AKENTEN APPIAH-MENKA
UNIVERSITY OF SKILLS TRAINING AND ENTREPRENEURIAL DEVELOPMENT
FACULTY OF APPLIED SCIENCES AND MATHEMATICS EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
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COURSE CODE	**MAT 124**
COURSE TITLE	**Geometry II**
DURATION	**Two (2) hours**
LECTURERS	**Mr. Ernest Larbi and Dr. Mohammed Kumah**
INSTRUCTION	**Answer *ALL* questions both in section A and section B.**

INDEX NO:..... **CLASS:**.....

SECTION A [30 MARKS]

***CIRCLE* the correct option/answer in questions 1 to 30.**

1. Find the gradient of the circle $x^2+y^2-10x+y-4=0$ at point (1,–4).
   (a) $-\frac{8}{9}$ (b) $-\frac{9}{8}$
   (c) $-\frac{12}{7}$ (d) $\frac{12}{7}$
2. A line touches a circle with centre $(1,-4)$ at a point $(2,1)$. Find the equation of the line.
   (a) $x-5y-9=0$ (b) $5x-y-9=0$
   (c) $5x-3y-9=0$ (d) $5x-y+9=0$
3. Find the area of a circle described by the equation $x^2+y^2-2x+4y-5=0$.
   (a) $\sqrt{10}\pi$ sq. units (b) 5π sq. units
   (c) 10π sq. units (d) $\sqrt{5}\pi$ sq. units
4. Determine the equation of the normal to the circle $x^2+y^2-2x+4y-5=0$ at point $(2,2)$.
   (a) $4x-y+6=0$ (b) $4x+y-6=0$
   (c) $4x-y-6=0$ (d) $x-4y-6=0$
5. Find the radius of the circle given by the equation $x^2+y^2-4x-6y-12=0$.
   (a) 5 (b) $\sqrt{12}$
   (c) 25 (d) 30
6. Find the equation of the parabola with vertex at the origin and has its focus at $(0,3)$.
   (a) $x^2=-12y$ (b) $x^2=12y$
   (c) $x^2=-3y$ (d) $x^2=3y$

The equation of a parabola is given by as $\left(x-\frac{1}{2}\right)^2=-\frac{1}{2}(y+4)$.

Use the equation to answer Questions 7 and 8.

7. Find the vertex of the parabola.
   (a) $(-\frac{1}{2},4)$ (b) $(-\frac{1}{2},-4)$
   (c) $(\frac{1}{2},-4)$ (d) $(\frac{1}{2},4)$

8. Determine the axis of the parabola.

(a) -4 (b) $x=\frac{1}{2}$

(c) 4 (d) $x=-\frac{1}{2}$

9. A parabola $y^2=12x$ passes through the point $P(3, 6)$. Find the equation of the normal.

(a) $x+y-9=0$ (b) $x+y+9=0$

(c) $x-y+9=0$ (d) $x-y-9=0$

10. Find the length of the latus rectum of the parabola $y^2=10x$.

(a) 4 units (b) 20 units

(c) 5 units (d) 10 units

The equation of an ellipse is given by $\frac{x^2}{9}+\frac{y^2}{4}=1$

Use the equation to answer Questions 11 to 15.

11. Find the vertices of the equation.

(a) $(\pm3,0)$ and $(\pm2,0)$ (b) $(\pm2,0)$ and $(\pm3,0)$

(c) $(\pm2,0)$ and $(\pm6,0)$ (d) $(\pm3,0)$ and $(\pm1,0)$

12. Determine the foci of the equation.

(a) $(\pm\sqrt{8},0)$ (b) $(\pm\sqrt{3},0)$

(c) $(\pm\sqrt{5},0)$ (d) $(\pm\sqrt{4},0)$

13. Find the gradient of the ellipse at point $(1, 4)$.

(a) -9 (b) $\frac{1}{9}$

(c) $-\frac{1}{9}$ (d) 9

14. Find the equation of the normal at point $(1, 4)$.

(a) $9x+y-5=0$ (b) $9x-y-5=0$

(c) $x+9y-5=0$ (d) $9x+y+5=0$

15. Determine the directrices of the ellipse.

(a) $\pm\frac{9}{5}\sqrt{5}$ (b) $\pm\frac{5}{9}\sqrt{5}$

(c) $\pm\frac{5}{14}\sqrt{5}$ (d) $\pm\frac{9}{8}\sqrt{5}$

16. Compute the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$.

(a) $\frac{16}{3}$ (b) $\frac{8}{3}$

(c) $\frac{32}{3}$ (d) $\frac{3}{16}$

17. Eliminate the parameter in the following equations $x=t^2-4$ and $y=t^3-4t$.

(a) $y^2=x^3-4x^2$ (b) $y^2=x^3+4x^2$

(c) $y^2=-x^3+4x^2$ (d) $y^2=-x^3-4x^2$

18. Convert the parameter $x = \cos\theta$ and $y = \sin\theta$ to rectangular coordinates.

(a) $x^2 - y^2 = 1$ (b) $x^2 + y^2 = -1$

(c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = -1$

19. Determine the direction of opening of the graph of the parabola $(y+1)^2 = 4(y-2)$.

(a) Opens to the left (b) Opens to the right

(c) Opens upwards (d) Opens downwards

20. Find the gradient of the tangent to the curve with the parametric equations $x = t^2 - 3$ and $y = 2t - 1$ when $t = 3$.

(a) $\frac{2}{3}$ (b) 1

(c) $\frac{1}{3}$ (d) –3

21. Find the equation of the normal to the curve $f(x) = x^2$ and $x = 3$.

(a) $x + 6y = 57$ (b) $6y - x = 57$

(c) $x - 6y = 57$ (d) $6y + x = -57$

22. What type of equation is the polar equation $r = 4$?

(a) A line (b) A parabola

(c) An ellipse (d) A circle

23. Find the equations of the tangent and the normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$.

(a) $yt + x = at^2$ (b) $yt - x = at^2$

(c) $y + tx = 2at + at^3$ (d) $y - tx = 2at + at^3$

24. Obtain the polar equation of the Cartesian equation $3x + 5y - 2 = 0$.

(a) $\dfrac{-2}{3\cos\theta + 5\sin\theta}$ (b) $\dfrac{20}{3\cos\theta + 5\sin\theta}$

(c) $\dfrac{2}{3\cos\theta + 5\sin\theta}$ (d) $\dfrac{2}{3\cos\theta - 5\sin\theta}$

25. Which of the following equations is the standard equation of the parabola if the directrix is parallel to the y-axis in the positive quadrant?

(a) $y^2 = 4ax$ (b) $x^2 = 4ay$

(c) $x^2 = -4ay$ (d) $y^2 = -4ax$

26. Convert $r = \dfrac{2}{1 + \sin\theta}$ to rectangular form.

(a) $x^2 - 4y - 4 = 0$ (b) $x^2 + 4y + 4 = 0$

(c) $-x^2 + 4y - 4 = 0$ (d) $x^2 + 4y - 4 = 0$

27. Conv2ert the Cartesian coordinates $(2,\ 2)$ to polar coordinates.

(a) $\left(2\sqrt{2},\ \frac{\pi}{4}\right)$ (b) $\left(2\sqrt{2},\ \frac{\pi}{2}\right)$

(b) $\left(\sqrt{2},\ \frac{\pi}{4}\right)$ (d) $\left(2,\ \frac{\pi}{3}\right)$

28. Find the equation of a hyperbola with foci at (0, 3) and (0, -3) and vertices at (0, 2) and (0, −2).

(a) $\frac{x^2}{5}-\frac{y^2}{4}=1$ (b) $\frac{y^2}{5}-\frac{x^2}{4}=1$

(b) $\frac{x^2}{4}-\frac{y^2}{5}=1$ (d) $\frac{y^2}{4}-\frac{x^2}{5}=1$

29. Find the coordinates of the focus and the equation of the directrix of the parabola $y^2=2x$.

(a) F(2,0), $x=-2$ (b) F(2,0), $x=2$

(b) F(−2,0), $x=-2$ (d) F(−2,0), $x=2$

30. Find the equation of the asymptotes of the hyperbola $\frac{x^2}{4}-\frac{y^2}{3}=1$.

(a) $y=\pm\frac{2}{\sqrt{3}}x$ (b) $y=\pm\frac{3}{4}x$

(b) $y=\pm\frac{4}{3}x$ (d) $y=\pm\frac{\sqrt{3}}{2}x$

SECTION B [30 MARKS]

Answer ALL questions in this section

16. (a) The line $2y+3x-6=0$ intersects the parabola $y^2=6x$ at K and M respectively. Find:

(i) the coordinates of K and M; **[5 marks]**

(ii) the equation of the normal to the parabola at the point $\left(\frac{2}{3},2\right)$. **[5 marks]**

(b) Eliminate the parameter in the following equation and identify the name of the conic section.

$x=4\cos t+3$ and $y=2\sin t+1$ **[5 marks]**

17. (a) Write the equation $9x^2-4y^2+18x+16y=43$ in a standard form and identify the equation. **[4 marks]**

(b) Hence, sketch the graph of the equation in **17(a).** **[6 marks]**

(c) Write the polar equation $r=6\cos\theta-8\sin\theta$ in rectangular coordinates and describe the equation obtained. **[5 marks]**