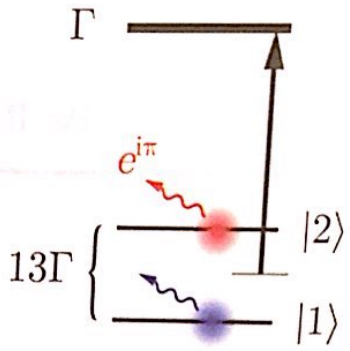

Figure 9.1: Illustration of spin-sensitive light scattering.

ently by the atoms. In ~~these~~ THIS regime the electric field scattered by each of the atoms is exactly in phase with the electric field of the incident light, and inelastic light scattering (which results in a field with a random phase) can be neglected. Demonstrations of Bragg scattering with ultracold atoms typically operate in this regime [140, 144, 142]. There has also been theoretical work which suggests ways to exploit the characteristics of the scattered light to probe the many body states that form in an optical lattice [58, 145]; however, those results are also in the context of a very weak probe and do not consider the effects of saturation of the atomic transition.

When performing Bragg scattering to detect the spin ordering of atoms in a lattice, one must have a spin-sensitive probe, i.e. light that scatters differently from atoms in states $|1\rangle$ and $|2\rangle$. The phase shift δ of the scattered light is related to the detuning Δ (in units of the linewidth) from the atomic transition as $e^{i\delta} \propto \Delta^{-1}$. If one sets the light detuning right in between the two states, light scattered from one state will have a π phase shift with respect to light scattered from the other one (see Fig. 9.1. We find experimentally that, to obtain a measurable scattering signal, we must use probe parameters that are not in the far-detuned or weak intensity ideal scenarios. The detuning is fixed by the requirement to have a spin-sensitive measurement, the power of the probe is determined by the signal to noise ratio of our detection setup, and the duration is constrained by the effects on the center of mass state of the atoms due to the recoil from every photon scattered.

This chapter starts out with a detailed derivation of the equations for light scattering

- $\Lambda = k' \times (k' \times \mathbf{d}_{ge})/(k'^2|\mathbf{d}_{ge}|)$ is the polarization vector of the field (dipole radiation pattern)

- $\mathbf{d}_{ge} = \langle g|\mathbf{d}|e\rangle$ is the transition matrix element between the ground $|g\rangle$ and excited $|e\rangle$ electronic states

- $\varsigma_{n-} = e^{i\omega_p t}|g\rangle\langle e|$ is the off-diagonal matrix element of the atomic density matrix in the rotating frame

The two atomic states, $|\uparrow\rangle$ and $|\downarrow\rangle$, only differ in the projection of the nuclear spin, so $\mathbf{d}_{ge}$ is the same for either state. For this reason, we make no distinction of the spin part of the wavefunction in states $|e\rangle$ and $|g\rangle$. Both states $|\uparrow\rangle$, $|\downarrow\rangle$ have an electronic angular momentum projection $m_J = -1/2$ and the respective excited states have $m_J = -3/2$. The matrix element can then be written as $\mathbf{d}_{ge} = \mathbf{d}_{ge}^* = |\mathbf{d}_{ge}|\hat{e}_{-1}$ (as for any $\Delta m_J = -1$ transition).

The intensity at the detector, for a momentum transfer $\boldsymbol{Q} = \boldsymbol{k}' - \boldsymbol{k}$, can be obtained by summing the field contributions from the individual atoms and squaring the total field:

$$I_{\boldsymbol{Q}} = 2\epsilon_0 c \,\langle\Psi|\left(\sum_m \boldsymbol{E}_m^-\right)\cdot\left(\sum_n \boldsymbol{E}_n^+\right)|\Psi\rangle. \tag{9.2}$$

Here $|\Psi\rangle$ is the product state of the array of atoms: $|\Psi\rangle = \prod_n |u\rangle_n|\sigma\rangle_n$ where u and σ represent the center of mass and electronic states of the atom respectively. The wavefunction for the system of atoms in the lattice (which in general can be a highly entangled many-body state) must be projected into a product state (like $|\Psi\rangle$) before performing the measurement. This is achieved by quickly ramping the lattice depth up to $20\,E_r$, which projects the state of the system into a state with a well defined atom number in each site, and prevents further tunneling during the measurement.

The projection to a product state helps simplify the interpretation of the scattered intensity measurements. The projected state in a deep lattice will allow us to neglect changes of the center of mass state of the system during scattering. Calculating the properties of

9.1.1 Non-interfering term and incoherent scattering

The non-interfering term on the right hand side of Eq. 9.6 consists of a combination of coherently and incoherently scattered light. The word "incoherent" is used here to define any light that is scattered by the atoms with a random phase. A random phase arises due to quantum fluctuations of the electric dipole moment, which occur if the atom is driven with large enough intensity(see for instance §V.D.2b in [146]). We can identify the coherent and incoherent contributions to the non-interfering by writing

$$\langle \varsigma_{n+}\varsigma_{n-}\rangle = \langle \varsigma_{n+}\rangle\langle \varsigma_{n-}\rangle + \langle \delta\varsigma_{n+}\rangle\langle \delta\varsigma_{n-}\rangle \tag{9.12}$$

where $\varsigma_{n\pm}$ is expressed as a sum of its expectation value plus quantum fluctuations, $\varsigma_{n\pm} = \langle \varsigma_{n\pm}\rangle + \delta\varsigma_{n\pm}$.

The spectrum of the incoherently scattered light exhibits sidebands at a frequency different from the frequency of the incident light [148, 149] and for that reason this light is also referred to as the inelastically scattered light. If the atomic transition is driven with a large saturation parameter ($s_0 \gg \Delta$), most of the light scattered by the atoms will have a random phase and will not result in interference at the detector. In this strong saturation limit, the non-interfering term in Eq. 9.6 overwhelms the interference term, resulting in

$$I_Q = \frac{3}{8\pi}\frac{hck\Gamma}{r_D^2}|A|^2\sum_n \rho_n^{ee} \tag{9.13}$$

One can see that the same limit would be found for an uncorrelated sample. For instance, a sample where the wavefunctions of neighboring atoms overlap significantly and $e^{-2W_Q(\tau)} \to 0$. In either case, uncorrelated or non-interfering, the total photon scattering rate can be evaluated by integrating I_Q along all directions:

$$\Gamma_{\text{scatt}} = \frac{1}{hck}\int I_Q r_D^2 \mathrm{d}\Omega, \tag{9.14}$$

and using $\int |A|^2 \mathrm{d}\Omega = \frac{8\pi}{3}$ we find

$$\Gamma_{\text{scatt}} = \Gamma\sum_n \rho_n^{ee} \tag{9.15}$$

just as we expect for a collection of uncorrelated atoms or in the limit of a strong probe.

9.1.2 Coherent scattering

In the limit of a weak probe $s_0 \ll \Delta$, the interference term can be the dominant contribution to the scattered intensity. Here I say "can be" because, depending on the value of the momentum transfer $\boldsymbol{Q}$, interference could be completely destructive (which would make the interference term zero) or completely constructive (which would make the interference term dominate). Here we are mostly interested in values of the momentum transfer which satisfy the Bragg condition[1], so in the case of an ordered sample the interference term will be constructive.

The interference term of the scattered intensity is

$$\begin{aligned} I_{\text{interf.}} &= e^{-2W_{\boldsymbol{Q}}(\tau)} \sum_{\substack{m,n \\ m \neq n}} e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n - \boldsymbol{R}_m)} 2\frac{\rho_m^{ee}\rho_n^{ee}}{s_0}(2\Delta_n - i)(2\Delta_m + i) \\ I_{\text{interf.}} &= e^{-2W_{\boldsymbol{Q}}(\tau)} \sum_{\substack{m,n \\ m \neq n}} e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n - \boldsymbol{R}_m)} \frac{\rho_m^{ee}\rho_n^{ee}}{s_0}(8\Delta_n\Delta_m + 4i\Delta_n - 4i\Delta_m + 2) \end{aligned} \tag{9.16}$$

To get an idea of the contributions of the four different terms inside the last parenthesis we split up the double sum over m and n explicitly for each of them:

$$8\sum_m e^{i\boldsymbol{Q}\cdot\boldsymbol{R}_n}\rho_n^{ee}\Delta_n \sum_{\substack{m,n \\ m \neq n}} e^{-i\boldsymbol{Q}\cdot\boldsymbol{R}_m}\rho_m^{ee}\Delta_m \tag{9.17}$$

$$4i\sum_m e^{i\boldsymbol{Q}\cdot\boldsymbol{R}_n}\rho_n^{ee}\Delta_n \sum_{\substack{m,n \\ m \neq n}} e^{-i\boldsymbol{Q}\cdot\boldsymbol{R}_m}\rho_m^{ee} \tag{9.18}$$

$$-4i\sum_m e^{i\boldsymbol{Q}\cdot\boldsymbol{R}_n}\rho_n^{ee} \sum_{\substack{m,n \\ m \neq n}} e^{-i\boldsymbol{Q}\cdot\boldsymbol{R}_m}\rho_m^{ee}\Delta_m \tag{9.19}$$

$$2\sum_m e^{i\boldsymbol{Q}\cdot\boldsymbol{R}_n}\rho_n^{ee} \sum_{\substack{m,n \\ m \neq n}} e^{-i\boldsymbol{Q}\cdot\boldsymbol{R}_m}\rho_m^{ee} \tag{9.20}$$

[1] We will consider the Bragg condition for scattering from the crystal lattice and also from the magnetic sublattice of a spin ordered sample.

9.1.3 Crystal structure

For a large detuning with respect to both states, such that $\Delta_m \approx \Delta$ (independent of the spin of the atom in question[2]) and such that $\Delta \gg \Gamma$, the terms in 9.18 and 9.19 cancel each other out, and the one in 9.20 can be neglected in comparison to the first. We then have the following result, including the interfering and non-interfering parts of the intensity:

$$\begin{aligned} I_{\boldsymbol{Q}}^{\text{crystal}}/A &= e^{-2W_{\boldsymbol{Q}}(\tau)}\frac{\delta(\rho^{ee})^2\Delta^2}{s_0}\sum_{\substack{m,n\\ m\neq n}} e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n-\boldsymbol{R}_m)} + \sum_n \rho^{ee} \\ &= e^{-2W_{\boldsymbol{Q}}(\tau)}\frac{2s_0\Delta^2}{(4\Delta^2+s_0)^2}\sum_{\substack{m,n\\ m\neq n}} e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n-\boldsymbol{R}_m)} + N\frac{s_0/2}{4\Delta^2+s_0} \end{aligned} \tag{9.21}$$

If $\boldsymbol{Q}$ is a reciprocal lattice vector, then coherent scattering from all atoms will interfere constructively at the detector. The off-diagonal sum becomes

$$\sum_{\substack{m,n\\ m\neq n}} e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n-\boldsymbol{R}_m)} = N(N-1) \tag{9.22}$$

and $I_{\boldsymbol{Q}}^{\text{crystal}}/A \sim N^2$

The crystal structure factor, a measure of the spatial ordering of the ensemble of atoms, is defined as

$$S_{\boldsymbol{Q}}^{\text{crystal}} = \frac{1}{N}\sum_{m,n} e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n-\boldsymbol{R}_m)} \tag{9.23}$$

and we find that it can be related to the scattered intensity as

$$I_{\boldsymbol{Q}}^{\text{crystal}}/A = e^{-2W_{\boldsymbol{Q}}(\tau)}\frac{2s_0\Delta^2}{(4\Delta^2+s_0)^2}N(S_{\boldsymbol{Q}}^{\text{crystal}}-1) + N\frac{s_0/2}{4\Delta^2+s_0} \tag{9.24}$$

9.1.4 Magnetic structure

As was mentioned before, to have sensitivity to the magnetic ordering of spins in the lattice the light must be detuned in between the two spin states. We then have $\Delta_\uparrow = -\Delta_\downarrow$, and $\Delta_n^2 \equiv \Delta^2$. We can replace the detuning for the n^{th} atom with the projection of its spin

[2] For Δ_m to be the same for atoms in either spin, one must necessarily have $\Delta \gg \Gamma$

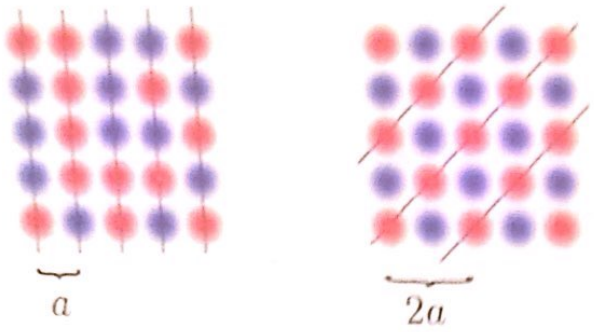

Figure 9.2: Illustration of disordered spins in a square optical lattice (left). When the spins order antiferromagnetically, a face-centered square lattice (right) is formed, which has twice the lattice spacing as the underlying optical lattice. Planes relevant for Bragg scattering are indicate by the gray lines. The situation is analogous in three dimensions.

along z as

$$\Delta_n = 2\langle\sigma_z\rangle\Delta \tag{9.25}$$

The intensity at the detector (including interfering and non-interfering components) then becomes:

$$I_{\boldsymbol{Q}}^{\text{spin}}/A = e^{-2W_{\boldsymbol{Q}}(\tau)}\frac{2s_0\Delta^2}{(4\Delta^2+s_0)^2}\sum_{\substack{m,n\\m\neq n}} 4\langle\sigma_z\rangle_m\langle\sigma_z\rangle_n e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n-\boldsymbol{R}_m)} + N\frac{s_0/2}{4\Delta^2+s_0} \tag{9.26}$$

For arbitrary $\boldsymbol{Q}$, the terms in Eqs. 9.18-9.20 can be neglected on the basis that $|\Delta| = 6.5 \gg 1$. When the spins are antiferromagnetically ordered, a magnetic face-centered cubic sublattice is formed, which has twice the lattice spacing of the crystal lattice. This situation is illustrated in Fig. 9.2.

If $\boldsymbol{Q}$ is a reciprocal lattice vector of the magnetic sublattice, then the sum

$$\sum_{\substack{m,n\\m\neq n}} 4\langle\sigma_z\rangle_m\langle\sigma_z\rangle_n e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n-\boldsymbol{R}_m)} = N(N-1) \tag{9.27}$$

and $I_{\boldsymbol{Q}}^{\text{spin}}/A \sim N^2$. In that case, the terms in Eqs. 9.18-9.20 become negligible, since at least one of the sums in each product does not add constructively.

In an analogous way to the crystal structure factor we define the spin structure factor, which is a measure of the spin order of the ensemble, as

$$S_{\boldsymbol{Q}} = \sum_{m,n} 4\langle\sigma_z\rangle_m\langle\sigma_z\rangle_n e^{i\boldsymbol{Q}\cdot(\boldsymbol{R}_n-\boldsymbol{R}_m)} \tag{9.28}$$