Circuit Theory and Electronics Fundamentals

Department of Electrical and Computer Engineering, Técnico, University of Lisbon

Laboratory Two Report

Francisco Batista, 95792.
Miguel Jesus, 95833.
Pedro Monteiro, 96465.

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1 Introduction

The objective of this laboratory assignment is to study the circuit that can be seen in Figure 1.

The circuit is composed by eleven components. These include an independent sinusoidal voltage source, a current-controlled voltage source, one current source (voltage-controlled), seven resistors and one capacitor. The indepedent source's behaviour is showcased in the following equation:

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{1}$$

Furthermore, the circuit contains thirteen meshes (four of which are essential) and eight nodes. The values for the resistors and the the capacitor, as well as the values of the constansts related with the dependent sources and the initial value(corresponding to t¡0) of the independent voltage source, are provided by the Python script.

In Section 2, a theoretical analysis of the circuit is presented. The analysis was divided in 6 different parts, where each of them represented a step in solving the complex problem that was presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. In this section, , we The conclusions of this study are outlined in Section 4.

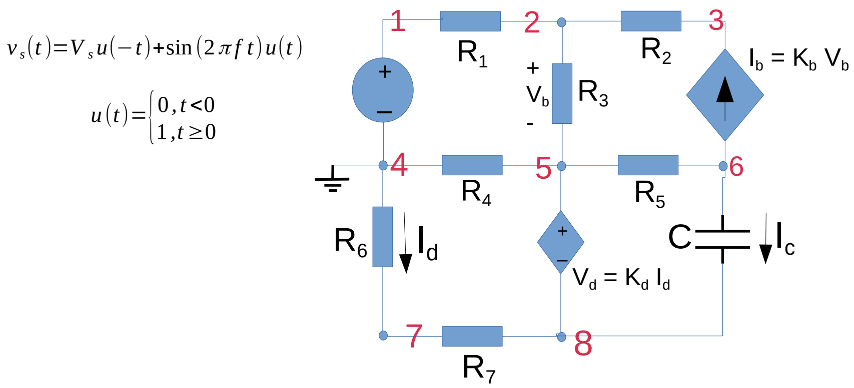

Figure 1: Studied circuit.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically in six steps.

In step (1), first of all, we used the nodal method to determine the voltages in all nodes and currents in all branches for t¡0. The nodal method yields the following equations:

$$V1 = Vs \tag{2}$$

$$-V1(\frac{1}{R1}) + V2(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}) - V3\frac{1}{R2} - V5\frac{1}{R3} = 0 \tag{3}$$

$$V2(Kb + \frac{1}{R2}) - V3\frac{1}{R2} - V5Kb = 0 \tag{4}$$

$$V2\frac{1}{R3} + V5(-\frac{1}{R3} - \frac{1}{R4} - \frac{1}{R5}) + V6\frac{1}{R5} + V7\frac{1}{R7} - V8\frac{1}{R7} = 0 \tag{5}$$

$$-V2Kb + V5(\frac{1}{R5} + Kb) - V6\frac{1}{R5} = 0 \tag{6}$$

$$V7(\frac{1}{R6} + \frac{1}{R7}) - V8\frac{1}{R7} = 0 \tag{7}$$

$$V5 + V7\frac{Kd}{R6} - V8 = 0 \tag{8}$$

$$V1(\frac{1}{R1}) - V2\frac{1}{R1} + Ia = 0 \tag{9}$$

$$V2\frac{1}{R2} - V3\frac{1}{R2} + Ib = 0 \tag{10}$$

$$V7\frac{1}{R6} + Id = 0 \tag{11}$$

$$-V5\frac{1}{R5} + Ib + Ic + V6\frac{1}{R5} = 0 \tag{12}$$

The results are shown in Table 1.

Nodes and branches	Value [V]/[A]
v1	5.134164
v2	4.929601
v3	4.510168
v4	0.000000
v5	4.957651
v6	5.588276
v7	-2.095712
v8	-3.151534

Table 1: Theoretical voltage values for each node, expressed in Volt, and current values for each branch, expressed in Ampere.

After this, in step (2), we determined the equivalent resistance as seen from the capacitor terminals. In order to do so, we followed the professor's sugestion. Therefore, in this section of the analysis, the capacitor was replaced by a voltage source Vx. Ix and Vx were calculated using Octave. The equations to be solved by octave were the following:

$$-V2\frac{1}{R1} - V5\frac{1}{R4} - V7\frac{1}{R6} = 0 \tag{13}$$

$$-V2(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}) + V3\frac{1}{R2} + V5\frac{1}{R3} = 0 \tag{14}$$

$$-V2(Kb + \frac{1}{R2}) + V3\frac{1}{R2} + V5Kb = 0 \tag{15}$$

$$V5 + V7(\frac{Kd}{R6}) - V8 = 0 \tag{16}$$

$$V7(\frac{1}{R6} + \frac{1}{R7}) - V8\frac{1}{R7} = 0 \tag{17}$$

$$V6 - V8 = Vx \tag{18}$$

$$V2(Kb) - V5(\frac{1}{R5} + Kb) + V6\frac{1}{R5} + Ix = 0 \tag{19}$$

After this, the value of the equivalent resistance was computed and determined. These procedures were necessary because they allowed us to calculate the time constant (τ) without which we would no be able to performe all the theoretical analisys in the following sections.

$$\tau = R_{eq}C, \tag{20}$$

The computed results can be found in Table 2.

Later, in step (3), we used the value of the equivalent resistant calculated in point (2) to find the natural solution of v6. Knowing that "τ" is calculated by the equation **??**, in Equation 21 we find the formula required to calculate the natural solution we wanted.

$$V_{6n}(t) = V_x e^{-\frac{t}{\tau}}, \tag{21}$$

Also the solution is ploted in Figure **??** where the x-axis corresponds to time, t, expressed in [ms] and the y-axis corresponds to the natural solution of v6, 'v6n', expressed in [V].

Computed Results	Values
v1	0.000000
v2	0.000000
v3	0.000000
v4	0.000000
v5	0.000000
v6	8.739810
v7	0.000000
v8	0.000000
Ix	-0.002826
Vx	8.739810
Req	-3092.796241

Table 2: Computed results: voltage expressed in Volt, current in Ampere and resistence in Ohm.

In step (4), the forced solution v6f(t) was determined for f=1KHz and for t in the interval [0,20]ms. Moreover, the capacitance was replaced by the impedance, 'Z', and Vs was considered equal to one, representing its amplitude. The system of equations derived to compute the complex amplitudes is given by:

$$V1 = 1 \tag{22}$$

$$V1\frac{1}{R1} - V2(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}) + V3\frac{1}{R2} + V5\frac{1}{R3} = 0 \tag{23}$$

$$-V2(Kb + \frac{1}{R2}) + V3\frac{1}{R2} + V5Kb = 0 \tag{24}$$

$$V5 + V7(\frac{Kd}{R6}) - V8 = 0 \tag{25}$$

$$-V7(\frac{1}{R6} + \frac{1}{R7}) + V8\frac{1}{R7} = 0 \tag{26}$$

$$V2(\frac{1}{R3}) - V5(\frac{1}{R3} + \frac{1}{R5} + \frac{1}{R4}) + V6(\frac{1}{R5} + \frac{1}{Zc}) + V7\frac{1}{R7} - V8(frac1R7 + \frac{1}{Zc}) = 0 \tag{27}$$

$$V2(Kb) - V5(\frac{1}{R5} + Kb) + V6(\frac{1}{R5} + \frac{1}{Zc}) - V8\frac{1}{Zc} = 0 \tag{28}$$

The complex amplitudes in the nodes are shown in Table 3.

Then, in step (5), we determined the final total solution v6(t) by converting the phasors to real time functions for f=1KHz, and superimposing the natural and forced solutions, just like it was asked by the professor. In Figure 3 are plotted both vs(t) and v6(t) in the interval [-5, 20]ms.

Finally, in step (6) of our theoretical analysis, we attempted to figure out the frequency response of the circuit, computing the magnitude and the phase changes with the frequency of both the source voltage, the voltage across the capacitor and the voltage in node 6, and then proceding to plot them. The source's magnitude and phase do not hinge on the frequency values, so they are expected to remain constant as it is verified in Figure 4 and Figure 5.

At low frequencies, the capacitor is able to charge up until its voltage reaches a value close to the input's. Therefore, the potential difference across the capacitor rises to a significant value. The voltage at node 6 and the voltage in the capacitor are showcased in the Figure 4 When

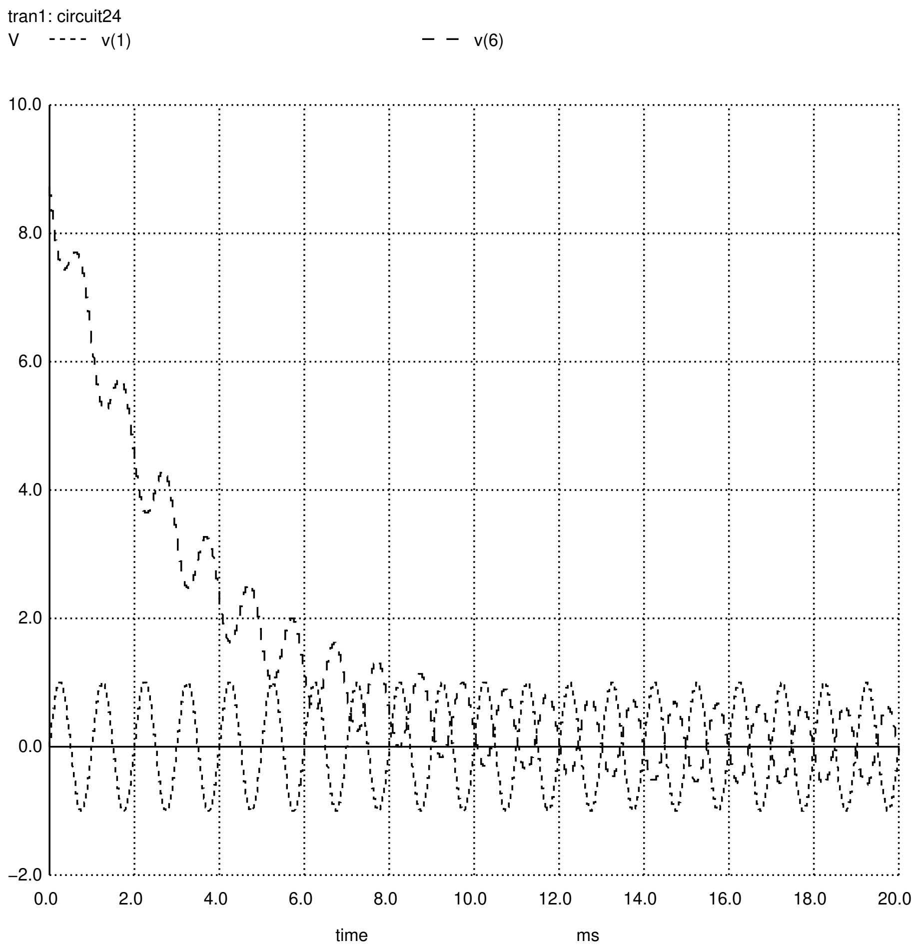

Figure 2: Plot of v6n(t) in the interval [0, 20]ms.

this is the case, the current flow to the capacitor plummets to a value close to zero, meaning that the circuit will behave as an open circuit. By the same reasoning, it can be concluded that the phase will also behave in the same way as the input's, without showing great disparity, as is shown in Figure 5. On the other hand, as the frequency values increase, the circuit will begin to act as a short circuit (shunt). At this point, the voltage drop in the capacitor's terminals is almost null. When converting the amplitude values from V into dB, a number much smaller than 1 turns into a magnitude that is negative despite having a great absolute value. This is the reason that leads to the visible slump corresponding to vc in Figure 4. Due to the huge frequency of the source, the phase difference of vc and v6 also become notable, as shown in Figure 5.

3 Simulation Analysis

Following our theoretical analysis, the circuit was simulated in the Ngspice software. In step (1), we were asked to simulate the operating point for t¡0, in order to find the voltages in all nodes and the currents in all branches. The results are shown in Table 4.

Then, in step (2), we simulated the operating point considering that the voltage of the source in the instant t=0 was null and replacing the capacitor with a voltage source that was equal to V6-V8, being this values the voltages in the capacitor's nodes that were obtained in (1). This step was needed because... The results are shown in Table 5.

Nodes	Complex Amplitudes
—v1—	1.000000
—v2—	0.960156
—v3—	0.878462
—v4—	0.000000
—v5—	0.965620
—v6—	-0.609606
—v7—	-0.408190
—v8—	-0.613836

Table 3: Complex amplitudes in the nodes, expressed in Volt.

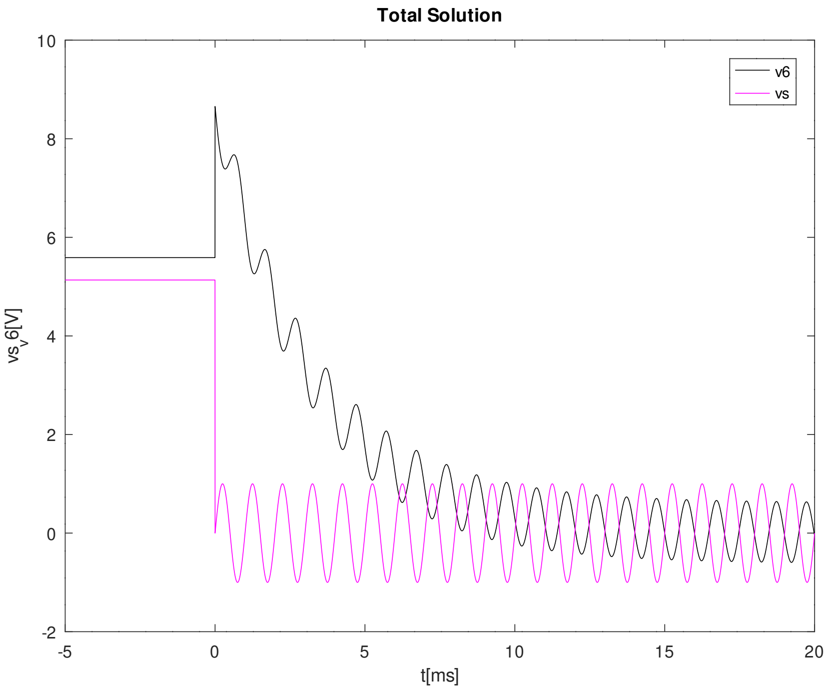

Figure 3: Plot of v6(t) and vs(t) in the interval [-5, 20]ms.

In step (3), we simulated the natural response of the circuit. In Figure 6 we can find the plot of v6 in the interval [0, 20]ms.

Later, in step (4), we were asked to simulate the natural and forced response on node 6 by repeating step (3) with vs(t) as given in Equation 29 and f=1kHz. In Figure 7 we can see the plot of both the stimulus and the response.

$$V_s(t) = sin(2\pi ft), \tag{29}$$

At last, in step (5), the frequency response in node 6 was simulated for the frequency range 0.1 Hz to 1 MHz. In Figure 8 and Figure **??** we plotted the amplitude frequency response and the phase response, respectively. It is important to notice that the frequency logscale as its magnitude in dB units and the phase is presented in degrees.

Just like was predicted in our theoretical analysis, by simulating we came to the conclusion that v6 and vs differ because of the reasons that were mentioned in the previous chapter.

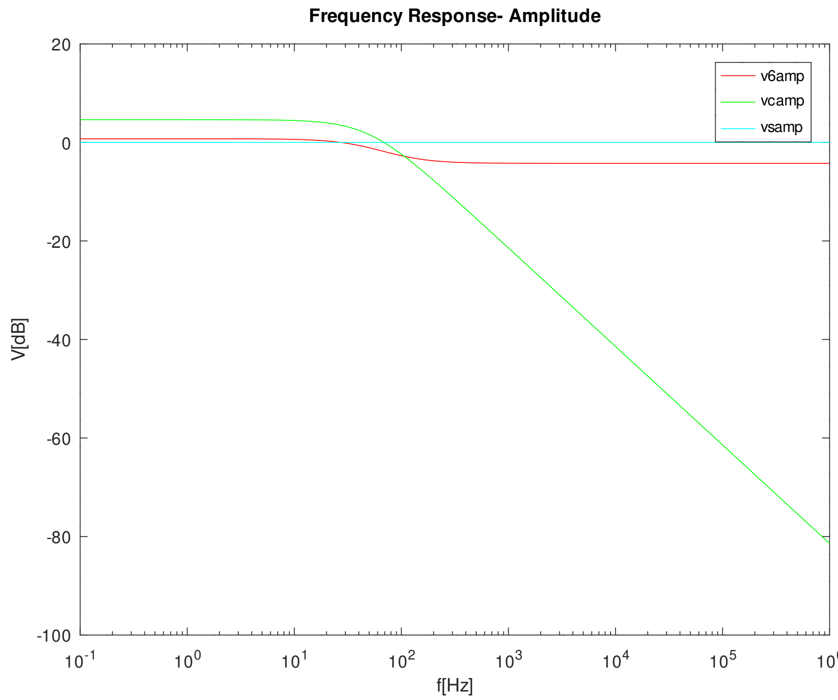

Figure 4: Plot of amplitude frequency response (for vs(f), vc(f) and v6(f)) in the interval [0.1, 1M]Hz.

Name	Value [A or V]
v(1)	5.134164e+00
v(2)	4.929601e+00
v(3)	4.510168e+00
v(4)	-2.09571e+00
v(5)	4.957651e+00
v(6)	5.588277e+00
v(7)	-2.09571e+00
v(8)	-3.15153e+00

Table 4: Operating point for t¡0. Simulated Values for voltage (V) and current (A) using Ngspice.

4 Conclusion

To sum up, in this assignment, the main goal was successfully achieved. This objective was to analyze a circuit with varius components such as resistors, a voltage source that changes over time and a capacitor.

To perform this study we used the Octave and Ngspice tools for the theoretical and simulation analysis, respectively.

The values that were obtained in both analysis are very similar. nevertheless, there are small discrepancies between the sets of values. These differences are due to approximations, made particularly by Ngspice, as this software has a different level of digit precision when compared to Octave. Whereas some values in Octave are presented as 0, in Ngspice the same values are simulated with the order of 1e-14 or 1e-15 which we can consider to be also zero. Therefore the relative error between the theoretical and simulated results are very close to 0%.

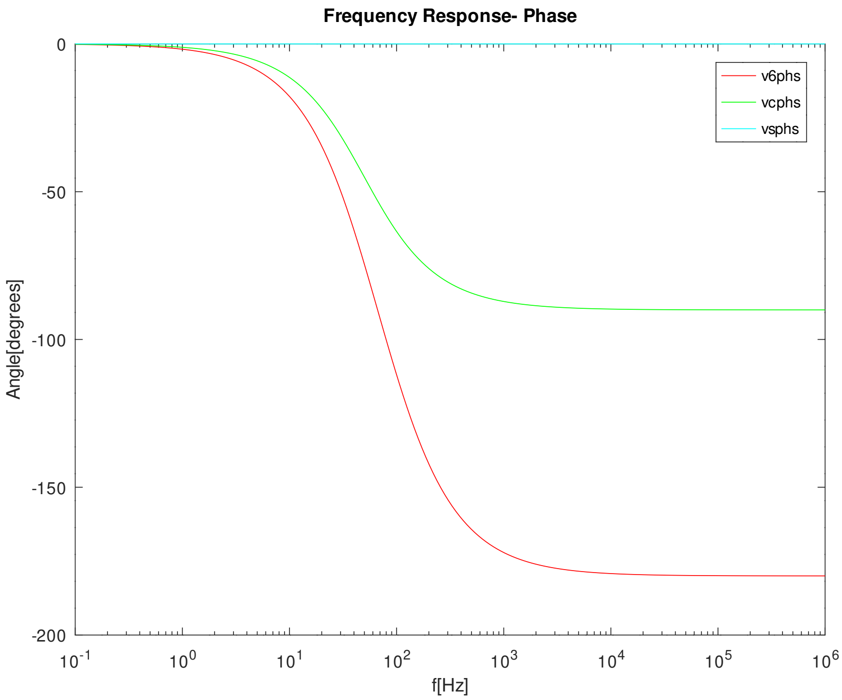

Figure 5: Plot of phase response (for vs(f), vc(f) and v6(f)) in the interval [0.1, 1M]Hz.

The proximity between the values obtained in the static, time and frequency analysis in the theoretical and simulation section can be explained with the simplicity of the circuit that is only made of linear components and one capacitor.

Name	Value [A or V]
@gb[i]	-4.85593e-18
@r1[i]	4.643578e-18
@r2[i]	-4.85593e-18
@r3[i]	-2.12353e-19
@r4[i]	1.020215e-18
@r5[i]	-2.82586e-03
@r6[i]	-7.43500e-19
@r7[i]	-2.20490e-19
v(1)	0.000000e+00
v(2)	-4.87169e-15
v(3)	-1.48605e-14
v(4)	1.545457e-15
v(5)	-4.20366e-15
v(6)	8.739810e+00
v(7)	1.545457e-15
v(8)	1.776357e-15
v(6)-v(8)	8.739810e+00

Table 5: Operating point for Vs=0 (instant t=0). Simulated Values for voltage (V) and current (A) using Ngspice.

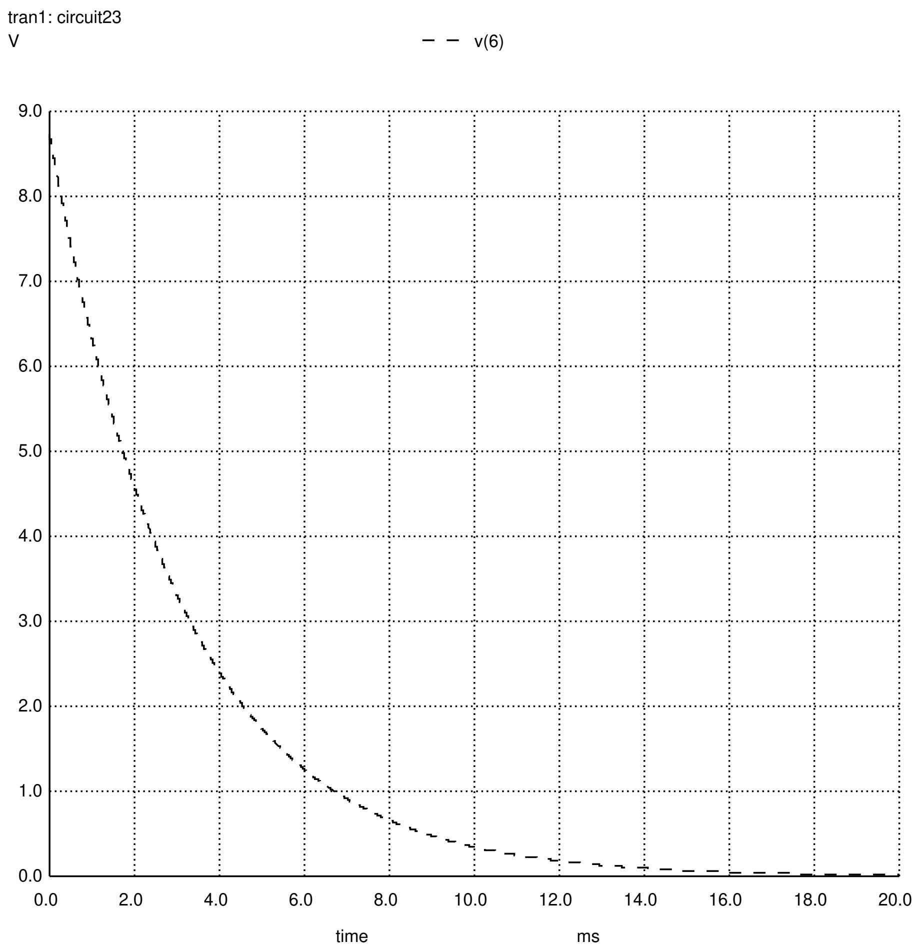

Figure 6: Plot of v6(t) in the interval [0, 20]ms.

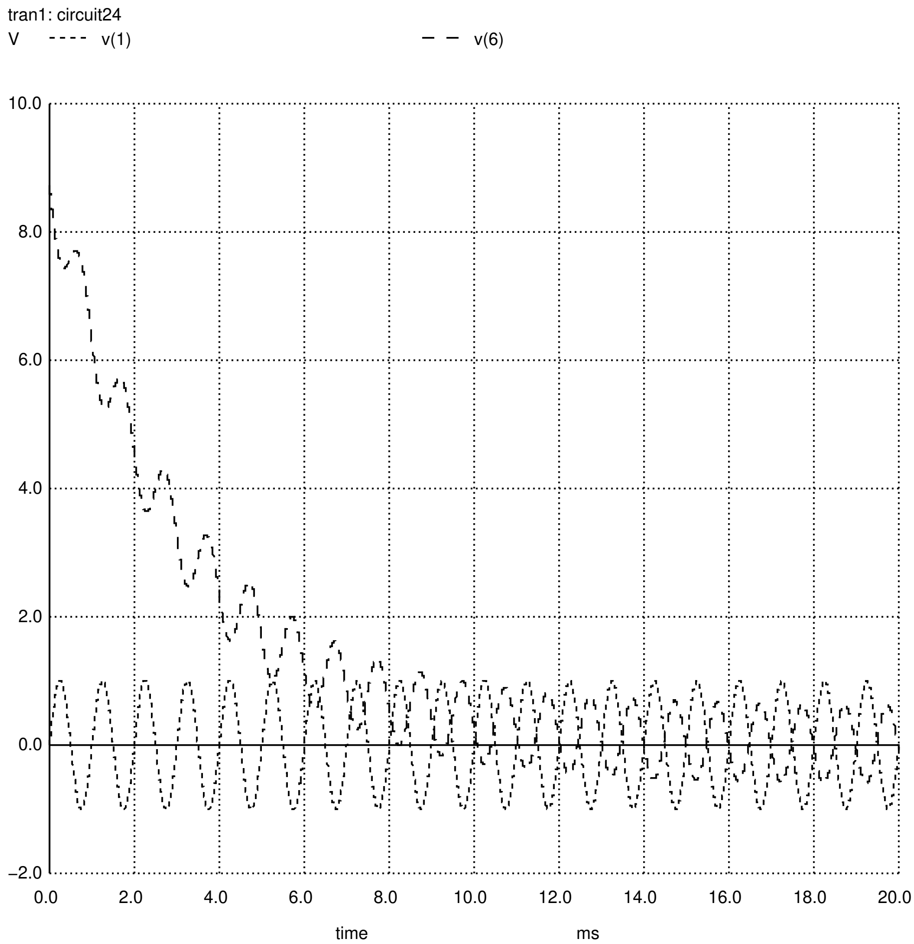

Figure 7: Plot of v6(t) in the interval [0, 20]ms.

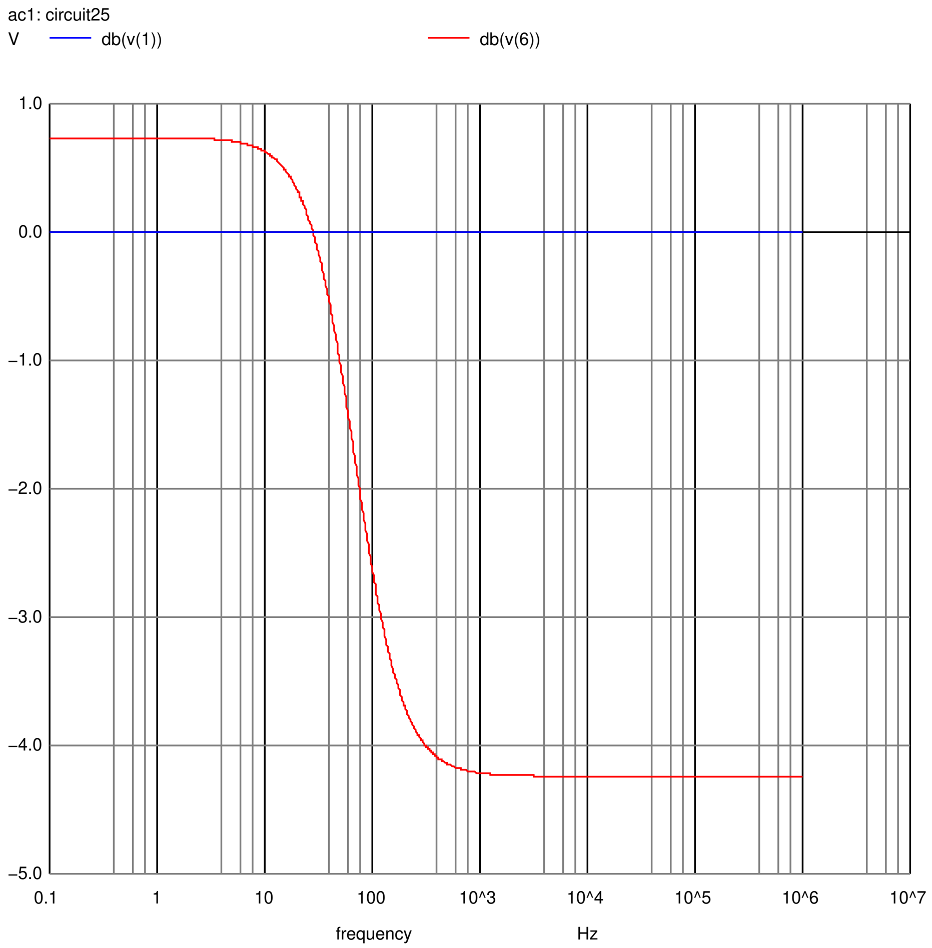

Figure 8: Plot of amplitude frequency response (for vs(f) and v6(f)) in the interval [0.1, 1M]Hz.

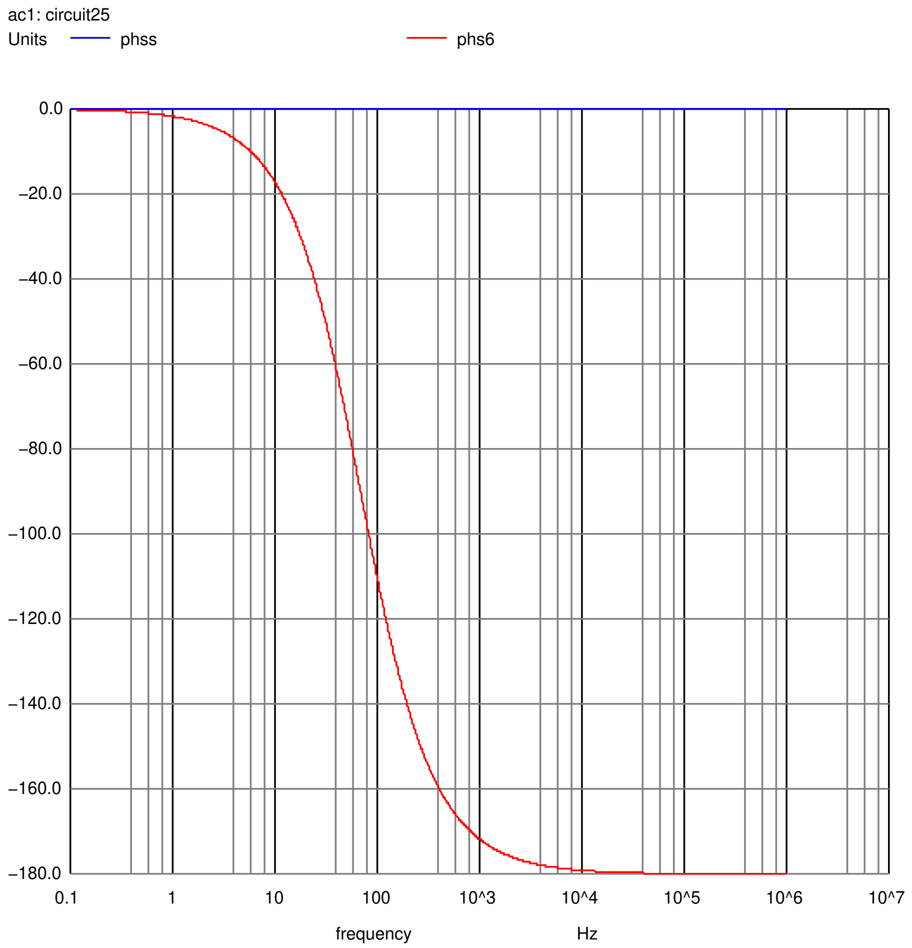

Figure 9: Plot of phase response (for vs(f) and v6(f)) in the interval [0.1, 1M]Hz.