A model for the MOTT featured Gaussian Column Density Profile

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In order to derive the density and atom number properly when the density profile of the cloud has a "MOTT" feature, we need to use a fit function which is a project of a 3D Gaussian with a constant core.
Before go into the spacial "MOTT" case. Let's review the normal Gaussian distribution first.

1 3D Gaussian Density Profile

In a normal case, we assume the atom cloud has a density profile of Gaussian in 3D:

$$n(\vec{r}) = n_0 \times e^{-(\frac{x^2}{w_x^2}+\frac{y^2}{w_y^2}+\frac{z^2}{w_z^2})} \tag{1}$$

The column density of it is:

$$n_{col}(\vec{r}) = \int_{-\infty}^{\infty} n(\vec{r})dz = w_z \times \sqrt{\pi} \times n_0 \times e^{-(\frac{x^2}{w_x^2}+\frac{y^2}{w_y^2})} \tag{2}$$

The total atom number is:

$$N_{tot} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n_{col}(\vec{r})dxdy = w_x w_y w_z \times \pi^{\frac{3}{2}} \times n_0 \tag{3}$$

In the experiment, we will get column density of the atom from either phase contrast of absorption image. We then fit the column density to the equation:

$$n_{col}(x,y) = A \times e^{-(\frac{x^2}{w_x^2}+\frac{y^2}{w_y^2})} \tag{4}$$

Compare this with equation (2) we find that the peak density n_0 and total number can be derived from fitting parameter A,w_x,w_x:

$$n_0 = \frac{A}{\sqrt{\pi} \times w_z} = \frac{A}{\sqrt{\pi} \times \sqrt{w_x w_y}} \tag{5}$$

$$N_{tot} = A \times \pi \times w_x \times w_y \tag{6}$$

Here since we don't have the information of w_z. We make an assumption that $w_z = \sqrt{w_x w_y}$.

2 3D Gaussian with A Constant Core Density Profile

We define the density profile of this case as:

$$n(\vec{r}) = \begin{cases} n_0 \cdot e^{-{r_d}^2}, r_d > r_0 \\ n_0 \cdot e^{-{r_0}^2}, else \end{cases} \tag{7}$$

$$r_d \equiv (\frac{x^2}{w_x^2} + \frac{y^2}{w_y^2} + \frac{z^2}{w_z^2})^{\frac{1}{2}} \tag{8}$$

Where r_0 is a constant. If $r_0 = 0$, the profile is a regular Gaussian. The column density of it is:

$$n_{col}(x, y) = \int_{-\infty}^{\infty} n(\vec{r}) dz = \begin{cases} w_z \cdot \sqrt{\pi} \cdot n_0 \cdot e^{-{r_{xy}}^2}, r_{xy} > r_0 \\ w_z \cdot \sqrt{\pi} \cdot n_0 \cdot e^{-{r_{xy}}^2} \cdot [1 - erf({r_{xy}}^*) + \frac{2}{\sqrt{\pi}} \cdot r_{xy}^* \cdot e^{-(r_{xy}^*)^2}], else \end{cases} \tag{9}$$

$$r_{xy} \equiv (\frac{x^2}{w_x^2} + \frac{y^2}{w_y^2})^{\frac{1}{2}} \tag{10}$$

$$r_{xy}^* \equiv ({r_0}^2 - {r_{xy}}^2)^{\frac{1}{2}} \tag{11}$$

$$erf(x) = \frac{2}{\sqrt{\pi}} \cdot \int_0^x e^{-t^2} dt \tag{12}$$

The peak density and numbers:

$$A \equiv w_z \cdot \sqrt{\pi} \cdot n_0 \tag{13}$$

$$n_0 = \frac{A \cdot e^{-{r_0}^2}}{\sqrt{\pi} \times w_z} = \frac{A \cdot e^{-{r_0}^2}}{\sqrt{\pi} \times \sqrt{w_x w_y}} \tag{14}$$

$$N_{tot} = A \cdot \pi \cdot w_x w_y \times [1 - erf(r_0) + \frac{2 \cdot r_0 \cdot e^{-{r_0}^2}}{\sqrt{\pi}} (1 + \frac{2r_0^2}{3})] \tag{15}$$