# Chapter 7
# Sampling and Sampling Distributions

**Learning Objectives**

1. Understand the importance of sampling and how results from samples can be used to provide estimates of population characteristics such as the population mean, the population standard deviation and / or the population proportion.

2. Understand the difference between sampling from a finite population and sampling from an infinite population.

3. Know what simple random sampling is and how simple random samples are selected.

4. Understand the concept of a sampling distribution.

5. Understand the central limit theorem and the important role it plays in sampling.

6. Know the characteristics of the sampling distribution of the sample mean ( $\bar{x}$ ) and the sampling distribution of the sample proportion ( $\bar{p}$ ).

7. Learn about a variety of sampling methods including stratified random sampling, cluster sampling, systematic sampling, convenience sampling and judgment sampling.

8. Know the definition of the following terms:

    parameter
    sampled population
    sample statistic
    simple random sampling
    sampling without replacement
    sampling with replacement
    point estimator
    point estimate
    target population
    sampling distribution
    finite population correction factor
    standard error
    central limit theorem
    unbiased

**Solutions:**

1. a. AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

   b. With 10 samples, each has a 1/10 probability.

   c. E and C because 8 and 0 do not apply.; 5 identifies E; 7 does not apply; 5 is skipped since E is already in the sample; 3 identifies C; 2 is not needed since the sample of size 2 is complete.

2. Using the last 3-digits of each 5-digit grouping provides the random numbers:

   601, 022, 448, 147, 229, 553, 147, 289, 209

   Numbers greater than 350 do not apply and the 147 can only be used once. Thus, the simple random sample of four includes 22, 147, 229, and 289.

3. 459, 147, 385, 113, 340, 401, 215, 2, 33, 348

4. a. 5, 0, 5, 8

   Bell South, LSI Logic, General Electric

   b. $\dfrac{N!}{n!(N-n)!} = \dfrac{10!}{3!(10-3)!} = \dfrac{3,628,800}{(6)(5040)} = 120$

5. 283, 610, 39, 254, 568, 353, 602, 421, 638, 164

6. 2782, 493, 825, 1807, 289

7. 108, 290, 201, 292, 322, 9, 244, 249, 226, 125, (continuing at the top of column 9) 147, and 113.

8. Random numbers used: 13, 8, 27, 23, 25, 18

   The second occurrence of the random number 13 is ignored.

   Companies selected: ExxonMobil, Chevron, Travelers, Microsoft, Pfizer, and Intel

9. 102, 115, 122, 290, 447, 351, 157, 498, 55, 165, 528, 25

10. a. Finite population. A frame could be constructed obtaining a list of licensed drivers from the New York State driver's license bureau.

    b. Infinite population. Sampling from an infinite population. The sample is taken from the production line producing boxes of cereal.

    c. Infinite population. Sampling from an infinite population. The sample is taken from the ongoing arrivals to the Golden Gate Bridge.

    d. Finite population. A frame could be constructed by obtaining a listing of students enrolled in the course from the professor.

    e. Infinite population. Sampling from an infinite population. The sample is taken from the ongoing orders being processed by the mail-order firm.

11. a. $\bar{x} = \Sigma x_i / n = \frac{54}{6} = 9$

b. $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\Sigma(x_i - \bar{x})^2 =$ (-4)$^2$ + (-1)$^2$ + 1$^2$ (-2)$^2$ + 1$^2$ + 5$^2$ = 48

$s = \sqrt{\frac{48}{6-1}} = 3.1$

12. a. $\bar{p} =$ 75/150 = .50

b. $\bar{p} =$ 55/150 = .3667

13. a. $\bar{x} = \Sigma x_i / n = \frac{465}{5} = 93$

b.

| | $x_i$ | $(x_i - \bar{x})$ | $(x_i - \bar{x})^2$ |
|---|---|---|---|
| | 94 | +1 | 1 |
| | 100 | +7 | 49 |
| | 85 | -8 | 64 |
| | 94 | +1 | 1 |
| | 92 | -1 | 1 |
| Totals | 465 | 0 | 116 |

$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{116}{4}} = 5.39$

14. a. Eighteen of the 40 funds in the sample are load funds. Our point estimate is

$\bar{p} = \frac{18}{40} = .45$

b. Six of the 40 funds in the sample are high risk funds. Our point estimate is

$\bar{p} = \frac{6}{40} = .15$

c. The below average fund ratings are low and very low. Twelve of the funds have a rating of low and 6 have a rating of very low. Our point estimate is

$\bar{p} = \frac{18}{40} = .45$

15. a. $\bar{x} = \Sigma x_i / n = \frac{\$45{,}500}{10} = \$4{,}550$

b. $s = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\dfrac{9{,}068{,}620}{10-1}} = \$1003.80$

16. a. We would use the sample proportion for the estimate.

$$\bar{p} = \frac{5}{50} = .10$$

(Authors' note: The actual proportion from New York is $p = \dfrac{52}{500} = .104.$)

b. The sample proportion from Minnesota is

$$\bar{p} = \frac{2}{50} = .04$$

Our estimate of the number of Fortune 500 companies from New York is (.04)500 = 20.

(Authors' note: The actual number from Minnesota is 18.)

c. Fourteen of the 50 in the sample come from these 4 states. So 36 do not.

$$\bar{p} = \frac{36}{50} = .72$$

(Authors' note: The actual proportion from Minnesota is $p = \dfrac{366}{500} = .732.$)

17. a. 409/999 = .41

b. 299/999 = .30

c. 291/999 = .29

18. a. $E(\bar{x}) = \mu = 200$

b. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 50 / \sqrt{100} = 5$

c. Normal with $E(\bar{x}) = 200$ and $\sigma_{\bar{x}} = 5$

d. It shows the probability distribution of all possible sample means that can be observed with random samples of size 100. This distribution can be used to compute the probability that $\bar{x}$ is within a specified ± from $\mu$.

19. a. The sampling distribution is normal with

$E(\bar{x}) = \mu = 200$

$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 50 / \sqrt{100} = 5$

For ± 5, $195 \le \bar{x} \le 205$

Using Standard Normal Probability Table:

At $\bar{x} = 205$, $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{5}{5} = 1$ $\quad P(z \le 1) = .8413$

At $\bar{x} = 195$, $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{-5}{5} = -1$ $\quad P(z < -1) = .1587$

$P(195 \le \bar{x} \le 205) = .8413 - .1587 = .6826$

b. For $\pm$ 10, $190 \le \bar{x} \le 210$

Using Standard Normal Probability Table:

At $\bar{x} = 210$, $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{10}{5} = 2$ $\quad P(z \le 2) = .9772$

At $\bar{x} = 190$, $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{-10}{5} = -2$ $\quad P(z < -2) = .0228$

$P(190 \le \bar{x} \le 210) = .9772 - .0228 = .9544$

20. $\sigma_{\bar{x}} = \sigma / \sqrt{n}$

$\sigma_{\bar{x}} = 25 / \sqrt{50} = 3.54$

$\sigma_{\bar{x}} = 25 / \sqrt{100} = 2.50$

$\sigma_{\bar{x}} = 25 / \sqrt{150} = 2.04$

$\sigma_{\bar{x}} = 25 / \sqrt{200} = 1.77$

The standard error of the mean decreases as the sample size increases.

21. a. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 10 / \sqrt{50} = 1.41$

b. $n / N = 50 / 50{,}000 = .001$

Use $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 10 / \sqrt{50} = 1.41$

c. $n / N = 50 / 5000 = .01$

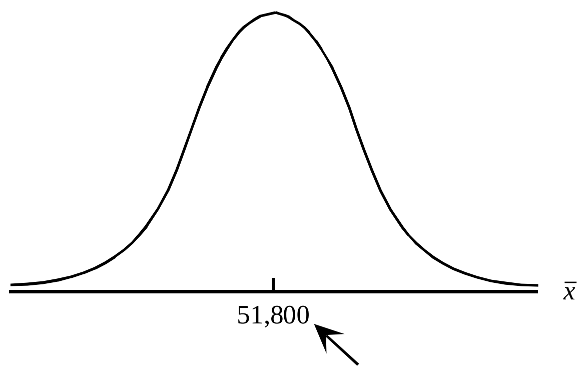
Use $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 10 / \sqrt{50} = 1.41$

d. $n / N = 50 / 500 = .10$

Use $\sigma_{\bar{x}} = \sqrt{\dfrac{N-n}{N-1}}\dfrac{\sigma}{\sqrt{n}} = \sqrt{\dfrac{500-50}{500-1}}\dfrac{10}{\sqrt{50}} = 1.34$

Note: Only case (d) where $n / N = .10$ requires the use of the finite population correction factor.

22. a.




$E(\bar{x})$

$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 4000 / \sqrt{60} = 516.40$

The normal distribution for $\bar{x}$ is based on the Central Limit Theorem.

b. For $n = 120$, $E(\bar{x})$ remains \$51,800 and the sampling distribution of $\bar{x}$ can still be approximated by a normal distribution. However, $\sigma_{\bar{x}}$ is reduced to $4000 / \sqrt{120} = 365.15$.

c. As the sample size is increased, the standard error of the mean, $\sigma_{\bar{x}}$, is reduced. This appears logical from the point of view that larger samples should tend to provide sample means that are closer to the population mean. Thus, the variability in the sample mean, measured in terms of $\sigma_{\bar{x}}$, should decrease as the sample size is increased.

23. a. With a sample of size 60 $\sigma_{\bar{x}} = \dfrac{4000}{\sqrt{60}} = 516.40$

At $\bar{x} = 52{,}300$, $z = \dfrac{52{,}300 - 51{,}800}{516.40} = .97$

$P(\bar{x} \le 52{,}300) = P(z \le .97) = .8340$

At $\bar{x} = 51{,}300$, $z = \dfrac{51{,}300 - 51{,}800}{516.40} = -.97$

$P(\bar{x} < 51{,}300) = P(z < -.97) = .1660$

$P(51{,}300 \le \bar{x} \le 52{,}300) = .8340 - .1660 = .6680$

b. $\sigma_{\bar{x}} = \dfrac{4000}{\sqrt{120}} = 365.15$

At $\bar{x}$ = 52,300, $z = \dfrac{52{,}300 - 51{,}800}{365.15} = 1.37$

$P(\bar{x} \le 52{,}300) = P(z \le 1.37) = .9147$

At $\bar{x}$ = 51,300, $z = \dfrac{51{,}300 - 51{,}800}{365.15} = -1.37$

$P(\bar{x} < 51{,}300) = P(z < -1.37) = .0853$

$P(51{,}300 \le \bar{x} \le 52{,}300) = .9147 - .0853 = .8294$

24. a. Normal distribution, $E(\bar{x}) = 17.5$

$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 4 / \sqrt{50} = .57$

b. Within 1 week means $16.5 \le \bar{x} \le 18.5$

At $\bar{x}$ = 18.5, $z = \dfrac{18.5 - 17.5}{.57} = 1.75$ $P(z \le 1.75) = .9599$

At $\bar{x}$ = 16.5, $z$ = -1.75. $P(z < -1.75) = .0401$

So $P(16.5 \le \bar{x} \le 18.5) = .9599 - .0401 = .9198$

c. Within 1/2 week means $17.0 \le \bar{x} \le 18.0$

At $\bar{x}$ = 18.0, $z = \dfrac{18.0 - 17.5}{.57} = .88$ $P(z \le .88) = .8106$

At $\bar{x}$ = 17.0, $z$ = -.88 $P(z < -.88) = .1894$

$P(17.0 \le \bar{x} \le 18.0) = .8106 - .1894 = .6212$

25. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 100 / \sqrt{90} = 10.54$ This value for the standard error can be used for parts (a) and (b) below.

a. $z = \dfrac{512 - 502}{10.54} = .95$ $P(z \le .95) = .8289$

$z = \dfrac{492 - 502}{10.54} = -.95$ $P(z < -.95) = .1711$

probability = .8289 - .1711 =.6578

b. $z = \dfrac{525-515}{10.54} = .95$ $P(z \le .95) = .8289$

$z = \dfrac{505-515}{10.54} = -.95$ $P(z < -.95) = .1711$

probability = .8289 - .1711 =.6578

The probability of being within 10 of the mean on the Mathematics portion of the test is exactly the same as the probability of being within 10 on the Critical Reading portion of the SAT. This is because the standard error is the same in both cases. The fact that the means differ does not affect the probability calculation.

c. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 100/\sqrt{100} = 10.0$ The standard error is smaller here because the sample size is larger.

$z = \dfrac{504-494}{10.0} = 1.00$ $P(z \le 1.00) = .8413$

$z = \dfrac{484-494}{10.0} = -1.00$ $P(z < -1.00) = .1587$

probability = .8413 - .1587 =.6826

The probability is larger here than it is in parts (a) and (b) because the larger sample size has made the standard error smaller.

26. a. $z = \dfrac{\bar{x}-939}{\sigma/\sqrt{n}}$

Within $\pm$ 25 means $\bar{x}$ - 939 must be between -25 and +25.

The *z* value for $\bar{x}$ - 939 = -25 is just the negative of the *z* value for $\bar{x}$ - 939 = 25. So we just show the computation of *z* for $\bar{x}$ - 939 = 25.

$n = 30$ $z = \dfrac{25}{245/\sqrt{30}} = .56$ $P(-.56 \le z \le .56) = .7123 - .2877 = .4246$

$n = 50$ $z = \dfrac{25}{245/\sqrt{50}} = .72$ $P(-.72 \le z \le .72) = .7642 - .2358 = .5284$

$n = 100$ $z = \dfrac{25}{245/\sqrt{100}} = 1.02$ $P(-1.02 \le z \le 1.02) = .8461 - .1539 = .6922$

$n = 400$ $z = \dfrac{25}{245/\sqrt{400}} = 2.04$ $P(-2.04 \le z \le 2.04) = .9793 - .0207 = .9586$

b. A larger sample increases the probability that the sample mean will be within a specified distance of the population mean. In the automobile insurance example, the probability of being within ±25 of $\mu$ ranges from .4246 for a sample of size 30 to .9586 for a sample of size 400.

27. a. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 40,000/\sqrt{40} = 6324.56$

At $\bar{x}$ = 178,000, $z = \dfrac{178,000-168,000}{6324.56} = 1.58$ $\quad P(z \leq 1.58) = .9429$

At $\bar{x}$ = 158,000, $z$ = -1.58

$P(z < -1.58) = .0571$, thus

$P(158,000 \leq \bar{x} \leq 178,000) = .9429 - .0571 = .8858$

b. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 25,000/\sqrt{40} = 3952.85$

At $\bar{x}$ = 127,000, $z = \dfrac{127,000-117,000}{3952.85} = 2.53$ $\quad P(z \leq 2.53) = .9943$

At $\bar{x}$ = 107,000, $z$ = -2.53, $P(z < -2.53) = .0057$, thus

$P(107,000 \leq \bar{x} \leq 127,000) = .9943 - .0057 = .9886$

c. In part (b) we have a higher probability of obtaining a sample mean within $10,000 of the population mean because the standard error is smaller.

d. With $n$ = 100, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 40,000/\sqrt{100} = 4000$

At $\bar{x}$ = 164,000, $z = \dfrac{164,000-168,000}{4000} = -1$

$P(\bar{x} < 164,000) = P(z < -1) = .1587$

28. a. This is a graph of a normal distribution with $E(\bar{x}) = 95$ and

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 14/\sqrt{30} = 2.56$

b. Within 3 strokes means $92 \leq \bar{x} \leq 98$

$z = \dfrac{98-95}{2.56} = 1.17$ $\qquad z = \dfrac{92-95}{2.56} = -1.17$

$P(92 \leq \bar{x} \leq 98) = P(-1.17 \leq z \leq 1.17) = .8790 - .1210 = .7580$

The probability the sample means will be within 3 strokes of the population mean of 95 is .7580.

c. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 14 / \sqrt{45} = 2.09$

Within 3 strokes means $103 \leq \bar{x} \leq 109$

$z = \dfrac{109 - 106}{2.09} = 1.44$ $\qquad$ $z = \dfrac{103 - 106}{2.09} = -1.44$

$P(103 \leq \bar{x} \leq 109) = P(-1.44 \leq z \leq 1.44) = .9251 - .0749 = .8502$

The probability the sample means will be within 3 strokes of the population mean of 106 is .8502.

d. The probability of being within 3 strokes for female golfers is higher because the sample size is larger.

29. $\mu = 2.34 \quad \sigma = .20$

a. $n = 30$

$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{.03}{.20 / \sqrt{30}} = .82$

$P(2.31 \leq \bar{x} \leq 2.37) = P(-.82 \leq z \leq .82) = .7939 - .2061 = .5878$

b. $n = 50$

$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{.03}{.20 / \sqrt{50}} = 1.06$

$P(2.31 \leq \bar{x} \leq 2.37) = P(-1.06 \leq z \leq 1.06) = .8554 - .1446 = .7108$

c. $n = 100$

$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{.03}{.20 / \sqrt{100}} = 1.50$

$P(2.31 \leq \bar{x} \leq 2.37) = P(-1.50 \leq z \leq 1.50) = .9332 - .0668 = .8664$

d. None of the sample sizes in parts (a), (b), and (c) are large enough. At $z = 1.96$ we find $P(-1.96 \leq z \leq 1.96) = .95$. So, we must find the sample size corresponding to $z = 1.96$. Solve

$$\frac{.03}{.20 / \sqrt{n}} = 1.96$$

$$\sqrt{n} = 1.96\left(\frac{.20}{.03}\right) = 13.0667$$

$$n = 170.73$$

Rounding up, we see that a sample size of 171 will be needed to ensure a probability of .95 that the sample mean will be within $\pm$ \$.03 of the population mean.

30. a. $n / N = 40 / 4000 = .01 < .05$; therefore, the finite population correction factor is not necessary.

b. With the finite population correction factor

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\frac{\sigma}{\sqrt{n}} = \sqrt{\frac{4000-40}{4000-1}}\frac{8.2}{\sqrt{40}} = 1.29$$

Without the finite population correction factor

$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 1.30$$

Including the finite population correction factor provides only a slightly different value for $\sigma_{\bar{x}}$ than when the correction factor is not used.

c. $z = \dfrac{\bar{x}-\mu}{1.30} = \dfrac{2}{1.30} = 1.54$ $\quad P(z \le 1.54) = .9382$

$P(z < -1.54) = .0618$

Probability = .9382 - .0618 = .8764

31. a. $E(\bar{p}) = p = .40$

b. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.40(.60)}{100}} = .0490$

c. Normal distribution with $E(\bar{p}) = .40$ and $\sigma_{\bar{p}} = .0490$

d. It shows the probability distribution for the sample proportion $\bar{p}$.

32. a. $E(\bar{p}) = .40$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.40(.60)}{200}} = .0346$$

Within ± .03 means $.37 \le \bar{p} \le .43$

$z = \dfrac{\bar{p}-p}{\sigma_{\bar{p}}} = \dfrac{.03}{.0346} = .87$ $\quad P(z \le .87) = .8078$

$P(z < -.87) = .1922$

$P(.37 \le \bar{p} \le .43) = .8078 - .1922 = .6156$

b. $z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.05}{.0346} = 1.44 \quad P(z \leq 1.44) = .9251$

$P(z < -1.44) = .0749$

$P(.35 \leq \bar{p} \leq .45) = .9251 - .0749 = .8502$

33. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}}$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{100}} = .0497$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{200}} = .0352$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{500}} = .0222$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{1000}} = .0157$

The standard error of the proportion, $\sigma_{\bar{p}}$, decreases as $n$ increases

34. a. $\sigma_{\bar{p}} = \sqrt{\dfrac{(.30)(.70)}{100}} = .0458$

Within ± .04 means $.26 \leq \bar{p} \leq .34$

$z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0458} = .87 \quad P(z \leq .87) = .8078$

$P(z < -.87) = .1922$

$P(.26 \leq \bar{p} \leq .34) = .8078 - .1922 = .6156$

b. $\sigma_{\bar{p}} = \sqrt{\dfrac{(.30)(.70)}{200}} = .0324$

$z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0324} = 1.23 \quad P(z \leq 1.23) = .8907$

$P(z < -1.23) = .1093$

$P(.26 \leq \bar{p} \leq .34) = .8907 - .1093 = .7814$

c. $\sigma_{\bar{p}} = \sqrt{\dfrac{(.30)(.70)}{500}} = .0205$

$z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0205} = 1.95 \quad P(z \leq 1.95) = .9744$

$P(z < -1.95) = .0256$

$P(.26 \leq \bar{p} \leq .34) = .9744 - .0256 = .9488$

d. $\sigma_{\bar{p}} = \sqrt{\dfrac{(.30)(.70)}{1000}} = .0145$

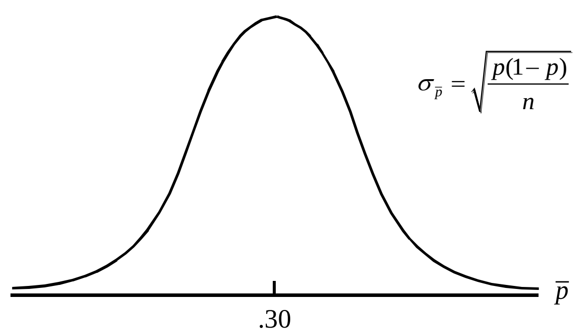
$z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0145} = 2.76 \quad P(z \leq 2.76) = .9971$

$P(z < -2.76) = .0029$

$P(.26 \leq \bar{p} \leq .34) = .9971 - .0029 = .9942$

e. With a larger sample, there is a higher probability $\bar{p}$ will be within $\pm$ .04 of the population proportion $p$.

35. a.




$\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.30(.70)}{100}} = .0458$

The normal distribution is appropriate because $np = 100(.30) = 30$ and $n(1 - p) = 100(.70) = 70$ are both greater than 5.

b. $P(.20 \leq \bar{p} \leq .40) = ?$

$z = \dfrac{.40 - .30}{.0458} = 2.18 \quad P(z \leq 2.18) = .9854$

$P(z < -2.18) = .0146$

$P(.20 \leq \bar{p} \leq .40) = .9854 - .0146 = .9708$

c. $P\,(.25 \le \bar{p} \le .35) = ?$

$$z = \frac{.35 - .30}{.0458} = 1.09 \quad P(z \le 1.09) = .8621$$

$P(z < -1.09) = .1379$

$P(.25 \le \bar{p} \le .35) = .8621 - .1379 = .7242$

36. a. This is a graph of a normal distribution with a mean of $E(\bar{p}) = .66$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.66(1-.66)}{300}} = .0273$$

b. Within ± .04 means $.62 \le \bar{p} \le .70$

$$z = \frac{.70 - .66}{.0273} = 1.47 \qquad z = \frac{.62 - .66}{.0273} = -1.47$$

$P(.62 \le \bar{p} \le .70) = P(-1.47 \le z \le 1.47) = .9292 - .0708 = .8584$

c. $$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.87(1-.87)}{300}} = .0194$$

Within ± .04 means $.83 \le \bar{p} \le .91$

$$z = \frac{.91 - .87}{.0194} = 2.06 \qquad z = \frac{.83 - .87}{.0194} = -2.06$$

$P(.83 \le \bar{p} \le .91) = P(-2.06 \le z \le 2.06) = .9803 - .0197 = .9606$

d. Yes, the probability of being within $\pm$ .04 is higher for the sample of youth users. This is because the standard error is smaller for the population proportion as it gets closer to 1.

e. For $n = 600$, $\sigma_{\bar{p}} = \sqrt{\frac{.66(1-.66)}{600}} = .0193$

Within ± .04 means $.62 \le \bar{p} \le .70$

$$z = \frac{.70 - .66}{.0193} = 2.07 \qquad z = \frac{.62 - .66}{.0193} = -2.07$$

$P(.62 \le \bar{p} \le .70) = P(-2.07 \le z \le 2.07) = .9808 - .0192 = .9616$

The probability is larger than in part (b). This is because the larger sample size has reduced the standard error.

37. a. Normal distribution

$E(\bar{p}) = .12$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.12)(1-.12)}{540}} = .0140$$

b. $z = \dfrac{\bar{p}-p}{\sigma_{\bar{p}}} = \dfrac{.03}{.0140} = 2.14$ $\quad P(z \leq 1.94) = .9838$

$P(z < -2.14) = .0162$

$P(.09 \leq \bar{p} \leq .15) = .9838 - .0162 = .9676$

c. $z = \dfrac{\bar{p}-p}{\sigma_{\bar{p}}} = \dfrac{.015}{.0140} = 1.07$ $\quad P(z \leq 1.07) = .8577$

$P(z < -1.07) = .1423$

$P(.105 \leq \bar{p} \leq .135) = .8577 - .1423 = .7154$

38. a. Normal distribution

$E(\bar{p}) = .56$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.56)(.44)}{400}} = .0248$$

b. $z = \dfrac{.02}{.0248} = .81$ $\quad P(z \leq .81) = .7910$

$P(z < -.81) = .2090$

$P(.54 \leq \bar{p} \leq .58) = .7910 - .2090 = .5820$

c. $z = \dfrac{.04}{.0248} = 1.61$ $\quad P(z \leq 1.61) = .9463$

$P(z < -1.61) = .0537$

$P(.52 \leq \bar{p} \leq .60) = .9463 - .0537 = .8926$

39. a. Normal distribution with $E(\bar{p}) = p = .75$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.75(1-.75)}{450}} = .0204$$

b. $z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0204} = 1.96$ $\quad P(z \le 1.96) = .9750$

$P(z < -1.96) = .0250$

$P(.71 \le \bar{p} \le .79) = P(-1.96 \le z \le 1.96) = .9750 - .0275 = .9500$

c. Normal distribution with $E(\bar{p}) = p = .75$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.75(1-.75)}{200}} = .0306$$

d. $z = \dfrac{\bar{p} - p}{\sqrt{\dfrac{.75(1-.75)}{200}}} = \dfrac{.04}{.0306} = 1.31$ $\quad P(z \le 1.31) = .9049$

$P(z < -1.31) = .0951$

$P(.71 \le \bar{p} \le .79) = P(-1.31 \le z \le 1.31) = .9049 - .0951 = .8098$

e. The probability of the sample proportion being within .04 of the population mean was reduced from .9500 to .8098. So there is a gain in precision by increasing the sample size from 200 to 450. If the extra cost of using the larger sample size is not too great, we should probably do so.

40. a. $E(\bar{p}) = .76$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.76(1-.76)}{400}} = .0214$$

Normal distribution because $np = 400(.76) = 304$ and $n(1 - p) = 400(.24) = 96$

b. $z = \dfrac{.79-.76}{.0214} = 1.40$ $\quad P(z \le 1.40) = .9192$

$P(z < -1.40) = .0808$

$P(.73 \le \bar{p} \le .79) = P(-1.40 \le z \le 1.40) = .9192 - .0808 = .8384$

c. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.76(1-.76)}{750}} = .0156$

$z = \dfrac{.79-.76}{.0156} = 1.92$ $\quad P(z \le 1.92) = .9726$

$P(z < -1.92) = .0274$

$P(.73 \le \bar{p} \le .79) = P(-1.92 \le z \le 1.92) = .9726 - .0274 = .9452$

41. a. $E(\bar{p}) = .17$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.17)(1-.17)}{800}} = .0133$$

Distribution is approximately normal because $np = 800(.17) = 136 > 5$ and $n(1 - p) = 800(.83) = 664 > 5$

b. $z = \dfrac{.19-.17}{.0133} = 1.51$ $\quad P(z \leq 1.51) = .9345$

$P(z < -1.51) = .0655$

$P(.15 \leq \bar{p} \leq .19) = P(-1.51 \leq z \leq 1.51) = .9345 - .0655 = .8690$

c. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{(.17)(1-.17)}{1600}} = .0094$

$z = \dfrac{.19-.17}{.0094} = 2.13$ $\quad P(z \leq 2.13) = .9834$

$P(z < -2.13) = .0166$

$P(.15 \leq \bar{p} \leq .19) = P(-2.13 \leq z \leq 2.13) = .9834 - .0166 = .9668$

42. a. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.25(.75)}{n}} = .0625$

Solve for $n$

$$n = \frac{.25(.75)}{(.0625)^2} = 48$$

b. Normal distribution with $E(\bar{p}) = .25$ and $\sigma_{\bar{p}} = .0625$

(Note: $(48)(.25) = 12 > 5$, and $(48)(.75) = 36 > 5$)

c. $P(\bar{p} \geq .30) = ?$

$z = \dfrac{.30-.25}{.0625} = .80$ $\quad P(z \leq .80) = .7881$

$P(\bar{p} \geq .30) = 1 - .7881 = .2119$

43. a. Normal distribution with $E(\bar{p}) = .15$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.15)(.85)}{150}} = .0292$$

b. $P(.12 \le \bar{p} \le .18) = ?$

$$z = \frac{.18 - .15}{.0292} = 1.03 \qquad P(z \le 1.03) = .8485$$

$P(z < -1.03) = .1515$

$P(.12 \le \bar{p} \le .18) = P(-1.03 \le z \le 1.03) = .8485 - .1515 = .6970$

44. a. $$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.40)(1-.40)}{380}} = .0251$$

Within ± .04 means $.36 \le \bar{p} \le .44$

$$z = \frac{.44 - .40}{.0251} = 1.59 \qquad z = \frac{.36 - .40}{.0251} = -1.59$$

$P(.36 \le \bar{p} \le .44) = P(-1.59 \le z \le 1.59) = .9441 - .0559 = .8882$

b. We want $P(\bar{p} \ge .45)$

$$z = \frac{\bar{p} - p}{\sigma_{\bar{p}}} = \frac{.45 - .40}{.0251} = 1.99$$

$P(\bar{p} \ge .45) = P(z \ge 1.99) = 1 - .9767 = .0233$

45. $$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.40)(.60)}{400}} = .0245$$

$P(\bar{p} \ge .375) = ?$

$$z = \frac{.375 - .40}{.0245} = -1.02 \qquad P(z < -1.02) = .1539$$

$P(\bar{p} \ge .375) = 1 - .1539 = .8461$

46. $p = .28$

a. This is the graph of a normal distribution with $E(\bar{p}) = p = .28$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.28(1-.28)}{240}} = .0290$$

b. Within ± .04 means $.24 \le \bar{p} \le .32$

$$z = \frac{.32 - .28}{.0290} = 1.38 \qquad z = \frac{.24 - .28}{.0290} = -1.38$$

$P(.24 \le \bar{p} \le .32) = P(-1.38 \le z \le 1.38) = .9162 - .0838 = .8324$

c. Within ± .02 means $.26 \le \bar{p} \le .30$

$$z = \frac{.30 - .28}{.0290} = .69 \qquad z = \frac{.26 - .28}{.0290} = -.69$$

$P(.26 \le \bar{p} \le .30) = P(-.69 \le z \le .69) = .7549 - .2451 = .5098$

47. a. $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{500}{\sqrt{n}} = 20$$

$\sqrt{n} = 500/20 = 25$ and $n = (25)^2 = 625$

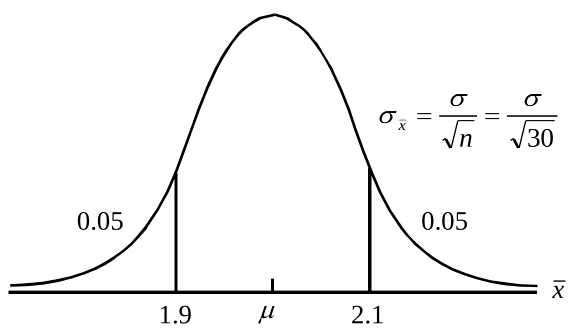
b. For ± 25,

$$z = \frac{25}{20} = 1.25 \qquad P(z \le 1.25) = .8944$$

$P(z < -1.25) = .1056$

Probability $= P(-1.25 \le z \le 1.25) = .8944 - .1056 = .7888$

48. Sampling distribution of $\bar{x}$




$$\mu = \frac{1.9 + 2.1}{2} = 2$$

The area below $\bar{x} = 2.1$ must be 1 - .05 = .95. An area of .95 in the standard normal table shows $z = 1.645$.

Thus,

$$z = \frac{2.1 - 2.0}{\sigma / \sqrt{30}} = 1.645$$

Solve for σ.

$$\sigma = \frac{(.1)\sqrt{30}}{1.645} = .33$$

49. $\mu = 27{,}175 \quad \sigma = 7400$

a. $\sigma_{\bar{x}} = 7400/\sqrt{60} = 955$

b. $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{0}{955} = 0$

$P(\bar{x} > 27{,}175) = P(z > 0) = .50$

Note: This could have been answered easily without any calculations ; 27,175 is the expected value of the sampling distribution of $\bar{x}$ .

c. $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{1000}{955} = 1.05 \qquad P(z \le 1.05) = .8531$

$P(z < -1.05) = .1469$

$P(26{,}175 \le \bar{x} \le 28{,}175) = P(-1.05 \le z \le 1.05) = .8531 - .1469 = .7062$

d. $\sigma_{\bar{x}} = 7400/\sqrt{100} = 740$

$z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{1000}{740} = 1.35 \qquad P(z \le 1.35) = .9115$

$P(z < -1.35) = .0885$

$P(26{,}175 \le \bar{x} \le 28{,}175) = P(-1.35 \le z \le 1.35) = .9115 - .0885 = .8230$

50. a. $\sigma_{\bar{x}} = \sqrt{\dfrac{N-n}{N-1}}\dfrac{\sigma}{\sqrt{n}}$

$N = 2000$

$$\sigma_{\bar{x}} = \sqrt{\frac{2000-50}{2000-1}}\frac{144}{\sqrt{50}} = 20.11$$

$N = 5000$

$$\sigma_{\bar{x}} = \sqrt{\frac{5000-50}{5000-1}}\frac{144}{\sqrt{50}} = 20.26$$

$N = 10{,}000$

$$\sigma_{\bar{x}} = \sqrt{\frac{10{,}000-50}{10{,}000-1}}\frac{144}{\sqrt{50}} = 20.31$$

Note: With $n / N \leq .05$ for all three cases, common statistical practice would be to ignore the finite population correction factor and use $\sigma_{\bar{x}} = \dfrac{144}{\sqrt{50}} = 20.36$ for each case.

b. $N = 2000$

$$z = \frac{25}{20.11} = 1.24 \qquad P(z \leq 1.24) = .8925$$

$P(z < -1.24) = .1075$

Probability = $P(-1.24 \leq z \leq 1.24) = .8925 - .1075 = .7850$

$N = 5000$

$$z = \frac{25}{20.26} = 1.23 \qquad P(z \leq 1.23) = .8907$$

$P(z < -1.23) = .1093$

Probability = $P(-1.23 \leq z \leq 1.23) = .8907 - .1093 = .7814$

$N = 10{,}000$

$$z = \frac{25}{20.31} = 1.23 \qquad P(z \leq 1.23) = .8907$$

$P(z < -1.23) = .1093$

Probability = $P(-1.23 \leq z \leq 1.23) = .8907 - .1093 = .7814$

All probabilities are approximately .78 indicating that a sample of size 50 will work well for all 3 firms.

51. a. Normal distribution because of central limit theorem ($n > 30$)

$E(\bar{x}) = 115.50$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{35}{\sqrt{40}} = 5.53$$

b. $$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{10}{35/\sqrt{40}} = 1.81 \qquad P(z \leq 1.81) = .9649$$

$P(z < -1.81) = .0351$

$P(105.50 \leq \bar{x} \leq 125.50) = P(-1.81 \leq z \leq 1.81) = .9649 - .0351 = .9298$

c. At $\bar{x} = 100$, $z = \dfrac{100 - 115.50}{35/\sqrt{40}} = -2.80$

$P(\bar{x} \leq 100) = P(z \leq -2.80) = .0026$

Yes, this is an unusually low spending group of 40 alums. The probability of spending this much or less is only .0026.

52. With $n$ = 60 the central limit theorem allows us to conclude the sampling distribution is approximately normal.

a. This means $14 \leq \bar{x} \leq 16$

At $\bar{x} = 16$, $z = \dfrac{16-15}{4/\sqrt{60}} = 1.94$ $\quad P(z \leq 1.94) = .9738$

$P(z < -1.94) = .0262$

$P(14 \leq \bar{x} \leq 16) = P(-1.94 \leq z \leq 1.94) = .9738 - .0262 = .9476$

b. This means $14.25 \leq \bar{x} \leq 15.75$

At $\bar{x} = 15.75$, $z = \dfrac{15.75-15}{4/\sqrt{60}} = 1.45$ $\quad P(z \leq 1.45) = .9265$

$P(z < -1.45) = .0735$

$P(14.25 \leq \bar{x} \leq 15.75) = P(-1.45 \leq z \leq 1.45) = .9265 - .0735 = .8530$

53. The random numbers corresponding to the first seven universities selected are

122, 99, 25, 55, 115, 102, 61

The third, fourth and fifth columns of Table 7.1 were needed to find 7 random numbers of 133 or less without duplicate numbers.

Author's note: The universities identified are: Clarkson U. (122), U. of Arizona (99), UCLA (25), U. of Maryland (55), U. of New Hampshire (115), Florida State U. (102), Clemson U. (61).

54. a. Normal distribution because $n$ = 50

$E(\bar{x}) = 6883$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2000}{\sqrt{50}} = 282.84$$

b. $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{300}{2000/\sqrt{50}} = 1.06$ $\quad P(z \leq 1.06) = .8554$

$P(z < -1.06) = .1446$

$P(6583 \leq \bar{x} \leq 7183) = P(-1.06 \leq z \leq 1.06) = .8554 - .1446 = .7108$

c. At 7500, $z = \dfrac{7500-6883}{2000/\sqrt{50}} = 2.18$

$P(\bar{x} \geq 7500) = P(z \geq 2.18) = 1 - P(z < 2.18) = 1 - .9854 = .0146$

Yes, I would question the consulting firm. A sample mean this large is unlikely if the population mean is $6883.