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THEORY OF FERROMAGNETIC HYSTERESIS †

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A mathematical model of the hysteresis mechanism in ferromagnets is presented. This is based on existing ideas of domain wall motion including both bending and translation. The anhysteretic magnetization curve is derived using a mean field approach in which the magnetization of any domain is coupled to the magnetic field H and the bulk magnetization M. The anhysteretic emerges as the magnetization which would be achieved in the absence of domain wall pinning. Hysteresis is then included by considering the effects of pinning of magnetic domain walls on defect sites. This gives rise to a frictional force opposing the movement of domain walls. The impedance to motion is expressed via a single parameter k, leading to a simple model equation of state. This exhibits all of the main features of hysteresis such as the initial magnetization curve, saturation of magnetization, coercivity, remanence, and hysteresis loss.

1. Introduction

In the majority of applications of ferromagnetic materials the magnetic properties are most conveniently expressed as magnetization curves or families of hysteresis loops. The absence of an adequate quantitative model of the behavior of these materials has been a serious disadvantage both in understanding the processes involved and in describing the variation of magnetization as a function of other parameters such as stress or temperature. For this reason, there has been, until recently, no reliable theory of the magnetomechanical effect [1].

Although there is no totally general form of hysteresis loop for ferromagnetic materials, there does exist a shape of hysteresis loop which occurs frequently in practice. This is called the 'sigmoid' and has been discussed by Craik and Tebble [2]. Its general form is shown in fig. 1. It is the equation of hysteresis loops of this form which the present work is concerned with.

1.1. Early attempts to explain hysteresis

Early investigators in the field of magnetism considered several possible explanations for the phenomenon of ferromagnetic hystersis. These hypotheses fell broadly into two categories, one of which suggested that a frictional type force was responsible and the other which considered hysteresis as due entirely to the strong mutual interac-

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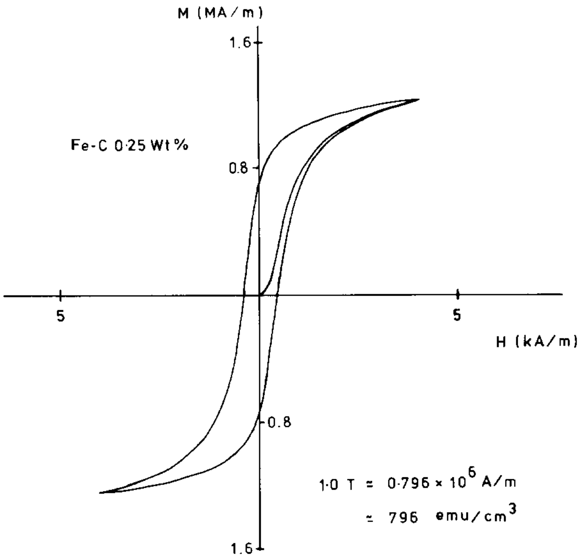

Fig. 1. General form of sigmoid shaped hysteresis loop for a specimen of iron containing 0.25 wt% carbon mostly in the form of iron carbide (Fe_3C) inclusions in lamellae. The specimen was obtained by air cooling from 820°C.

tions between the individual magnetic moments. The original suggestion of a restoring force which tried to maintain the moments in their initial unmagnetized state was due to Weber [3]. However, although this would be able to explain the shape of the initial magnetization curve, it would be unable to explain residual magnetization once the applied field was removed. Wiedemann [4] postulated the existence of a frictional resistance to rotation of the "magnetic molecules", as he called them, an idea which lends quite well to account for the most obvious effects of magnetic hysteresis.

Maxwell [5] appears to have been the first to suggest that hysteresis could be explained in terms of the mutual interactions of an array of magnetic moments. This idea was pursued further by Ewing [6] who believed that the mutual magnetic interactions could entirely account for the phenomenon. Ewing's concept was supported by some simple calculations which showed that the essential features of hysteresis would be obtained from a small number of strongly interacting moments. Ewing's influence was such that the concept of mutual interactions being totally responsible for hysteresis was accepted readily and Wiedemann's idea of the "frictional force" was discarded. It will be shown later that in fact an elementary form of hysteresis loop can be obtained simply on the basis of strongly interacting moments, fig. 2, but that the form of this loop is somewhat different from the experimentally observed loops. In particular, sides of the loop have infinite slope givin a very square loop with rapid "switching" of the magnetization between the two extreme states.

In practice, most hysteresis loops show a fairly smooth change in magnetization with field (some notable exceptions with square loops do occur however) and it is believed that this is due to a frictional force of the type envisaged by Wiedemann which opposes the changes in magnetization. The frictional force is due to pinning of domain walls by defect sites inside the solid, which causes an opposing force to resist any changes in magnetization.

1.2. Modelling and curve fitting of hysteresis

In the past there have been many attempts to fit equations to actual magnetization data, however no single satisfactory equation has been developed to describe the processes involved in this. Consequently, attempts to describe the behavior of ferromagnets have always been handicapped by their restriction to only narrow ranges of field. According to the review by Cullity [7] in only three instances have algebraic expressions been obtained for the curves. These are, high field magnetization curves of single crystals, as in the work of Williams [8], high field magnetization curves of polycrystals which are governed by the law of approach to saturation as indicated by Chikazumi [9] and low field magnetization curves and hysteresis loops of polycrystalline specimens which exhibit Rayleigh loops [10].

The recurring problem in previous attempts to obtain a hysteresis function has been that either an extremely complicated mathematical function could be used to describe the behavior to an arbitrary level of accuracy but with little or no theoretical basis, or alternatively, a simple function, obtained from first principles, could be used which, although it has a good theoretical foundation did not characterize real materials with sufficient accuracy.

The usual approach to the empirical curve fitting method has been the use of power series as in the work of Brauer [11] or the use of rational polynominals as in the work of Fischer and Moser [12], Trutt [13] and Widger [14]. A more recent application of the rational function method has been made by Rivas, Zamarro, Martin and Pereira [15] in which families of major hysteresis loops have been generated and the results have shown good agreement with experimental data. It is noticeable that Rivas et al. used two generating functions which were added to give the correct behavior. These functions bear a marked resemblance to the two terms of the equation derived on the basis of the present model, the anhysteretic function and its derivative. Although Rivas et al. [15] had no theoretical justification for summing two such functions, the present work indicates why such an approach is correct.

The theoretical approach to the problem of hysteresis has involved generally one of two methods. Calculations based on the Preisach–Néel model [16–18], two of the most recent of which have been by Del Vecchio [19] and by Rahman, Poloujadoff, Jackson, Perrard and Gowda [20]. However a serious drawback of the Preisach model is its aribitrary nature. The other method, which is theoretically sounder, is based on the micromagnetics theory of Brown [21] and Aharoni [22]. However this method does not yield a simple equation of state for a ferromagnet which was the objective of the present work and consequently this method will not be considered further.

Globus [23] and Globus and Duplex [24–26] developed a model of domain wall motion which was able to explain qualitatively the general shape of hysteresis curves of ferromagnetic and ferrimagnetic materials. Their model assumed that domain walls were pinned only on grain boundaries by a type of frictional force. Under the action of a field the domain walls underwent first a reversible motion due to domain wall bulging and later an irreversible motion due to domain wall displacement. These ideas are very much in accord with the model presented here, except that we believe pinning of domain walls on grain boundaries only is too restricted a concept. Domain walls are also pinned by inhomogeneities within a grain, for example tangles of dislocations, regions of inhomogeneous strain and any precipitates or nonmagnetic inclusions within a grain. In addition the locations at which the domain walls are planar we believe are on the anhysteretic magnetization curve, not at remanence as suggsted by Globus. For materials with high anhysteretic susceptibility, of course, these locations will be very close to the remanence point on the M, H plane.

More recently the problem of modelling magnetization curves of ferromagnets has received attention from Porteseil and Vergne [27] who studied the magnetization process from consideration of Bloch wall motion in polycrystalline ferromagnets and the interactions of these walls with structural defects. They then calculated the magnetization curves. However this model only took into account irreversible domain wall motion in the low field region, it did not fully consider reversible domain wall motion. Further experimental investigations were undertaken by Astie, Degauque, Porteseil and Vergne [28], Degauque and Astie [29] and Astie, Degauque, Porteseil and Vergne [30]. In one of these papers [29] the authors state that in low magnetic fields the magnetization is due to the displacement of the magnetic domain walls. Although it is true that at low fields domain wall motion dominates the magnetization process compared with domain rotation, we consider, in accordance with Globus, that domain wall bulging (a reversible process) and domain wall displacement (an irreversible process) are both important in this region and that an irreversible mechanism alone is unrealistic.

The present paper describes a model of hysteresis which generates the familiar sigmoid-shaped hysteresis loops by considering impedences to domain wall motion caused by pinning sites encountered by the domain walls as they move. The existence of such pinning sites was first suggested by Kersten [31,32] and by Becker and Döring [33]. For the purposes of the model no distinction has been made between the different types of pinning sites. A mean pinning energy per site is used and the pinning sites are assumed to be uniformly distributed throughout the solid. Such assumptions allow the hysteresis equation to take its simplest form. The model applies at present to isotropic ferromagnets only, and so is applicable to polycrystalline materials or crystals with low anisotropy. However extension of the equations, in particular eq. (6), to include anisotropy has been achieved by Furlani [34].

2. Anhysteretic magnetization

2.1. Coupling of domain magnetization to the magnetic field H

Consider the energy per unit volume of a typical domain with magnetic moment per unit volume m in a magnetic field H. By this H is meant the actual internal magnetic field experienced by the domain within the solid, and not the applied field. If there is no preferred direction, that is if the solid is polycrystalline and behaves isotropically, then,

$$E = -\mu_0 \boldsymbol{m} \cdot \boldsymbol{H}. \tag{1}$$

2.2. Coupling of domain magnetization to the bulk magnetization M

Inside a ferromagnetic solid there will be coupling between the domains. This may be expressed in the simplest terms as a coupling to the bulk magnetization

$$E = -\mu_0 \boldsymbol{m} \cdot (\boldsymbol{H} + \alpha \boldsymbol{M}), \tag{2}$$

where α is a mean field parameter representing interdomain coupling, which has to be determined experimentally. The energy per unit volume may then be expressed as

$$E = -\mu_0 \boldsymbol{m} \cdot \boldsymbol{H}_e, \tag{3}$$

where $\boldsymbol{H}_e = \boldsymbol{H} + \alpha \boldsymbol{M}$ is an effective field, and is analogous to the Weiss mean field experienced by the individual magnetic moments within a domain. Such an effective field for domains has been used by Callen, Liu and Cullen [35].

The response of the magnetization to this effective field may in the case of an isotropic material be expressed as,

$$M = M_s f(H_e), \tag{4}$$

where f is an arbitrary function of the effective field which takes the value zero when H_e is zero and takes the value unity as H_e tends to infinity. M_s is the saturation magnetization.

This expression for the magnetization so far only takes into account the response to the magnetic field and some averaged interaction with the magnetization of the rest of the solid included in the form of the mean field term αM. It represents only a statistical distribution of domains which corresponds to an optimum energy state without taking into account any features relating to the structure of the material such as impurity sites or nonmagnetic inclusions.

This expression can only be used to model the magnetization state of a ferromagnet at its global equilibrium state. This applies only in the case of an ideal or perfect solid in which there are no impedences to the changes in magnetization, such as pinning of domain wall motion. In a real solid it applies to the anhysteretic or ideal magnetization curve, along which the domain walls achieve positions of true equilibrium under the prevailing value of the field H, as described by Tebble and Craik [2]. A method of obtaining the anhysteretic magnetization by superimposing a decaying ac field on the steady dc field H has been described by Bozorth [38].

Therefore we may write

$$M_{an}(H_e) = M_s f(H_e), \tag{5}$$

where M_{an} is now the anhysteretic magnetization. f is the arbitrary function of field and M_s the saturation magnetization.

2.3. Equation for the anhysteretic magnetization

For purposes of modelling the anhysteretic magnetization we have chosen a modified Langevin expression $L(H_e)$ [36] as the arbitrary function $M_s f(H_e)$. This therefore leads to an expression for the anhysteretic magnetization [37]

$$M_{an}(H_e) = M_s(\coth(H_e/a) - (a/H_e)), \tag{6}$$

where a is a parameter with dimensions of mag-

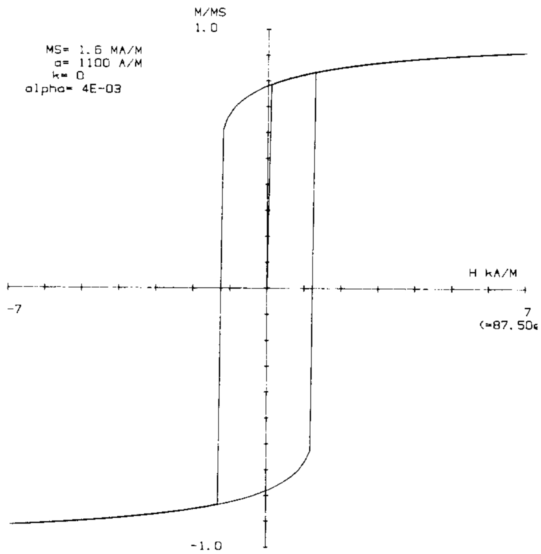

Fig. 2. Elementary form of hysteresis loop with switching from extreme magnetization states. The curves were obtained as the solution of eq. (6) with the following values of the parameters: $M_s = 1.6 \times 10^6$ A/m, $a = 1100$ A/m, $\alpha = 4 \times 10^{-3}$.

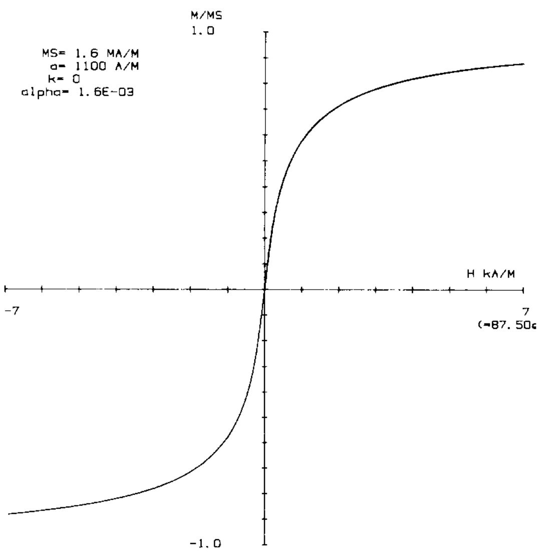

Fig. 3. Solution of the anhysteretic magnetization eq. (6) with the following values of the parameters: $M_s = 1.6 \times 10^6$ A/m, $a = 1100$ A/m, $\alpha = 1.6 \times 10^{-3}$.

netic field which characterizes the shape of the anhysteretic magnetization.

The modified Langevin eq. (6) can give rise to an elementary form of hysteresis loop if the coefficient α is sufficiently large. This has been referred to in the introduction and is the type of hysteresis that Ewing [6] found from his calculations based solely on strong mutual interactions between the magnetic moments. An example of the solution of eq. (6) showing hysteresis is given in fig. 2. However, for values of the coefficient α which have been found in practice for samples used in the present work, the general form of the solutions to eq. (6) was as shown in fig. 3. This represents the continuous, single-valued anhysteretic curve.

3. Normal magnetization

Although the Langevin equation works quite well for describing the magnetization of a paramagnet, the modified Langevin equation does not give such a good description of the normal dc magnetization of a ferromagnet because the model ignores the possibility of the change of magnetization being impeded, as for example when the

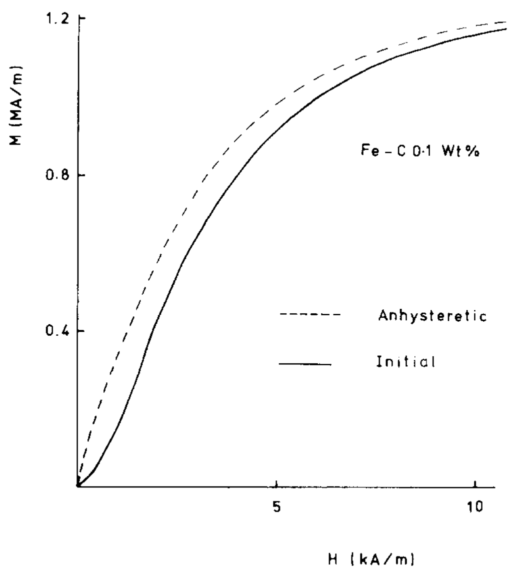

Fig. 4. Experimental initial and anyhysteretic magnetization curves for a sample of iron containing 0.1 wt% carbon mainly as Fe_3C precipitates.

motion of domain walls is inhibited by pinning sites.

The initial magnetization curve of a ferromagnet always lies below the anhysteretic. However, it does approach the anhysteretic asymptotically at high fields. In the high field regions therefore the magnetization is described well by the modified Langevin equation. Examples of experimentally determined anhysteretic and initial magnetization curves for a sample of ferromagnetic steel are given in fig. 4.

3.1. Domain wall motion of rigid, planar domain walls

If a domain wall is displaced in a constant potential, then no change in wall energy will occur and therefore when the field is removed, the wall will remain in its final position, as discussed by Chikazumi [9]. In order to have reversible wall displacement as would occur in an ideal or unpinned specimen, it is necessary to invoke a potential which increases with the magnetization. The domain boundary will then come to rest when the work done by the field is balanced by the magnetization energy of the sample as indicated by Hoselitz [39], and when the field is removed the domain wall will return to its original location.

Consider the total work done per unit volume by a magnetic field,

$$E = \int H \, \mathrm{d}B = \frac{1}{\mu_0} \int B \, \mathrm{d}B - \int M \, \mathrm{d}B, \tag{7}$$

where the second term on the right-hand side is the work done on the sample.

In the absence of pinning the domain walls of a ferromagnet are acted upon by what may be envisioned as a pressure which tends to move them in such a way that the magnetization reaches equilibrium at the anhysteretic. Using the effective field B_e for the ferromagnet

$$\int M \, \mathrm{d}B_e = \int M_{an}(H_e) \, \mathrm{d}B_e, \tag{8}$$

It should be noted that here M represents the bulk magnetization of the solid and should not be confused with the spontaneous magnetization within a domain.

3.2. Domain wall pinning

The motion of domain walls under the influence of an applied magnetic field is however impeded by the presence of pinning sites in the solid such as non magnetic inclusions and voids [31,32] or regions of inhomogeneous stress [33]. The present work is not concerned with the nature of these imperfections and they will be referred to collectively as pinning sites. These pinning sites have the effect of causing a decrease in the initial permeability of a ferromagnetic material and an increase in its coercive force. We are in agreement with the conclusions of Globus [23] that irreversible changes in magnetization are caused during wall displacement by the pinning process.

Consider a pinning site on a perfectly rigid domain wall between domains with magnetic moment per unit volume $\boldsymbol{m}$ and $\boldsymbol{m}'$ where $\boldsymbol{m}$ is aligned, for simplicity, along the field direction and is the growing domain, and $\boldsymbol{m}'$ is aligned at some arbitrary angle θ to the field. The energy required to overcome the pinning site will depend on two factors, the nature of the pinning site itself and the relative orientations of the moments in the domains on either side of the wall.

Suppose that the energy required to overcome the pinning site is proportional to the change in energy per unit volume of the $\boldsymbol{m}'$ domain caused by rotating its moments into the field direction.

$$\Delta E = \boldsymbol{m} \cdot \boldsymbol{B}_e - \boldsymbol{m}' \cdot \boldsymbol{B}_e \tag{9}$$

and consequently ϵ_{pin}, the pinning energy of the site is proportional to

$$\epsilon_{pin} \propto m B_e (1 - \cos\theta). \tag{10}$$

Let the pinning energy of the site for 180° domain walls be ϵ_π. This is then a characteristic of the pinning site only and $\epsilon_\pi \propto 2mB_e$.

$$\epsilon_{pin} = \tfrac{1}{2}\epsilon_\pi (1 - \cos\theta), \tag{11}$$

where this expression includes both the characteristics of the pinning site ϵ_π and the relative orientations of the domains, θ.

If n is the average density of pinning sites throughout the solid and $\langle \epsilon_\pi \rangle$ is the average pinning energy of all these sites for 180° walls, then

$$\langle\epsilon_{\text{pin}}\rangle = \tfrac{1}{2}\langle\epsilon_{\pi}\rangle(1-\cos\theta). \tag{12}$$

The total energy dissipated through pinning when a domain wall of area A is moved through a distance x between domains whose moments lie at an angle θ, $E_{\text{pin}}(x)$ is given by

$$E_{\text{pin}}(x) = \int_0^x \frac{n\langle\epsilon_{\pi}\rangle}{2}(1-\cos\theta)A\,\mathrm{d}x. \tag{13}$$

The net change in magnetization of the ferromagnet (remembering that by symmetry there will be a number of domains at an angle θ to the field direction such that the component of magnetization perpendicular to the field due to these domain will be zero) will be

$$\mathrm{d}M = m(1-\cos\theta)A\,\mathrm{d}x, \tag{14}$$

substituting into equation (13) leads to

$$E_{\text{pin}}(M) = \frac{n\langle\epsilon_{\pi}\rangle}{2m}\int_0^M \mathrm{d}M. \tag{15}$$

Replacing $k = n\langle\epsilon_{\pi}\rangle/2m$ therefore gives

$$E_{\text{pin}}(M) = k\int_0^M \mathrm{d}M. \tag{16}$$

3.3. The irreversible magnetization process

Hence under the assumptions of a uniform distribution of pinning sites, and treating each one as having the mean pinning energy, the total work done against pinning is proportional to the change in magnetization. The magnetization energy $\int M\mathrm{d}B_{\mathrm{e}}$ from equation (8) is now the difference between the energy which would be obtained in the ideal or lossless case $\int M_{\text{an}}(H_{\mathrm{e}})\mathrm{dB}_{\mathrm{e}}$ minus the loss due to hysteresis, $k\int\mathrm{d}M$

$$\int M\,\mathrm{d}B_{\mathrm{e}} = \int M_{\text{an}}(H_{\mathrm{e}})\,\mathrm{d}B_{\mathrm{e}} - k\int\left(\frac{\mathrm{d}M}{\mathrm{d}B_{\mathrm{e}}}\right)dB_{\mathrm{e}} \tag{17}$$

and consequently, differentiating with respect to B_{e}.

$$M = M_{\text{an}} - \delta k(\mathrm{d}M/\mathrm{d}B_{\mathrm{e}}). \tag{18}$$

This is the model differential equation of hysteresis. The parameter δ takes the value $+1$ when H increases in the positive direction, $\mathrm{d}H/\mathrm{d}t > 0$, and -1 when H increases in the negative direction, $\mathrm{d}H/\mathrm{d}t < 0$, ensuring that the pinning opposes changes in magnetization.

This equation of state for a ferromagnet under the given conditions has been given previously [37]. The coefficient k is not constrained to be constant and may vary as a function of M and H. Nevertheless, the form of the solution remains the same whether k is constant or not, only the shape is modified by variable k.

It should be noticed then that the form of eq. (18) is free from any limitations which may be imposed on its generality by a choise of a specific function to model the anhysteretic. In order to make use of it, however, some assumptions have to be made as to the form of M_{an} as a function of field. We have found that an empirical form of Langevin function models most forms of anhysteretic curve very well.

The differential eq. (18) may be rewritten in a more convenient form as

$$\frac{\mathrm{d}M}{\mathrm{d}H} = \frac{1}{\delta k/\mu_0 - \alpha(M_{\text{an}} - M)}(M_{\text{an}} - M) \tag{19}$$

which shows that apart from the perturbation due to the coupling of magnetization, expressed through the coefficient α, the rate of change of magnetization M with field is proportional to the displacement from the anhysteretic, $M_{\text{an}} - M$, which is a useful result.

3.4. Domain wall motion of flexible domain walls

One of the central assumptions of the models of Becker and Döring [33], Kersten [32] and Kondorsky [40] is that the domain walls are planar and rigid. Among other features of these models the hypothesis of planar walls was criticized by Néel [41] who noted that the domain walls should be flexible in order that the walls could be displaced without ever being completely unattached from pinning sites. Later Kersten [42] published a revised theory which included the expansion, or bending, of domain walls under the action of a magnetic field.

When the domain walls bend while being held for example on two pinning sites as shown in fig. 5

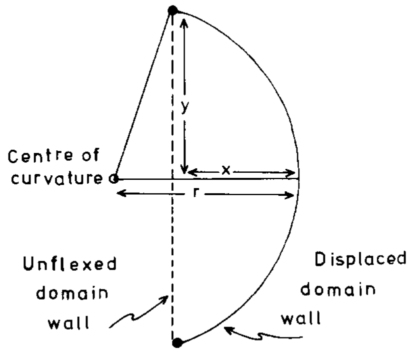

Fig. 5. Bending of a magnetic domain wall between two pinning sites under the influence of a magnetic field, after Globus, Duplex and Guyot [42].

this results initially in a reversible change in magnetization. The reversible process continues until the domain wall either encounters another nearby pinning site which it becomes attached to, or until it has expanded sufficiently to break away from the present pinning sites and moves discontinuously and irreversibly until it encounters further pinning sites.

The amount of domain wall bending which occurs depends on three factors. Two of these are intrinsic, being dependent solely on the properties of the material, the domain wall surface energy and the strength of the pinning sites. If the surface energy is low and the pinning energy high then the domain walls will undergo more bending before breaking away from the sites. If the surface energy is high and the pinning energy low then the domain walls undergo less bending before breaking away from the sites.

The third factor is extrinsic and depends on the magnetic field H. Clearly in the demagnetized state the domain walls are planar since they do not experience any net force tending to move them and hence there is no reason for bending. On the basis of the model presented here, in which the anhysteretic magnetization is the state of optimum configuration of the domains (i.e. lowest energy), it follows that at any given field H, if the magnetization M is greater than the anhysteretic magnetization M_{an}, the domain walls will experience a force which tends to reduce the magnetization. However if the magnetization M is less than the anhysteretic magnetization M_{an} then the domain walls will experience a force which tends to increase the magnetization. Because the pinning sites themselves exert a force on the domain walls which impedes their motion, the existence of the force on the domain walls when above or below the anhysteretic can only be demonstrated by breaking the walls away from the sites and observing the resulting change in magnetization. This has been reported recently [1] using stress cycles to unpin the domain walls.

It follows from this that a domain wall between two domains, aligned for example parallel and antiparallel to the field direction, bends one way when $M > M_{an}$ and the other way when $M < M_{an}$. When $M = M_{an}$ therefore the wall will be planar since there is no longer any net force on it.

Consequently the amount of domain wall bending depends on a third factor which is the difference between the prevailing magnetization $M(H)$ and the anhysteretic magnetization at the same field $M_{an}(H)$.

3.5. Reversible component of domain wall motion

As indicated above and elsewhere [1] the anhysteretic magnetization at a given field H represents the global minimum energy state as described by Tebble and Craik [2]. Consequently the domain walls bend in such a way as to reduce the difference between the prevailing magnetization and the anhysteretic magnetization.

If the magnetization M is expressed as the sum of a reversible component M_{rev} due to domain wall bending and an irreversible component due to wall displacement, then

$$M = M_{irr} + M_{rev}. \tag{20}$$

Consequently, if $M < M_{an}$ then $M_{rev} > 0$, but if $M > M_{an}$ then $M_{rev} < 0$. Finally if $M = M_{an}$ it follows that $M_{rev} = 0$.

3.6. Bulging of domain walls due to displacement of magnetization from the anhysteretic

Consider the bulging of a domain wall between two pinning sites as shown in fig. 5. The dotted line gives the position of the unflexed wall. x is the linear displacement, r is the radius of curvature and $2y$ is the distance between pinning sites.

By simple geometry

$$x = r - \sqrt{(r^2 - y^2)} \tag{21}$$

and if E is the surface energy of the domain wall and P is the excess pressure caused by application of the field,

$$P = 2E/r, \tag{22}$$

so that

$$x = 2E/P - \sqrt{(2E/P)^2 - y^2} \tag{23}$$

and using a binominal expansion on the square root term,

$$x = \frac{2E}{P} - \frac{2E}{P}\left(1 - \tfrac{1}{2}y^2\left(\frac{P}{2E}\right)^2 + \dots\right), \tag{24}$$

$$x \approx \tfrac{1}{2}y^2 P/2E, \tag{25}$$

for small displacements this expression relating x to the excess pressure P becomes exact.

Now consider the actual form of the pressure P on the domain wall. The fundamental idea of the model presented so far is that the force experienced by the domain walls is not simply due to the applied field, but is due to the applied field minus a contribution due to the tendency of a ferromagnet towards a random orientation of its domain configuration.

It is implicit in the model presented so far that the effective magnetostatic energy is given to within a coefficient of proportionality by,

$$E = \tfrac{1}{2}(M - M_{an})^2, \tag{26}$$

and that the force on the domain walls is

$$F = (M_{an} - M). \tag{27}$$

The pressure on the domain walls will then be simply

$$P = C'(M_{an} - M), \tag{28}$$

where C' is a constant and substituting this into eq. (25) gives,

$$x = (y^2/4E)C'(M_{an} - M) \tag{29}$$

as the expression for the bending of the domain wall.

It is now necessary to make some assumptions concerning the dependence of M_{rev} upon x. It is not possible to take into account every possible situation and geometry. Naturally there will be an almost infinite number of possible configurations in which domain walls interact with defects, be they regions of inhomogeneous strain, point defects, dislocations, nonmagnetic inclusions or grain boundaries.

Therefore consider the situation in fig. 5 and suppose, like Globus and Duplex [42] that a domain wall bisects a spherical grain and is pinned at the grain boundary. Under the action of a field the domain wall is deformed reversibly through a distance x. The volume swept out by the domain wall is $\Delta V(\pi/6)x(3y^2 + x^2)$ and the change in magnetization will be $M_{rev} = 2\Delta V m$ if the moments in the two domains are parallel and antiparallel to the field.

Consequently, substituting from eq. (29)

$$M_{rev} = \frac{m\pi y^4 C'}{4E}(M_{an} - M) + m\frac{\pi}{3}\left(\frac{y^2}{4E}C'(M_{an} - M)\right)^3 \tag{30}$$

and as in the Globus analysis [42] neglecting terms of order greater than x^2 leaves

$$M_{rev} = c(M_{an} - M), \tag{31}$$

where now the coefficient of proportionality is $c = (m\pi y^4/4E)C'$. The value of the coefficient c is determined experimentally by the ratio of the initial differential susceptibilities of the normal and anhysteretic magnetization curves, $c = \chi'_{0norm}/\chi'_{0anhys}$. Consequently the amount by which the domain walls bulge before breaking away from their pinning sites, and hence the reversible component of magnetization, is for small displacement linearly dependent on $M_{an} - M$.

3.7. Extension of the equation of hysteresis to include reversible changes in magnetization due to domain wall bulging

The magnetization M can be calculated as the sum of two components, an irreversible component M_{irr} and a reversible component M_{rev}, as given in eq. (20).

The irreversible component of magnetization is now given by the solution of eq. (19)

$$\frac{\mathrm{d}M_{\mathrm{irr}}}{\mathrm{d}H}=\frac{1}{\delta k/\mu_0-\alpha(M_{\mathrm{an}}-M_{\mathrm{irr}})}(M_{\mathrm{an}}-M_{\mathrm{irr}}), \tag{32}$$

and differentiation of eq. (31) gives the rate of changes of the reversible component.

$$\mathrm{d}M_{\mathrm{rev}}/\mathrm{d}H=c(\mathrm{d}M_{\mathrm{an}}/\mathrm{d}H-\mathrm{d}M/\mathrm{d}H). \tag{33}$$

Summing these leads to,

$$\frac{dM}{dH}=\frac{1}{(1+c)}\frac{1}{\delta k/\mu_0-\alpha(M_{\mathrm{an}}-M)}(M_{\mathrm{an}}-M) +\frac{c}{(1+c)}\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}. \tag{34}$$

Solutions may be obtained either by solving eq. (34) directly or alternatively by solving eq. (19) and then adding M_{rev} using eq. (31).

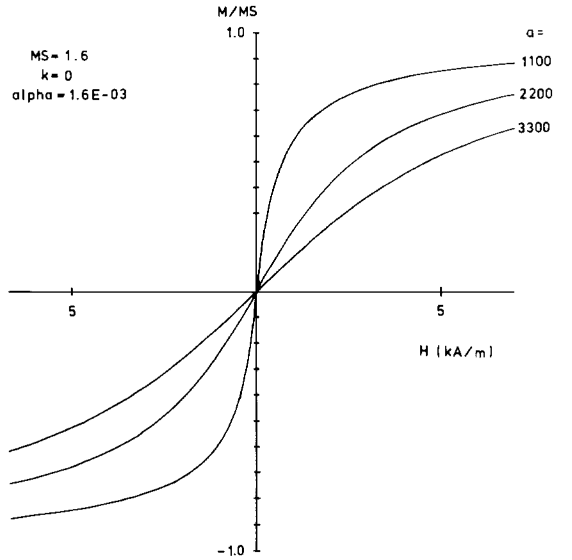

Fig. 6. Theoretical anhysteretic magnetization curves obtained as solutions of eq. (6) with the following values of the parameters: $M_s=1.6\times10^6$ A/m, $k=0$, $\alpha=1.6\times10^{-3}$. Values of a are given on the figure.

4. Results

The comparison between experimental results and theoretical predictions will be discussed in relation to the following types of magnetization curves (i) the anhysteretic curve, (ii) the initial magnetization curve, and (iii) major hysteresis loops (i.e. loops which are symmetric with respect to rotation of 180° about the origin), in that order.

The measurements were taken on bar shaped samples of dimensions $6\times1\times1$ cm^3 magnetized along the long axis. The H field was measured locally on the surface of the sample at the center of the long axis, making use of the fact that H tangential is continuous across the surface and that H is uniform inside. This avoided the need for demagnetizing field calculations.

4.1. Anhysteretic curve

Although the anhysteretic curve is the most difficult of the four curves to obtain experimentally it is the most fundamental and has the simplest form. The anhysteretic curve represents the locus of points in B, H space at which the domain walls have achieved positions of true equilibrium under the prevailing field H [2], and as such it is a reversible single valued function. The equation which has been used to model the anhysteretic

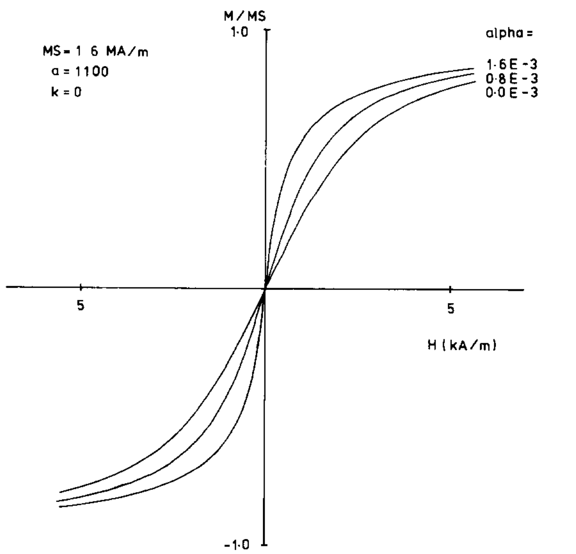

Fig. 7. Theoretical anhysteretic magnetization curves obtained as solutions of eq. (6) with the following values of the parameters: $M_s=1.6\times10^6$ A/m, $a=1100$ A/m, $k=0$. Values of α are given in the figure.

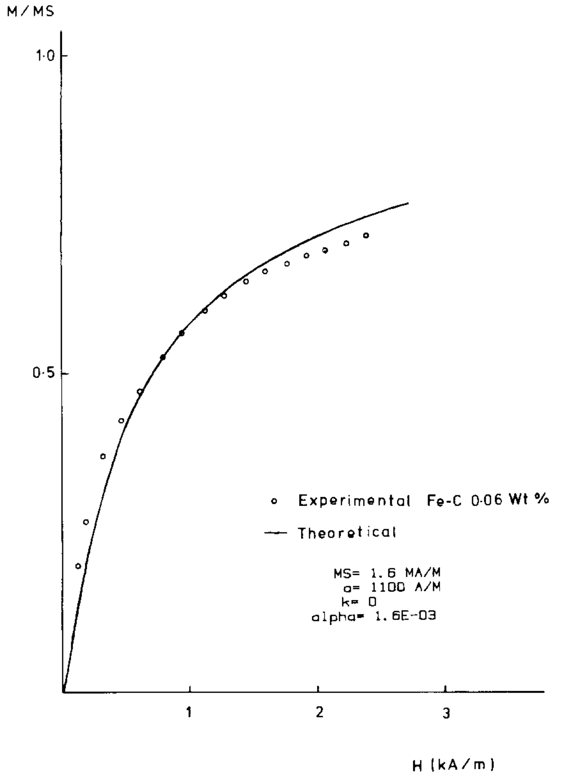

Fig. 8. Comparison of experimental and theoretical anhysteretic magnetization curves. The experimental results were obtained on a specimen of Fe–C 0.06 wt%. The theoretical results were obtained with following values of parameters: $M_s = 1.6 \times 10^6$ A/m, $a = 1100$ A/m, $k = 0$, $\alpha = 1.6 \times 10^{-3}$.

curve is given by the eq. (6). Solutions of eq. (6) for various of the parameters M_s, a and α are given in figs. 6 and 7. In fig. 6, the effect of different values of the parameter a are shown while M_s and α are held constant, while in fig. 7 the effects of different values of the parameter α are shown while M_s and a are held constant.

Experimentally the anhysteretic curve is obtained by applying a known steady field H_{dc} and then superimposing upon it a large alternating field of low frequency ($< \approx 0.2$ Hz) whose initial amplitude is sufficient to virtually saturate the magnetization. The amplitude of the alternating field is then slowly reduced to zero and the flux density B (and consequently the magnetization M) converges to the anhysteretic value at the given field H, reaching the anhysterestic only when the alternating field amplitude reaches zero. An example of an experimental anhysteretic curve for a sample of steel used in the present work is given in fig. 8. This is compared in the figure with theoretical anhysteretic curve data obtained by using a curve fitting routine to determine the values of the parameters M_s, a and α in eq. (6). A large number of anhysteretic curves have been determined experimentally both for different samples of steel and under different constant applied uniaxial stress. In all cases, the solutions of eq. (6) with different values of the three parameters, gave an excellent fit to the curve.

4.2. Initial magnetization curve

Solutions of eq. (34) for monotonically increasing field H yield theoretical initial magnetization curves. The simplest solutions, involving constant k are shown in fig. 9 where the effect of different values of k on the initial magnetization curve with constant M_s, a and α are shown. Fig. 10 represents the experimental results.

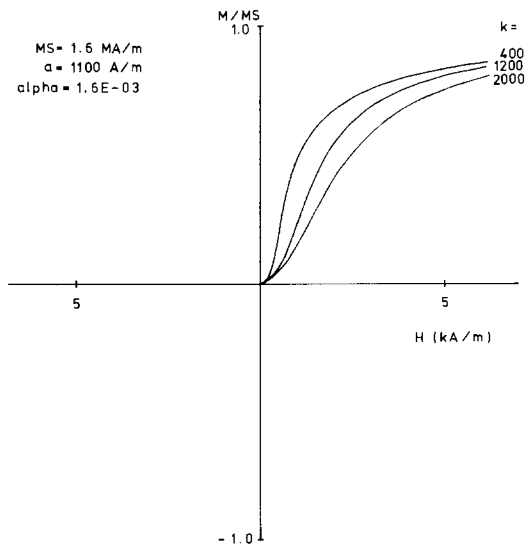

Fig. 9. Theoretical initial magnetization curves obtained as solutions of eq. (19) with $M_s = 1.6 \times 10^6$, $a = 1100$ A/m, $\alpha = 1.6 \times 10^{-3}$ and various values of k as shown on the figure. The reversible component given in eq. (31) was obtained with a value $c = 0.2$.

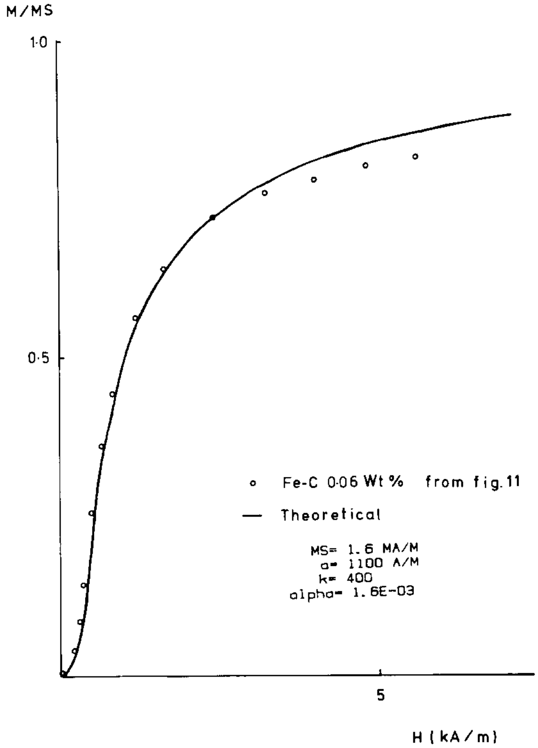

Fig. 10. Comparison of experimental and theoretical initial magnetization curves. The experimental results were obtained on a specimen of Fe–C 0.06 wt%. The theoretical results were obtained with the following values of parameters: $M_s = 1.6 \times 10^6$ A/m, $a = 1100$ A/m, $k = 400$, $\alpha = 1.6 \times 10^{-3}$, $c = 0.2$.

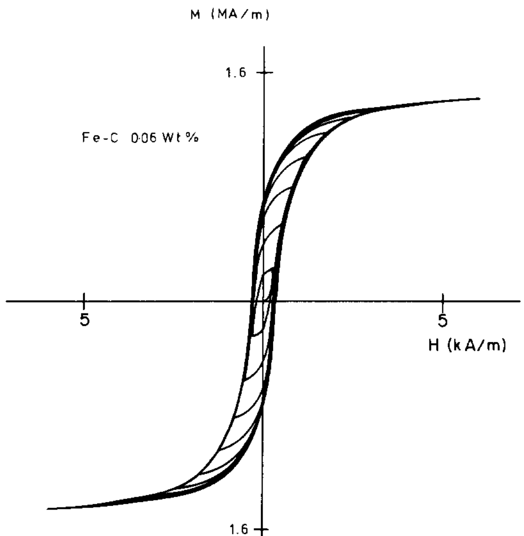

Fig. 11. Experimental magnetic hysteresis loops for a specimen of Fe–C 0.06 wt%.

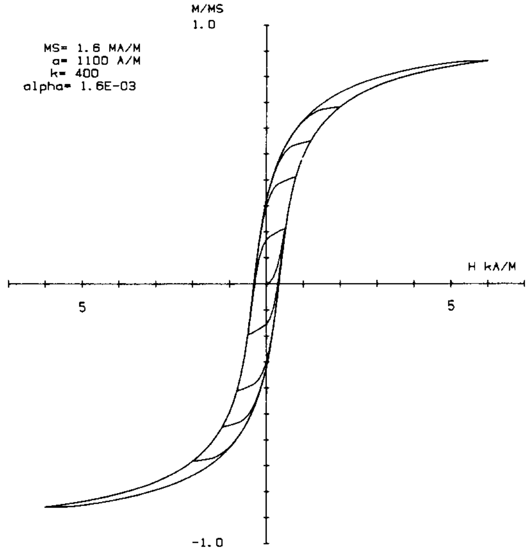

Fig. 12. Theoretical magnetic hysteresis loops obtained as solutions of eq. (19) with $M_s = 1.6 \times 10^6$, $a = 1100$ A/m, $\alpha = 1.6 \times 10^{-3}$ and $k = 400$. The reversible components was obtained from eq. (31) with a value $c = 0.2$.

4.3. *Major hysteresis loops*

Major hysteresis loops are obtained by cycling the H field at progressively increasing amplitudes starting from the demagnetized state. These are used widely to define the magnetic properties of a ferromagnet and their general shape is well known. A family of experimental major hysteresis loops for a sample of steel is shown in fig. 11.

Theoretical major hysteresis loops are obtained from solutions of eq. (34) also with progressively increasing H_{max} starting from the demagnetized state $H_{max} = 0$. From this the theoretical major hysteresis loops of fig. 12 were obtained.

5. Conclusions

A theoretical equation has been derived which describes ferromagnetic hysteresis in the case of sigmoid-shaped hysteresis loops. The theoretical results are able to reproduce the initial magnetization curve and families of major (symmetric) hysteresis loops. From the theoretical equation, which is in the form of a simple differential eq. (18), the significance of the anhysteretic curve, which is the locus of global equilibrium states, emerges. An equation for the anhysteretic curve, which is particularly simple is also given.

The model is based on a mean field approximation in which each domain is assumed to interact with the field H and a weighted mean of the bulk magnetization. The impedance to the changes in magnetization, the equivalent of Wiedemann's "frictional type" force is provided by pinning sites inside the solid in the form of imperfections, inclusions and regions of inhomogeneous strain, which oppose the motion of domain walls. This is the fundamental cause of hysteresis behavior in ferromagnets.

The model at present applies to isotropic solids, that is polycrystalline materials or single crystals with low anisotropy. However by modifying the form of eq. (6) to account for anisotropy this could be included into the model as indicated by Furlani [34]. The impedance to changes in magnetization is assumed to be uniform, that is a mean pinning energy per site only has been used. In practice a distribution of pinning energies would be more exact however the form of the solution is likely to remain much the same.

The model includes at present irreversible changes in magnetization due to displacement of domain walls and reversible changes in magnetization due to bending of domain walls. Although the model does not yet take into account less significant contributions such as rotational processes these can not be entirely excluded and must ultimately be incorporated into a more realistic model.

The results of the theory have been compared with experimental results for ferromagnetic steel by considering the various types of magnetization curves, the anhysteretic, initial magnetization curve, and families of major hysteresis loops. In all cases the agreement is excellent.

References

[1] D.C. Jiles and D.L. Atherton, J. Phys. D 17 (1984) 1265.
[2] R.S. Tebble and D.J. Craik, Magnetic Materials (Wiley, New York, 1969).
[3] W. Weber, Abhandlungen der Kg. Sächs Gesellschaft der Wissens (1852) 572.
[4] G. Wiedemann, Phil. Mag. (1886) 52.
[5] J.C. Maxwell, Electricity and Magnetism (Oxford, Univ. Press, Oxford, 1873).
[6] J.A. Ewing, Phil. Mag. 5 (1890) 205.
[7] B.D. Cullity, Introduction to Magnetic Materials, (Addison-Wesley, New York, 1972).
[8] H.J. Williams, Phys. Rev. 52 (1937) 747.
[9] S. Chikazumi, Physics of Magnetism (Wiley, New York, 1964).
[10] L. Rayleigh, Phil. Mag. 23 (1887) 225.
[11] J.R. Brauer, IEEE Trans. Magn. MAG-11 (1975) 81.
[12] J. Fischer and H. Moser, Archiv. für Electrotechnik 42 (1956) 286.
[13] F.C. Trutt, E.A. Erdelyi and R.E. Hopkins, IEEE Trans. Power App. Sys. PAS-87 (1968) 665.
[14] G.F.T. Widger, Proc. Inst. Elect. Engg. 116 (1969) 156.
[15] J. Rivas, J.M. Zamarro, E. Martin and C. Pereira, IEEE Trans. Magn. MAG-17 (1981) 1498.
[16] F. Preisach, Zeit. für Physik 94 (1935) 277.
[17] L. Néel, J. de Phys. Rad. 11 (1950) 49.
[18] L. Néel, Advan. Phys. 4 (1955) 191.
[19] R.M. Del Vecchio, IEEE Trans. Magn. MAG-16 (1980) 809.
[20] M.A. Rahman, M. Poloujadoff, R.D. Jackson, J. Perrard and S.D. Gowda, IEEE Trans. Magn. MAG-17 (1981) 3253.
[21] W.F. Brown, J. Appl. Phys. 30 (1959) 62S.
[22] A. Aharoni, J. Appl. Phys. 30 (1959) 70S.
[23] A. Globus, Comptes Rendus, Acad. Seances 255 (1962) 1709.
[24] A. Globus and P. Duplex, Phys. Stat. Sol. 31 (1969) 765.
[25] A. Globus and P. Duplex, Phys. Stat. Sol. (a) 3 (1970) 53.
[26] A. Globus and P. Duplex, IEEE Trans. Magn. MAG-2 (1966) 441.
[27] J.L. Porteseil and R. Vergne, J. de Phys. 40 (1979) 871.
[28] B. Astie, J. Degauque, J.L. Porteseil and R. Vergne, IEEE Trans. Magn. MAG-17 (1981) 2929.
[29] J. Degauque and B. Astie, Phys. Stat. Sol. (a) 74 (1982) 201.
[30] B. Astie, J. Degauque, J.L. Porteseil and R. Vergne, J. Magn. Magn. Mat. 28 (1982) 149.
[31] M. Kersten, Problem der Technischer Magnetisierungskurve (Springer-Verlag, Berlin, 1938).
[32] M. Kersten, Grundlagen Theorie der Ferromagnetischer Hysterese und der Koerzitivkraft (Hirzel, 1943).
[33] R. Becker and W. Döring, Ferromagnetismus (Springer-Verlag, Berlin, 1939).
[34] E. Furlani, private communication.
[35] E. Callen, Y.J. Liu and J.R. Cullen, Phys. Rev. B16 (1977) 263.
[36] M. Langevin, Ann. de Chem. et Phys. 5 (1905) 70.
[37] D.C. Jiles and D.L. Atherton, IEEE Trans. Magn. MAG-19 (1983) 2183.
[38] R.M. Bozorth, Ferromagnetism (Van Nostrand, New York, 1951).
[39] K. Hoselitz, Ferromagnetic Properties of Metals and Alloys (Oxford Univ. Press, Oxford, 1952).
[40] E.I. Kondorsky, Physik. Zeit. Sow 11 (1937) 597.
[41] L. Néel, Annales Univ. Grenoble 22 (1946) 321.
[42] A. Globus, P. Duplex and M. Guyot, IEEE Trans. Magn. MAG-7 (1971) 617.