Prova calculo 2A universidade federal fluminense gabarito

Cálculo

Universidade Federal Fluminense (UFF)

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GABARITO da P2 de Cálculo 2-A (2016-2, turmas B1 e F1, Prof. Toscano)

1) <u>1,3 ponto</u> $=$ $13 \times 0{,}1$

$$\lambda(4xy^2 + 1/x^3)\,dx + \lambda 2x^2y\,dy = 0 \quad \Rightarrow \quad \frac{\partial}{\partial y}\Big[\lambda(4xy^2 + 1/x^3)\Big] = \frac{\partial}{\partial x}\Big[\lambda 2x^2 y\Big] .$$

Se $\lambda(x)$: $\quad \lambda' 2x^2y + \lambda 4xy = \lambda 8xy \quad \Rightarrow \quad \lambda' + \dfrac{-4xy}{2x^2y}\lambda(x) = 0 \quad \Rightarrow \quad \lambda' - \dfrac{2}{x}\lambda(x) = 0 .$

$$f = e^{-\int \frac{2}{x}dx} = e^{-2\ln x} = x^{-2} \Rightarrow \quad \lambda = c/f = cx^2 .$$

$$dU = (4x^3y^2 + 1/x)\,dx + 2x^4y\,dy = 0 .$$

$$\left.\begin{array}{l} U_y = 2x^4y \quad \Rightarrow \quad U(x,y) = x^4y^2 + B(x) \\ \therefore U_x = 4x^3y^2 + 1/x = 4x^3 + B'(x) \end{array}\right\rangle \quad \Rightarrow \quad B'(x) = \frac{1}{x} \quad \Rightarrow \quad B(x) = \ln x + c_1 .$$

<u>Resposta</u>: $x^4y^2 + \ln x = const.$

2) <u>0,9 ponto</u> $=$ $9 \times 0{,}1$

$$r^3(r^2 + 2r + 2)(r-3) = 0 \Rightarrow \begin{cases} r^3(r-3) = 0 \Rightarrow r = 0 \text{ (tripla) ou } 3 \\ r^2 + 2r + 2 \Rightarrow r = \dfrac{-2 \pm \sqrt{4-8}}{2} = \dfrac{-2 \pm 2\mathrm{i}}{2} = -1 \pm \mathrm{i} \end{cases}$$

<u>Resposta</u>:

$$y(x) = c_1 + c_2x + c_3x^2 + c_4e^{3x} + e^{-x}(c_5\cos x + c_6 \operatorname{sen} x) .$$

3) <u>1,0 ponto</u> $=$ $10 \times 0{,}1$

$$\{x,1\} \times \{e^{-x}\} \cup \{\operatorname{sen} x, \cos x\} = \underbrace{\{xe^{-x},\, e^{-x}\}}_{\substack{\searrow \cdot x^2 \\ \{x^3e^{-x},\, x^2e^{-x}\}}} \cup \underbrace{\{\operatorname{sen} x,\, \cos x\}}_{\substack{\searrow \cdot x \\ \{x \operatorname{sen} x,\, x\cos x\}}} = \{x^3e^{-x},\, x^2e^{-x},\, x\operatorname{sen} x,\, x\cos x\} .$$

<u>Resposta</u>: $y_P(x) = Ax^3e^{-x} + Bx^2e^{-x} + Cx\operatorname{sen} x + Dx\cos x$.

4) 1,3 ponto $=$ $13 \times 0{,}1$

$$t = \ln x \; (\Leftrightarrow x = e^t) \quad \Rightarrow \quad y'' - y' + y' + y(t) = y'' + y(t) = 3 - 4e^t .$$

$$r^2 + 1 = 0 \Rightarrow r = \pm \mathrm{i} \Rightarrow y_H(t) = c_1\cos t + c_2 \operatorname{sen} t \Rightarrow \quad \boxed{y_H(x) = c_1 \cos \ln x + c_2 \operatorname{sen} \ln x} .$$

$$y_P(t) = A + Be^t \Rightarrow \quad 2Be^t + A = 3 - 4e^t \quad \Rightarrow \quad A = 3 \text{ e } B = -2 .$$

$$y_P(t) = 3 - 2e^t \quad \Rightarrow \quad \boxed{y_P(x) = 3 - 2x} .$$

<u>Resposta</u>: $y(x) = y_H(x) + y_P(x)$.

5) <u>1,3 ponto</u> $=$ $13 \times 0{,}1$

$$x^2 y'' + x y' - y(x) = 2x^2\sqrt{x} \quad \overset{t=\ln x}{\Rightarrow} \quad y'' - y' + y' - y(t) = y'' - y(t) = 2e^{5t/2} \quad .$$

$$r^2 - 1 = 0 \quad \Rightarrow \quad r = \pm 1 \quad \Rightarrow \quad y_H(t) = c_1 e^t + c_2 e^{-t} \quad \Rightarrow \quad \boxed{y_H(x) = c_1 x + c_2 x^{-1}} \quad .$$

$$\boxed{y_P(x) = A(x)\,x + B(x)\,x^{-1}} \quad .$$

$$\begin{cases} A'x + B'x^{-1} = 0 \\ A' - B'x^{-2} = 2\sqrt{x} \end{cases} \Rightarrow \quad A' = \sqrt{x} \quad \Rightarrow \quad \boxed{A(x) = \frac{2}{3}x^{3/2}} \quad .$$

$$B' = -A'x^2 = -x^{5/2} \quad \Rightarrow \quad \boxed{B(x) = -\frac{2}{7}x^{7/2}} \quad . \qquad (**)$$

<u>Resposta</u>: $y(x) = y_H(x) + y_P(x) = c_1 x + c_2 x^{-1} + \dfrac{8}{21}x^{5/2}$.

$(**)$ Ou equivalentemente:

$$y_P = A(t)\,e^t + B(t)\,e^{-t} = A(x)\,x + B(x)\,x^{-1} \quad .$$

$$\begin{cases} A'e^t + B'e^{-t} = 0 \\ A'e^t - B'e^{-t} = 2e^{5t/2} \end{cases} \Rightarrow \quad A'e^t = e^{5t/2} \quad \Rightarrow \quad A' = e^{3t/2} \quad \Rightarrow \quad \boxed{A = \frac{2}{3}e^{3t/2} = \frac{2}{3}x^{3/2}} \quad .$$

$$B' = -A'e^{2t} = -e^{7t/2} \quad \Rightarrow \quad \boxed{B(x) = -\frac{2}{7}e^{7t/2} = -\frac{2}{7}x^{7/2}} \quad .$$

6) <u>1,4 ponto</u> $=$ $14 \times 0{,}1$

$$y = v\,e^x \quad \Rightarrow \quad x\,(v'' + 2v' + v) + (1-2x)(v' + v) + (x-1)\,v = 2x \quad .$$

$$x\,v'' + (\underbrace{2x + 1 - 2x}_{1})\,v' + (\underbrace{2 + 1 - 2x + x - 1}_{0})\,v = 2x \quad .$$

$$(v')' + \frac{1}{x}(v') = 2 \quad \Rightarrow \quad f = e^{\int \frac{1}{x}dx} = e^{\ln x} = x \quad .$$

$$\left(v'x\right)' = 2x \quad \Rightarrow \quad v'x = x^2 + c_1 \quad \Rightarrow \quad v' = x + \frac{c_1}{x} \quad \Rightarrow \quad \boxed{v(x) = \frac{x^2}{2} + c_1 \ln x + c_2} \quad .$$

<u>Resposta</u>: $y(x) = v(x)\,e^x$.

7) <u>0,8 ponto</u> $=$ $8 \times 0{,}1$

$$\int 2y\,dy = \int \frac{4x^3}{1+x^4}dx \quad \Rightarrow \quad y^2 = \ln(1+x^4) + c \quad \xrightarrow{x=0 \;\text{ e }\; y=-1} \quad 1 = 0 + c \quad .$$

<u>Resposta</u>: $y(x) = -\sqrt{1 + \ln(1+x^4)}$.