Barabasi-Albert model

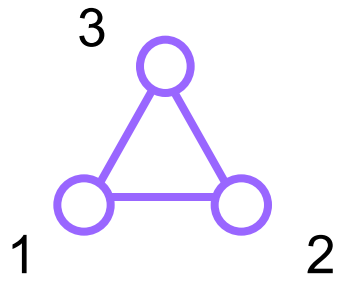
- First used to describe skewed degree distribution of the World Wide Web
- Each node connects to other nodes with probability proportional to their degree
  - the process starts with some initial subgraph
  - each new node comes in with *m* edges
  - probability of connecting to node *i*

$$\Pi(i) = m \frac{k_i}{\sum_j k_j}$$

- Results in power-law with exponent α = 3

Basic BA-model

- Very simple algorithm to implement
  - start with an initial set of m_0 fully connected nodes
    - e.g. $m_0 = 3$



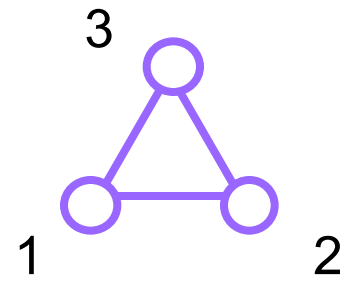
  - now add new vertices one by one, each one with exactly m edges
  - each new edge connects to an existing vertex in proportion to the number of edges that vertex already has → ***preferential attachment***
  - easiest if you keep track of edge endpoints in one large array and select an element from this array at random
    - the probability of selecting any one vertex will be proportional to the number of times it appears in the array – which corresponds to its degree

1 1 2 2 2 3 3 4 5 6 6 7 8 ….

generating BA graphs – cont'd

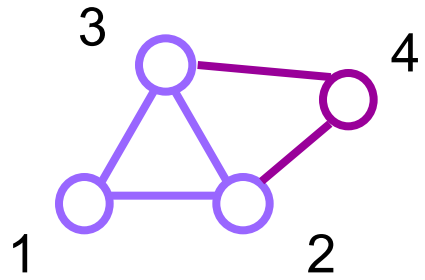
- To start, each vertex has an equal number of edges (2)
  - the probability of choosing any vertex is 1/3

1 1 2 2 3 3

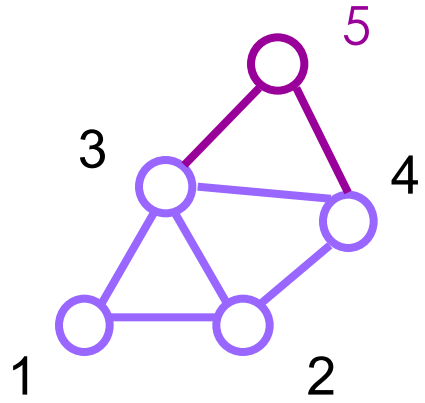


- We add a new vertex, and it will have m edges, here take m=2
  - draw 2 random elements from the array – suppose they are 2 and 3

1 1 2 2 2 3 3 3 4 4



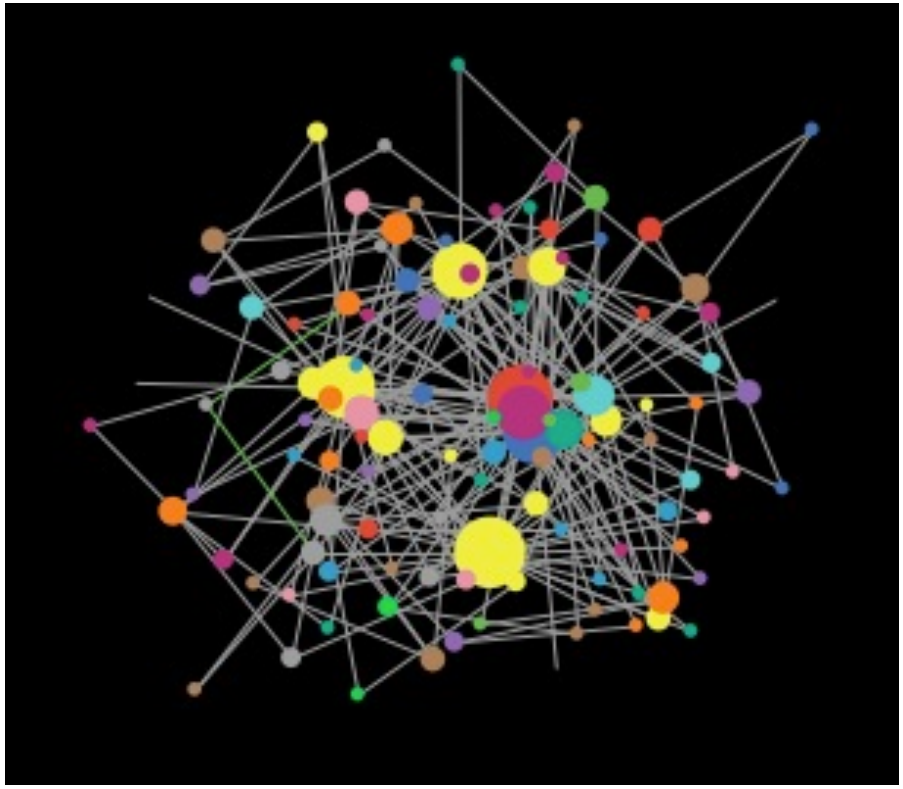
- Now the probabilities of selecting 1,2,3,or 4 are
  1/5, 3/10, 3/10, 1/5



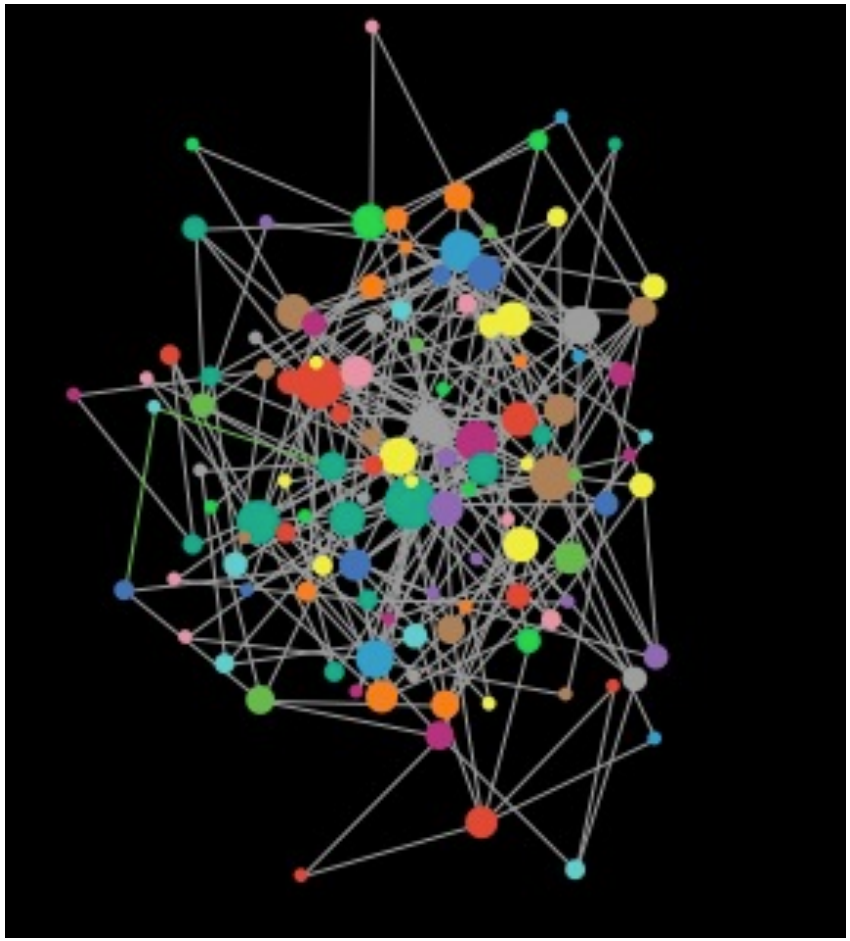
- Add a new vertex, draw a vertex for it to connect from the array
  - etc.

1 1 2 2 2 3 3 3 3 4 4 4 5 5

after a while...

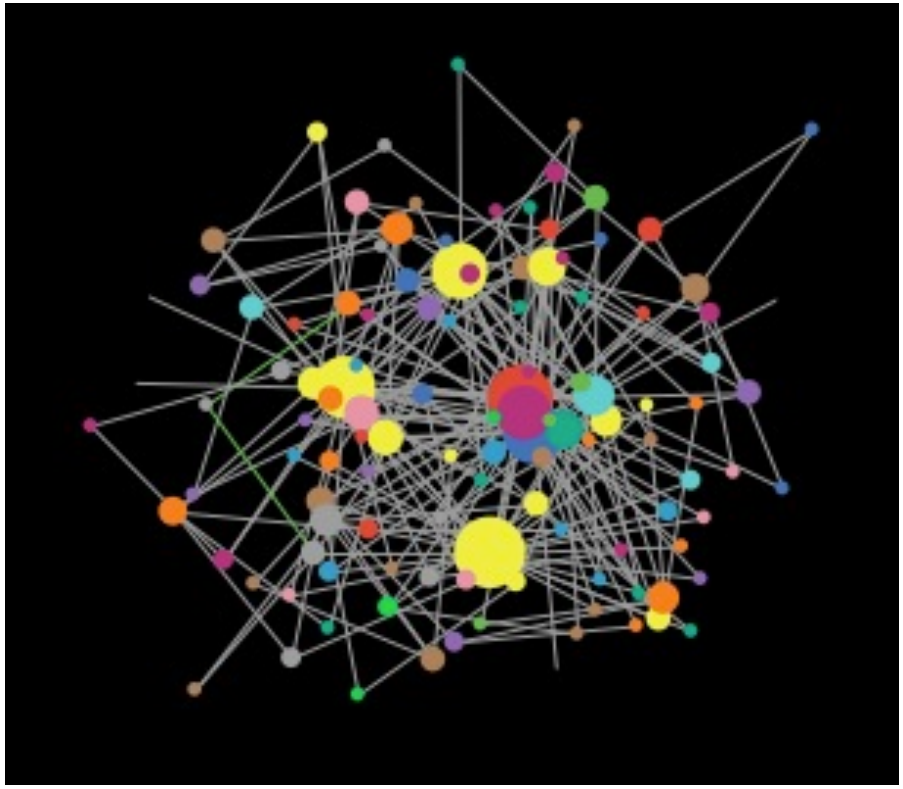


contrasting with random (non-preferential) growth



random

m = 2



preferential