atomic wavefunctions in the 20 E_r lattice. We have respectively (cf. Eq. 9.7)

$$e^{2W_\pi(\tau=0)} = 1.39 \simeq e^{\frac{3}{2V_L^{1/2}}}$$

$$e^{2W_\theta(\tau=0)} = 1.08$$

S_Q is obtained from the scattered intensities by using

$$\boxed{S_Q = 1 + C_Q \left(\frac{I_{Q0}}{I_{Q\infty}} - 1 \right)} \tag{12.2}$$

where $C_Q = e^{2W_Q(\tau=0)} \left(1 + \frac{s_0}{4\Delta^2}\right)$ is the correction factor, with values

$$C_\pi = 1.52$$

$$C_\theta = 1.18$$

for π and θ respectively.

12.1.1 Light collection.

We collect Bragg scattered light in the π direction over a full angular width of 110 mrad, given by a 2.5 cm diameter collection lens located 23 cm away from the atoms. In the θ direction, light is collected by a 2.5 cm diameter lens placed 8 cm away from the atoms, corresponding to a full angular width of 318 mrad. The scattered light in each of the directions is focused to a few pixels on the cameras, so no additional angular information is obtained. For $N = 1.8 \times 10^5$, $s_0 = 15.5$, $\Delta = 6.4\,\Gamma$ and a 1.7 μs pulse, the detector in the π direction collects approximately 1300 photons, whereas the detector in the θ direction collects approximately 10^4 photons. The noise floor from readout, dark current and background light per shot has a standard deviation equivalent to approximately 250 photons in the π direction and 1000 photons in the θ direction.

ARE THESE JUST OFFSETS OR ARE
THEY THE ACTUAL S.D.?
(SHOULD SAY).

12.1.2 Data averaging.

The signals we detect are small enough that an uncorrelated sample may, in a single shot, produce a scattering signal as large as the ones produced by samples with AFM correlations. To obtain a reliable measurement of S_π we average at least 40 *in-situ* shots to obtain I_{Q0} and at least 40 time-of-flight shots to obtain $I_{Q\infty}$.

We estimate the expected variance on S_π by considering a randomly ordered sample in which $e^{i\pi\cdot R_n}2\langle\sigma_z\rangle_n$ is equal to +1 or -1 with equal probability. S_π can be written as

$$S_\pi = \left|\sum_n e^{i\pi\cdot R_n}\frac{2\langle\sigma_z\rangle_n}{\sqrt{N}}\right|^2,$$

which is equivalent to the square of the distance traveled on an unbiased random walk with step size $1/\sqrt{N}$. The mean and standard deviation can then be readily calculated: $\overline{S_\pi} = 1$ and $\sqrt{\mathrm{Var}(S_\pi)} = \sqrt{2}$, where $\mathrm{Var}(S_\pi)$ denotes the variance of the random variable S_π. With a standard deviation that is larger than the mean value, a considerable number of shots needs to be taken in order to obtain an acceptable error in the mean. The standard error of the mean for 40 shots will be $\sqrt{2/40} = 0.22$, consistent with what we obtain in the experiment (cf. vertical error bars in Fig. 12.6). BUT WHAT IS σ FOR $\overline{S_\pi} > 1$?

THIS IS WHAT IS RELEVANT TO US.

12.1.3 Other considerations for Bragg scattering

Momentum transferred from the probe to the atoms. In the derivation of the relationship between the intensity and the structure factor, showed in Chapter 9, we assumed that the center of mass state of the atom remains unchanged after scattering a photon. For this assumption to be valid, the Lamb-Dicke parameter, η^2, which relates the energy of the photon to the harmonic oscillator spacing in a lattice site[1], needs to be $\eta^2 \ll 1$. In the locked 20 E_r lattice, $\eta^2 = 0.27$, meaning that approximately one out of every 4 photons

[1]
$$\eta^2 = \hbar\omega_{\rm p}/(2E_r\sqrt{v_0/E_r}) \tag{12.3}$$
where $\omega_{\rm in}$ is the angular frequency of the incident light, and v_0 is the lattice depth.

NOT REALLY: $E_{\rm PHOTON} = \hbar\omega \approx 2\,\mathrm{eV}$

$$\eta^2 = \frac{\omega_R(\mathrm{PROBE})}{\omega_{\rm LAT}} = \frac{K_P^2}{2K_L^2 V_L^{1/2}}$$

TO WITHIN NUMERICAL FACTORS THIS IS THE ARGUMENT OF THE DW FACTOR.

scattered will excite an atom to the second band of the lattice. An atom in the second band has larger position variance and therefore a smaller Debye-Waller factor, so it contributes less to the Bragg scattering signal.

+1 27.1 ns [McAlexander]

The total number of photons scattered is given by $t_{\mathrm{exp}}\Gamma\frac{s_0/2}{s_0+4\Delta^2}$, where the duration of the probe pulse is $t_{\mathrm{exp}} = 1.7\,\mu\mathrm{s}$ and the linewidth of the excited state is $\Gamma = 1/27\ \mathrm{ns}^{-1}$. For $s_0 = 15.5$ and $\Delta = 6.4$ this corresponds to 2.7 photons scattered per atom during the probe pulse. In a $20\,E_r$ lattice it is then justifiable to assume the atoms remain in the lowest band during the pulse. A BETTER ESTIMATE IS YOUR EQ. 9.29. YOUR ESTIMATE APPLIES TO THE 2ND TERM. WHAT ABOUT LIGHT SCATTERED ALONG A RECIPROCAL LATTICE VECTOR? DOES THAT LEAD TO HEATING?

For the Bragg scattering measurements performed after time-of-flight, the momentum transferred from the probe to the atoms plays a more significant role, since the atoms are not trapped and will recoil after every photon scatter. As we will show below (cf. Fig 12.2), we still see good agreement between the observed decay of the Bragg scattering signal in time-of-flight and the decay expected for a Heisenberg limited wavepacket (Eq. 9.33). A similar consideration arises for Bragg scattering off of the (0 1 0) lattice planes, where in Chapter 9 we saw that there was also good agreement with Eq. 9.33.

$\frac{2.7}{4}$ IMPLIES THAT MORE THAN HALF THE ATOMS ARE SCATTERED INTO THE 2ND BAND. SEEMS LIKE A PROBLEM

Optical density. A low optical density of the sample is important so that the probe is unattenuated through the atom cloud, and multiple scattering events of the Bragg scattered photons are limited [58]. The optical density can be approximated as

$$\mathrm{OD} \simeq \frac{\sigma_0|\hat{e}_{\mathrm{p}}\cdot\hat{e}_{-1}|^2}{4\Delta^2+s_0}\frac{1}{a^2}\left(\frac{3N}{4\pi}\right)^{1/3}$$

where $\sigma_0 = 3\lambda_0^2/2\pi$. With $s_0 = 15.5$, $\Delta = 6.4$ and $N = 1.8\times10^5$ atoms we have OD $\simeq$ 0.072. At this value we do not expect significant corrections to the spin structure factor measurement due to the attenuation of the probe. We do not include any corrections in our measurement due to finite optical density effects.

When the Bragg condition is satisfied, the coherent enhancement of the signal along $\boldsymbol{Q} = \pi$ suppresses the scattered intensity in other directions (there is a sum rule for the total scattered intensity with a fixed value of the wavevector of the input light), which leads to a further reduction of $I_{\theta 0}$ beyond the reduction due to the presence of doubly occupied sites.

ADD DISCUSSION FROM DAVID AND OUR EMAIL CORRESPONDENCE.

12.3 Numerical calculations

Within the local density approximation (LDA) we model the sample by considering each point in the trap as a homogeneous system in equilibrium at a temperature T, with local values of the chemical potential and the Hubbard parameters determined by the trap potential. The spin structure factor of the sample $S_{\boldsymbol{Q}}$ can then be expressed as the integral over the trap of the local spin structure factor per lattice site, $s_{\boldsymbol{Q}}$.

$$S_{\boldsymbol{Q}} = a^{-3}N^{-1}\int s_{\boldsymbol{Q}}(\mu/t, T/t, U/t)\,\mathrm{d}^3r \tag{12.4}$$

Figure 12.3a shows calculations of s_π at various temperatures in a homogeneous lattice with $U/t = 8$, close to where T_N is maximal [154]. The figure shows that s_π is sharply peaked around $n = 1$ and grows rapidly as T approaches T_N from above.

Figures. 12.3b and 12.3c show, respectively, the results of numerical calculations of the local density and the local spin structure factor in our trap, obtained as a function of distance from the center along a body diagonal of the lattice. For more details about these calculations refer to Appendix B. In Fig. 12.3c we can see that the local spin structure factor is maximized at the largest radius for which the density is $n \approx 1$.

The finite extent of the lattice beams causes the lattice depth to decrease with distance from the center, resulting in an increasing t, such that both U/t and T/t decrease with increasing radius for constant T, as shown in Fig. 12.4. The radial decrease in T/t causes $s_\pi(r)$ to maximize at the largest radius for which the density is $n \approx 1$. For large U_0/t_0, where the cloud exhibits an $n = 1$ Mott plateau, this is the outermost radius of the plateau.