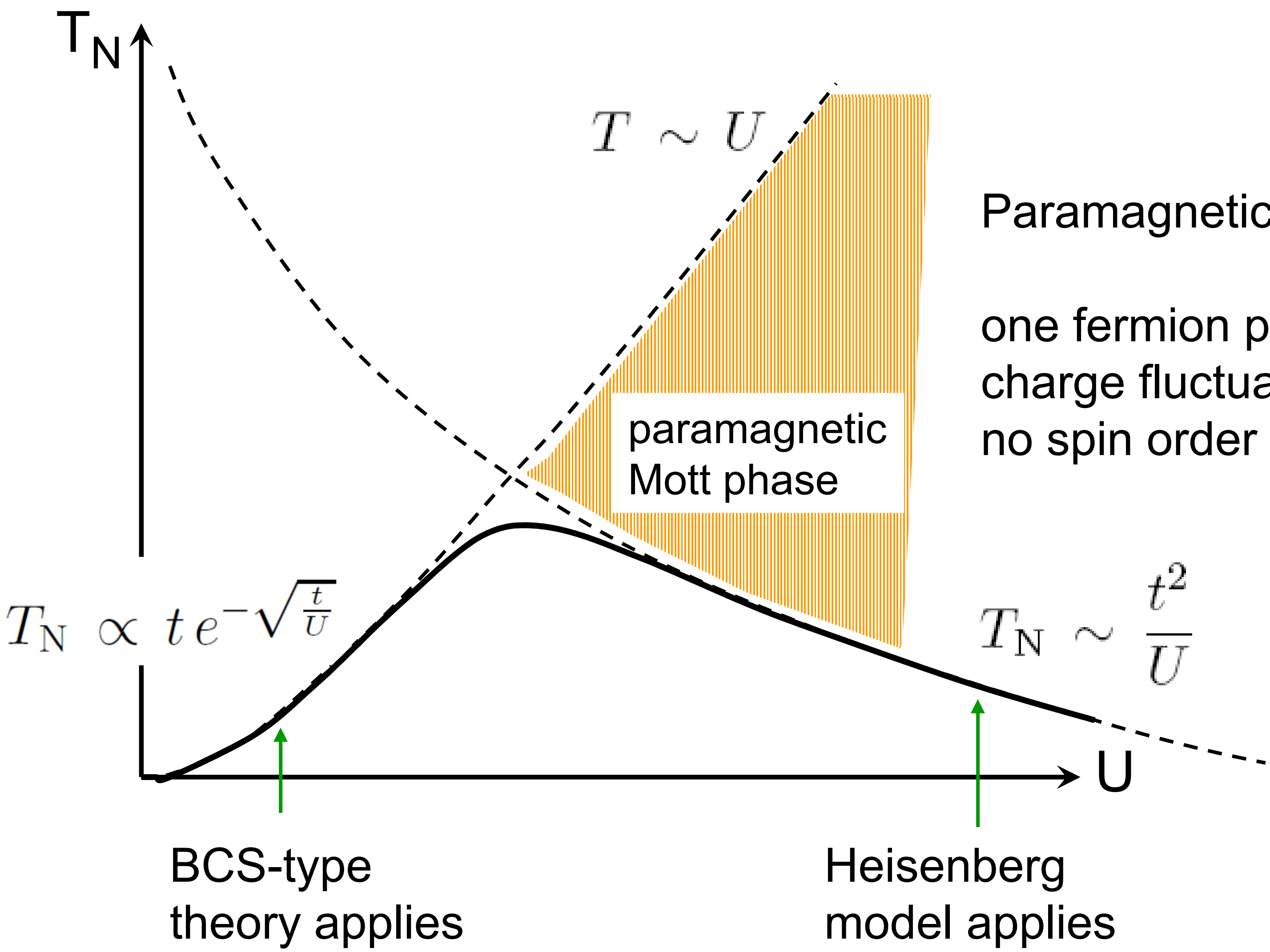


Fermi liquid state



**Do we have emergent universal phenomena in nonequilibrium dynamics of many-body quantum systems?**

# Hubbard model at half filling




Paramagnetic Mott phase:

one fermion per site
charge fluctuations suppressed
no spin order

# Hubbard model at half filling