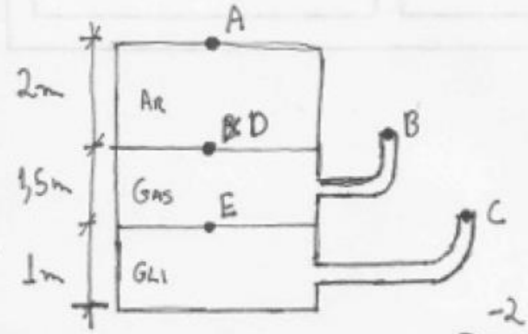
2ª Lista de Exercícios - Hidrostática

RESOLUÇÃO DOS EXERCÍCIOS

1ª QUESTÃO)



$$P_{A_m} = P_A - P_{atm} = 1{,}5\,kPa$$

$$\boxed{P_2 - P_1 = -\int_{z_1}^{z_2} \gamma\, dz} \xrightarrow[\text{FLUIDO}]{\text{NUM MESMO}} \boxed{P_2 - P_1 = -\gamma(z_2 - z_1)}$$

* DE A → D: $P_D - P_A = -\gamma_{AR}\cdot\overbrace{(z_D - z_A)}^{-2} = 2\gamma_{AR} \rightarrow P_D = P_A + 2\gamma_{AR}$

* DE D → B: $\underbrace{\cancel{P_B}}_{0} - P_D = -\gamma_{GAS}\cdot(\overbrace{z_B - z_D}^{2{,}5}) \rightarrow z_B = \dfrac{P_D}{\gamma_{GAS}} + 2{,}5 = \dfrac{P_A + 2\gamma_{AR}}{\gamma_{GAS}} + 2{,}5 \rightarrow$

à 1atm e 20°C:

$\rho_{AR} = 1{,}2\,kg/m^3$

$\rho_{GAS} = 680\,kg/m^3$

$\rho_{GLI} = 1.264\,kg/m^3$

$\gamma = \rho \cdot g$

$$\rightarrow z_B = \frac{1{,}5\cdot 10^3 + 2\cdot(1{,}2\cdot 9{,}8)}{680\cdot 9{,}8} + 2{,}5 \cong 2{,}73\,m$$

* DE D → E: $P_E - P_D = -\gamma_{GAS}\cdot\overbrace{(z_E - z_D)}^{-1{,}5} = 1{,}5\cdot\gamma_{GAS} \rightarrow P_E = P_D + 1{,}5\cdot\gamma_{GAS}$

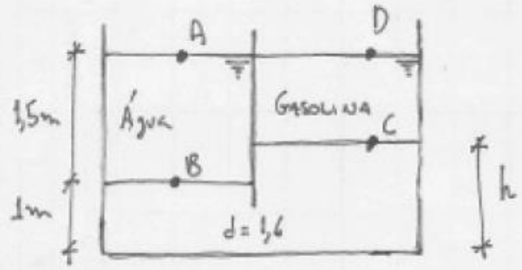
* DE E → C: $\underbrace{\cancel{P_C}}_{0} - P_E = -\gamma_{GLI}\cdot(\overbrace{z_C - z_E}^{1}) \rightarrow z_C = \dfrac{P_E}{\gamma_{GLI}} + 1 = \dfrac{P_D + 1{,}5\cdot\gamma_{GAS}}{\gamma_{GLI}} + 1 =$

$$= \frac{P_A + 2\gamma_{AR} + 1{,}5\gamma_{GAS}}{\gamma_{GLI}} + 1 \rightarrow$$

$$\rightarrow z_C = \frac{1{,}5\cdot 10^3 + 2\cdot 1{,}2\cdot 9{,}8 + 1{,}5\cdot 680\cdot 9{,}8}{1264\cdot 9{,}8} + 1 \cong 1{,}93\,m$$

2ª QUESTÃO)

$$P_2 - P_1 = -\int_{z_1}^{z_2} \gamma\, dz \xrightarrow[\text{FLUIDO}]{\text{MESMO}} P_2 - P_1 = -\gamma (z_2 - z_1)$$



$$\ast\ \underline{A \rightarrow B}: \quad P_B - \underbrace{\cancel{P_A}}_{0} = -\gamma_{Água} \cdot \overbrace{(z_B - z_A)}^{-1,5} \rightarrow P_B = 1,5\,\gamma_{Água}$$

$$\ast\ \underline{B \rightarrow C}: \quad P_C - \overbrace{P_B}^{1,5\gamma_{Água}} = -\gamma_d \cdot (\overbrace{z_C}^{h} - \overbrace{z_B}^{1}) \rightarrow P_C - P_B = -\gamma_d \cdot (h-1)$$

$$\ast\ \underline{C \rightarrow D}: \quad P_D - P_C = -\gamma_{GAS} \cdot (\underbrace{z_D}_{2,5} - \underbrace{z_C}_{h}) \rightarrow \underbrace{\cancel{P_D}}_{0} - P_C = -\gamma_{GAS} \cdot (2,5-h)$$

$$\Rightarrow +1,5\,\gamma_{Água} = +\gamma_d \cdot (h-1) + \gamma_{GAS} \cdot (2,5-h) \rightarrow h = \frac{1,5\,\gamma_{Água} + \gamma_d - 2,5 \cdot \gamma_{GAS}}{\gamma_d - \gamma_{GAS}}$$

$$\begin{cases} \gamma = \rho \cdot g \\ d = \dfrac{\rho}{\rho_{água}} \rightarrow \rho = d \cdot \rho_{água} \end{cases}$$

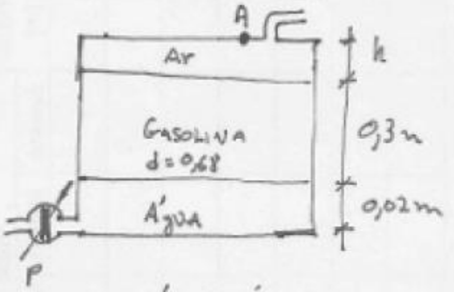
$$\Rightarrow h = \frac{1,5 \cdot 998 \cdot \cancel{g} + 1,6 \cdot 998 \cdot \cancel{g} - 2,5 \cdot 680 \cdot \cancel{g}}{1,6 \cdot 998 \cdot \cancel{g} - 680 \cdot \cancel{g}} \cong 1,52\ m$$

$$\begin{cases} \rho_{Água} = 998\ kg/m^3 \\ \rho_{GAS} = 680\ kg/m^3 \end{cases}$$

3ª QUESTÃO)

PARA VÁRIOS FLUIDOS $\rightarrow P_2 = P_1 + \sum \gamma h$

TANQUE COM ÁGUA:



(DESPREZÍVEL)

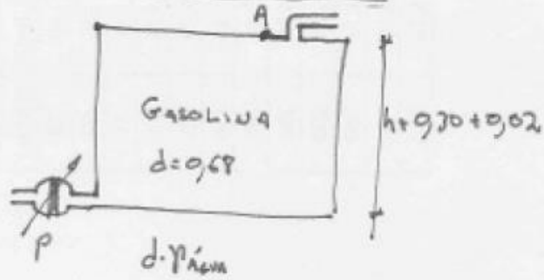
$$p = \cancel{p_A} + \cancel{\gamma_{Ar}} \cdot h + \underbrace{\gamma_{GAS}}_{d \cdot \gamma_{ÁGUA}} \cdot 0,3 + \gamma_{ÁGUA} \cdot 0,02$$

0

$$p = \gamma_{ÁGUA} \cdot (0,3d + 0,02) =$$

$= 998$

TANQUE REALMENTE CHEIO:



$$p = \cancel{p_A} + \overbrace{\gamma_{GAS}}^{d \cdot \gamma_{ÁGUA}} \cdot (h + 0,3 + 0,02)$$

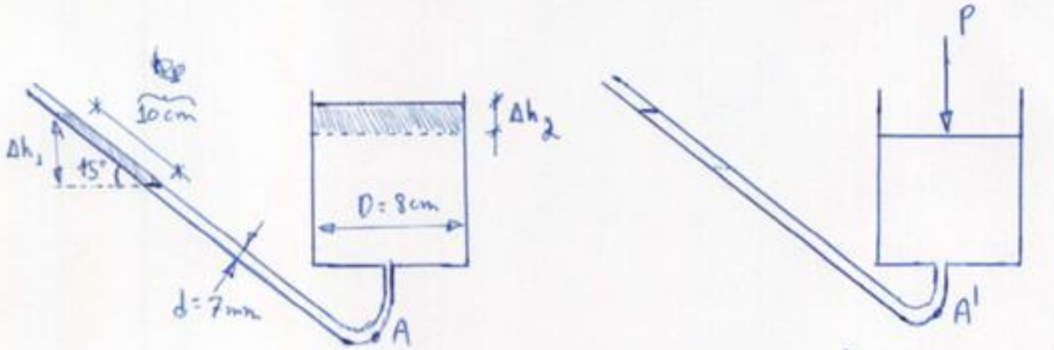
0

$$p = d \cdot \gamma_{ÁGUA} \cdot (h + 0,3 + 0,02)$$

$$\Rightarrow \cancel{\gamma_{ÁGUA}} \cdot (0,3d + 0,02) = d \cdot \cancel{\gamma_{ÁGUA}} \cdot (h + 0,3 + 0,02) \quad ; \text{ onde } d = 0,68$$

$$\rightarrow h \cong 0,0094 m = 9,4 mm$$

4ª QUESTÃO)



$$\Delta V_1 = \Delta V_2 \rightarrow 0{,}1 \cdot \frac{\cancel{\pi} d^2}{\cancel{4}} = \Delta h_2 \cdot \frac{\cancel{\pi} D^2}{\cancel{4}} \rightarrow \Delta h_2 = 0{,}1 \cdot \left(\frac{d}{D}\right)^2 = 0{,}1 \cdot \left(\frac{7}{80}\right)^2 = 0{,}766 \text{ mm}$$

$$\Delta h_1 = 0{,}1 \cdot \text{sen}\, 15° = 2{,}59 \text{ cm}$$

$$\Delta P_A = P'_A - P_A = \rho g \cdot \Delta h_1 = -\rho g \cdot \Delta h_2 + \frac{P}{\frac{\pi D^2}{4}} \rightarrow P = \frac{\pi D^2}{4} \cdot \rho g \cdot (\Delta h_1 + \Delta h_2)$$

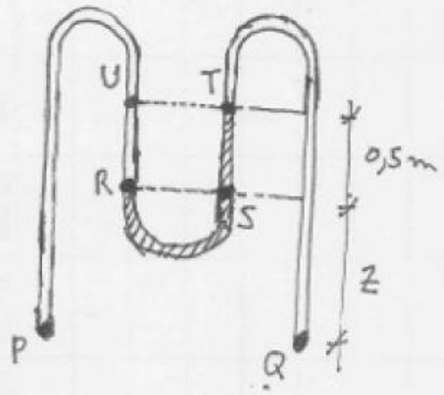
$$d = \rho / \rho_{água} \rightarrow \rho = d \cdot \rho_{água} \rightarrow P = \frac{\pi D^2}{4} \cdot d \rho_{água} \cdot g \cdot (\Delta h_1 + \Delta h_2) \rightarrow$$

$$\rightarrow P = \frac{\pi \cdot (0{,}08)^2}{4} \cdot 0{,}827 \cdot 998 \cdot 9{,}8 \cdot (2{,}59 + 0{,}0766) \cdot 10^{-2} = 1{,}08 \text{ Kg}$$

5ª QUESTÃO)

$$P_2 - P_1 = -\int \gamma\, dz \xrightarrow{\text{MESMO}} \xrightarrow[\text{FLUIDOS}]{\text{VÁRIOS}} P_2 - P_1 = -\sum \gamma\, \Delta z = \sum \gamma \overbrace{(-\Delta z)}^{h}$$

$$\Delta z: \begin{cases} \uparrow \rightarrow \Delta z > 0 \rightarrow h < 0 \\ \downarrow \rightarrow \Delta z < 0 \rightarrow h > 0 \end{cases}$$



* P→Q:

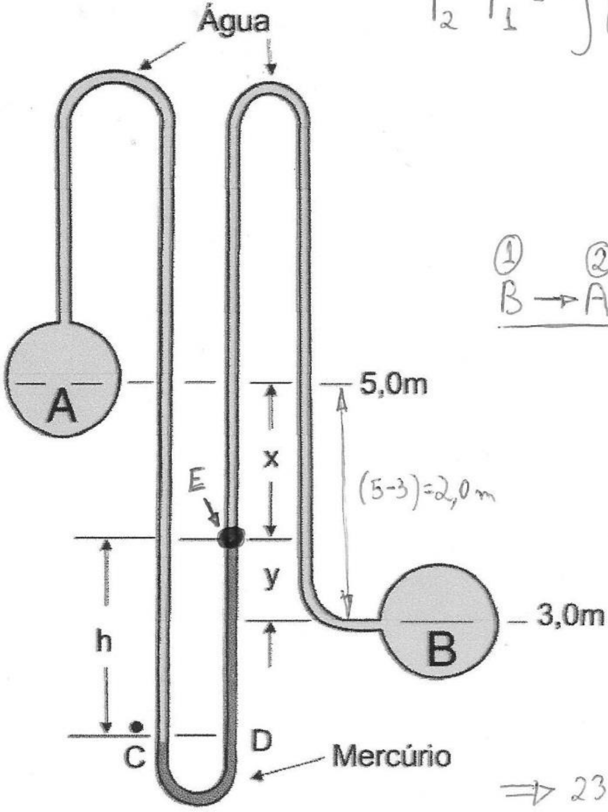
$$P_Q - P_P = \underbrace{-\cancel{\gamma_{Água}\cdot z}}_{P\to R} \overbrace{-\gamma_M \cdot 0{,}5}^{R\to T} + \underbrace{\gamma_{Água}\cdot(0{,}5+\cancel{z})}_{T\to Q} =$$

$$P_Q - P_P = -136\cdot 10^3 \cdot 0{,}5 + 10\cdot 10^3 \cdot 0{,}5 = -63\ kPa$$

$$kPa \xrightarrow{\div \gamma_{Água}} mca \rightarrow P_Q - P_P = -\frac{63}{10}\ m.c.a = -6{,}3\ m.c.a \rightarrow P_P - P_Q = 6{,}3\ mca$$

Resp.: Letra A.

6ª QUESTÃO)



$$P_2 - P_1 = -\int \gamma \, dz \xrightarrow[\text{INCOMPRESSÍVEIS}]{\text{P/ VÁRIOS FLUIDOS}} P_2 - P_1 = \sum \pm \gamma h$$

$$\begin{cases} \downarrow \Rightarrow +\gamma h \\ \uparrow \Rightarrow -\gamma h \end{cases}$$

$\underset{1}{B} \rightarrow \underset{2}{A}$:

$$P_A - P_B = \overbrace{-\gamma_a \cdot y}^{B \to E} + \overbrace{\gamma_m \cdot h}^{E \to C} - \overbrace{\gamma_a \cdot (h + x)}^{C \to A} =$$

$$P_A - P_B = -\gamma_a \cdot \underbrace{(y + x)}_{= (5-3)} + h \cdot (\gamma_m - \gamma_a) =$$

$$\underbrace{P_A}_{300\cdot 10^3} - \underbrace{P_B}_{68\cdot 10^3} = -2\gamma_a + h(\gamma_m - \gamma_a) \Rightarrow$$

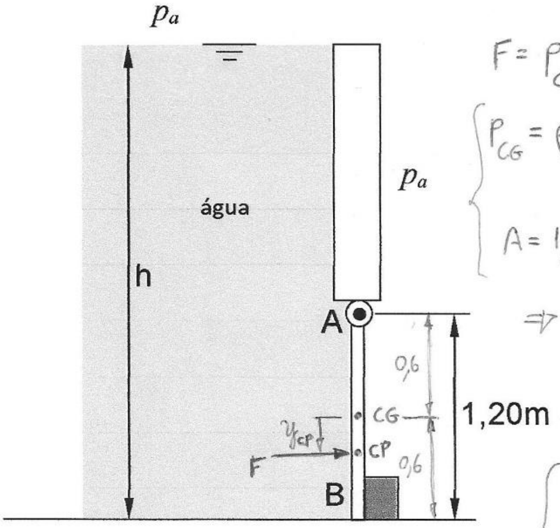
$\gamma_a = \rho_a \cdot g = 10^4$

$\gamma_m = \rho_m \cdot g = 13{,}6 \cdot 10^4$

$$\Rightarrow 232 \cdot \cancel{10^3} = -2 \cdot 10 \cdot \cancel{10^3} + h \cdot (136 - 10) \cdot \cancel{10^3} \Rightarrow$$

$$h = \frac{252}{126} = 2\,m$$

7ª QUESTÃO)



$$F = P_{CG} \cdot A$$

$$P_{CG} = \rho g \cdot h_{CG} = \rho g \cdot (h - 0{,}6) = 1000 \cdot 10 \cdot \overbrace{(2{,}85 - 0{,}6)}^{2{,}25} = 22{,}5\ kPa$$

$$A = 1{,}2 \times 1{,}5 = 1{,}8\ m^2$$

$$\Rightarrow F = 22{,}5 \cdot 10^3 \cdot 1{,}8 = 40{,}5\ kN$$

$$y_{cp} = -\gamma \cdot sen\theta \cdot \frac{I_{xx}}{F}$$

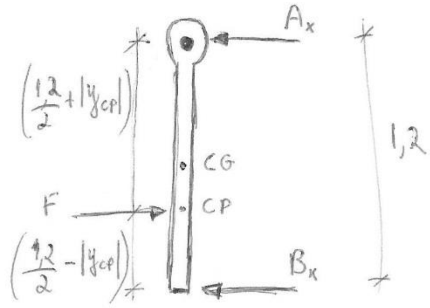
$$\theta = 90^\circ \rightarrow sen\theta = 1$$

$$I_{xx} = \frac{1{,}5 \cdot (1{,}2)^3}{12} = 0{,}216\ m^4$$

$$\gamma = \rho g = 1000 \cdot 10 = 10 \cdot 10^3 \frac{N}{m^3} = 10\ kN/m^3$$

$$\Rightarrow$$

$$\Rightarrow y_{cp} = -10 \cdot \cancel{10^3} \cdot 1 \cdot \frac{0{,}216}{40{,}5 \cdot \cancel{10^3}} \cong -5{,}33\ cm$$

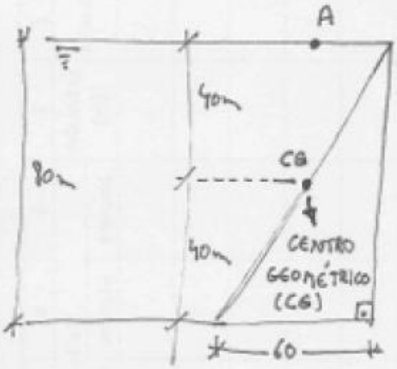


$$\sum M_A = 0 \rightarrow +F \cdot (0{,}6 + 0{,}0533) - B_x \cdot 1{,}2 = 0$$

$$\rightarrow B_x = \frac{F \cdot 0{,}6533}{1{,}2} = \frac{40{,}5 \cdot 0{,}6533}{1{,}2} \cdot 10^3 \cong 22{,}05\ kN \cong 22{,}0\ kN$$

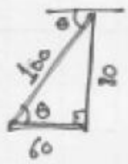
$$\sum F_x = 0 \Rightarrow -A_x + F - B_x = 0 \rightarrow A_x = F - B_x = (40{,}5 - 22{,}0) \cdot 10^3 = 18{,}5\ kN$$

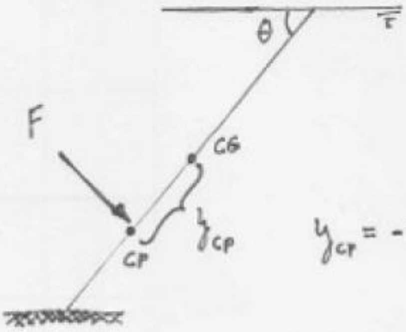
8ª QUESTÃO)



* $\underline{A \rightarrow CG}$: $P_{CG} = \cancel{P_A}^{\,0} + \gamma_{ÁGUA} \cdot 40 = 998 \cdot 9,8 \cdot 40 \cong 391,2\ kPa$

$$F = P_{CG} \cdot A = 391,2 \cdot 10^3 \cdot 30 \cdot 100 \cong 1,17 \cdot 10^9\ N$$

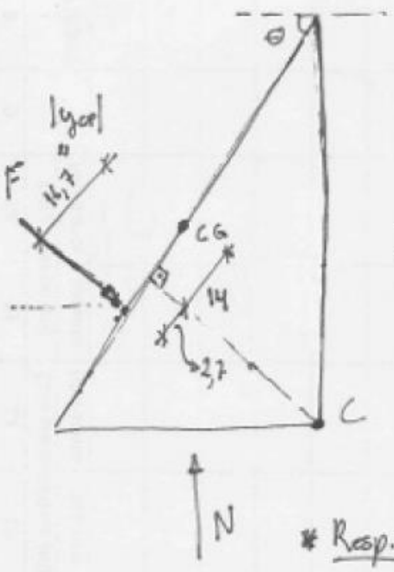




$$y_{cp} = -\gamma \operatorname{sen}\theta \frac{I_{xx}}{P_{CG} \cdot A} \qquad I_{xx} = \frac{bh^3}{12}$$

$$y_{cp} = -998 \cdot 9,8 \cdot \overbrace{\left(\frac{80}{100}\right)}^{\operatorname{sen}\theta} \cdot \frac{30 \cdot 100^3}{391,2 \cdot 10^3 \cdot 30 \cdot 100 \cdot 12}$$

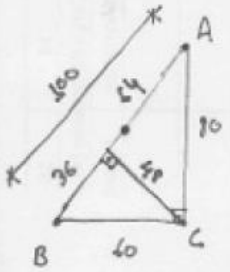
$$y_{cp} \cong -16,7\ m$$



$$P = \gamma \cdot V = d \cdot \gamma_{Água} \cdot V = 2,4 \cdot \overbrace{998 \cdot 9,8}^{\gamma_{Água}} \cdot \overbrace{\overbrace{\frac{60 \cdot 80}{2}}^{A} \cdot \overbrace{30}^{L}}^{V} \cong 1,69 \cdot 10^9\ N$$

$$\Sigma F_y = 0 \rightarrow N =$$

$$M_{F_o} = +F \cdot d_o = +1,17 \cdot 10^9 \cdot 2,7 \cong +3,16 \cdot 10^9\ N \cdot m$$

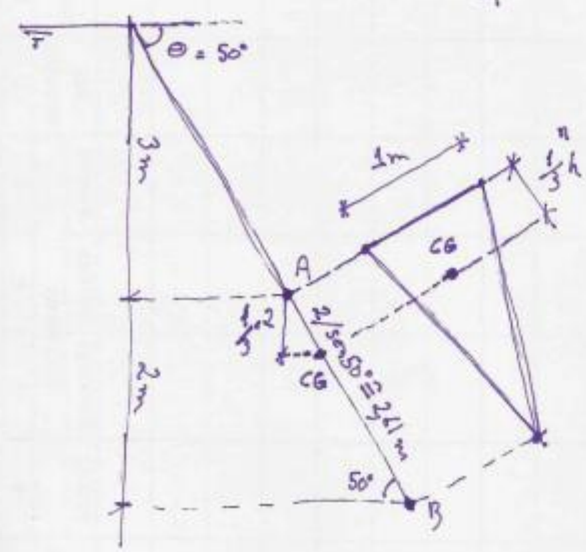


* Resp.: ESTA FORÇA NÃO PODERIA TOMBAR A BARRAGEM, POIS SEU MOMENTO É POSITIVO EM RELAÇÃO AO PONTO C, PONTO DE APOIO NO TOMBAMENTO.

9ª QUESTÃO)

9ª Questão

$$p_2 - p_1 = -\int_{z_1}^{z_2} \gamma\, dz = -\gamma \Delta z = \gamma h \rightarrow p_2 = p_1 + \gamma h$$



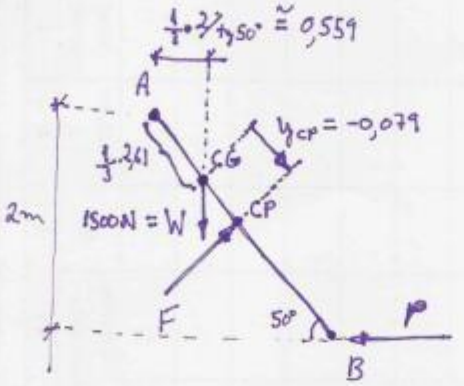
$$p_{CG} = 0 + \gamma \cdot h_{CG} = 998 \cdot 9{,}81 \cdot \overbrace{\left(3 + \frac{1}{3}\cdot 2\right)}^{h_{CG}}$$

$$p_{CG} \cong 35{,}9 \text{ kPa}$$

$$F = p_{CG}\cdot A = 35{,}9\cdot 10^3 \cdot \overbrace{\left(\frac{1\cdot 2{,}61}{2}\right)}^{A} \cong 46{,}8 \text{ kN}$$

$$y_{CP} = -\gamma \cdot \text{sen}\,\theta \, \frac{I_{xx}}{p_{CG}\cdot A} =, \qquad I_{xx} = \frac{bh^3}{36}$$

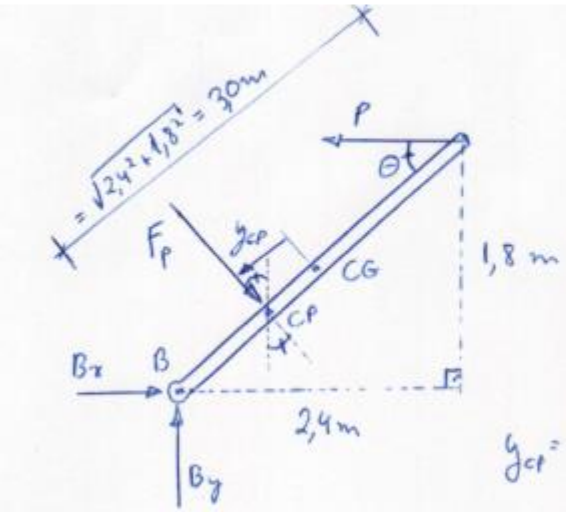
$$y_{CP} = -998 \cdot 9{,}81 \cdot \text{sen}\,50° \cdot \frac{\frac{1\cdot(2{,}61)^3}{36}}{35{,}9\cdot 10^3 \cdot \frac{1\cdot 2{,}61}{2}} \cong -0{,}079 \text{ m}$$



$$\sum M_A = 0 \rightarrow -W\cdot 0{,}559 + F\cdot \overbrace{\left(\frac{1}{3}\cdot 2{,}61 + 0{,}079\right)}^{0{,}949} - P\cdot 2 = 0$$

$$\rightarrow P = \frac{-1500\cdot 0{,}559 + 46{,}8\cdot 10^3\cdot 0{,}949}{2} \cong 21{,}8 \text{ kN}$$

10ª QUESTÃO)



$$F_P = P_{CG}\cdot A = (\rho g h_{CG} + \cancel{P_{atm}})\times A \quad \rightarrow 0$$

$$h_{CG} = 4,6 - 0,9 = 3,7\ m$$

$$F_P = 1025\times 9,8\times 3,7\times(3\times 1,5) = 167\ KN$$

$$y_{cp} = -\gamma\ sen\,\theta\cdot\frac{I_{xx}}{F_P} = -\cancel{1025\times 9,8}\times\left(\frac{1,8}{3}\right)\times\frac{\frac{1,5\times 3^3}{12}}{\cancel{1025\times 9,8}\times 16,65} =$$

$$y_{cp} = -12,2\ cm$$

$$* \sum M_B = 0 \rightarrow P\times 1,8 = F_P\times(1,5 - |y_{cp}|) \rightarrow P = \frac{167\cdot 10^3}{1,8}\times(1,5 - 0,122) = 127,8\ KN$$

$$* \sum F_x = 0 \rightarrow B_x + F_P\cdot sen\,\theta = P \rightarrow B_x = 127\cdot 10^3 - 167\cdot 10^3\cdot\frac{1,8}{3} = -27\ KN$$

$$* \sum F_y = 0 \rightarrow B_y = F_P\cdot\cos\theta = 167\cdot 10^3\cdot\frac{2,4}{3} = 134\ KN$$

11ª QUESTÃO)

a)

Aplicando

$$p_2 - p_1 = -\int_{z_1}^{z_2} \gamma dz$$

E adotando $p_1 = p_{atm}$ e $z_1 = 0m$ (nível do mar), então $z_2 = -2180m$ e

$$p_2 - p_1 = p_2 - p_{atm} = -\int_{z_1}^{z_2} \gamma dz = -\int_{z_1}^{z_2} \rho g dz = -g\int_{z_1}^{z_2} (-0{,}005275z + 1026)dz = g\int_{z_1}^{z_2} 0{,}005275zdz - g\int_{z_1}^{z_2} 1026dz$$

considerando g = 9,8 m/s²

$$= 9{,}8 \cdot 0{,}005275\frac{z^2}{2}\bigg|_{z_1}^{z_2} - 9{,}8 \cdot g1026z\Big|_{z_1}^{z_2} = 0{,}05169\frac{(-2180)^2}{2} - 10054(-2180) = -22{,}04MPa$$

$$\rightarrow p_2 = p_{atm} - 22{,}04MPa = 0 - 22{,}04MPa = 22{,}04MPa$$

b)

desprezando a variação da massa específica com a profundidade $\rho = \rho(0) = 1026$ kg/m³ e

$$p_2 - p_1 = \gamma h = \rho gh \rightarrow p_2 = \rho gh + p_1 = 1026 \cdot 9{,}8 \cdot 2180 + 0 = 21{,}92\,MPa$$

Erro = (21,92-22,04) / 22,04 = -0,544 %

c)

A força resultante da pressão em tubulações em curva de 90° é dada por

$$F_p = \sqrt{2}(p_i A_i - p_e A_e) = \frac{\sqrt{2}\pi}{4}(p_i D_i^2 - p_e D_e^2).$$

$$\left.\begin{aligned} D_e &= 18 \cdot 0{,}0254 = 0{,}4572m \\ D_i &= (18 - 2 \cdot 1{,}125) \cdot 0{,}0254 = 0{,}4000m \end{aligned}\right\} \Rightarrow F_p = \frac{\sqrt{2}\pi}{4}\left[33{,}0 \cdot (0{,}4)^2 - 22{,}04 \cdot (0{,}4572)^2\right]10^6 = 747{,}4kN$$