① $Y_i = \exp\{X_i\beta + \gamma Q_i + \varepsilon_i\}$

$Q_i \geq 0$ is school quality

X_i other observed characteristics

ε_i unobserved. $\gamma > 0$.

$U_i(C_i, Y_i) = \ln C_i + \delta_i \ln Y_i$

income M_i Q's price is q per unit. C_i price is 1.

ⓐ $M = C_i \cdot 1 + Q_i \cdot q$

$M = C_i + Q_i q$.

ⓑ $M_i = C_i - Q_i q$

$C_i = M_i - Q_i q$

$U(C_i, Y_i) = \ln C_i + \delta_i \ln(\exp\{X_i\beta + Q_i\gamma + \varepsilon_i\})$.

$U(C_i, Y_i) = \ln C_i + \delta_i(X_i\beta + \gamma Q_i + \varepsilon_i)$

$$\boxed{\begin{array}{l} U(C_i, Y_i) = \ln(M_i - Q_i q) + \delta_i(X_i\beta + \gamma Q_i + \varepsilon_i) \\ \text{subject to } C_i = M_i - Q_i q \end{array}}$$

ⓒ $\dfrac{\partial U}{\partial Q_i} = \dfrac{-q}{M_i - Q_i q} + \delta_i \gamma$

$$\boxed{0 = \delta_i \gamma - \frac{q}{M_i - Q_i q}}$$

ⓓ If $Q_i = 0$:

$$\boxed{\delta_i \gamma - \frac{q}{M_i}}$$

(e) $\frac{-q}{M_i - Q_i q} = -\delta_i \gamma$

$\frac{q}{M_i - Q_i q} = \delta_i \gamma$

$q = \delta_i \gamma M_i - \delta_i \gamma Q_i q$

$\delta_i \gamma Q_i q = \delta_i \gamma M_i - q$

$\boxed{Q_i = \frac{\delta_i \gamma M_i - q}{\delta_i \gamma q}}$

(f) $M_i \uparrow$ leads to $\uparrow Q_i$

$M_i \downarrow$ " " $\downarrow Q_i$

When the parent has higher income, it is willing to spend more on the child's education, with the opposite true as well.

$q \uparrow$ leads to $\downarrow Q_i$

$q \downarrow$ " " $\uparrow Q_i$

When the price of school quality increases, the parent is not willing to pay for the same level of education and will want to pay for less.

$\gamma\uparrow$ leads to $\downarrow Q_i$

$\gamma\downarrow$ " " $\uparrow Q_i$

When this parameter increases, that means the child can achieve same level of productivity as before, but with less education. Therefore the parent needs less school quality for the child.

$\delta_i\uparrow$ leads to $\downarrow Q_i$

$\delta_i\downarrow$ " " $\uparrow Q_i$

When the taste increases, that means the parent gets happier with the child's productivity level easier, so the parent's utility level can be achieved with smaller productivity of the child, so the child doesn't require as much quality as before.

(9) If you choose to run an OLS, there are high chances that unobserved characteristics will influence the coefficient of education and/or the Y_i, so we would get overestimations or underestimations. Intead we can do an IV regression, by doing so, we calculate the impact of school quality separately from other variables on the log earnings, because we expect no correlation between the instrument variable and the controls. Therefore I would recommend an IV with Q_i as the instrument.

(2) low, $D=0$ $\rightarrow .25$

moderate, $D=1$ $\rightarrow .5$

high, $D=2$ $\rightarrow .25$

$U_i(w,D) = w - \theta_i D^2$ w is wage, D is risk, θ_i is preference towards risk.

$\theta_i = 1 \rightarrow .5$ $\theta_i = 4 \rightarrow .5$.

(a) Workers with $\theta_i = 4$ are more risk averse, because they have smaller preference toward injury risk and therefore derive less utility from it.

(b) $U_i(w,0) = W_0 - 0$ $U_i(w,0) = W_0$

$U_i(w,1) = W_1 - 1$ $U_i(w,1) = W_1 - 4$

$U_i(w,2) = W_2 - 4$ $U_i(w,2) = W_2 - 16$

$D_2 = W_2 \cdot Q$

$D_1 = W_1 \cdot Q$

$D_0 = W_0 \cdot Q$

$.25 W_0 Q + .5 W_1 Q + .25 W_2 Q =$

$Q(W_0 + W_1 + W_2) = .5Q(W_0 + W_1 + W_2 - 5) + .5Q(W_0 + W_1 + W_2 - 20)$.

The workers must be divided such that the sum of utility from the risk averse is the same as the less risk averse across all jobs.

(c) $Q^d = Q^s$

$.25\,Q^s \cdot W_0 + .5\,Q^s \cdot W_1 + .25\,Q^s\,W_2 = .5\,Q_{\theta_1}\,W_0 + .5\,Q_{\theta_1} \cdot W_1 + .5\,Q_{\theta_2} \cdot W_1 + .25\,Q_{\theta_2} \cdot W_2$

$.25\,W_0 + .5\,W_1 + .25\,W_2 = .25(W_0) + .5(W_1 - 25) + .25(W_2 - 4).$

The quantity of firms for each type must be the same as the quantity of labor

(d) $W_1^* - W_0^* =$

(e) Then we would have more workers from type $\theta_i = 4$ on medium risk, while the quantity of workers from type $\theta_i = 1$ on medium companies would decrease

(f) They would change that if the wage difference from $W_1 - W_0$ was bigger than 3, because that way this new wage from safety measures W_s would follow: $W_0 < W_s < W_1$, so it would be cheaper to employ workers. That would increase the W_0, which in turn would lead to more workers desiring that position, specially from the risk averse group, while the other wages would remain the same. If the change of wage from W_1 to W_s is valued by the companies, then the share of "low risk" will increase, as well as the number of workers leaving "medium" who didn't adapt to that.

(g) It follows the same idea, but now it depends on whether the $W_1 - W_0 \geq 5$, to make it worth for medium firms