# Strongly correlated many-body systems: from electronic materials to ultracold atoms to photons

- Introduction. Systems of ultracold atoms.
- Bogoliubov theory. Spinor condensates.
- Cold atoms in optical lattices. Band structure and semiclasical dynamics.
- Bose Hubbard model and its extensions
- Bose mixtures in optical lattices

  Quantum magnetism of ultracold atoms.

  Current experiments: observation of superexchange
- Detection of many-body phases using noise correlations
- Fermions in optical lattices

  Magnetism and pairing in systems with repulsive interactions.

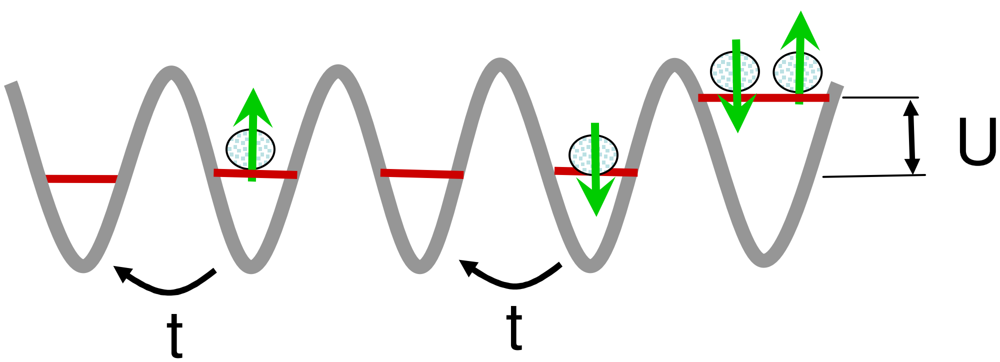
  Current experiments: Mott state
- Experiments with low dimensional systems

  Interference experiments. Analysis of high order correlations
- Probing topological states of matter with quantum walk

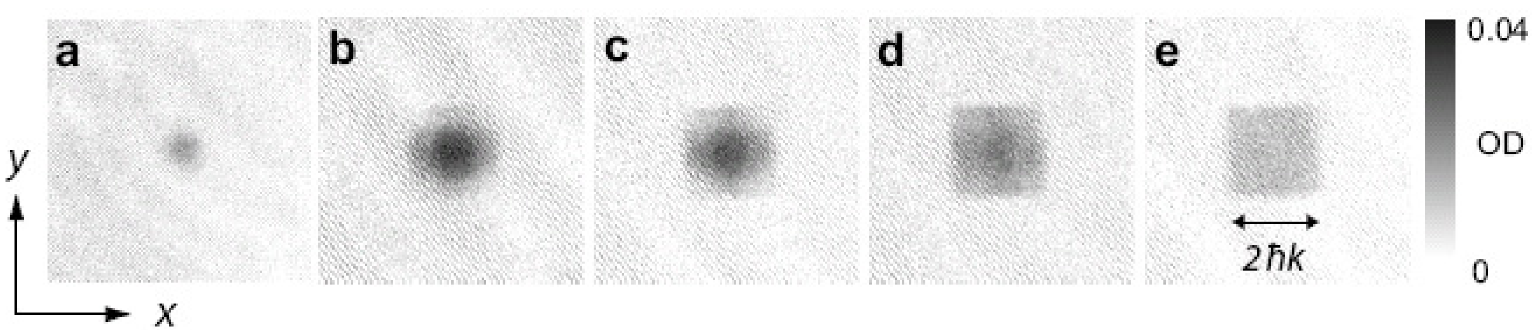
# Ultracold fermions in optical lattices

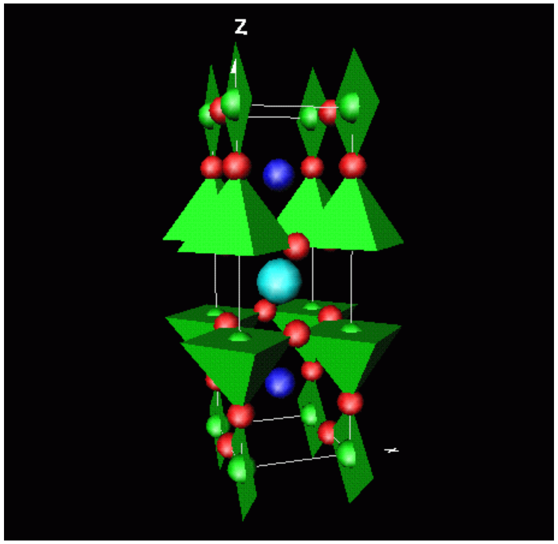
# Fermionic atoms in optical lattices

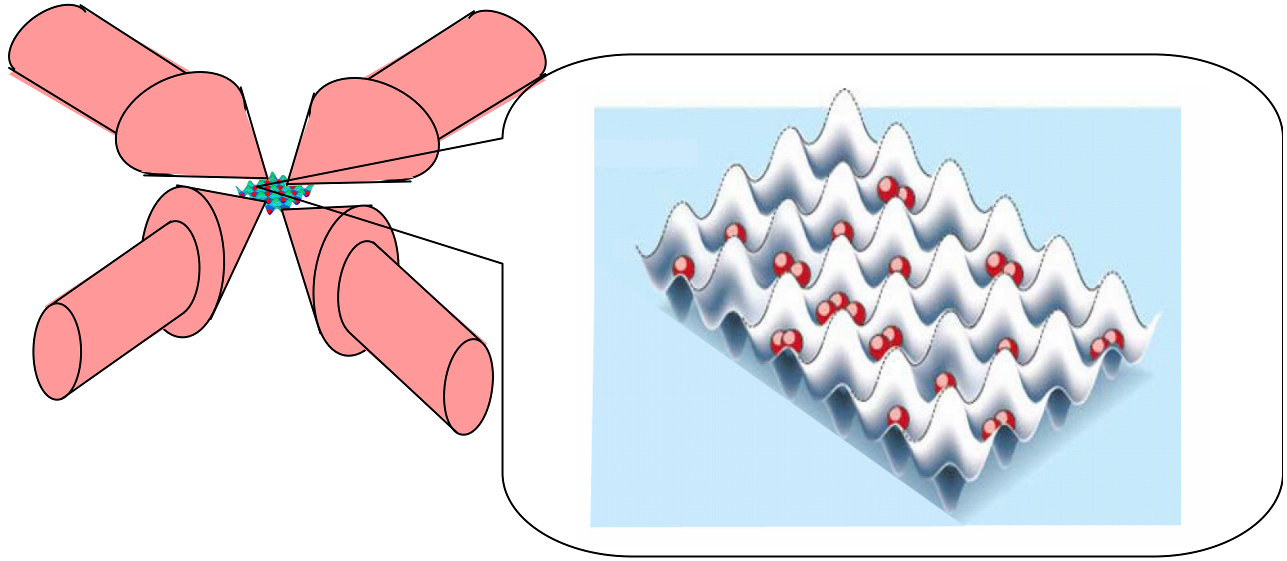
$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i}$$




Experiments with fermions in optical lattice, Kohl et al., PRL 2005





$YBa_2Cu_3O_7$

Atoms in optical lattice

Antiferromagnetic and
superconducting Tc
of the order of 100 K

Antiferromagnetism and
pairing at sub-micro Kelvin
temperatures

# Same microscopic model

$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i$$

# Fermionic Hubbard model

## Phenomena predicted

Superexchange and antiferromagnetism (P.W. Anderson, …)

Itinerant ferromagnetism. Stoner instability (J. Hubbard, …)

Incommensurate spin order. Stripes (Schulz, Zaannen, Emery, Kivelson, White, Scalapino, Sachdev, …)

Mott state without spin order. Dynamical Mean Field Theory (Kotliar, Georges, Giamarchi, …)

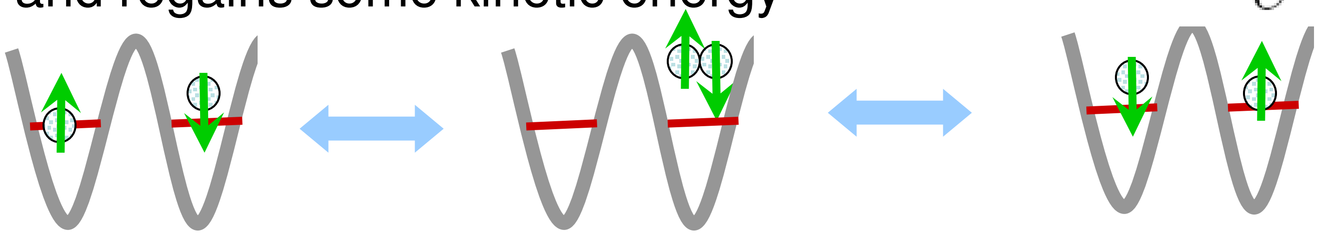
d-wave pairing
(Scalapino, Pines, Baeriswyl, …)

d-density wave (Affleck, Marston, Chakravarty, Laughlin,…)

# Superexchange and antiferromagnetism at half-filling. Large U limit

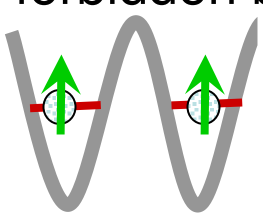
Singlet state allows virtual tunneling and regains some kinetic energy

$$E_{\mathrm{S}} = -\frac{4t^2}{U}$$



Triplet state: virtual tunneling forbidden by Pauli principle

$$E_{\mathrm{T}} = 0$$

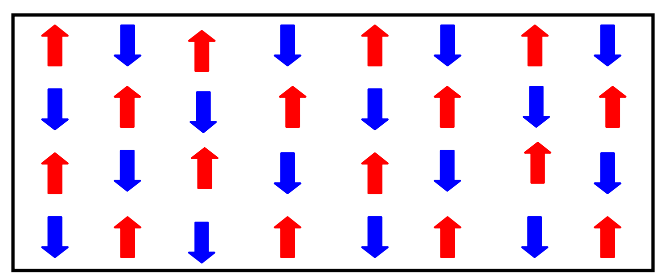


Effective Hamiltonian: Heisenberg model

$$\mathcal{H}_{\mathrm{eff}} = J\,\vec{S}_i\,\vec{S}_j$$

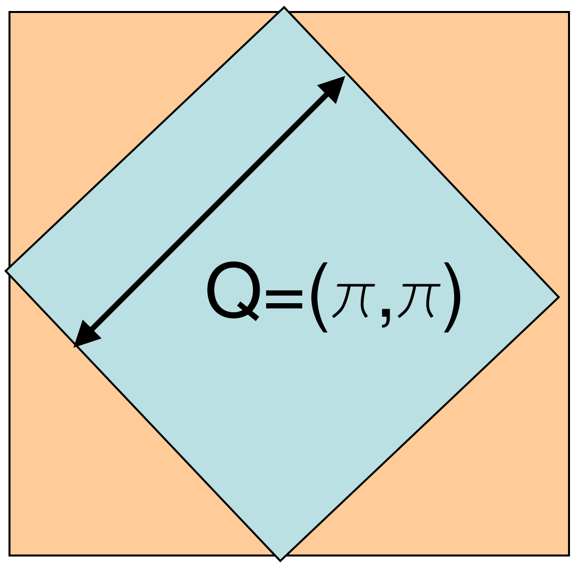
$$J = \frac{4\,t^2}{U}$$

Antiferromagnetic ground state



# Hubbard model for small U. Antiferromagnetic instability at half filling

Fermi surface for n=1




Analysis of spin instabilities.
Random Phase Approximation

$$\chi_{\mathrm{RPA}}(q,\omega) = \frac{\chi_0(q,\omega)}{1 - U\chi_0(q,\omega)}$$

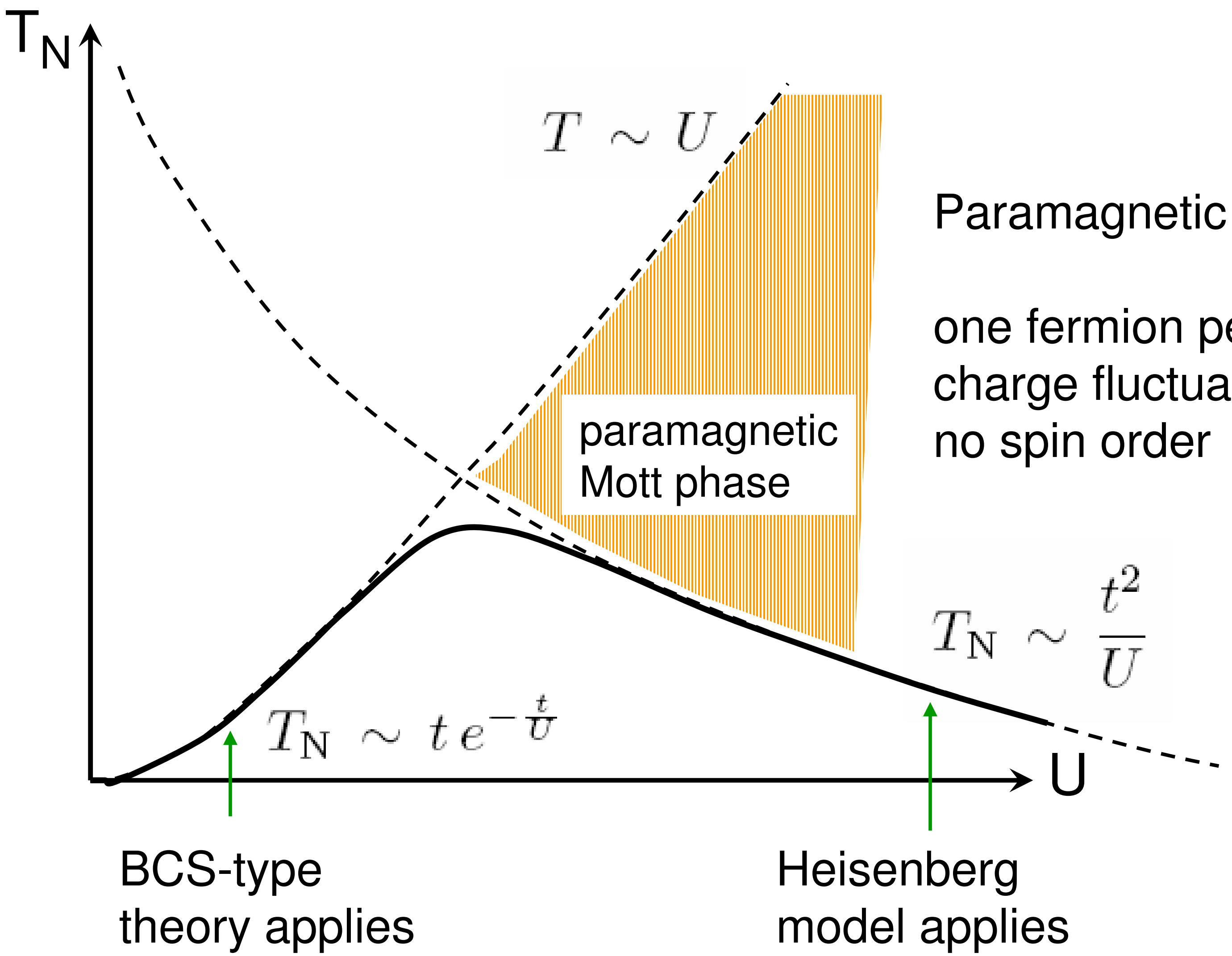
$$\chi_0(q,\omega) = \sum_p \frac{n_p - n_{p+q}}{\omega - \epsilon_p + \epsilon_{p+q}}$$

Nesting of the Fermi surface leads to singularity

$$\chi_0(q,\omega=0) \sim \frac{1}{t}\log(\frac{t}{T})$$

BCS-type instability for weak interaction $T_{\mathrm{N}} \sim t\, e^{-\frac{t}{U}}$

# Hubbard model at half filling

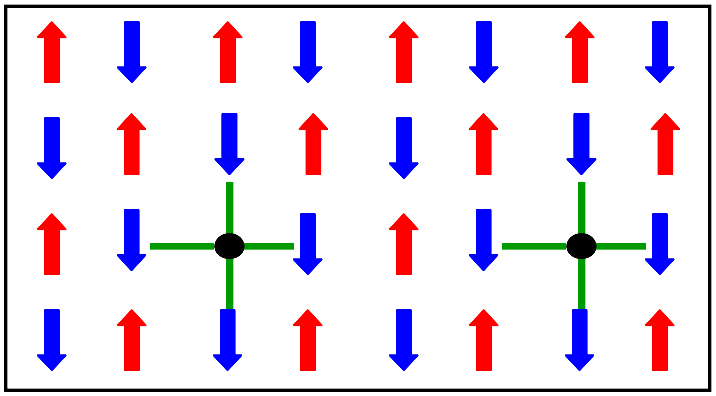


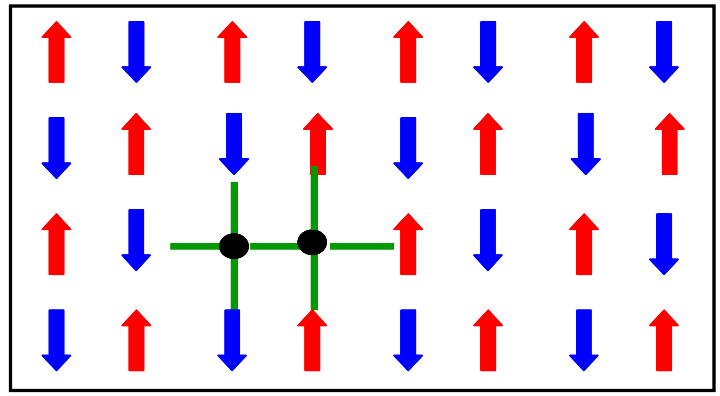

Paramagnetic Mott phase:

one fermion per site
charge fluctuations suppressed
no spin order

# Doped Hubbard model

# Attraction between holes in the Hubbard model



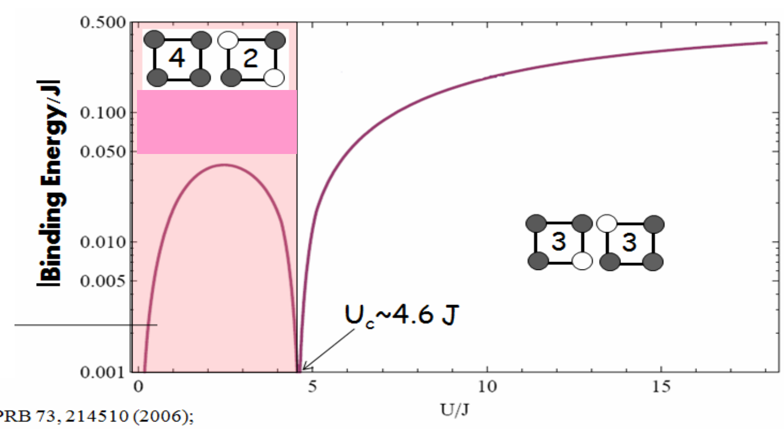


Loss of superexchange energy from 8 bonds

Loss of superexchange energy from 7 bonds

Single plaquette: binding energy

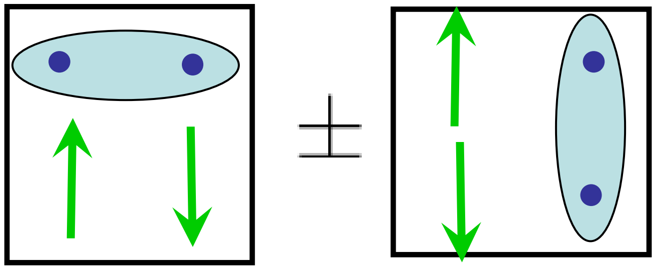
$$\Delta_b = 2E_g(N=3) - E_g(N=4) - E_g(N=2)$$



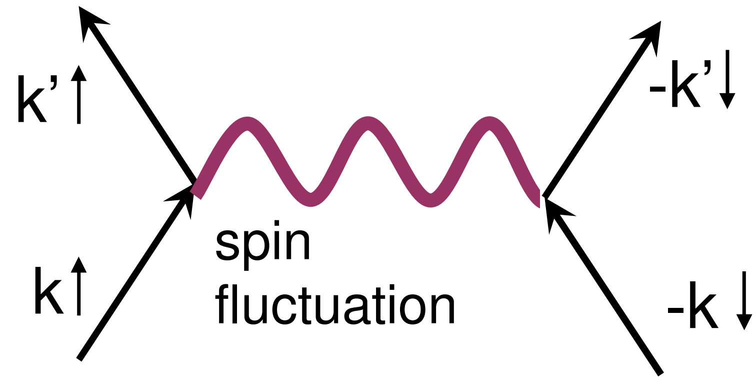


A.F Tsai *et al*, PRB 73, 214510 (2006);
S. Trebst *et al* PRL 96, 250402 (2006);
E. Altman *et al* PRL65, 104508 (2002)

# Pairing of holes in the Hubbard model

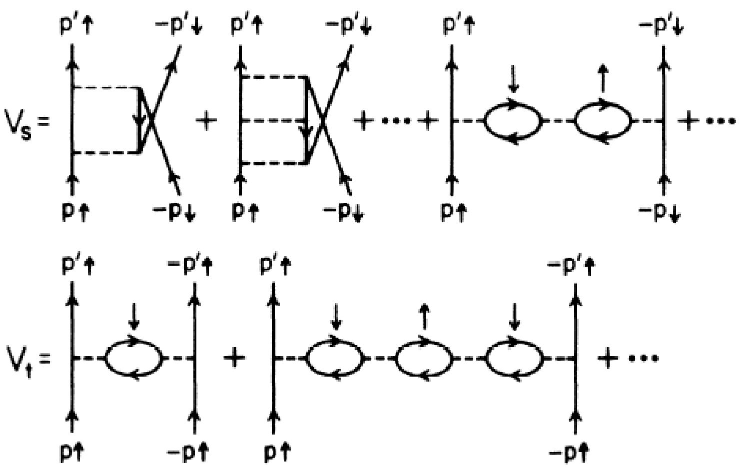


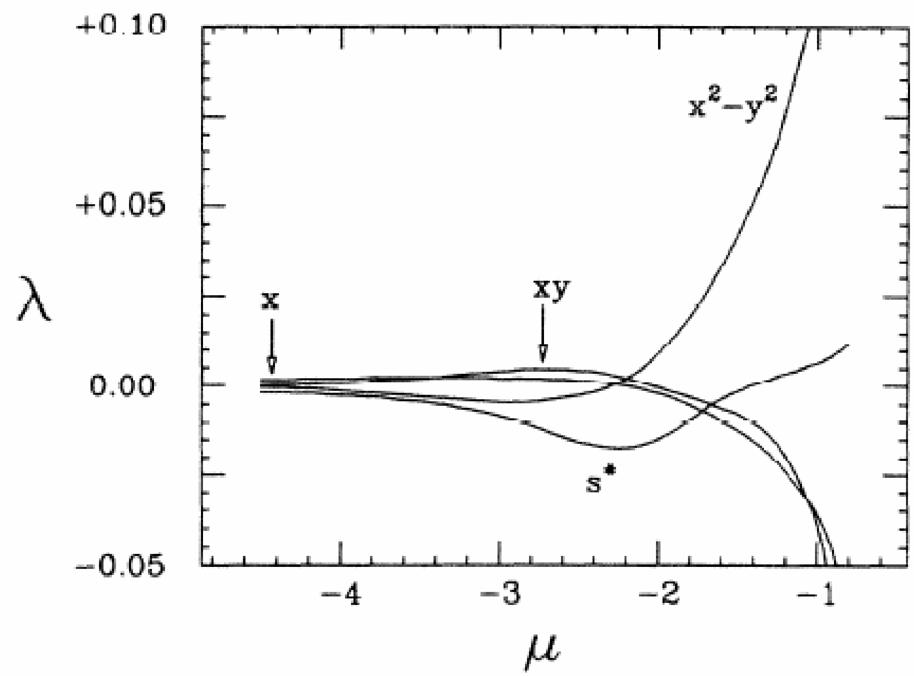

Non-local pairing of holes



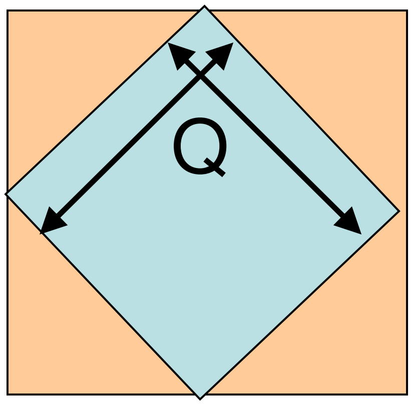

Leading istability:
d-wave

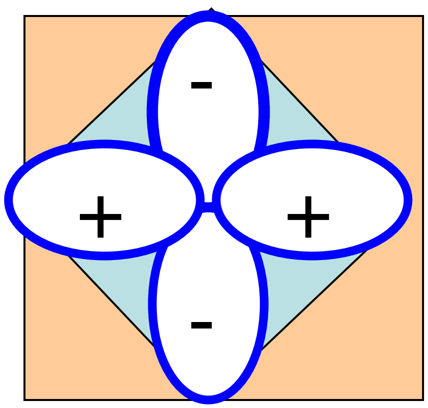
Scalapino et al, PRB (1986)





# Pairing of holes in the Hubbard model



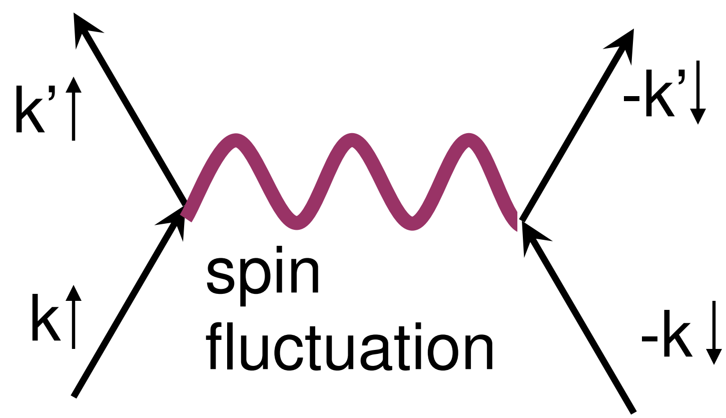




$d_{x2-y2}$

BCS equation for pairing amplitude

$$\Delta_k = -\sum_{k'} V_{kk'} \Delta_{k'}$$

$$V_{kk'} \sim \chi_S(k-k')$$

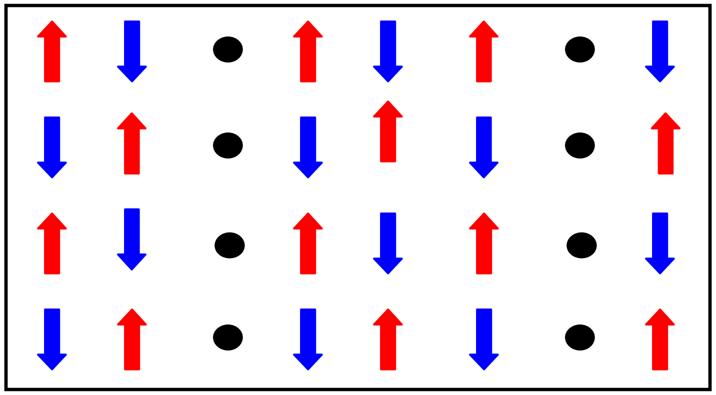



Systems close to AF instability:

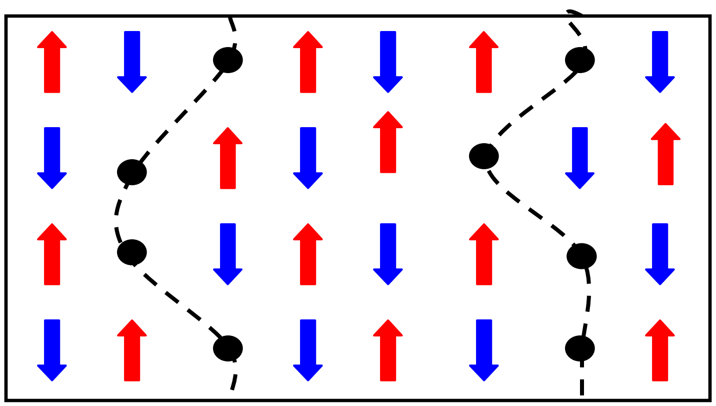
$\chi$(Q) is large and positive

$\Delta_k$ should change sign for k'=k+Q

# Stripe phases in the Hubbard model



**Stripes:**
Antiferromagnetic domains separated by hole rich regions



**Antiphase AF domains**
stabilized by stripe fluctuations

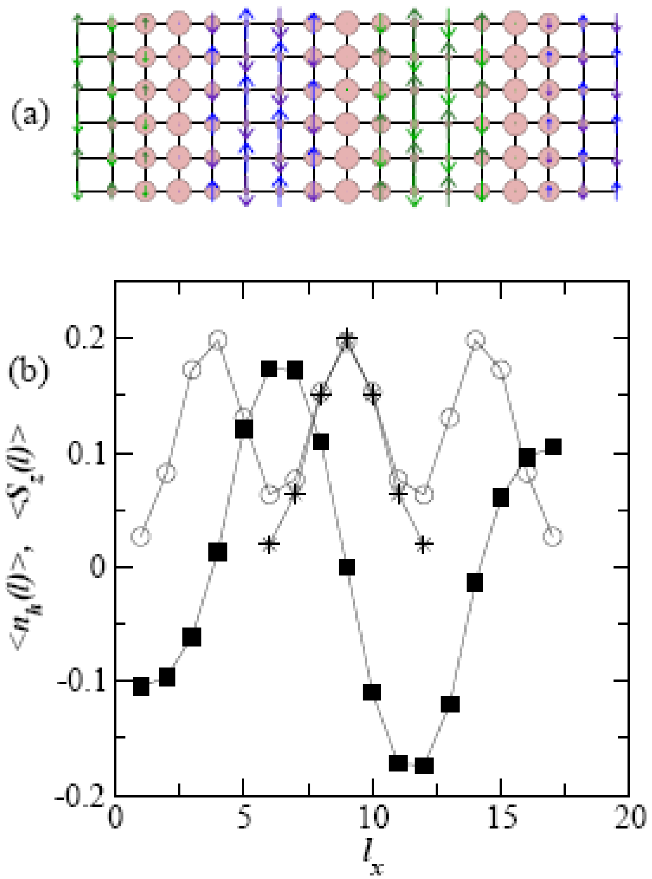
First evidence: Hartree-Fock calculations. Schulz, Zaannen (1989)

$$\mathcal{H} = -t \sum_{\langle ij \rangle} c^\dagger_{i\sigma} c_{j\sigma} - \frac{U}{4} \sum_i \langle \vec{S}_i \rangle c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

# Stripe phases in ladders

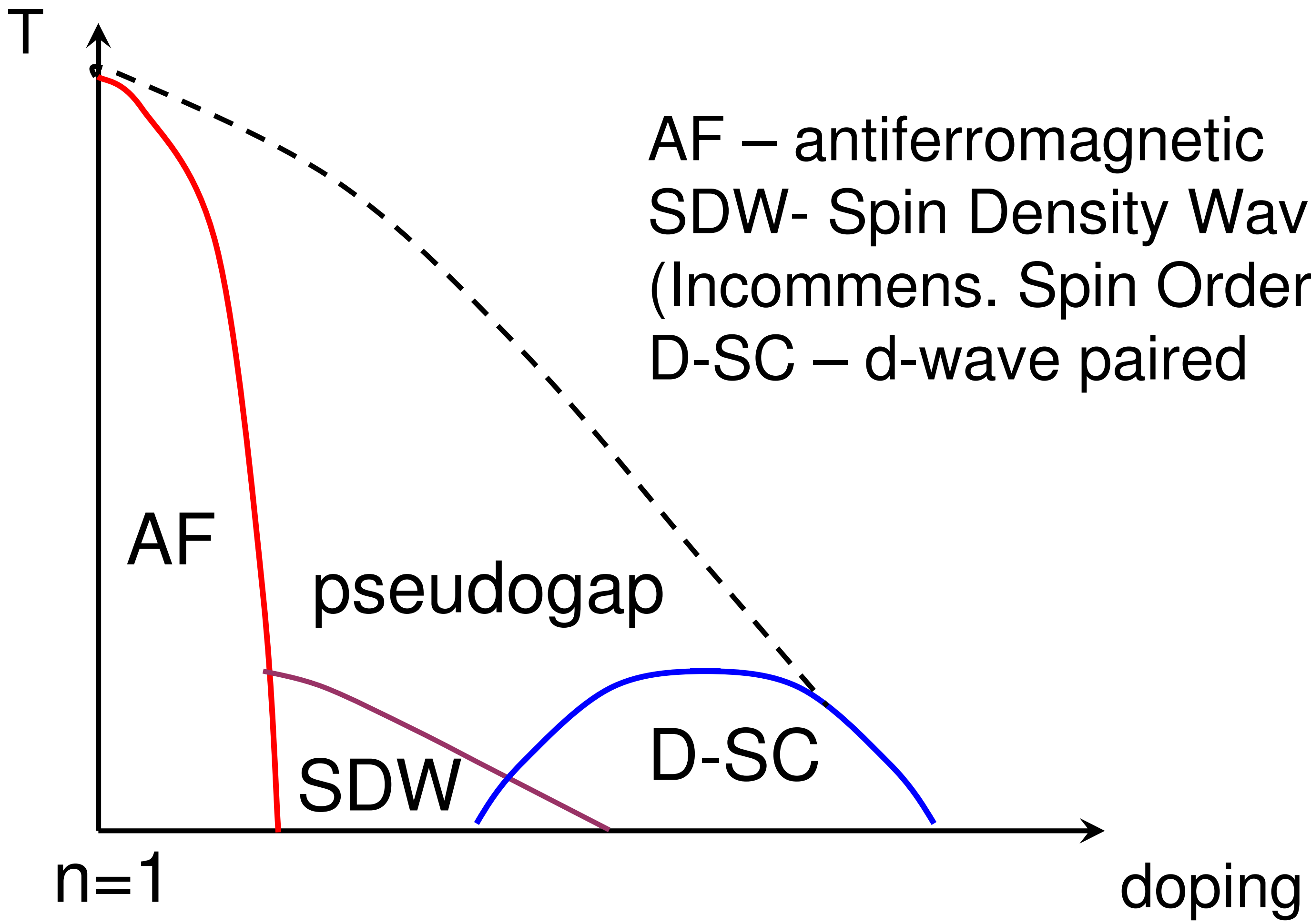
t-J model

$$\mathcal{H}_{tJ} = -t \sum_{\langle ij \rangle} P\, c^{\dagger}_{i\sigma}\, c_{j\sigma}\, P + J \sum_{i} \vec{S}_i\, \vec{S}_j$$




DMRG study of
t-J model on ladders
Scalapino, White, PRL 2003

# Possible Phase Diagram



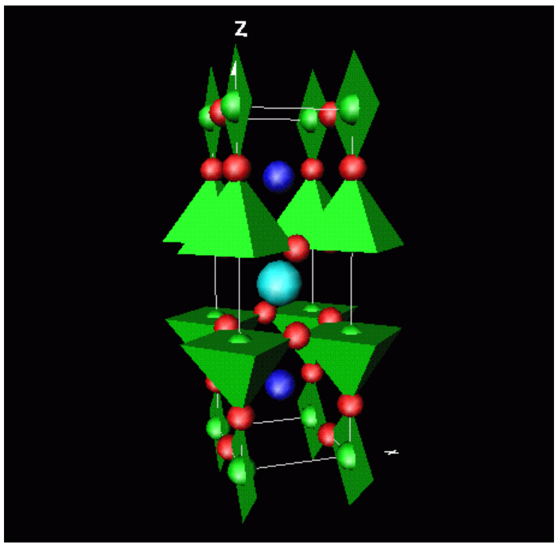

AF – antiferromagnetic
SDW- Spin Density Wave
(Incommens. Spin Order, Stripes)
D-SC – d-wave paired

After several decades we do not yet know the phase diagram

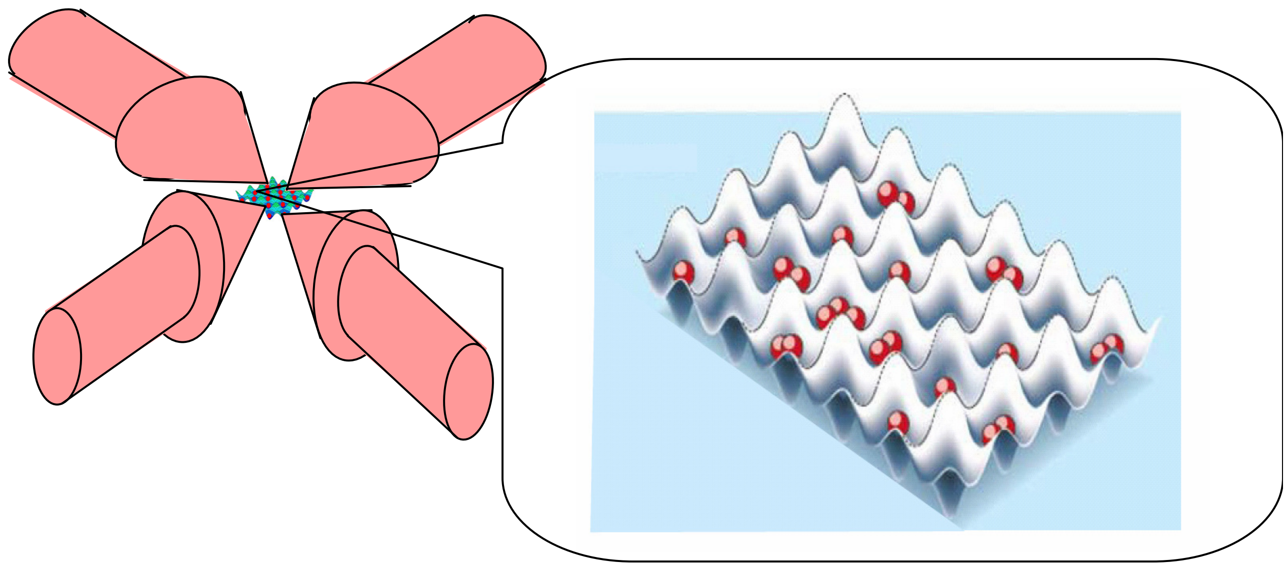
# Fermionic Hubbard model

From high temperature superconductors to ultracold atoms

$$\mathcal{H} = -t \sum_{\langle ij \rangle \, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i}$$




$YBa_2Cu_3O_7$



Atoms in optical lattice

Antiferromagnetic and superconducting Tc of the order of 100 K

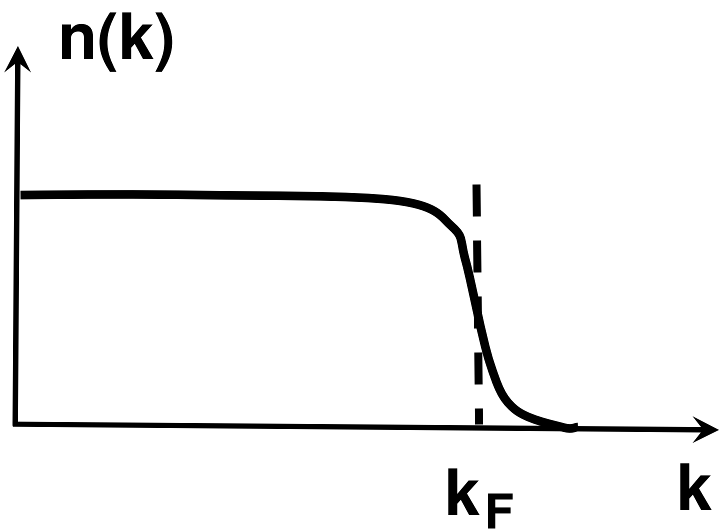
Antiferromagnetism and pairing at sub-micro Kelvin temperatures

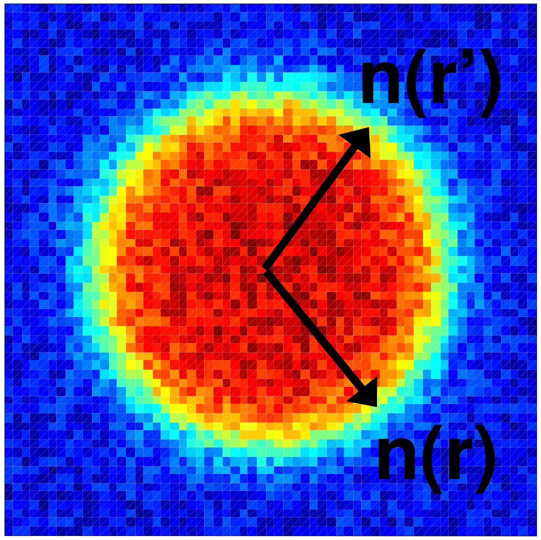
# How to detect fermion pairing

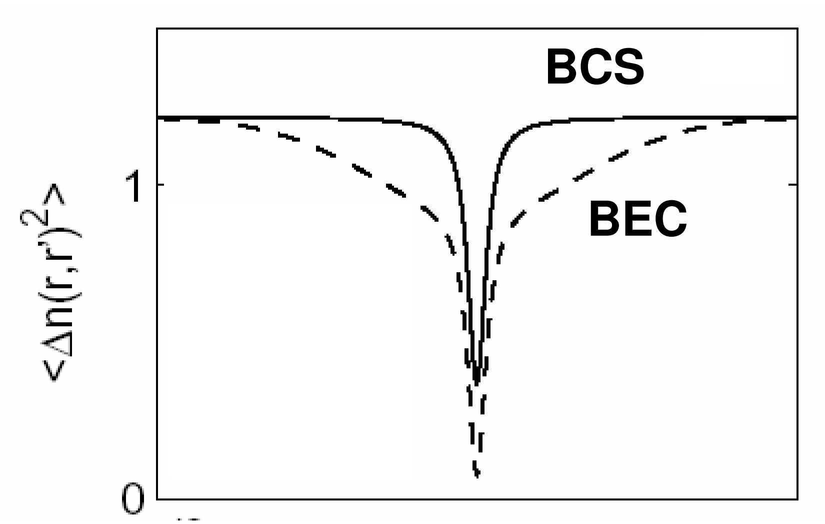
Quantum noise analysis of TOF images
is more than HBT interference

# Second order interference from the BCS superfluid

Theory: Altman et al., PRA (2004)



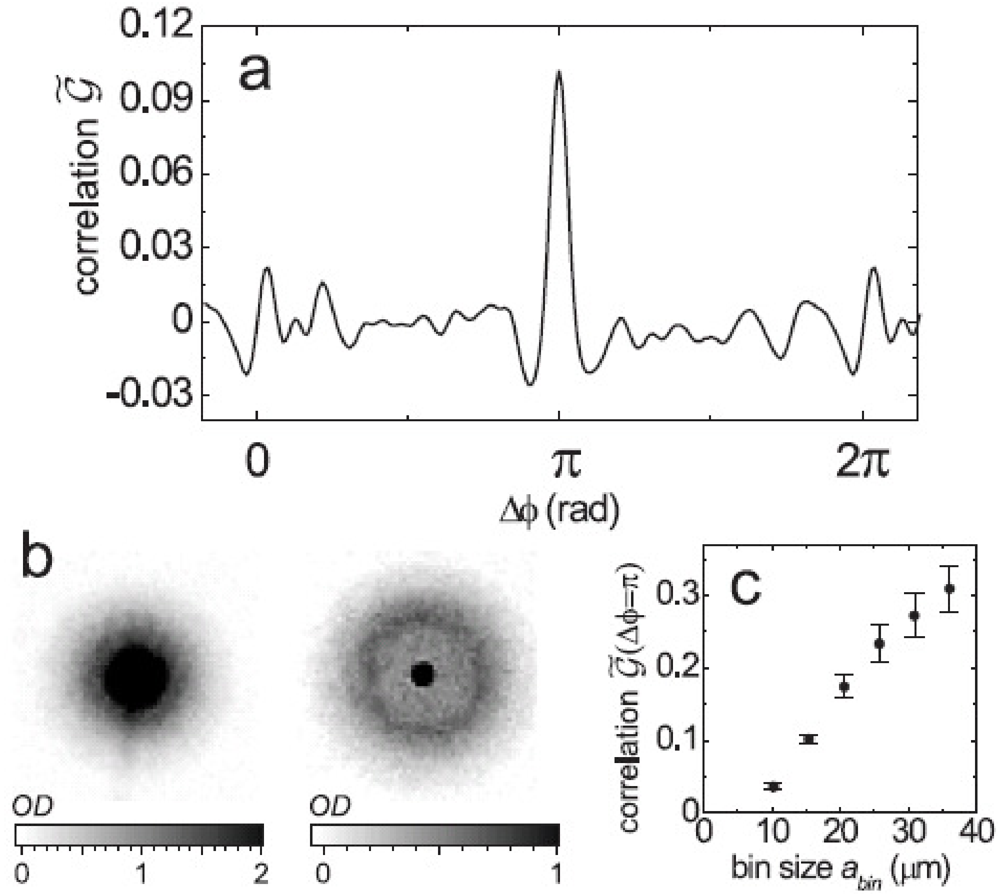




$$\Delta n(\mathbf{r},\mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}')$$

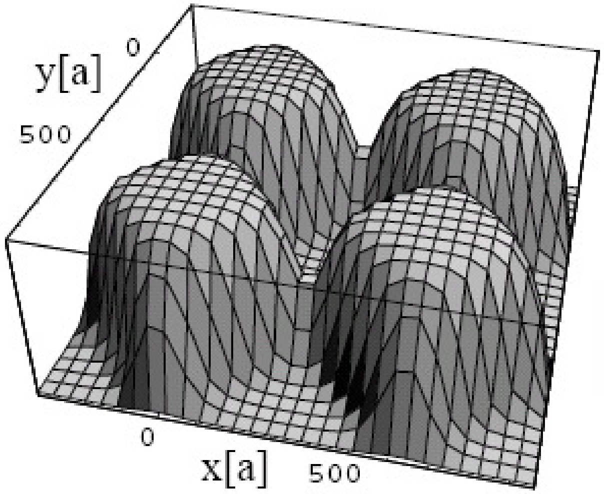
$$\Delta n(\mathbf{r},-\mathbf{r})\left|\Psi_{BCS}\right\rangle = 0$$

# Momentum correlations in paired fermions

**Greiner et al., PRL (2005)**



# Fermion pairing in an optical lattice




**Second Order Interference**
**In the TOF images**

$$G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$$

**Normal State**

$$G_{\mathrm{N}}(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m})$$

**Superfluid State**

$$G_{\mathrm{S}}(r_1, r_2) = G_{\mathrm{N}}(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m})$$

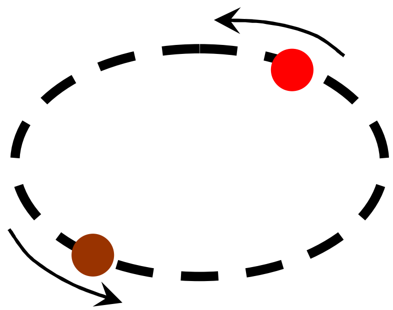
$\Psi(r) = |u(Q(r))v(Q(r))|^2$ **measures the Cooper pair wavefunction**

$$Q(r) = \frac{mr}{\hbar t}$$

**One can identify nodes in pairing amplitude but not the phase change**

# Phase-sensitive measurement of the Cooper pair wavefunction

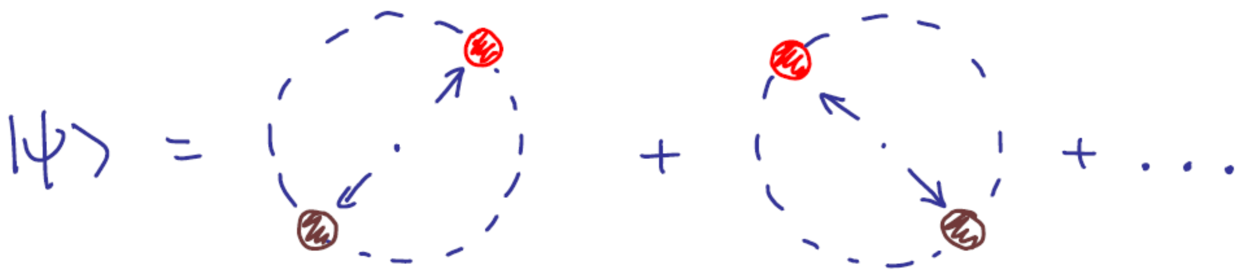
Kitagawa et al., 2010



Consider a single molecule first

$$|\Psi\rangle = \sum_p \psi(p)\, c^\dagger_{p\uparrow}\, c^\dagger_{-p\downarrow}$$

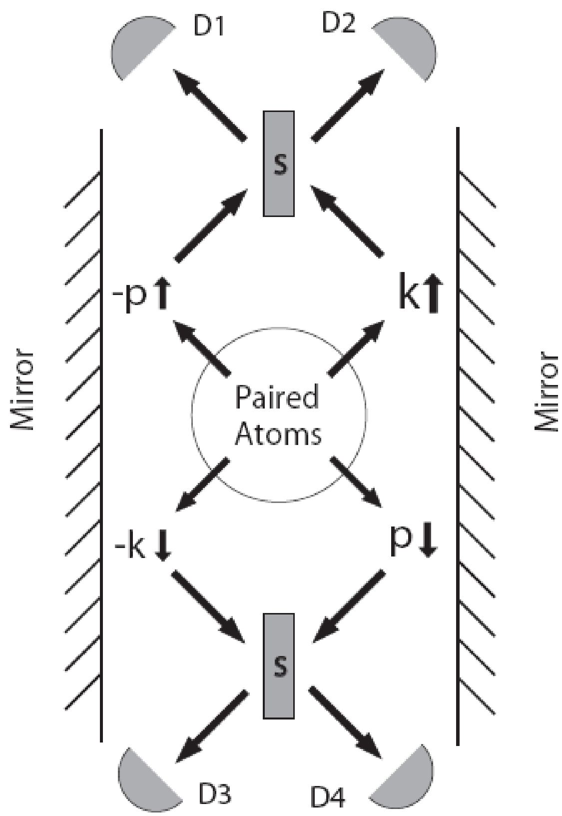
How to measure the non-trivial symmetry of $\psi$(p)?




$$|\Psi\rangle = \psi(p)\, c^\dagger_{p\uparrow}\, c^\dagger_{-p\downarrow} + \psi(k)\, c^\dagger_{k\uparrow}\, c^\dagger_{-k\downarrow} + \cdots$$

We want to measure the relative phase between components of the molecule at different wavevectors

# Two particle interference




Coincidence count on detectors measures two particle interference

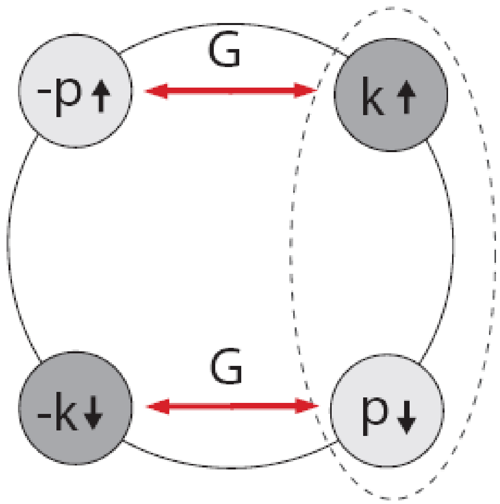
$$
\begin{aligned}
\langle n_1 n_3 \rangle_c &= |\Psi|^2 \sin^2(2\beta)\cos^2\left(\frac{\Phi_I}{2}\right) \\
\langle n_1 n_4 \rangle_c &= |\Psi|^2 \left[1 - \sin^2(2\beta)\cos^2\left(\frac{\Phi_I}{2}\right)\right] \\
\Phi_I &= \tilde{\phi}_k - \tilde{\phi}_p + \chi_\uparrow - \chi_\downarrow
\end{aligned}
$$

$\chi_\uparrow - \chi_\downarrow$

phase controlled by beam splitters and mirrors

# Two particle interference

Implementation for atoms: Bragg pulse before expansion




Bragg pulse mixes states
k and –p = k-G
-k and p =-k+G

$$\tilde{c}^{\dagger}_{k\uparrow} = \cos\beta\, c^{\dagger}_{k\uparrow} - i\,\sin\beta\, e^{i\chi_{\uparrow}}\, c^{\dagger}_{k-G\uparrow}$$

$$\tilde{c}^{\dagger}_{-k\downarrow} = \cos\beta\, c^{\dagger}_{-k\downarrow} - i\,\sin\beta\, e^{-i\chi_{\downarrow}}\, c^{\dagger}_{-k+G\downarrow}$$

Coincidence count for states k↑and p↓depends on two particle interference and measures phase of the molecule wavefunction

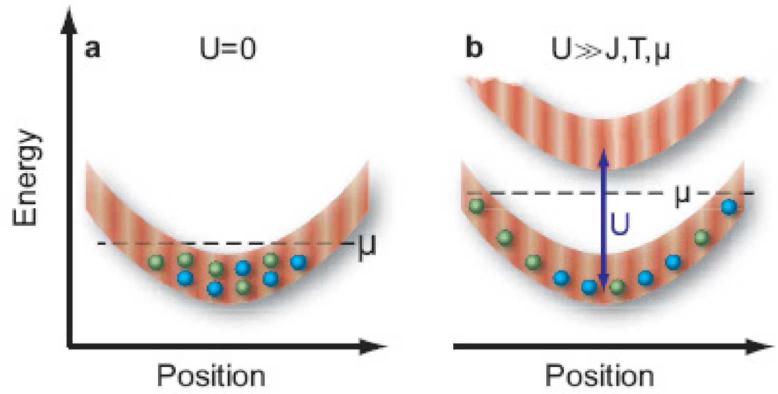
$$\langle\, n_{k\uparrow}\, n_{p\downarrow}\,\rangle = |\Psi|^2\, \sin^2(2\beta)\, \cos^2\left(\frac{\Phi}{2}\right)$$

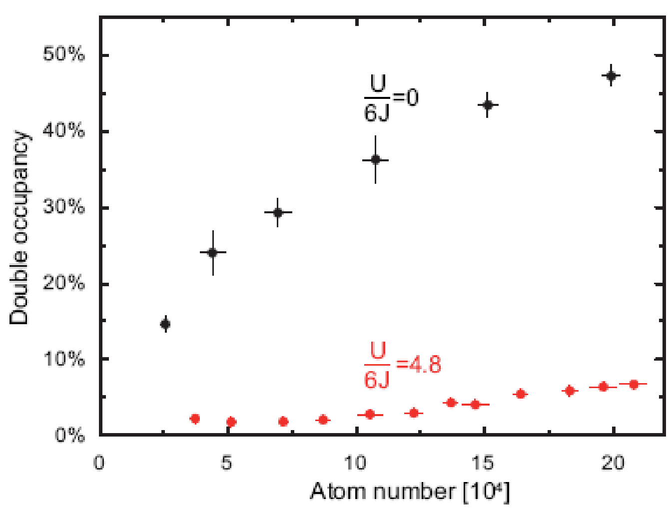
$$\Phi = \phi_k - \phi_p + \chi_{\uparrow} - \chi_{\downarrow}$$

# Experiments on the Mott state of ultracold fermions in optical lattices

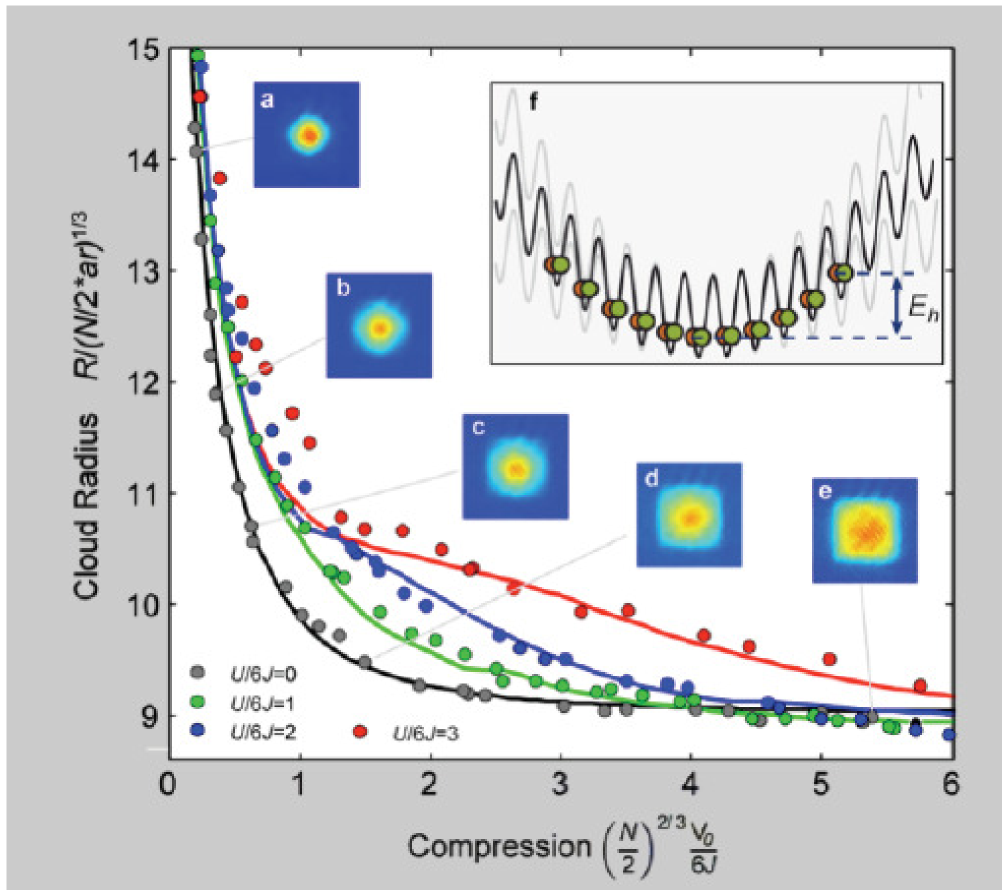
# Signatures of incompressible Mott state of fermions in optical lattice

Suppression of double occupancies
R. Joerdens et al., Nature (2008)

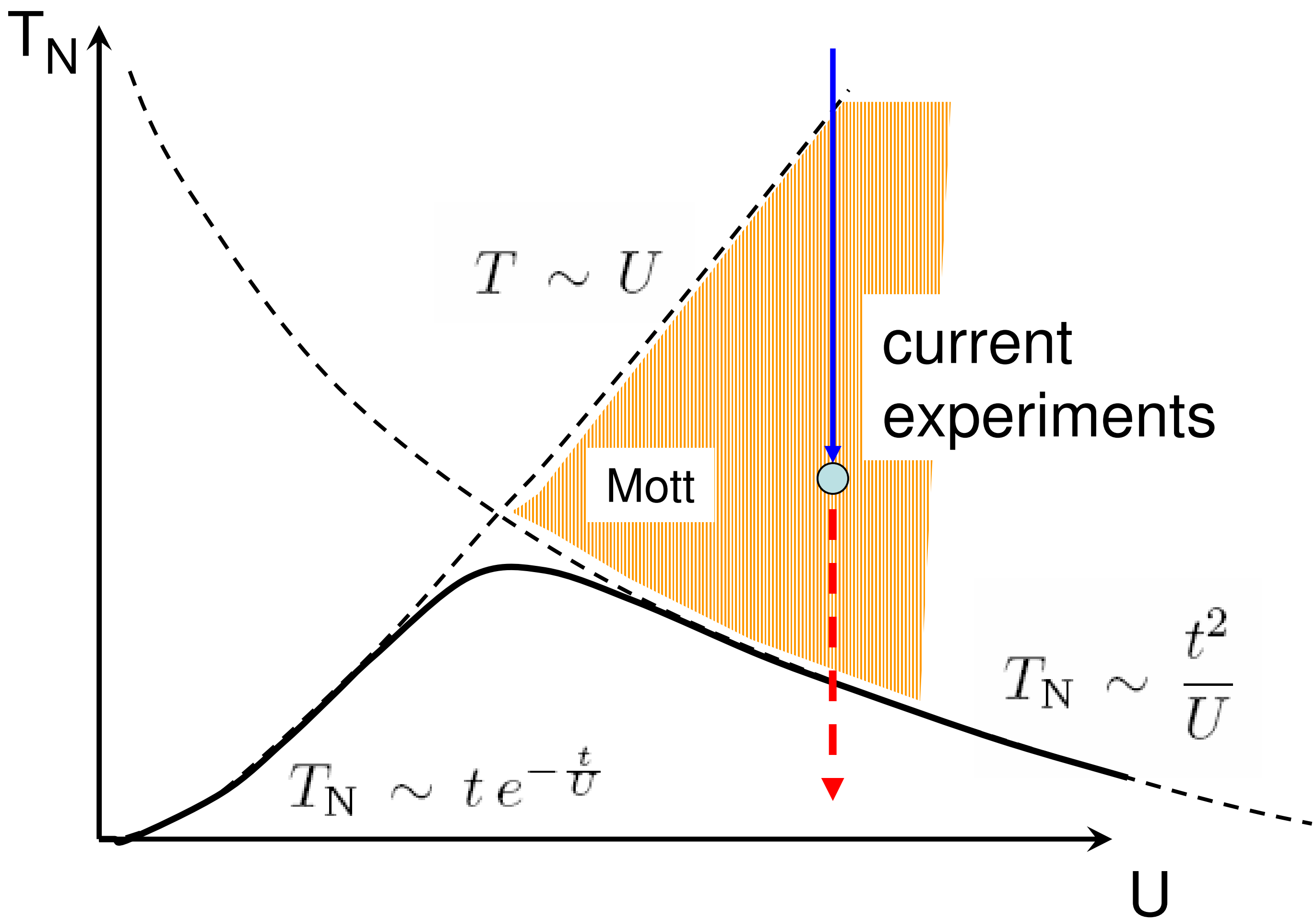



Compressibility measurements
U. Schneider et al., Science (2008)

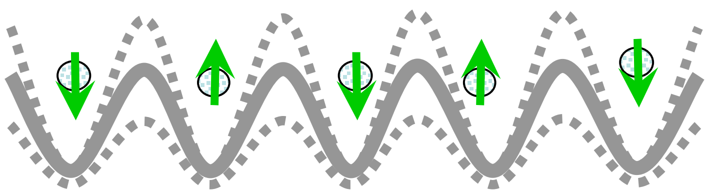


# Fermions in optical lattice. Next challenge: antiferromagnetic state



# Lattice modulation experiments with fermions in optical lattice.

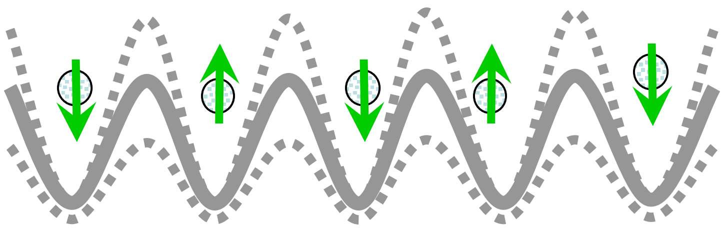
## Probing the Mott state of fermions



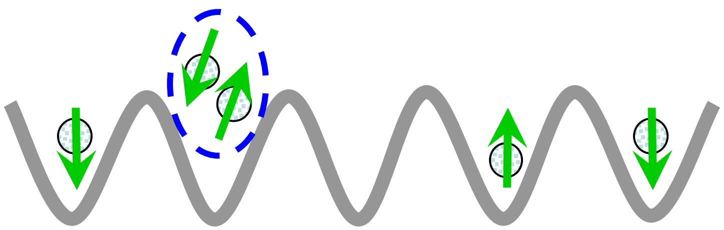
Theory: Kollath et al., PRA (2006)
Sensarma et al., PRL (2009)
Huber, Ruegg, PRB (2009)
Expts: Joerdens et al., Nature (2008)

# Lattice modulation experiments

## Probing dynamics of the Hubbard model



Modulate lattice potential $V_0$



Measure number of doubly occupied sites

$$t \sim \exp(-\sqrt{V_0/E_R}) \qquad U \sim \left(\frac{V_0}{E_R}\right)^{3/4}$$

Main effect of shaking: modulation of tunneling

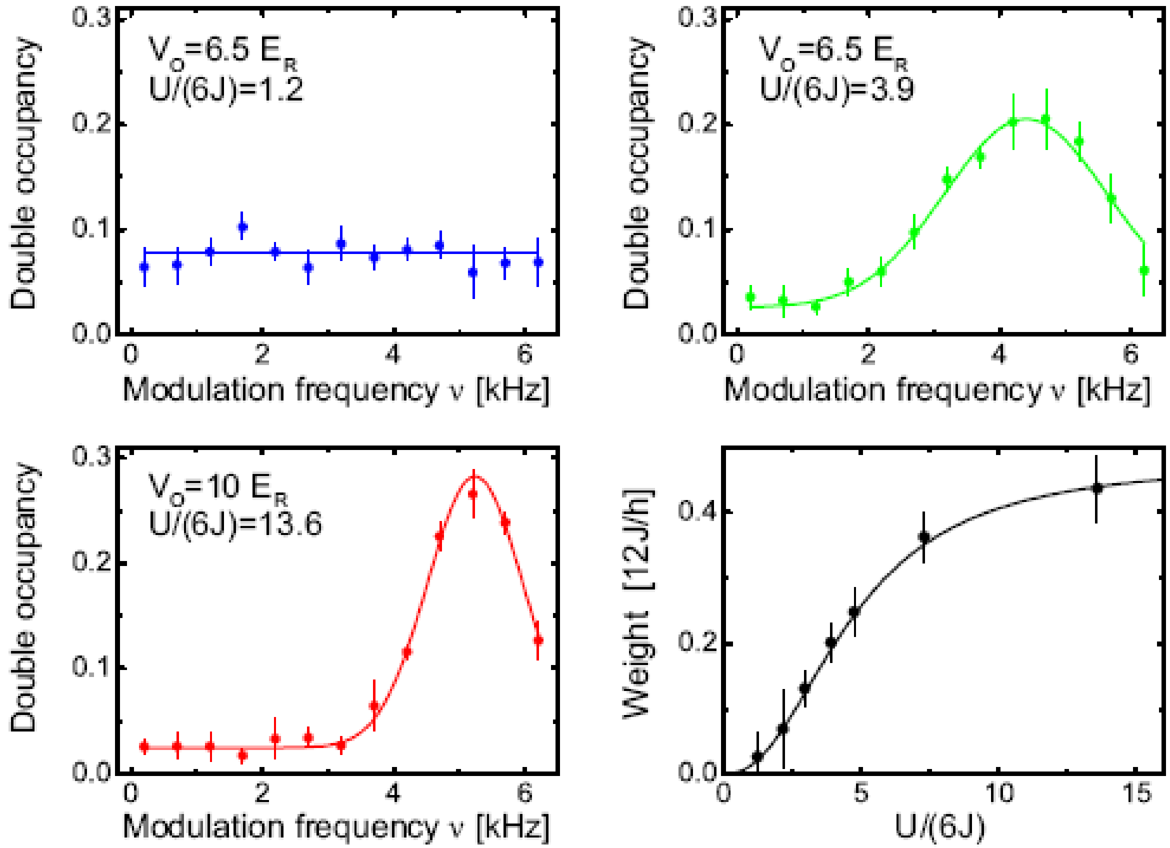
$$\mathcal{H}_{\text{pert}}(\tau) = \lambda t \cos\omega\tau \sum_{\langle ij\rangle\sigma} c_{i\sigma}^{\dagger} c_{j\sigma}$$

Doubly occupied sites created when frequency $\omega$ matches Hubbard $U$

# Lattice modulation experiments

## Probing dynamics of the Hubbard model

R. Joerdens et al., Nature 455:204 (2008)



# Mott state

Regime of strong interactions *U>>t.*

Mott gap for the charge forms at $T \sim U$

Antiferromagnetic ordering at $T_N \sim J = \frac{4t^2}{U}$

"High" temperature regime $T_N << T << U$

All spin configurations are equally likely.
Can neglect spin dynamics.

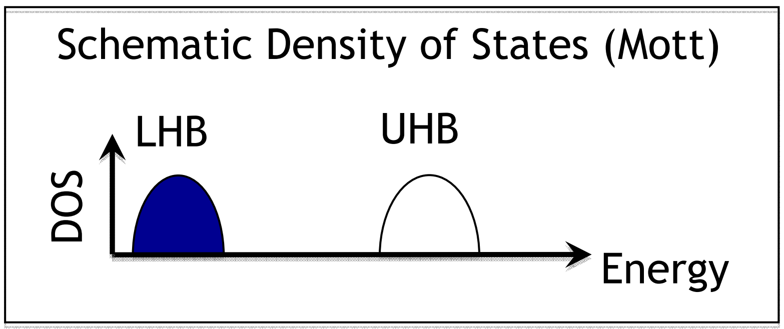
"Low" temperature regime $T \leq T_N$

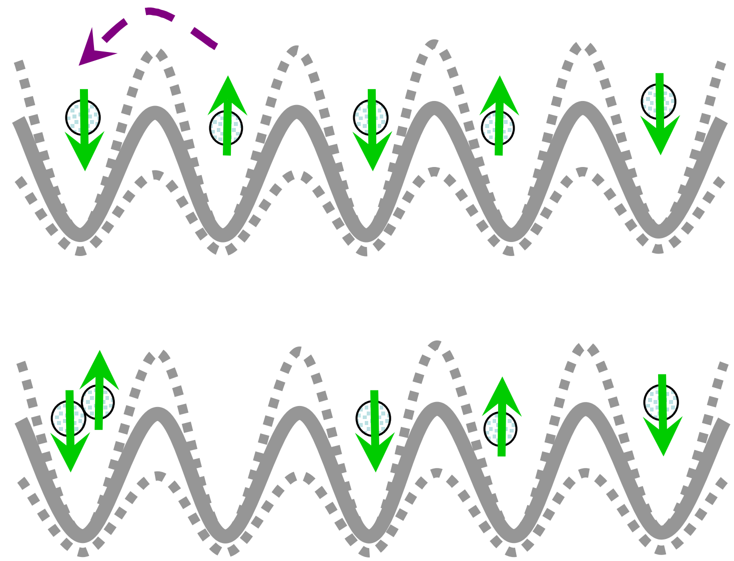
Spins are antiferromagnetically ordered
or have strong correlations

# Lattice Modulation

Experiment:

- Modulate lattice intensity
- Measure number Doublons



Golden Rule doublon/hole production rate

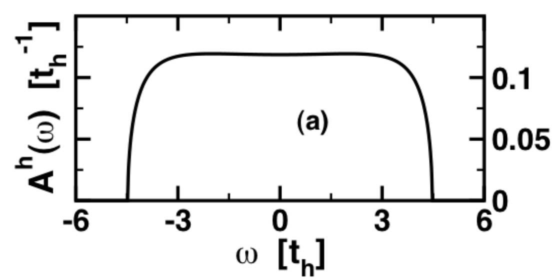
hole spectral function

doublon spectral function

$$P_d(\omega) = \frac{\pi}{2}(\delta t)^2 P_s \sum_{\langle ij\rangle\langle lm\rangle} \int d\omega' \mathcal{A}^d_{il}(\omega')\mathcal{A}^h_{jm}(\omega-\omega')$$

probability of singlet

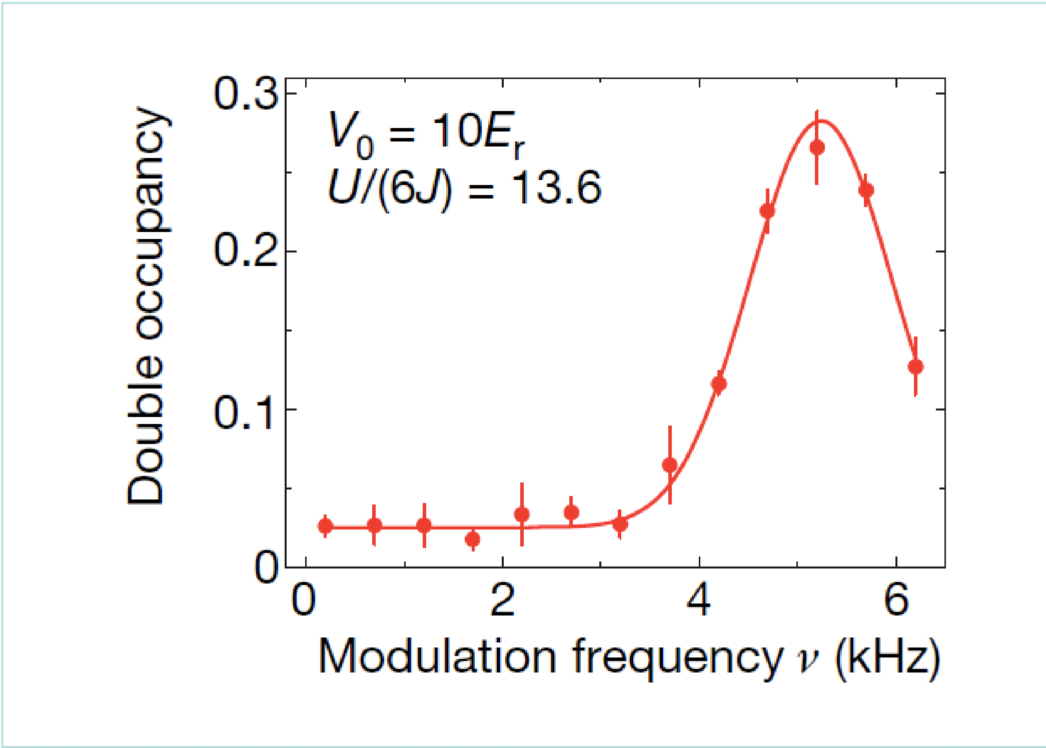
Spectral Function for doublons/holes
retracing path approximation




*Brinkman & Rice, 1970*

# Medium Temperature

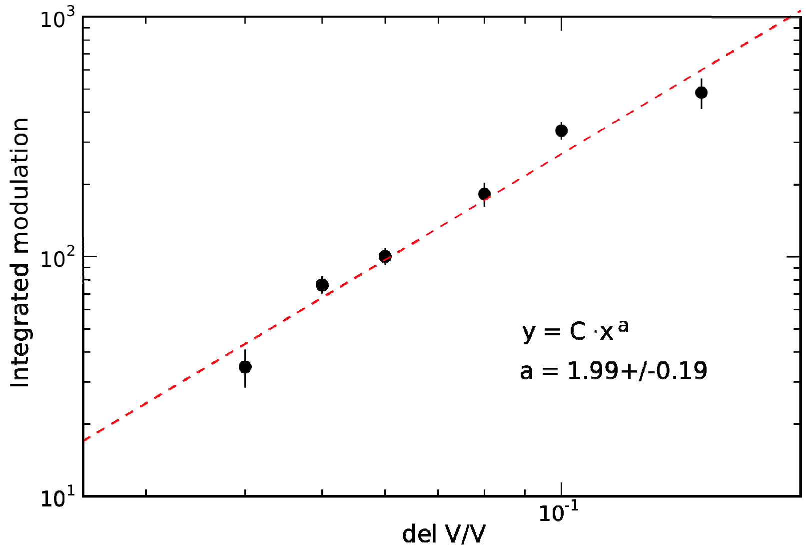
$$\boxed{J \ll T, t \ll U}$$

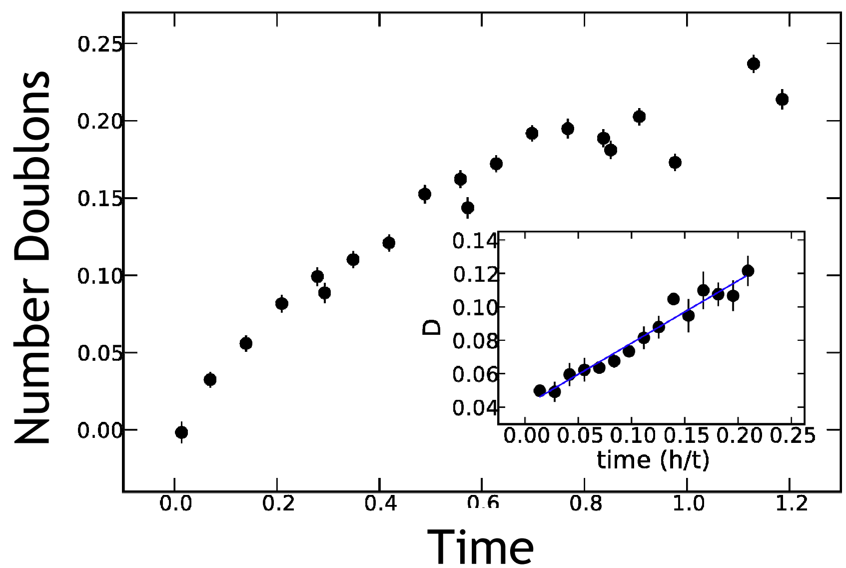



Original Experiment: R. Joerdens et al., Nature 455:204 (2008)

Theory: Sensarma, Pekker, Lukin, Demler,
PRL **103**, 035303 (2009)

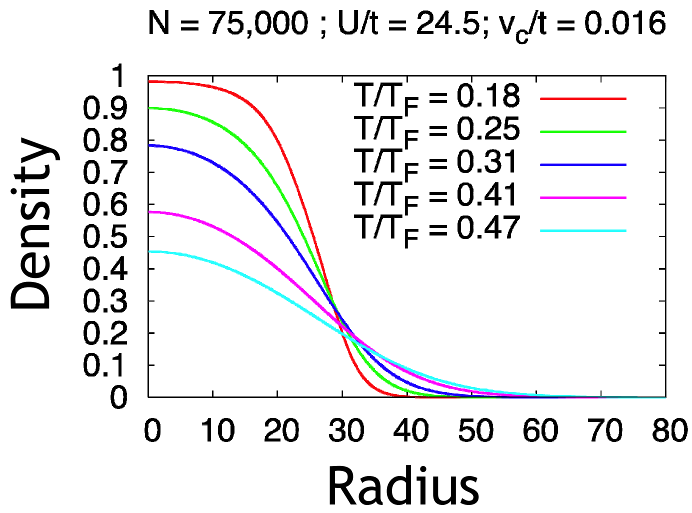
# Build up rate (preliminary data)

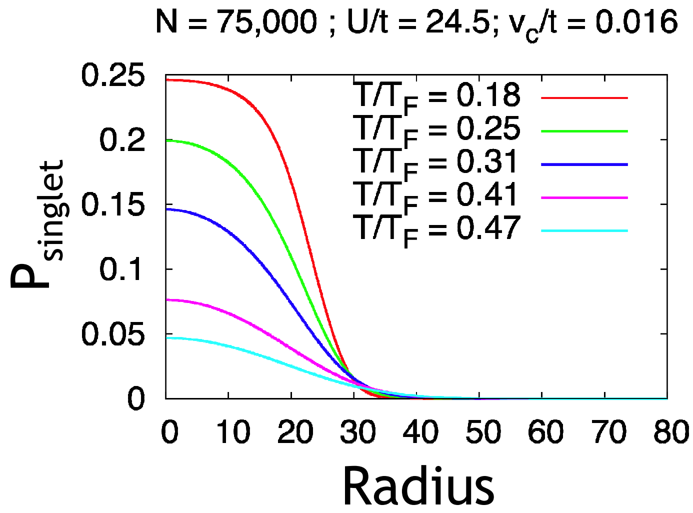


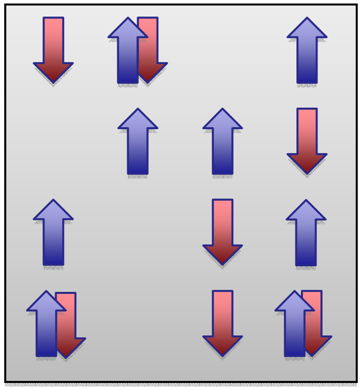



# Warmer than medium temperature

1. Decrease in density (reduced probability to find a singlet)



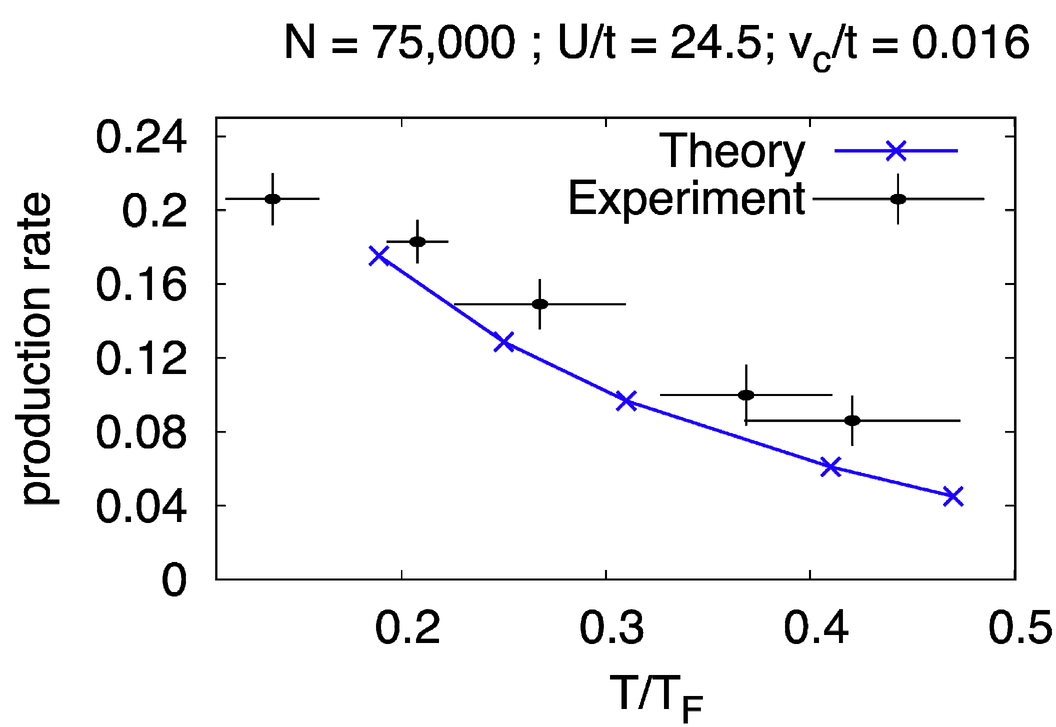



2. Change of spectral functions
   – Harder for doublons to hop (work in progress)

# Temperature dependence

Simple model: take doublon production rate at half-filling and multiply by the probability to find atoms on neighboring sites.

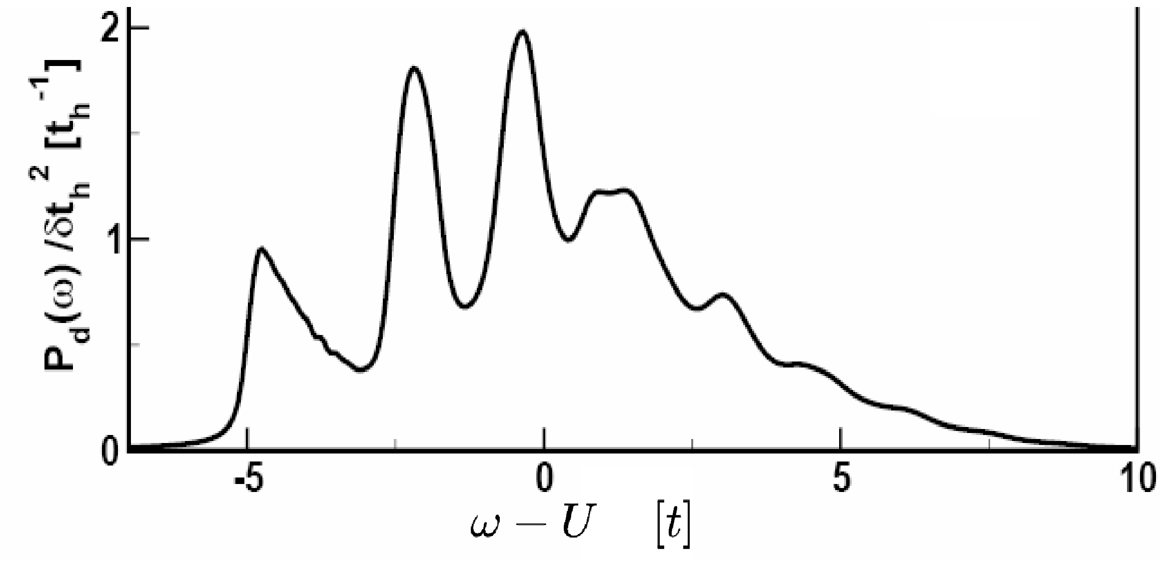
Experimental results: latest ETH data, unpublished, preliminary



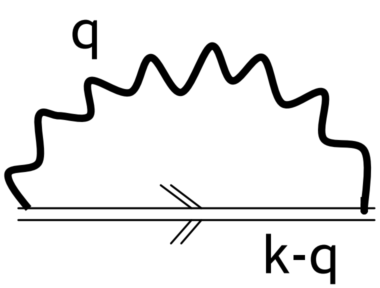
# Low Temperature

$$T < J$$

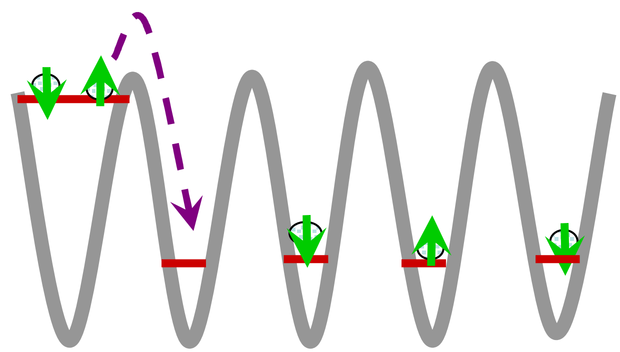
- Rate of doublon production in linear response approximation




- Fine structure due to spinwave shake-off
- Sharp absorption edge from coherent quasiparticles
- Signature of AFM!



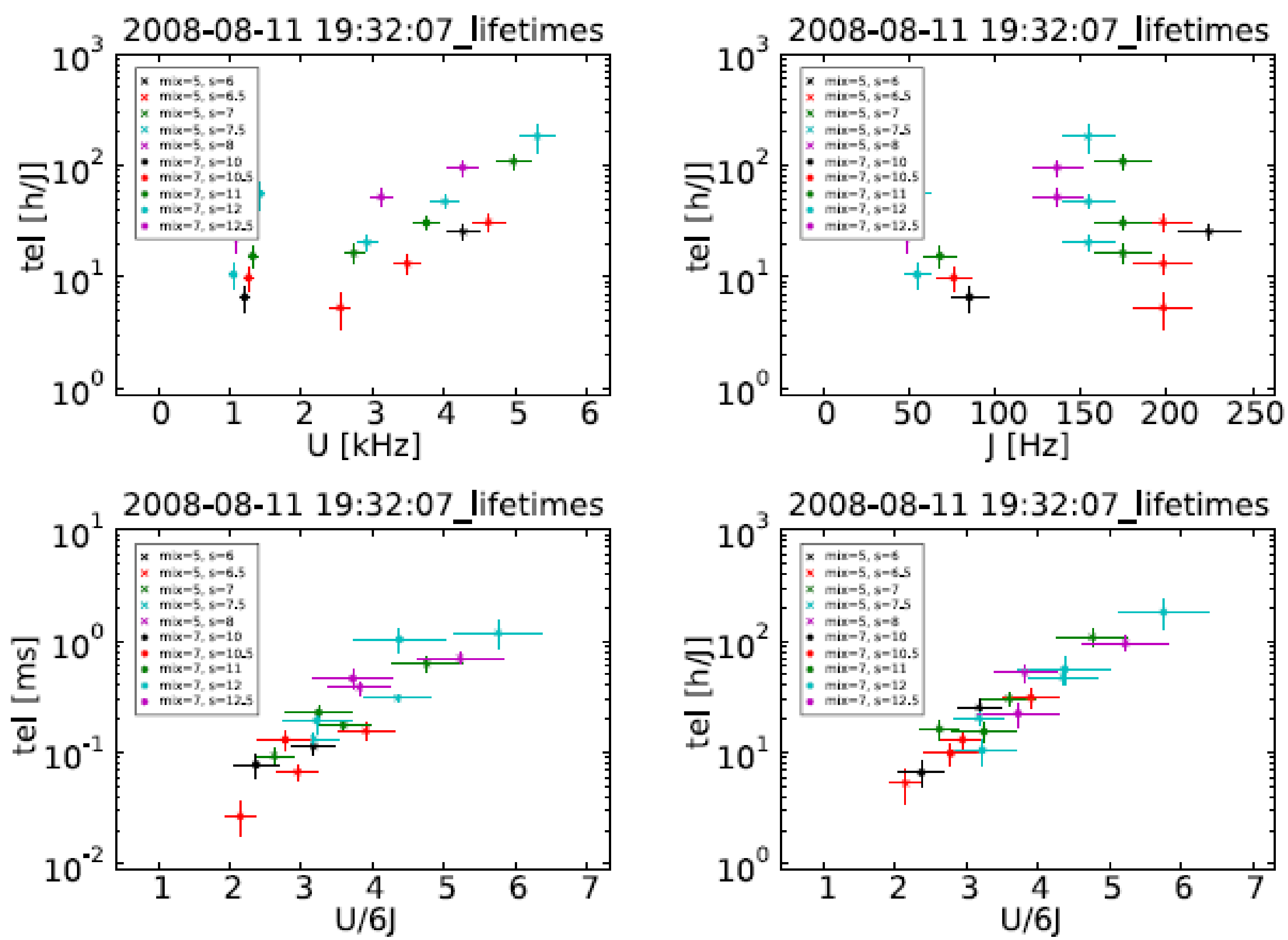
# Fermions in optical lattice. Decay of repulsively bound pairs



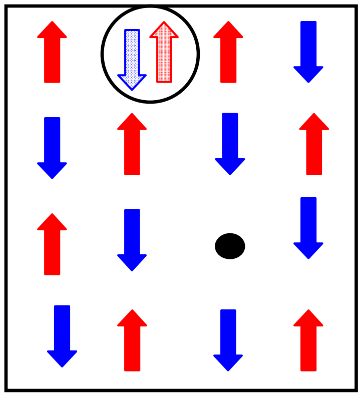
Ref: N. Strohmaier et al., arXiv:0905.2963
Experiment: T. Esslinger's group at ETH
Theory: Sensarma, Pekker, Altman, Demler

# Fermions in optical lattice. Decay of repulsively bound pairs

Experiments: N. Strohmaier et. al.



# Relaxation of doublon- hole pairs in the Mott state



Energy U needs to be absorbed by spin excitations

❖Energy carried by

spin excitations

~ $J = 4t^2/U$

❖ Relaxation requires

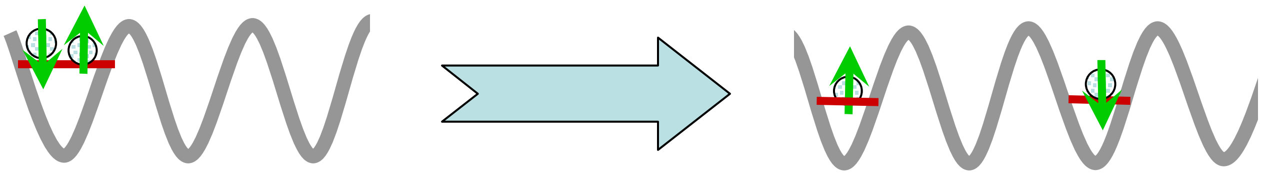
creation of $\sim U^2/t^2$

spin excitations

Relaxation rate $W \sim t(t/U)^{U^2/t^2}$

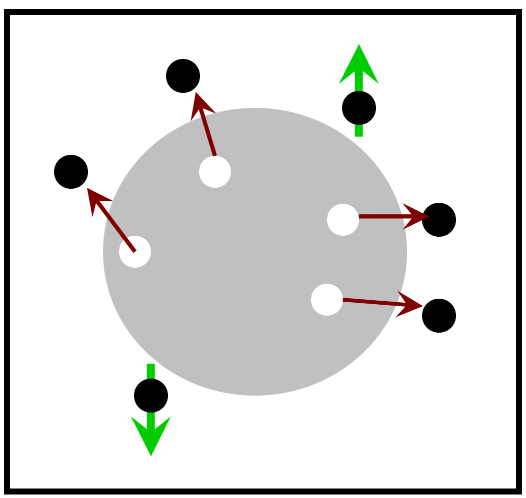
Very slow, not relevant for ETH experiments

# Doublon decay in a compressible state



Excess energy U is converted to kinetic energy of single atoms

## Compressible state: Fermi liquid description



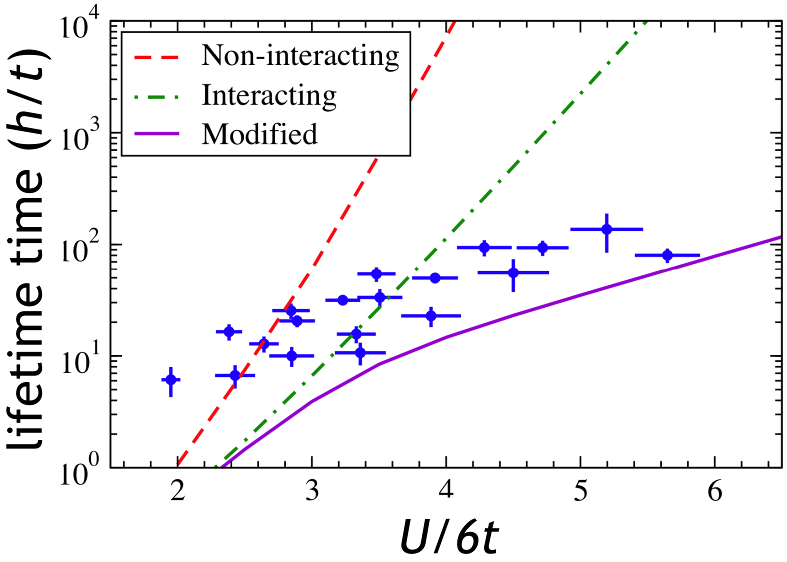
Perturbation theory to order n=U/6t
Decay probability

$$\mathrm{P} \sim \left(\frac{t}{U}\right)^{\mathrm{const}\,\cdot\,\frac{U}{6t}} \sim e^{-\,\mathrm{const}\,\cdot\,\frac{U}{6t}\,\cdot\,\log(\frac{U}{t})}$$

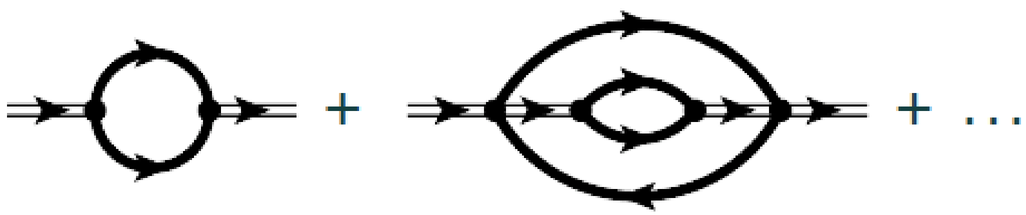
Doublon can decay into a pair of quasiparticles with many particle-hole pairs

# Diagramatic Flavors

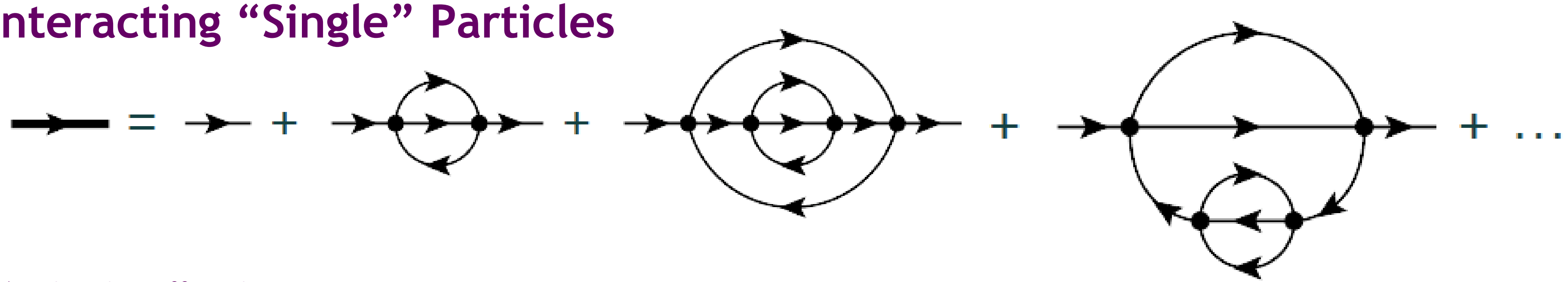
Comparison of approximations



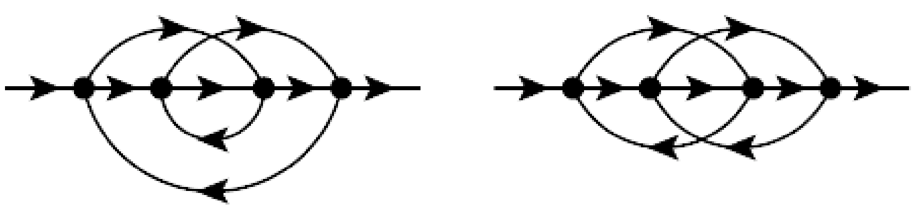

## Doublon Propagator



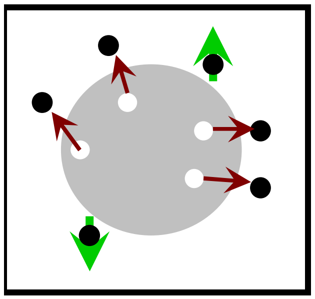
## Interacting "Single" Particles

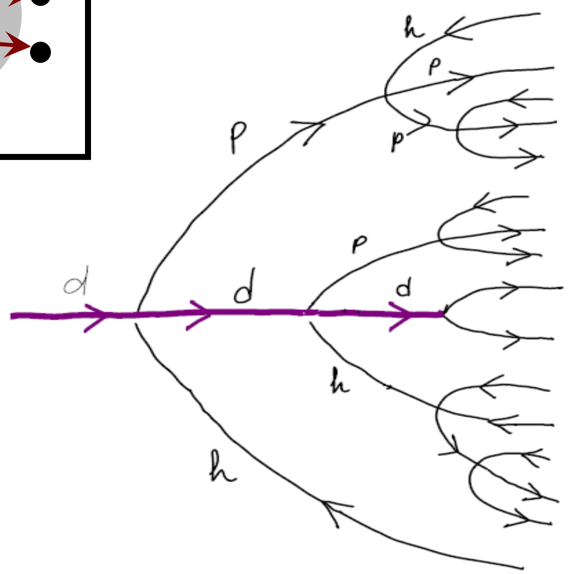


## "Missing" Diagrams

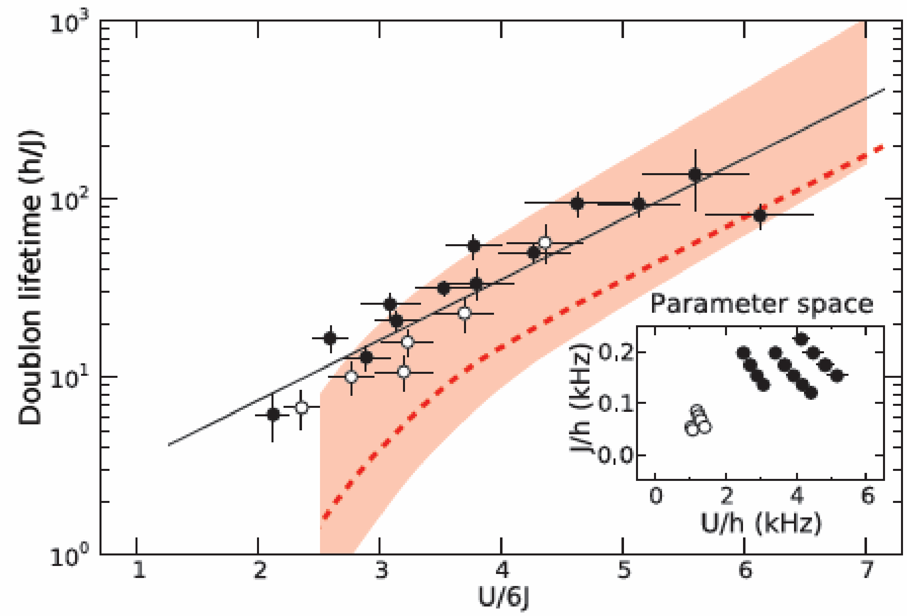


# Doublon decay in a compressible state






Changes of density around 30%




To calculate the rate: consider processes which maximize the number of particle-hole excitations

# Why understanding doublon decay rate is important

Prototype of decay processes with emission of many interacting particles.
Example: resonance in nuclear physics: (i.e. delta-isobar)

Analogy to pump and probe experiments in condensed matter systems

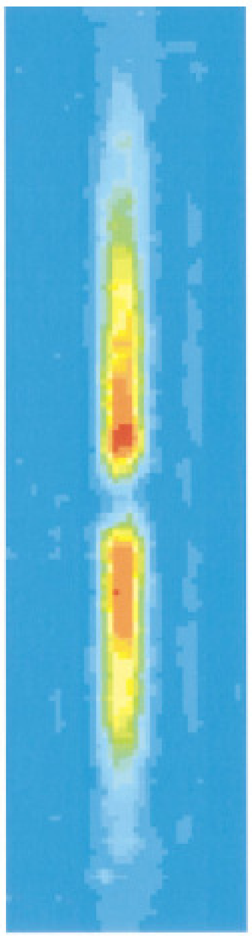
Response functions of strongly correlated systems at high frequencies. Important for numerical analysis.

Important for adiabatic preparation of strongly correlated systems in optical lattices

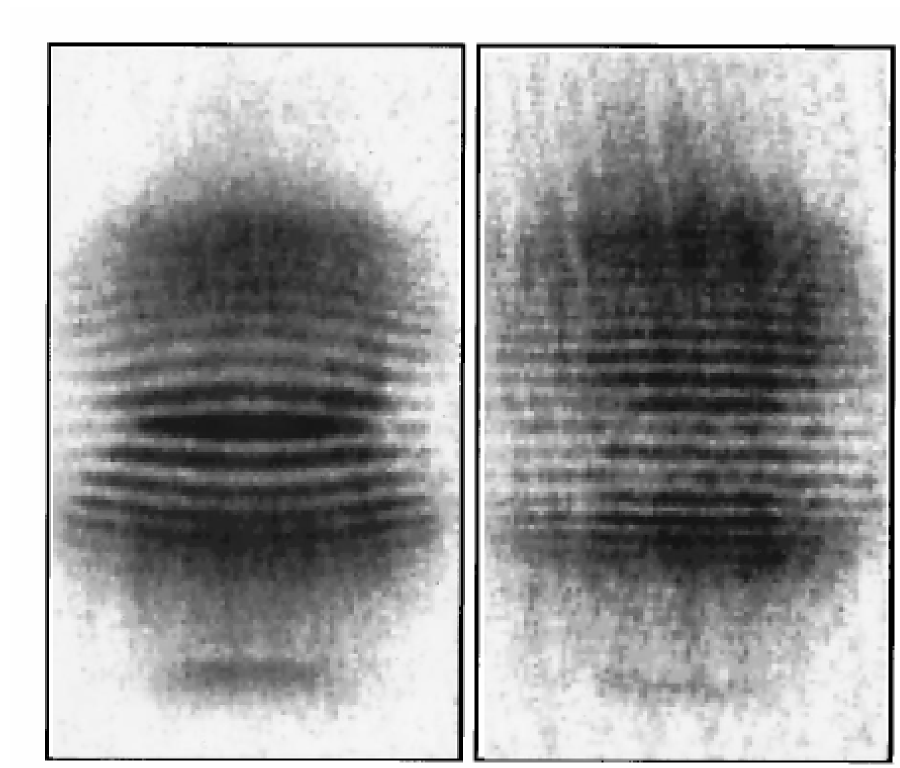
# Interference experiments with cold atoms

Probing fluctuations in low dimensional systems

# Interference of independent condensates



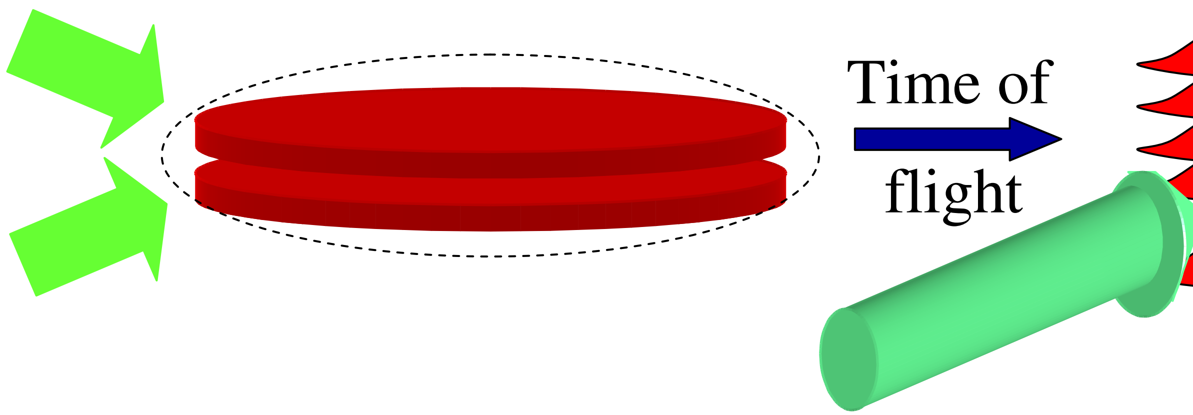
Experiments: Andrews et al., Science 275:637 (1997)

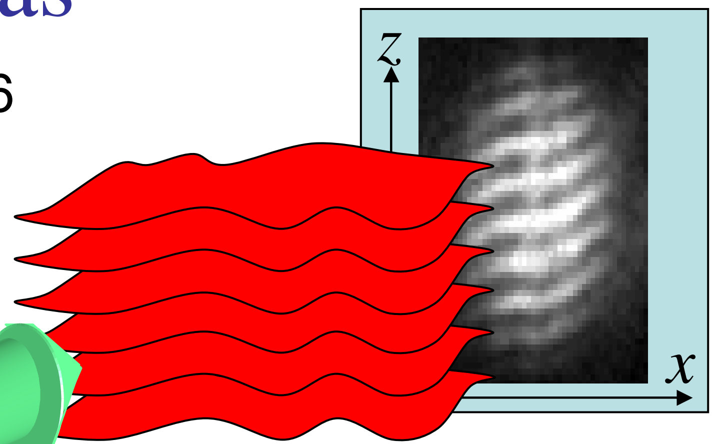


Theory: Javanainen, Yoo, PRL 76:161 (1996)
Cirac, Zoller, et al. PRA 54:R3714 (1996)
Castin, Dalibard, PRA 55:4330 (1997)
and many more

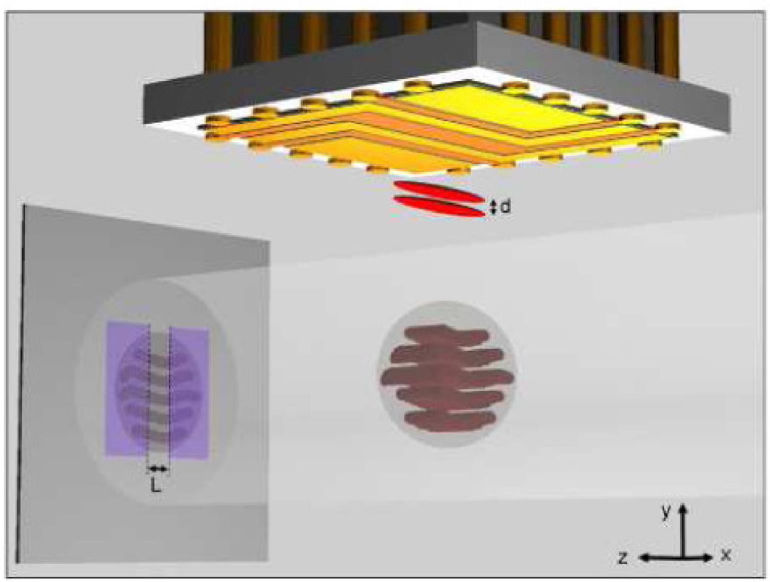
# Experiments with 2D Bose gas

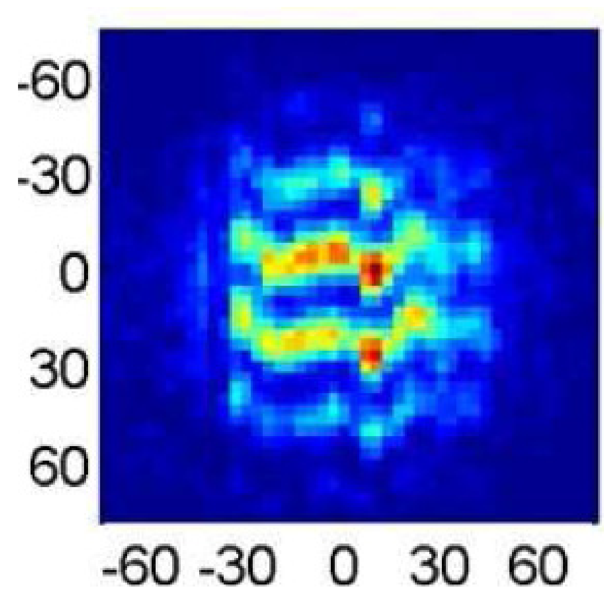
Hadzibabic, Dalibard et al., Nature 2006



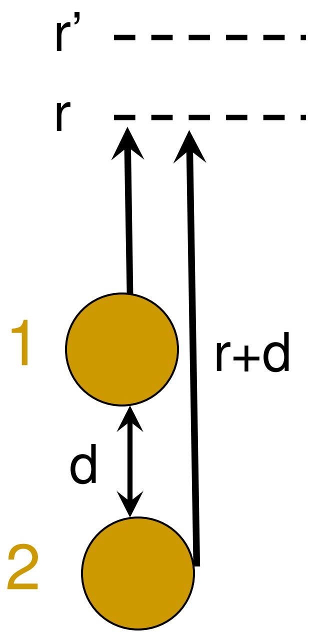




## Experiments with 1D Bose gas Hofferberth et al. Nat. Physics 2008



# Interference of two independent condensates




$$\psi(r) = \psi_1(r) + \psi_2(r)$$

$$\rho_{\rm int}(r) = \psi_1^\dagger(r)\,\psi_2(r) + {\rm c.c.}$$

Assuming ballistic expansion

$$\boxed{\rho_{\rm int}(r) = e^{i\,\frac{m\,d\,r}{\hbar\,t}}\, e^{i\,(\phi_2 - \phi_1)} + {\rm c.c.}}$$

Phase difference between clouds 1 and 2
is not well defined

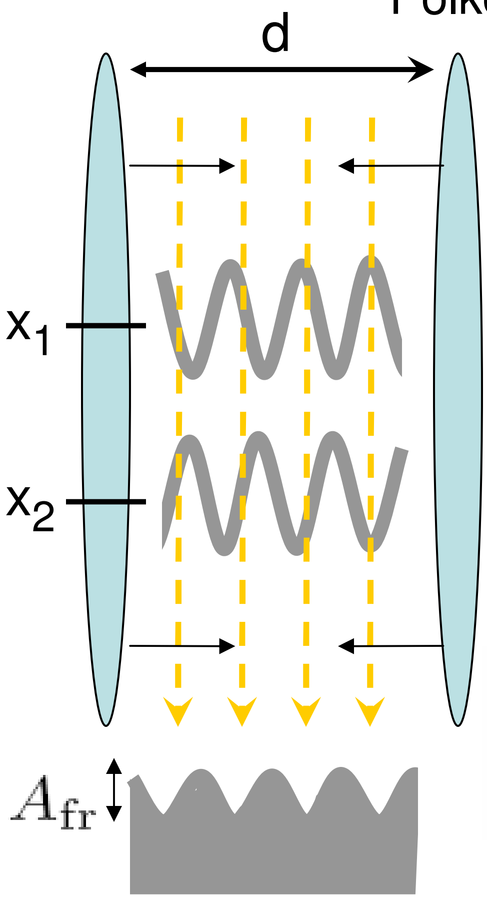
Individual measurements show interference patterns
They disappear after averaging over many shots

$$\langle \rho_{\rm int}(r) \rangle = 0$$

$$\langle \rho_{\rm int}(r)\, \rho_{\rm int}(r') \rangle = e^{i\,\frac{m\,d}{\hbar\,t}\,(r - r')} + {\rm c.c.}$$

# Interference of fluctuating condensates

Polkovnikov et al., PNAS (2006); Gritsev et al., Nature Physics (2006)




Amplitude of interference fringes, $A_{\rm fr}$

$$|A_{\rm fr}|\, e^{i\Delta\phi} = \int_0^L dx\, e^{i(\phi_1(x)-\phi_2(x))}$$

For independent condensates $A_{fr}$ is finite but $\Delta\phi$ is random

$$\langle\, |A_{\rm fr}|^2\,\rangle = \int_0^L dx_1 \int_0^L dx_2\, \langle\, e^{i(\phi_1(x_1)-\phi_2(x_1))}\, e^{-i(\phi_1(x_2)-\phi_2(x_2))} \rangle$$

$$\langle\, |A_{\rm fr}|^2\,\rangle \approx L \int_0^L dx\, \langle\, e^{i(\phi_1(x)-\phi_1(0))} \rangle \, \langle\, e^{-i(\phi_2(x)-\phi_2(0))}\,\rangle$$

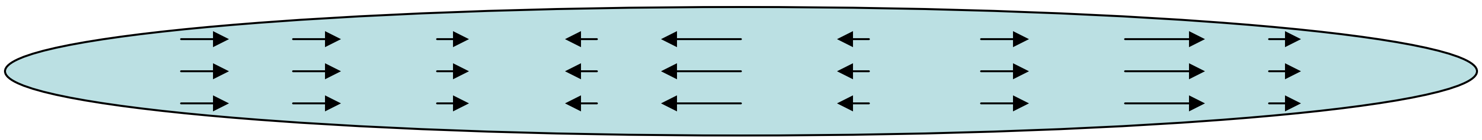
For identical condensates $\langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx\, (\, G(x)\,)^2$

Instantaneous correlation function $G(x) = \langle\, e^{i\phi(x)}\, e^{-\phi(0)}\,\rangle$

# Fluctuations in 1d BEC

For review see Thierry's book

## Thermal fluctuations

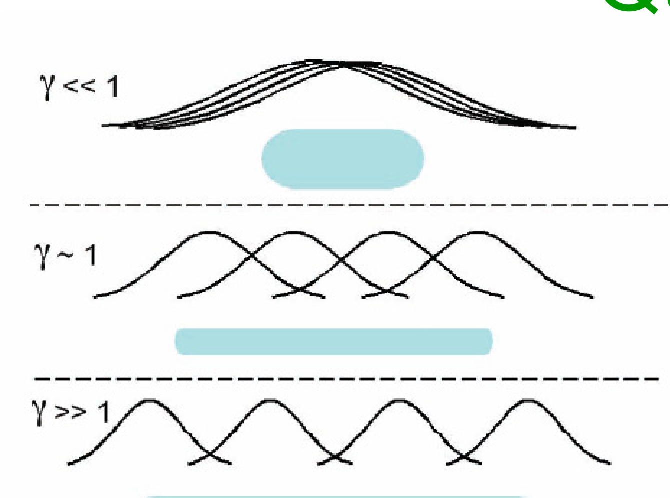


Thermally energy of the superflow velocity $\mathrm{v}_s = \nabla\phi(x)$

$$\boxed{\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)}\,\rangle \sim e^{-|x_1-x_2|/\xi_T}}$$

$$\xi_T = \sqrt{\frac{\hbar^2\, m}{T}}$$

## Quantum fluctuations




$$\boxed{\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)}\,\rangle \sim \left(\frac{\xi_h}{|x_1-x_2|}\right)^{1/2K}}$$

$$K = \sqrt{\frac{n}{g\, m}}$$

# Interference between Luttinger liquids

Luttinger liquid at T=0 $$G(x) \sim \rho \left( \frac{\xi_h}{x} \right)^{1/2K}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

*K* – Luttinger parameter

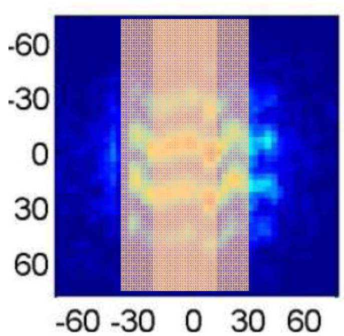
For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

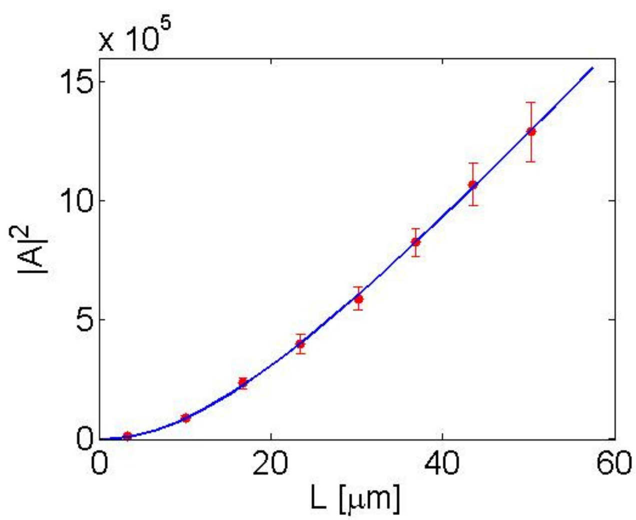
For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

Finite temperature

$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left( \frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

Experiments: Hofferberth, Schumm, Schmiedmayer



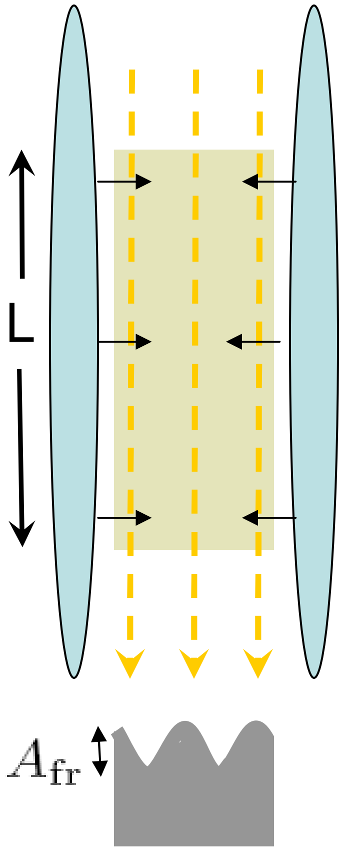

$$n_{1d} = 60 \mu\text{m}^{-1}$$

$$K = 47$$

$$\boxed{T_{fit} = 84 \pm 22 \text{ nK}}$$

# Distribution function of fringe amplitudes for interference of fluctuating condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics 2006
Imambekov, Gritsev, Demler, PRA (2007)




$A_{\rm fr}$ is a quantum operator. The measured value of $|A_{\rm fr}|$ will fluctuate from shot to shot.

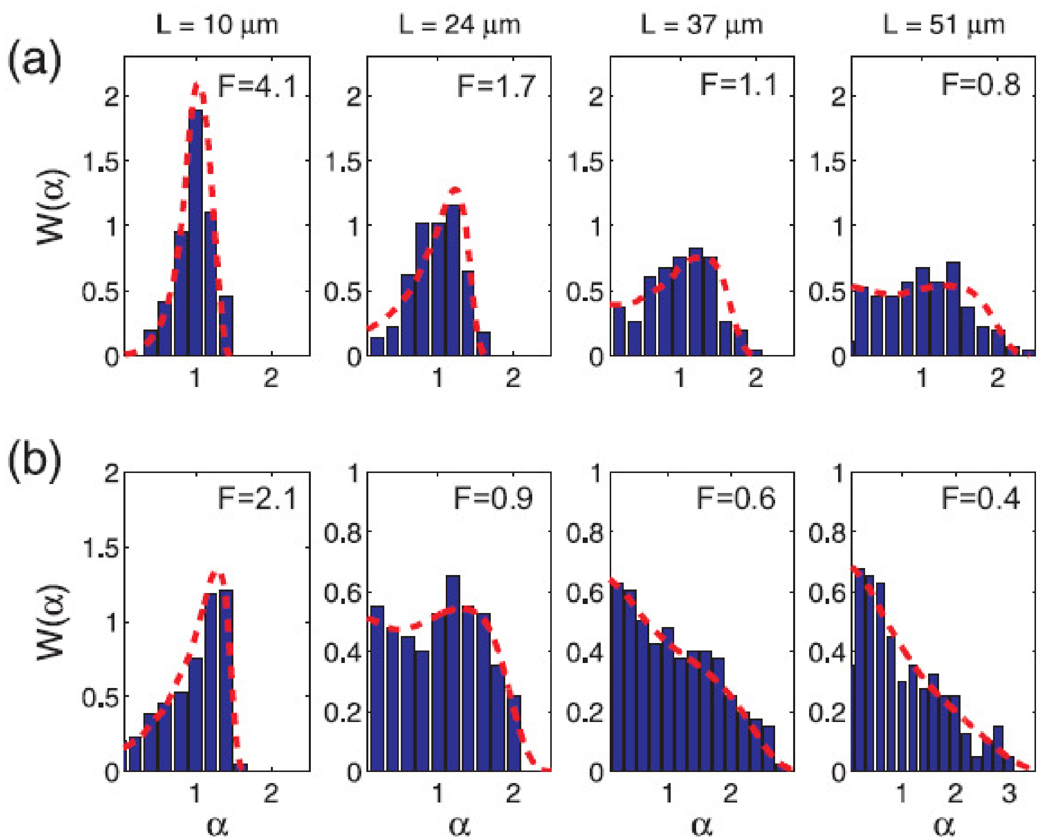
$$\langle\, |A_{\rm fr}|^{2n} \,\rangle = \int_0^L dz_1 \ldots dz_n' \, |\, \langle\, e^{i\phi(z_1)} \ldots e^{i\phi(z_n)} \, e^{-i\phi(z_1')} \ldots e^{-i\phi(z_n')} \,\rangle|^2$$

Higher moments reflect higher order correlation functions

We need the full distribution function of $|A_{\rm fr}|$

# Distribution function of interference fringe contrast

Hofferberth et al., Nature Physics 2009



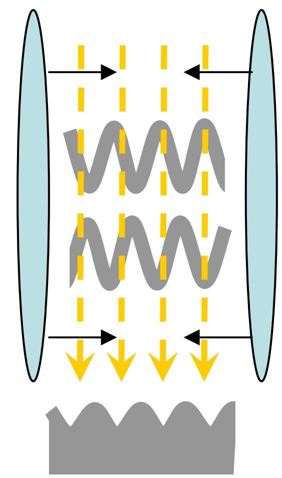

Quantum fluctuations dominate:
asymetric Gumbel distribution
(low temp. T or short length L)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. T or long length L)

Intermediate regime:
double peak structure

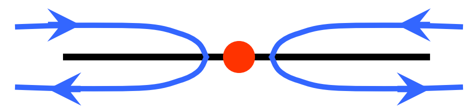
Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained

# Interference between interacting 1d Bose liquids. Distribution function of the interference amplitude

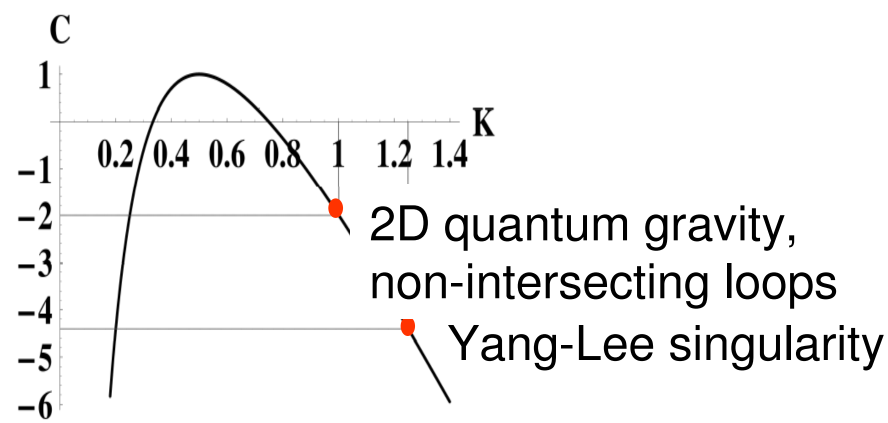


Distribution function of $|A_{\mathrm{fr}}|$

Quantum impurity problem: interacting one dimensional electrons scattered on an impurity

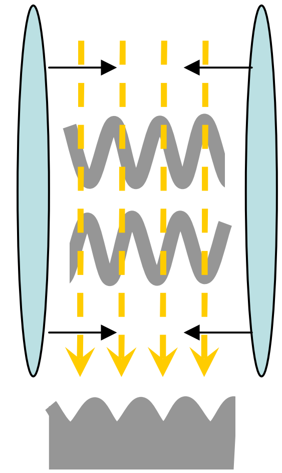


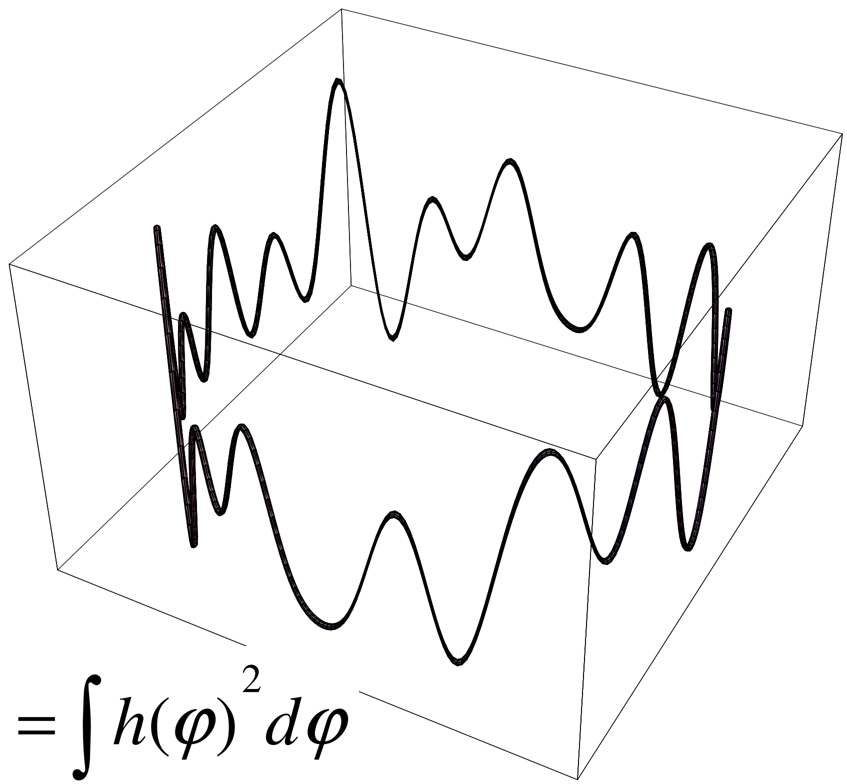
Conformal field theories with negative central charges: 2D quantum gravity, non-intersecting loop model, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …



# Fringe visibility and statistics of random surfaces

Distribution function of $|A_{\mathrm{fr}}|$





$h(\varphi)$

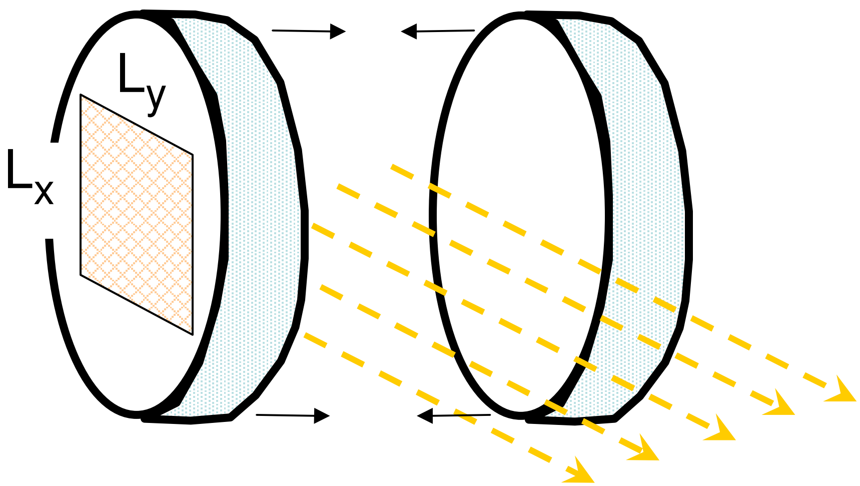
Roughness $= \int h(\varphi)^2 d\varphi$

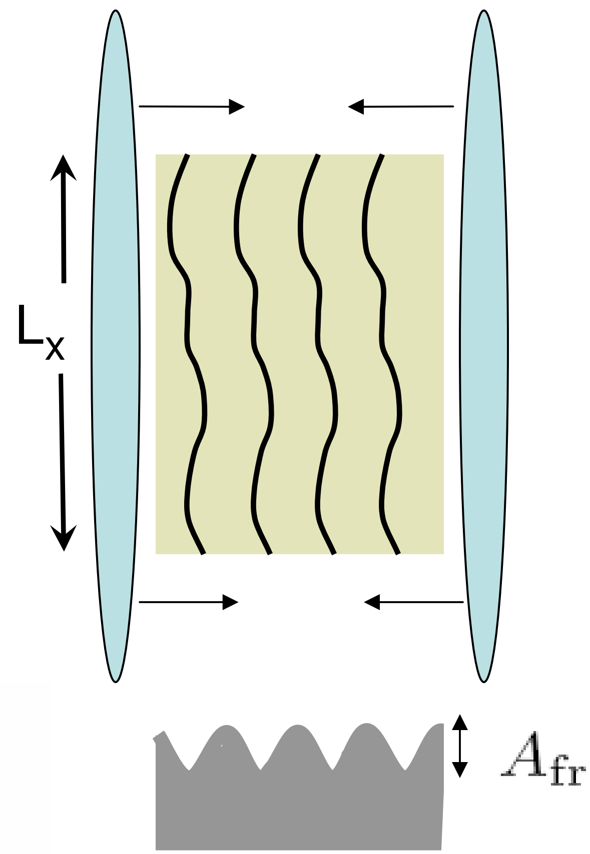
Mapping between fringe visibility and the problem of surface roughness for fluctuating random surfaces.
Relation to 1/f Noise and Extreme Value Statistics

# Interference of two dimensional condensates

Experiments: Hadzibabic et al. Nature (2006)

Gati et al., PRL (2006)



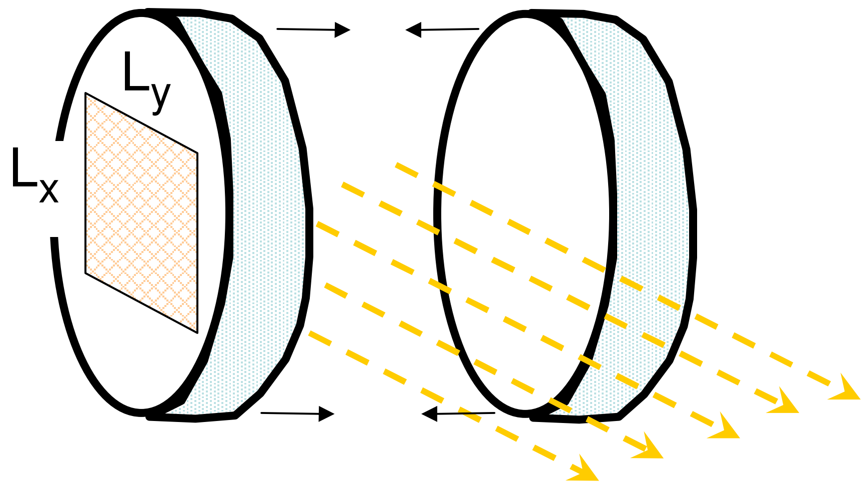




Probe beam parallel to the plane of the condensates

$$\langle\, |A_{fr}|^2 \rangle \;=\; L_x\, L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\;(\,G(\vec{r}\,)\,)^2$$

$$G(\vec{r}\,) \;=\; \langle\, a(\vec{r}\,)\, a^\dagger(0)\,\rangle$$

# Interference of two dimensional condensates. Quasi long range order and the KT transition




Above KT transition

$$G(r) \sim e^{-r/\xi}$$

$$\langle |A_{\text{fr}}|^2 \rangle \sim L_x L_y$$

$$\log \xi(T) \sim 1/\sqrt{T - T_{\text{KT}}}$$

Below KT transition

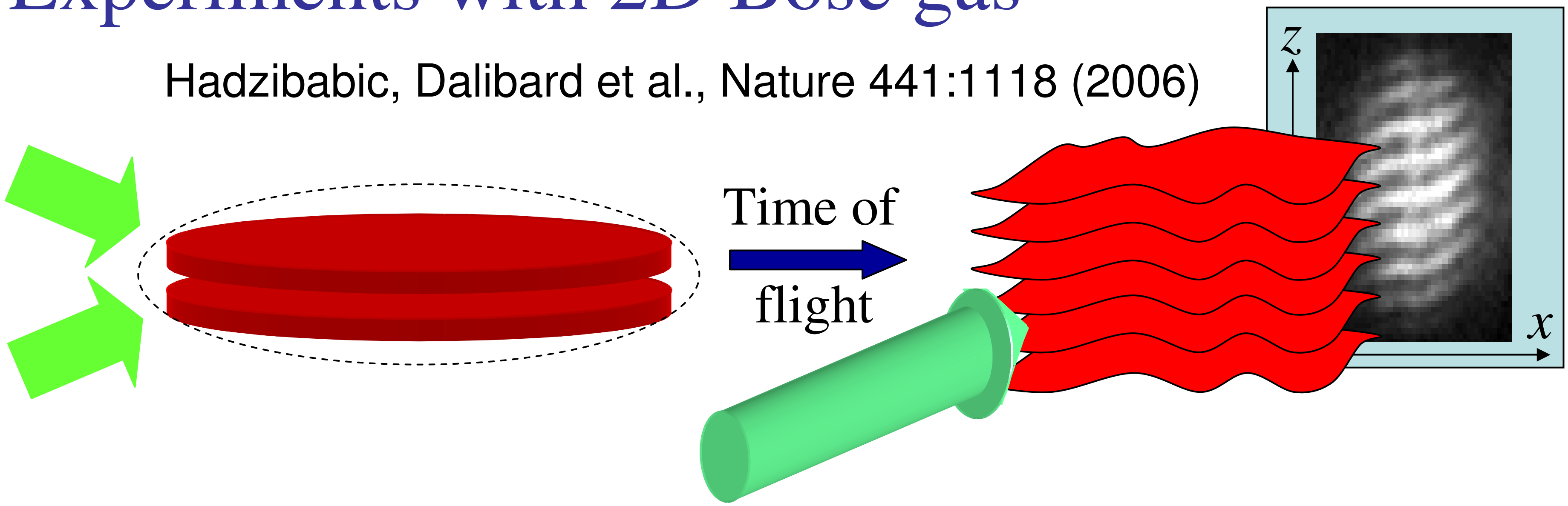
$$G(r) \sim \rho \left( \frac{\xi_h}{r} \right)^{\alpha}$$

$$\alpha(T) = \frac{m T}{2 \pi \rho_s(T) \hbar^2}$$

$$\langle |A_{\text{fr}}|^2 \rangle \sim (L_x L_y)^{2-\alpha}$$

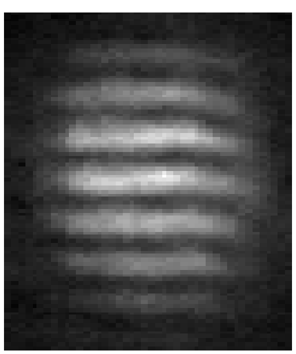
# Experiments with 2D Bose gas

Hadzibabic, Dalibard et al., Nature 441:1118 (2006)

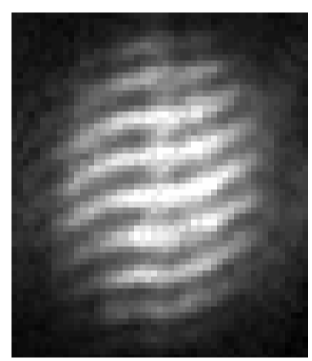



Typical interference patterns

low temperature

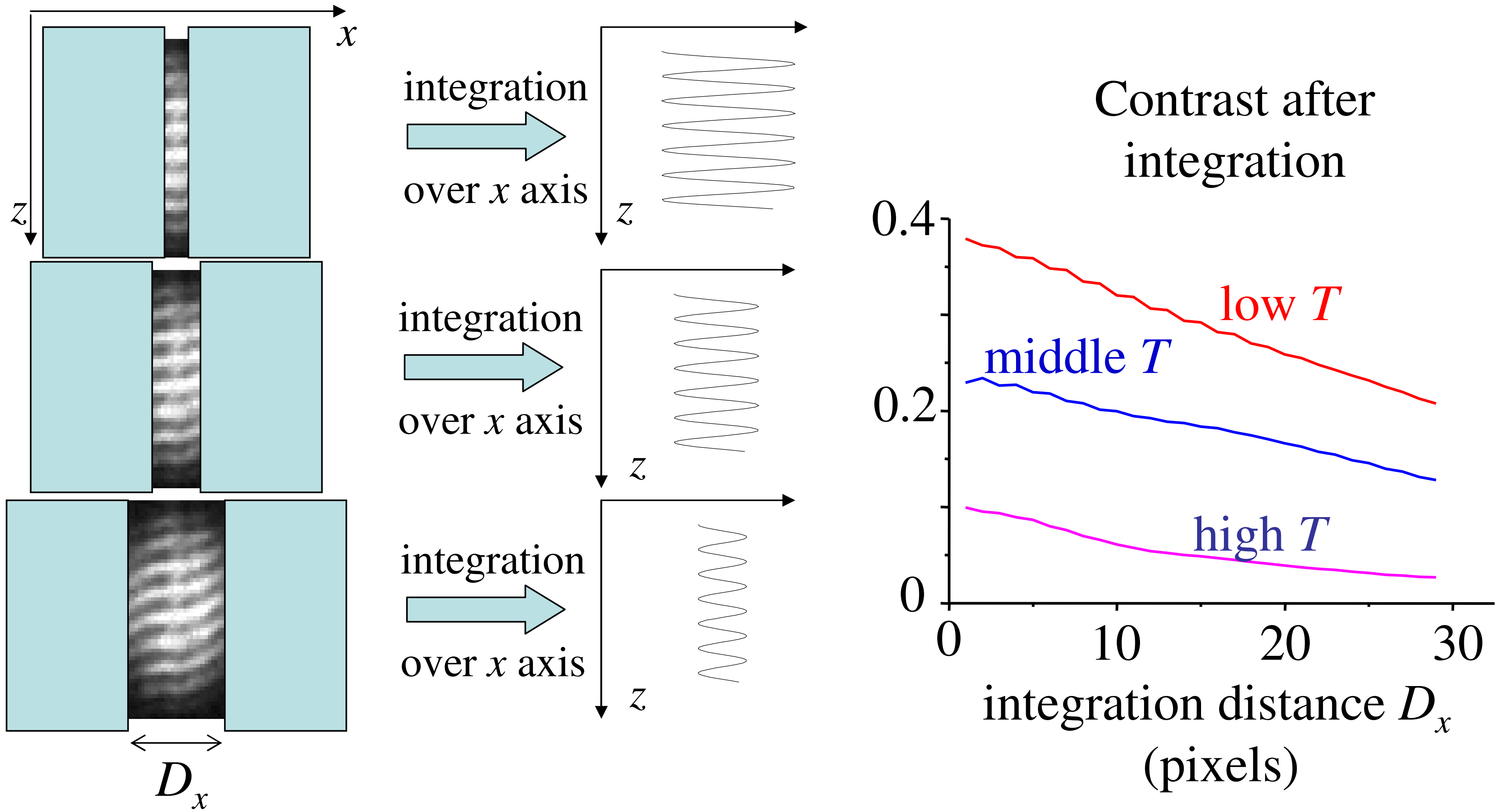


higher temperature



# Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)



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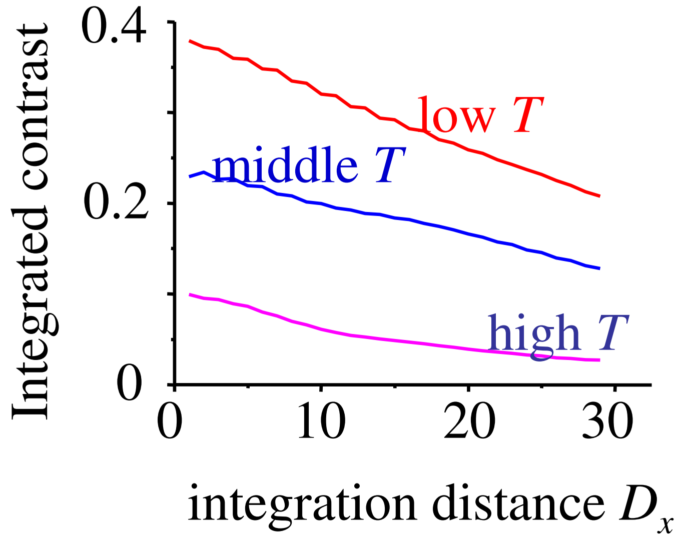
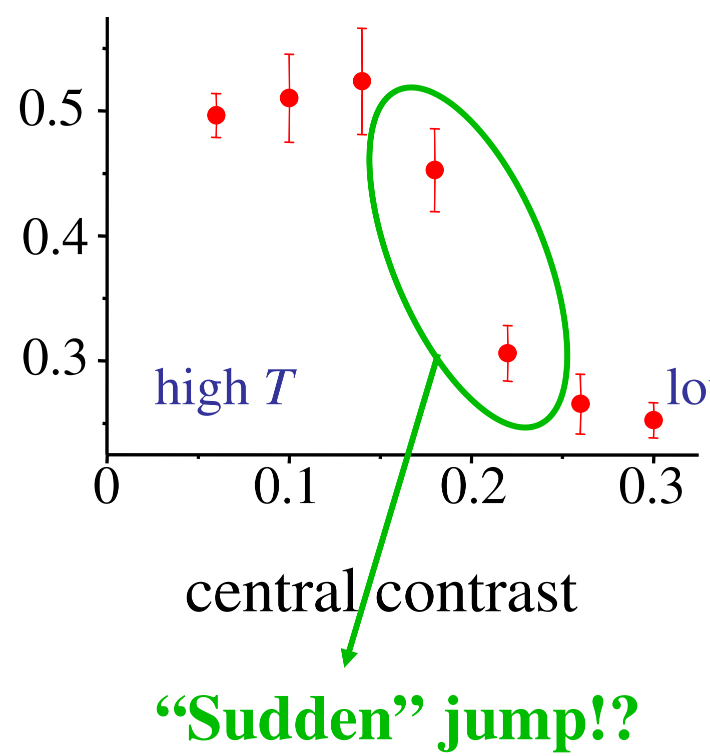
Hadzibabic et al., Nature 441:1118 (2006)




fit by: $C^2 \sim \frac{1}{D_x}\int^{D_x}[g_1(0,x)]^2 dx \sim \left(\frac{1}{D_x}\right)^{2\alpha}$

Exponent $\alpha$



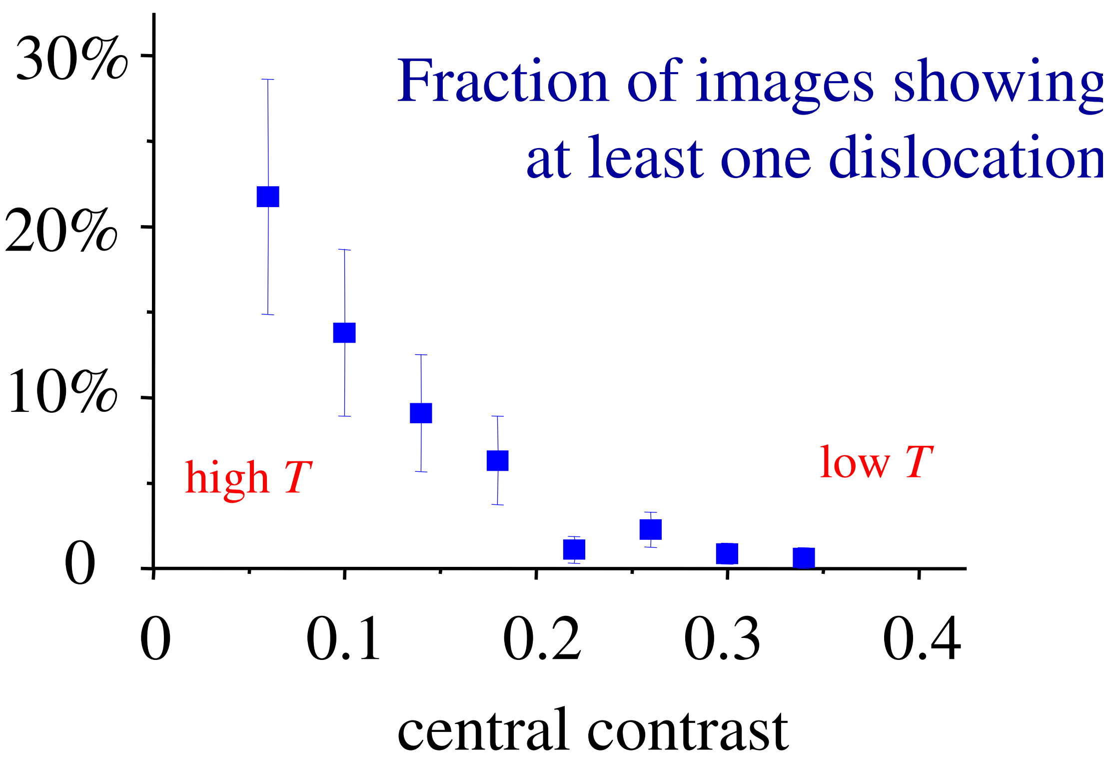

⇒ if $g_1(r)$ decays exponentially with $\ell_{\mathrm{coh}} \ll D_x$: $\alpha = 1/2$

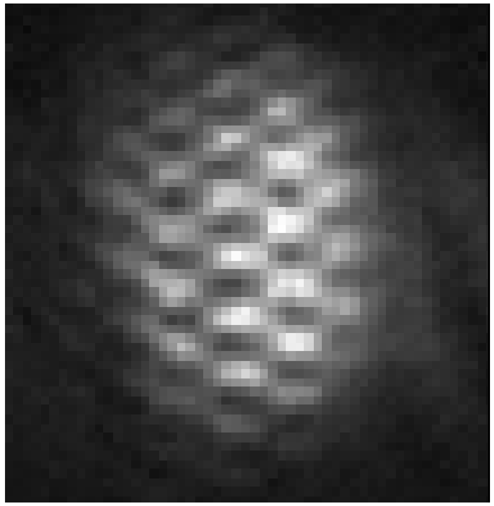
⇒ if $g_1(r)$ decays algebraically or exponentially with a large $\ell_{\mathrm{coh}}$:

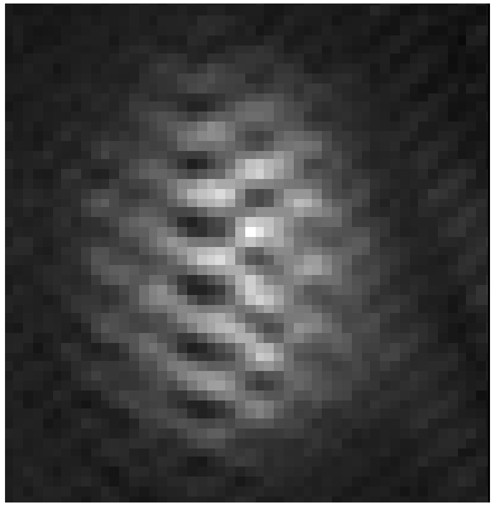
$$\alpha < 1/2$$

# Experiments with 2D Bose gas. Proliferation of thermal vortices

Haddzibabic et al., Nature (2006)



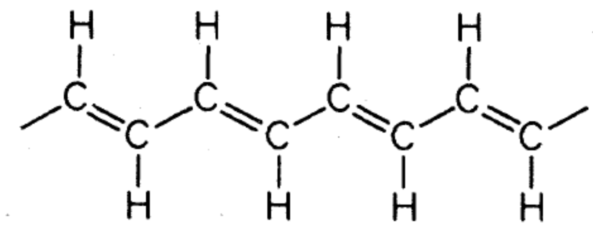


The onset of proliferation coincides with $\alpha$ shifting to 0.5!

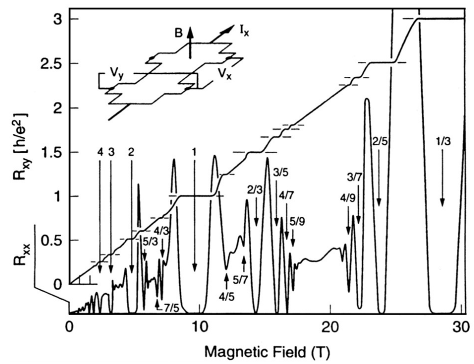
# Exploration of Topological Phases with Quantum Walks

Kitagawa, Rudner, Berg, Demler,
arXiv:1003.1729

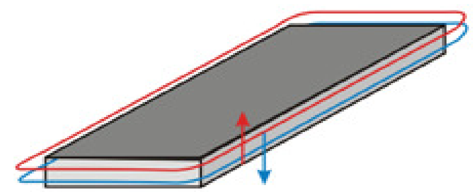
# Topological states of matter



Polyethethylene
SSH model




Integer and Fractional
Quantum Hall effects



Quantum Spin Hall effect

Exotic properties:
quantized conductance (Quantum Hall systems, Quantum Spin Hall Sysytems)
fractional charges (Fractional Quantum Hall systems, Polyethethylene)

Geometrical character of ground states:
Example: TKKN quantization of
Hall conductivity for IQHE

PRL (1982)

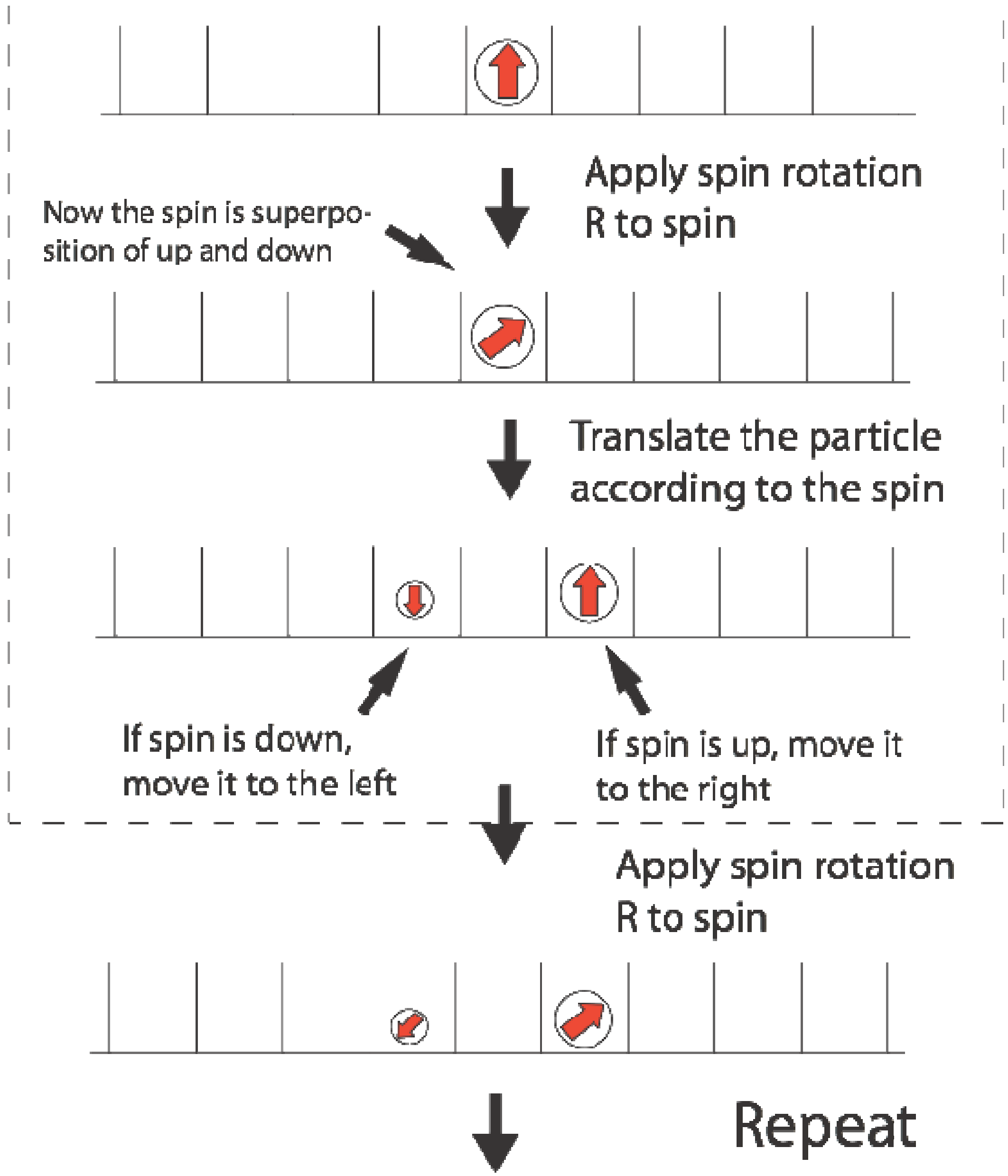
$$A(\mathbf{k}) = \sum_{\mathbf{E_n < E_F}} \langle\, \mathbf{u_n}(\mathbf{k}) \,|\, \partial_{\mathbf{k}} \,|\mathbf{u_n}(\mathbf{k})\rangle$$

$$F(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$$

$$\sigma_{xy} = \frac{1}{2\pi} \int_{T^2} d^2\mathbf{k}\, F(\mathbf{k})$$

# Discrete quantum walks

# Definition of 1D discrete Quantum Walk




1D lattice, particle starts at the origin

Spin rotation

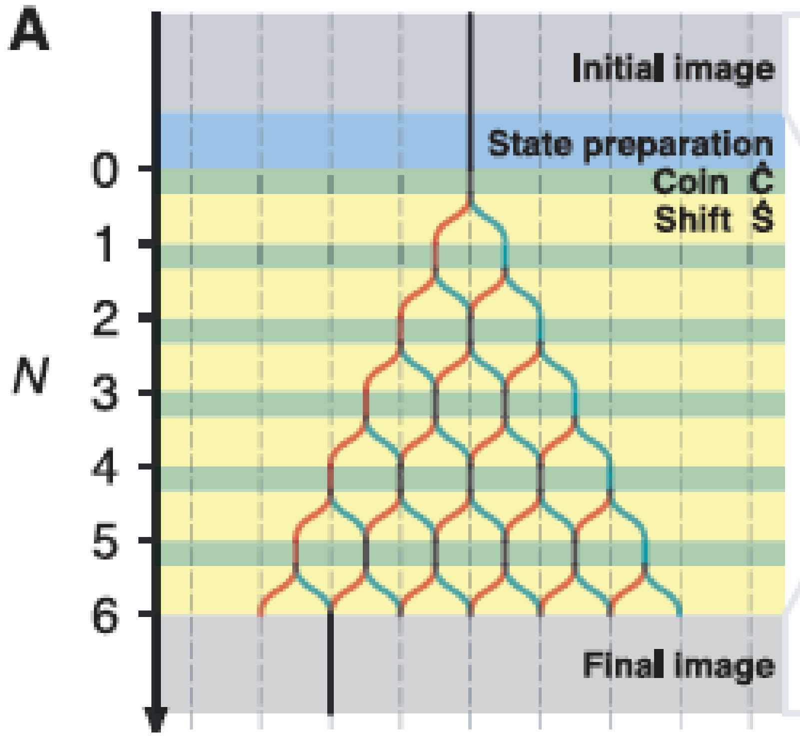
Spin-dependent Translation

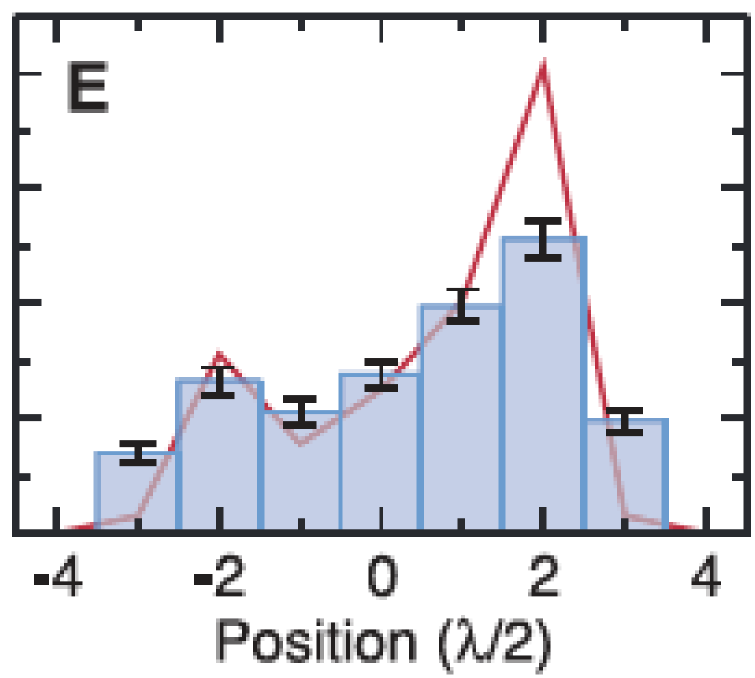
Analogue of classical random walk.
Introduced in quantum information:
Q Search, Q computations

# Quantum Walk in Position Space with Single Optically Trapped Atoms

Michal Karski,* Leonid Förster, Jai-Min Choi, Andreas Steffen, Wolfgang Alt,
Dieter Meschede, Artur Widera*

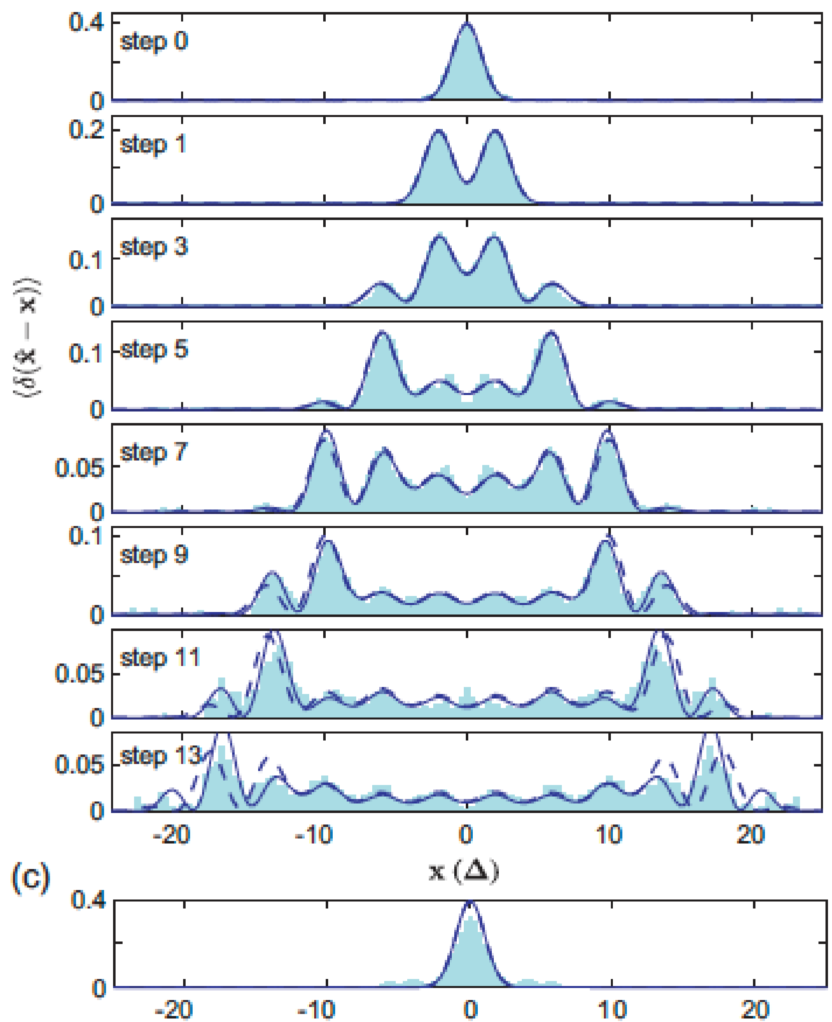
10 JULY 2009 VOL 325 SCIENCE



Realization of a quantum walk with one and two trapped ions

F. Zähringer[1,2], G. Kirchmair[1,2], R. Gerritsma[1,2], E. Solano[3,4], R. Blatt[1,2], and C. F. Roos[1,2]




**arXiv:0911.1876**

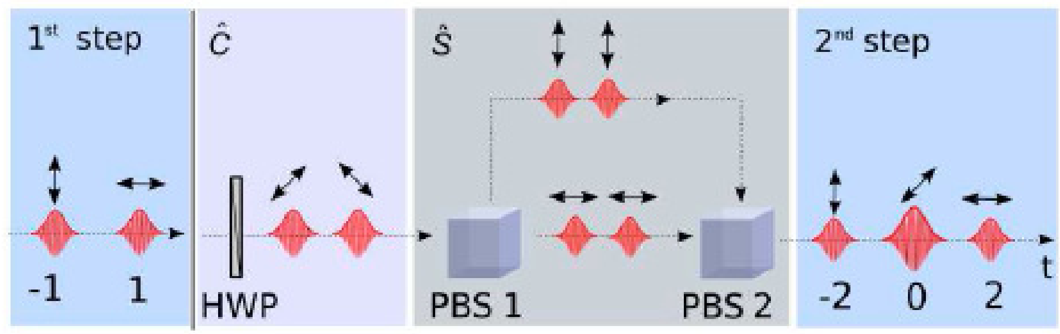
# Photons Walking the Line

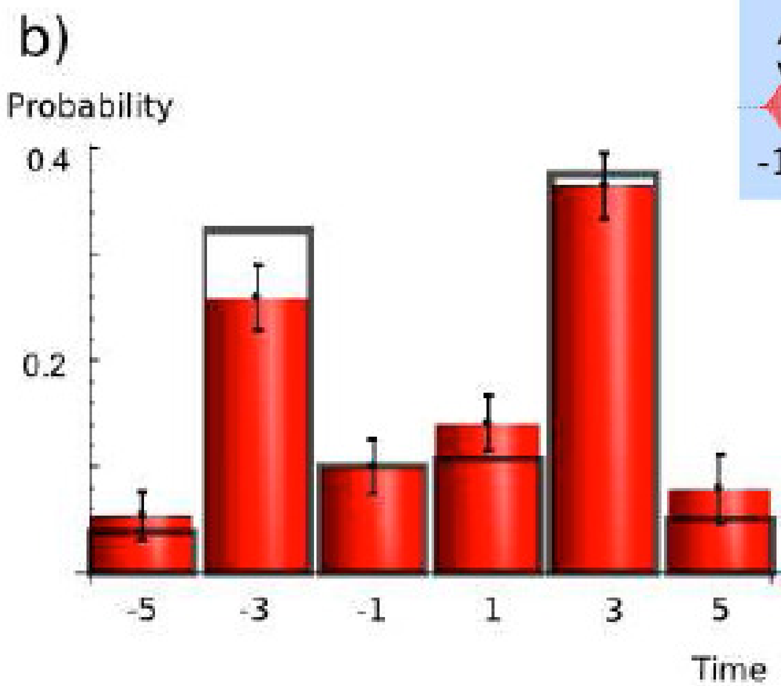
A. Schreiber,[1] K. N. Cassemiro,[1,*] V. Potoček,[2] A. Gábris,[3,2] P. Mosley,[1,†] E. Andersson,[4] I. Jex,[2] and Ch. Silberhorn[1]

[1]*Max Planck Institute for the Science of Light, Günther-Scharowsky-str. 1 / Bau 24, 91058 Erlangen, Germany.*
[2]*Department of Physics, FNSPE, Czech Technical University in Prague, Břehová 7, 115 19 Praha, Czech Republic.*
[3]*Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, H-1525 Budapest, P. O. Box 49, Hungary.*
[4]*SUPA, School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom.*

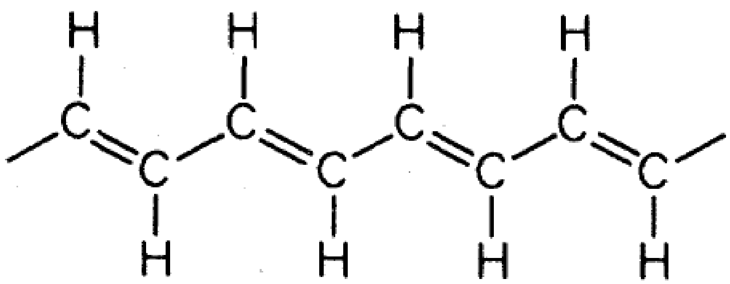
arXiv:0910.2197v1

(Dated: October 12, 2009)



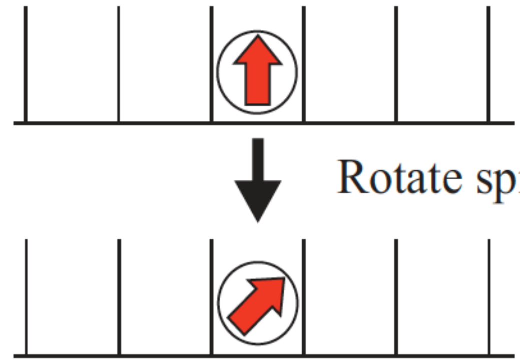
# Quantum walk in 1D: Topological phase



# Discrete quantum walk

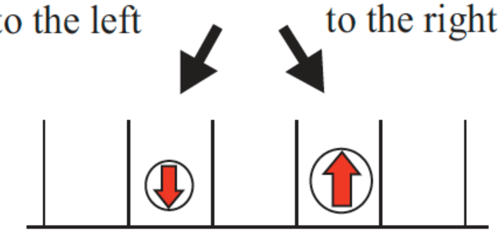
Spin rotation around y axis

$$
\begin{aligned}
R(\theta) &= e^{-i\theta\sigma_y} \\
&= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}
\end{aligned}
$$




Translation

$$
\begin{aligned}
T &= \sum_x |x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow| \\
&\quad + |x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow| \\
&= \sum_k e^{ik\sigma_z} \otimes |k\rangle\langle k|
\end{aligned}
$$




$$
\begin{aligned}
U_{onestep} &= TR(\theta) \\
&= \sum_k e^{ik\sigma_z} e^{-i\theta\sigma_y} \otimes |k\rangle\langle k|
\end{aligned}
$$

One step
Evolution operator

# Effective Hamiltonian of Quantum Walk

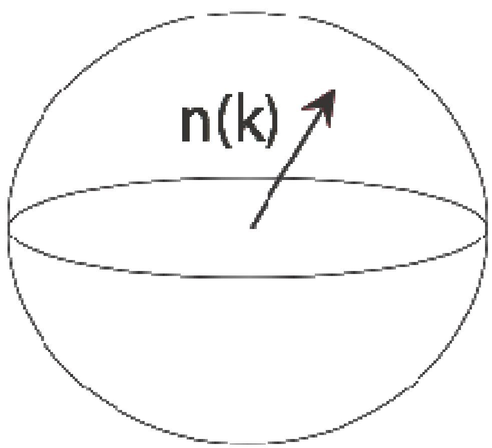
Interpret evolution operator of one step as resulting from Hamiltonian.

$$
\begin{aligned}
U_{onestep} &\equiv e^{-iH_{eff}(\theta)\delta t} \\
H_{eff}(\theta) &= \sum_k \{E_\theta(k)\mathbf{n}_\theta(k)\cdot\sigma\} \otimes |k\rangle\langle k|
\end{aligned}
$$

$$\cos E_\theta(k) = \cos\theta\ \cos k$$

Stroboscopic implementation of $H_{eff}$

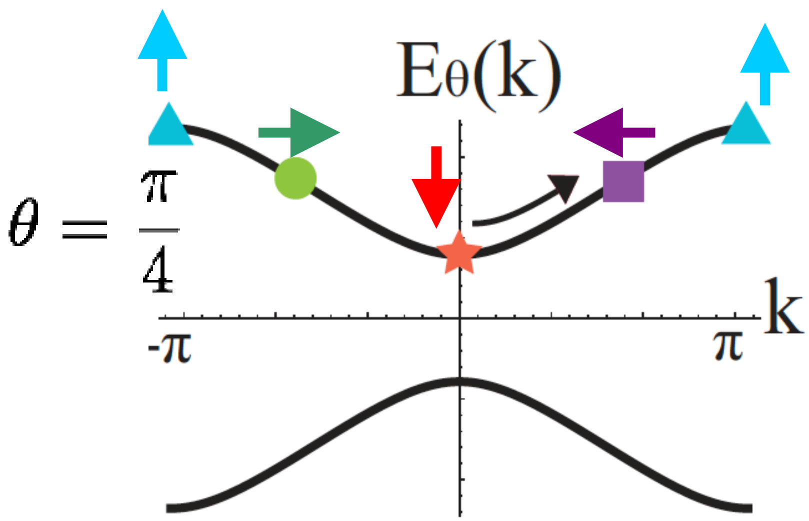
Spin-orbit coupling in effective Hamiltonian



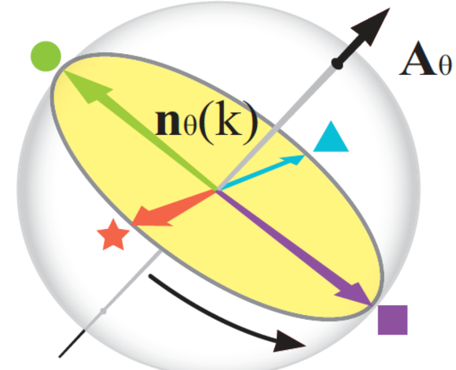
# From Quantum Walk to Spin-orbit Hamiltonian in 1d

k-dependent "Zeeman" field

$$\mathbf{n}_\theta(k) = \frac{(\sin\theta\sin k,\ \sin\theta\cos k,\ -\cos\theta\sin k)}{\sqrt{1-(\cos\theta\cos k)^2}}.$$




$$\mathbf{A}_\theta = (\cos\theta, 0, \sin\theta)$$




Winding Number Z on the plane defines the topology!

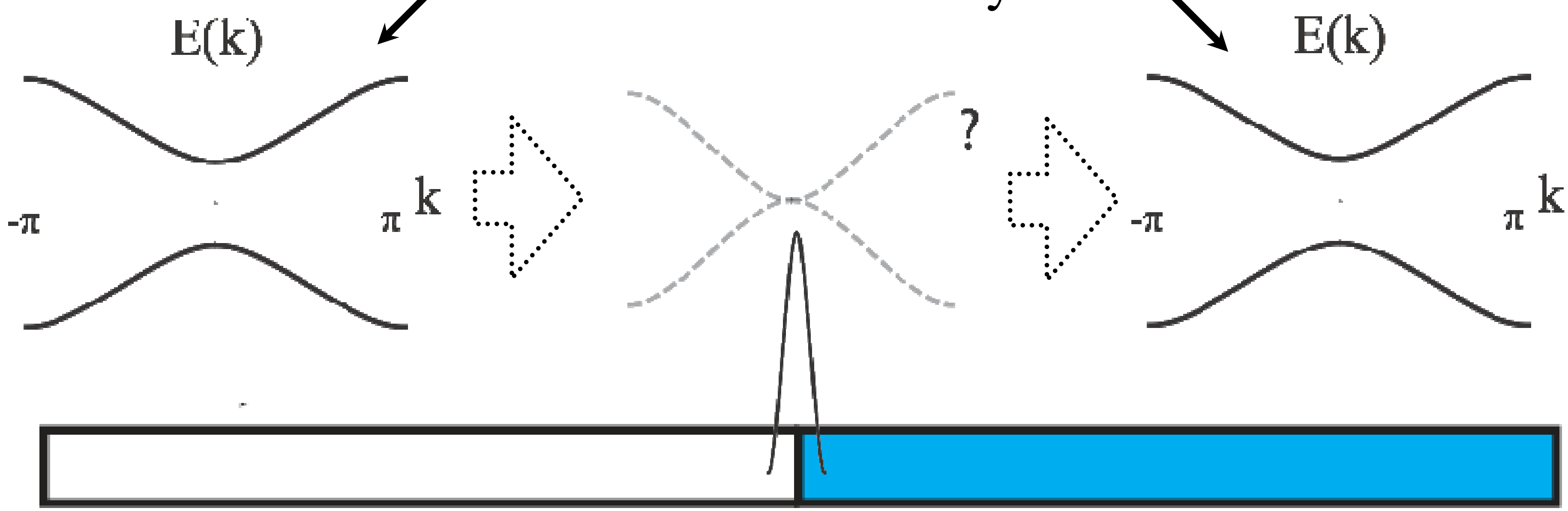
Winding number takes integer values, and can not be changed unless the system goes through gapless phase

# Detection of Topological phases: localized states at domain boundaries

# Phase boundary of distinct topological phases has bound states!

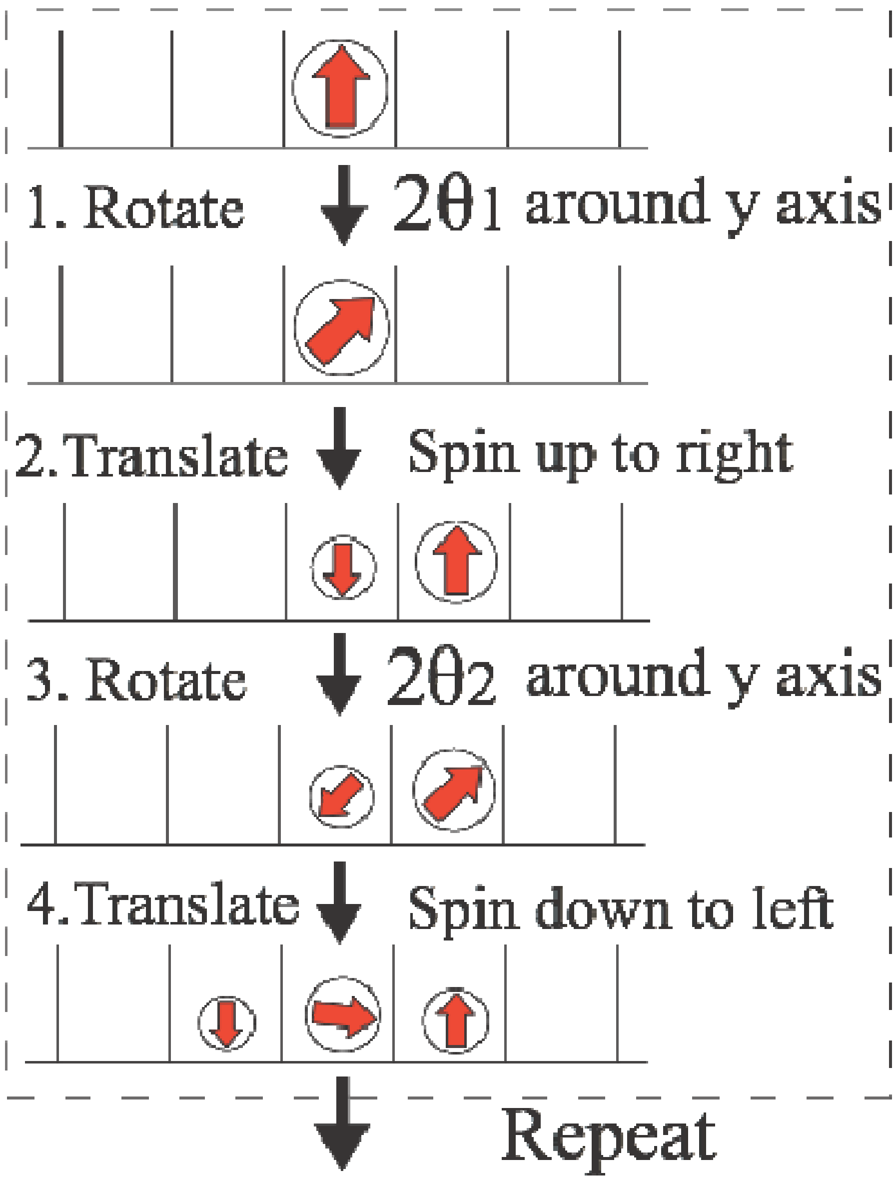
Bulks are insulators

Topologically distinct, so the “gap” has to close near the boundary




a localized state is expected

# Split-step DTQW




$$R(\theta_1) = e^{-i\theta_1 \sigma_y}$$

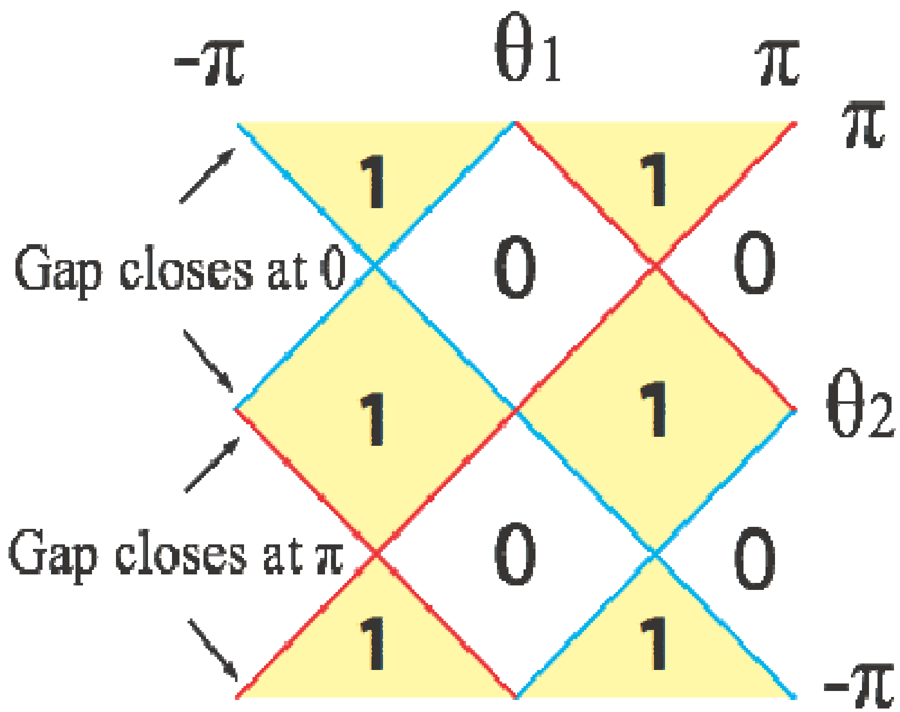
$$T_\uparrow = \sum_x |x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow| + \mathbf{1} \otimes |\downarrow\rangle\langle\downarrow|$$

$$R(\theta_2) = e^{-i\theta_2 \sigma_y}$$

$$T_\downarrow = \mathbf{1} \otimes |\uparrow\rangle\langle\uparrow| + \sum_x |x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow|$$

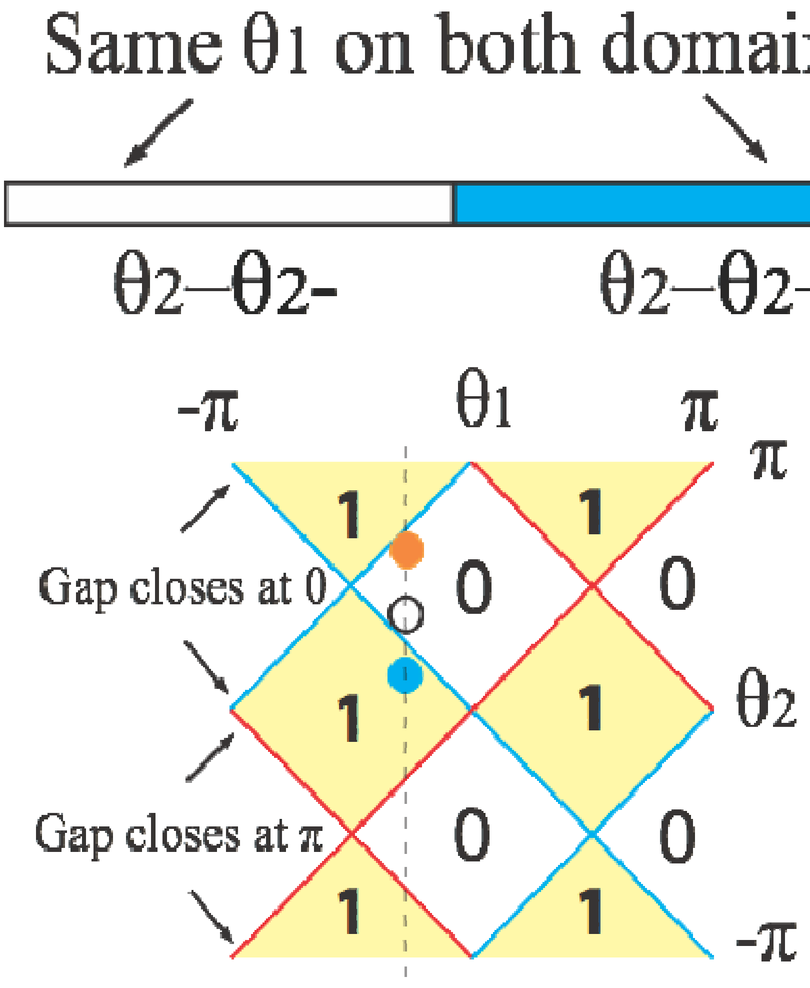
# Split-step DTQW

## Phase Diagram

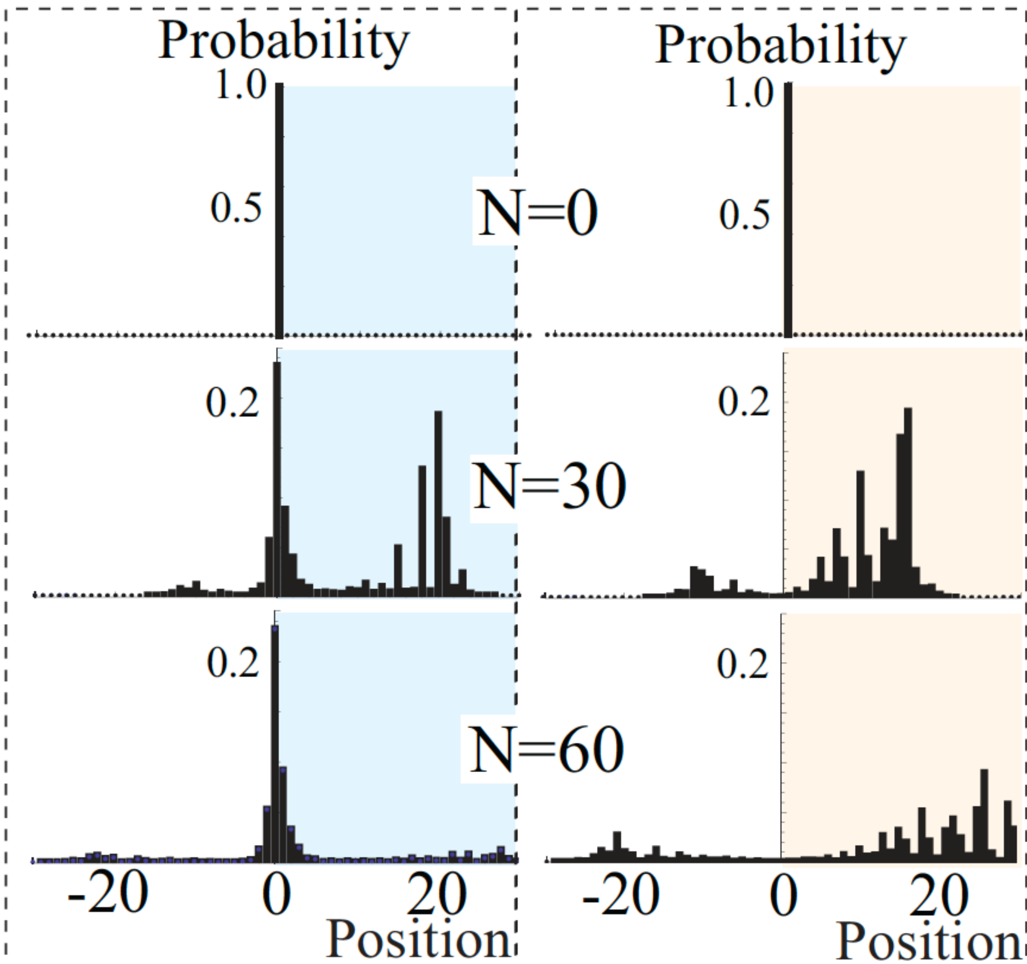


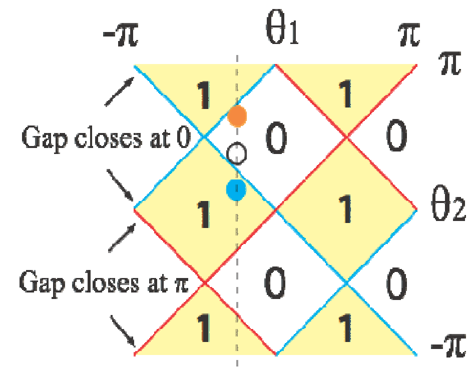
# Split-step DTQW with site dependent rotations

Apply site-dependent spin rotation for $\theta_2$

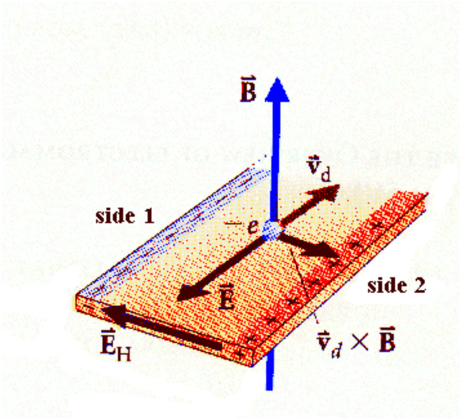


# Split-step DTQW with site dependent rotations: Boundary State



# Quantum Hall like states:
# 2D topological phase
# with non-zero Chern number



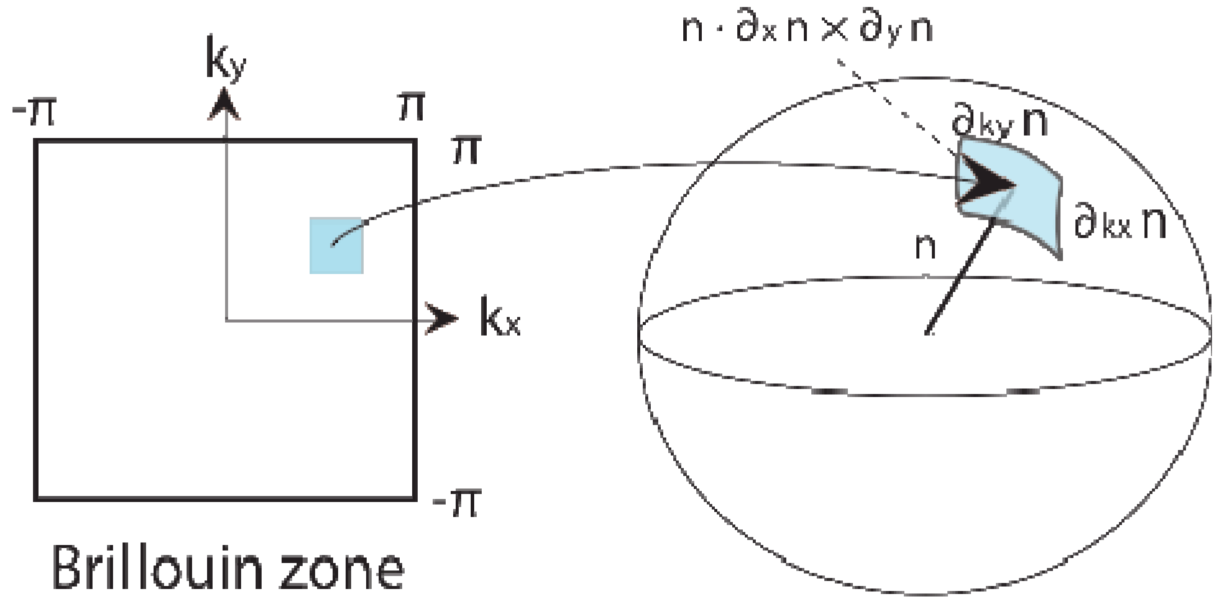

Quantum Hall system

# Chern Number

This is the number that characterizes the topology of the Integer Quantum Hall type states

$$H_{eff} = \sum_{\vec{k}} \left\{ E(\vec{k})\vec{n}(\vec{k}) \cdot \vec{\sigma} \right\} b_k^\dagger b_k$$

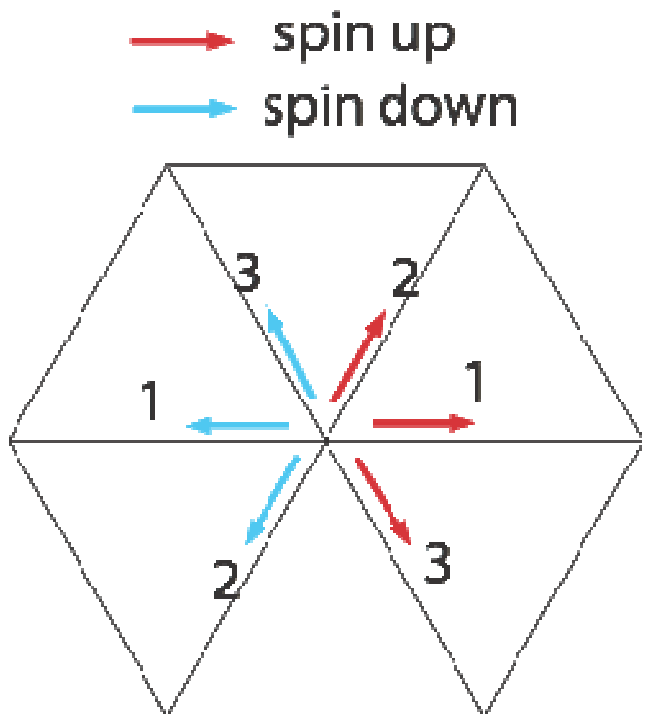
$$\begin{aligned} C &= -2\pi\sigma_{xy} \\ &= \frac{1}{4\pi}\int_{FBZ} dk_x dk_y \left( \vec{n}(\vec{k}) \cdot \partial_{k_x}\vec{n}(\vec{k}) \times \partial_{k_y}\vec{n}(\vec{k}) \right) \end{aligned}$$




Chern number is quantized to integers

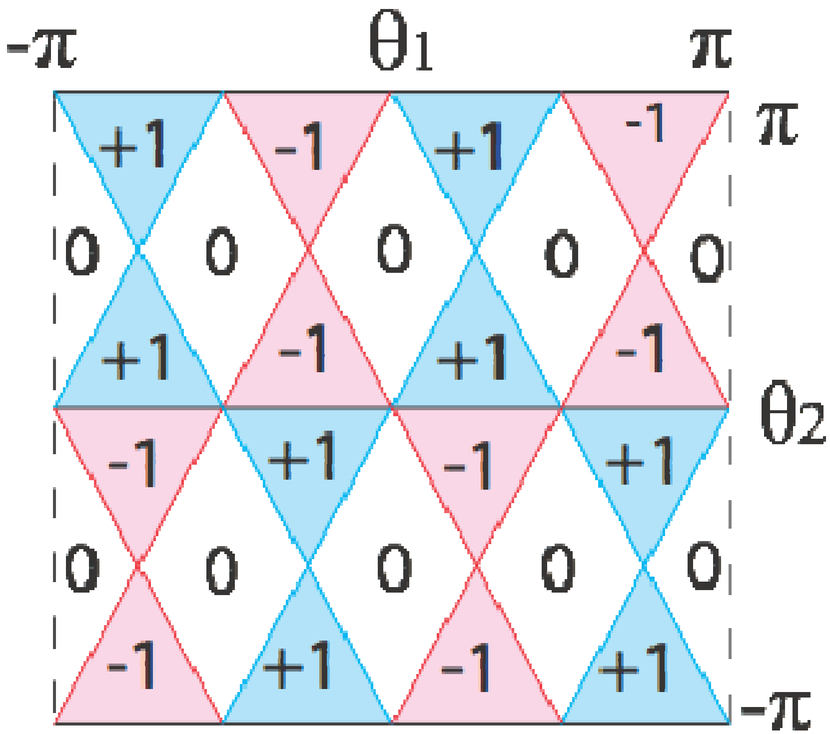
# 2D triangular lattice, spin 1/2

"One step" consists of three unitary and translation operations in three directions

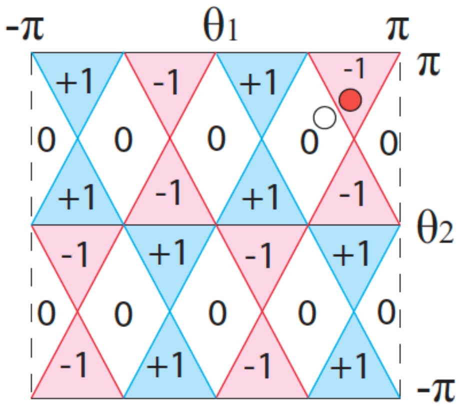



$$e^{-iH_{eff}} \equiv T_3 e^{-i\theta_1 \sigma_y} T_2 e^{-i\theta_2 \sigma_y} T_1 e^{-i\theta_1 \sigma_y}$$

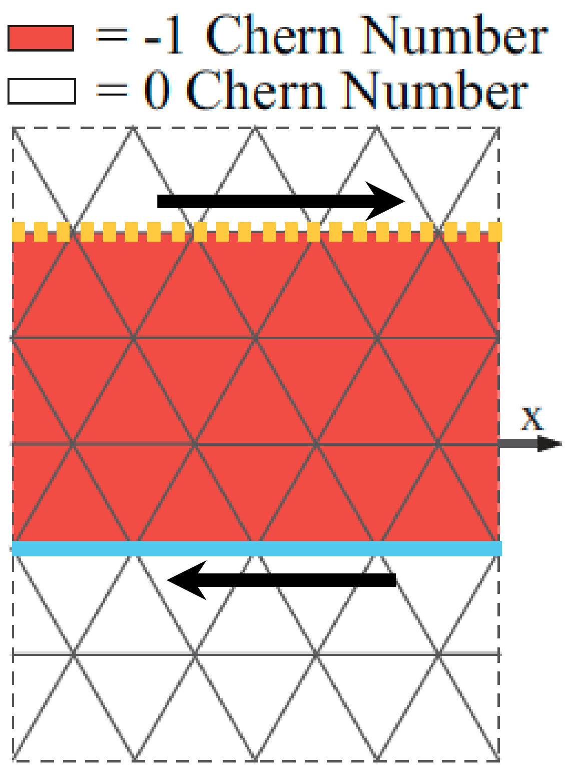
# Phase Diagram

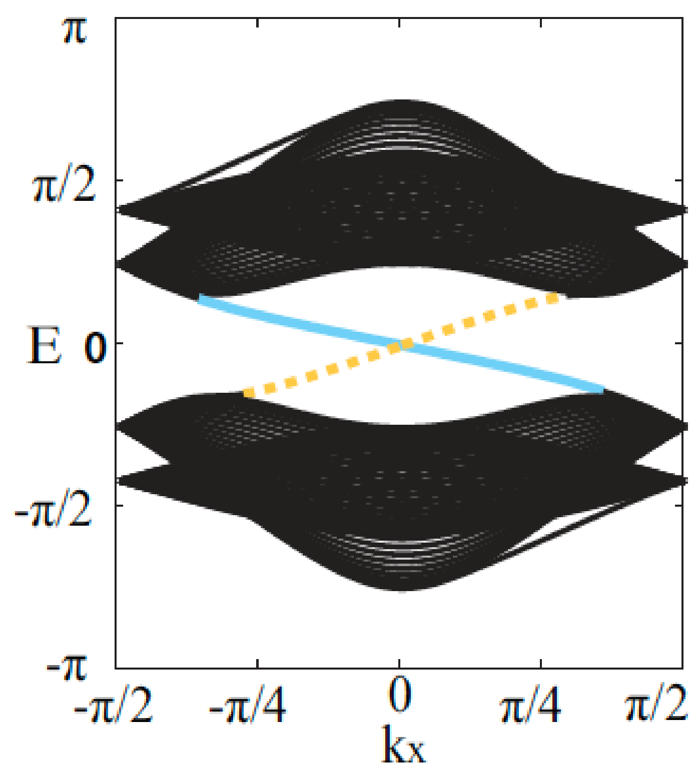



$$e^{-iH_{eff}} \equiv T_3 e^{-i\theta_1\sigma_y} T_2 e^{-i\theta_2\sigma_y} T_1 e^{-i\theta_1\sigma_y}$$




# Chiral edge mode



# Summary

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. They pose new questions about new strongly correlated states, their detection, and nonequilibrium many-body dynamics

# Strongly correlated many-body systems: from electronic materials to ultracold atoms to photons

- Introduction. Systems of ultracold atoms.
- Bogoliubov theory. Spinor condensates.
- Cold atoms in optical lattices. Band structure and semiclasical dynamics.
- Bose Hubbard model and its extensions
- Bose mixtures in optical lattices

  Quantum magnetism of ultracold atoms.

  Current experiments: observation of superexchange
- Detection of many-body phases using noise correlations
- Fermions in optical lattices

  Magnetism and pairing in systems with repulsive interactions.

  Current experiments: Mott state
- Experiments with low dimensional systems

  Interference experiments. Analysis of high order correlations
- Probing topological states of matter with quantum walk