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UNIVERSITY OF SKILLS TRAINING AND ENTREPRENEURIAL DEVELOPMENT

FACULTY OF TECHNICAL EDUCATION

BSC WOOD/MECHANICAL/AUTOMOTIVE/CONSTRUCTION

END OF SECOND SEMESTER EXAMINATION, AUGUST 2021

COURSE CODE	MAT 126
COURSE TITLE	ENGINEERING MATHEMATICS II
DURATION	2 HOURS
LECTURER(S)	DR. JOSEPH FRANK GORDON
INSTRUCTION(S)	1. Write your ***index number*** in the space provided **below.** 2. You ***must*** answer all questions by ***circling*** the ***best*** option on the question paper. 3. Where you think the answer is ***not*** in the options provided, kindly provide your ***own answer*** and label it **E.** 4. Submit the ***question paper*** provided to you after the ***exam.*** 5. Non-programmable calculators are ***not*** prohibited. 6. Please ***tick*** ☑ your ***programme*** of study from the list ***below.*** ☐ CONST 1A ☐ CONST 1B ☐ WOOD ☐ AUTO ☐ MEC

INDEX NUMBER: _____

1. Evaluate $\int_1^2 xe^x dx$

   A. e^{-2} B. e^2 C. e D. 0

2. Find the integral, $\int \frac{2x}{3x^2+9} dx$

   A. $\ln(3x^2+3)+c$ B. $3\ln(3x^2+3)+c$ C. $\frac{1}{3}\ln(3x^2+3)+c$ D. $\frac{1}{3}\ln(x^2+3)+c$

3. Find the equation of the curve with a tangent $y' = 3x^2+1$ at the point $(1,1)$.

   A. x^3+x+1 B. x^3+x-1 C. x^3-x+1 D. x^3-x-1

Use the following information to answer question 4 to 5

Given that

$$f(x,y) = \frac{2y+\ln x}{y^2}$$

4. Find the first partial derivative f_x

   A. $\frac{1}{xy^2}$ B. $\frac{1}{x^2y}$ C. $\frac{1}{x^2y^2}$ D. $-\frac{1}{xy^2}$

5. Find f_{xy}.

   A. $-\frac{2}{x^3y}$ B. $\frac{2}{x^3y}$ C. $-\frac{2}{xy^3}$ D. $\frac{2}{xy^3}$

6. Evaluate $\int \lambda dx$

   A. $\lambda+c$ B. $\lambda x+c$ C. 0 D. $x+c$

7. Differentiate the function $f(x) = (2x^{-3}+4x-2)^{\frac{1}{2}}$.

   A. $\frac{1}{2}(2x^{-3}+4x-2)^{\frac{1}{2}}(-6x^{-4}+4)$ B. $\frac{1}{2}(2x^{-3}+4x-2)^{-\frac{1}{2}}(-6x^{-4}+4)$ C. $\frac{1}{2}(2x^{-3}-4x-2)^{\frac{1}{2}}(-6x^{-4}+4)$ D. $\frac{1}{2}(2x^{-3}+4x+2)^{-\frac{1}{2}}(-6x^{-4}+4)$

8. Find $\frac{dy}{dx}$, if $2y^3-x^2y+4x^2=0$.

   A. $\frac{2xy-8x}{6y^2+x^2}$ B. $\frac{2xy+8x}{6y^2-x^2}$ C. $\frac{2xy+8x}{6y^2+x^2}$ D. $\frac{2xy-8x}{6y^2-x^2}$

9. Find the derivative of the function $y = x(15^x)$.

   A. $15^x+x(15^x\ln 15)$ B. $15^x+15^x\ln 15$ C. $15x+x(15^x\ln 15)$ D. $15^x+(15x\ln 15)$

10. The gradient of the curve $y = e^{-x}$ at $x = \ln 2$ is

    A. $\frac{1}{2}$ B. $-\ln 2$ C. $-\frac{1}{\ln 2}$ D. $-\frac{1}{2}$

11. Gasoline is pumped into a tank at a rate of $r(t) = 30(1-e^{-0.16t})$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 500 gallons of gasoline when the pump was turned on, how much gasoline, to the nearest gallon, is in the tank after 20 minutes?

    A. 530 gallons B. 520 gallons C. 519 gallons D. 529 gallons

12. A motorcycle moves along the x-axis. The velocity of the motorcycle at time t is $6t - t^2$. What is the total distance traveled by the motorcycle from time $t = 0$ to $t = 3$?

A. 19 B. 18 C. 15 D. 17

13. The derivative of a function means

A. the volume described by the function B. the area described by the function C. the slope of the tangent to the function at a point D. the slope of the secant to the function

14. Evaluate $\lim_{t\to\infty} \frac{2t+3}{9-u}$

A. 2 B. -2 C. 0 D. ∞

15. If $f'(x) < 0$ to the left of c and $f'(x) < 0$ to the right of c, the point at c is referred to as

A. Positive inflection B. Relative minimum C. Relative maximum D. Negative inflection

16. If -3 is a critical number of the curve $f(x) = x^3 + 3x^2 + 1$, determine the type of extremum.

A. relative minimum B. relative maximum C. increasing function D. decreasing function

17. Find the volume in cm^3 of the solid bounded above by the plane $f(x, y) = 8 - x - y$ and below by the rectangle $R(x, y) = \{(x, y) : 0 \le x \le 1, 0 \le y \le 2\}$, represented by the double integral $\int_0^2 \int_0^1 (8 - x - y)dxdy$

A. $12cm^3$ B. $15cm^3$ C. $17.5cm^3$ D. $13cm^3$

18. What is the area of the region in the first quadrant bounded by the graph of $y = e^{\frac{x}{2}}$ and the line $x = 2$?

A. $2e + 2$ B. $2e - 2$ C. $-2e + 2$ D. $2e$

19. Find the approximate value of $\int_0^{\frac{\pi}{2}} x\cos x dx$ by using three-panel Trapezium Rule.

A. 0.51158 B. 0.52058 C. 0.57081 D. 2.21519

20. Compute the second-order partial derivative f_{xx} of the function $f(x, y) = xy^3 + 5xy^2 + 2x + 1$.

A. $3y^2 + 10y$ B. $6xy + 10x$ C. $y^3 + 5y^2 - 2$ D. 0

21. The following table contains data of speed of a computer with respect to time.

Time(μs)	11.03	14.17	17.09	20.10	24.06	27.31
Speed (Ghz)	0.9	1.3	1.6	1.8	2.0	2.4

If you were going to use cubic interpolation to find the value of the speed at $t = 17.85$ micro seconds, what four data points of time would you choose for your interpolation?

A. 11.03, 14.17, 17.09, 20.10 B. 14.17, 17.09, 20.10, 24.06 C. 17.09, 20.10, 24.06, 27.31
D. 14.17, 17.09, 24.06, 27.31

22. Find the integral $\int(12x^5+8x^3-3x^2+5)dx$

A. $2x^6+2x^4-x^3+5x+c$ B. $2x^6+2x^4-3x^3+5x+c$ C. $2x^6+x^4-x^3+5x+c$ D. $x^6+2x^4-x^3+5x+c$

23. Find $\int x^{\pi}dx$.

A. $\frac{x^{\pi+1}}{\pi+1}$ B. $\frac{x^{\pi-1}}{\pi+1}$ C. $\frac{x^{\pi+1}}{\pi-1}$ D. $\frac{x^{\pi-1}}{\pi-1}$

Use the following information to answer question 24-26.

A reservoir contains 100 liters of pure water. Liquid containing 20 grams of pollutant per liter leaks into the reservoir at 2 liters per minute. The concentration of pollutant, $C(t)$ after t minutes (in g/L) in the reservoir is governed by the equation,

$$C(t)=\frac{20t}{50+t}.$$

24. Determine the concentration of pollutant in the reservoir after 50 minutes.

A. 25 g/L B. 20 g/L C. 15 g/L D. 12 g/L

25. What is the long-term concentration of pollutant.

A. 200 g/L B. 12 g/L C. 15 g/L D. 20 g/L

26. How long in hours, will it take the concentration of pollutant in the reservoir to reach 15 g/L?

A. 2.5 hour B. 2 hours C. 1.5 hours D. 3 hours

27. Find $\lim_{x\to 3}(7x^2+3x-10)$.

A. 62 B. 60 C. 40 D. 50

28. Evaluate $\lim_{x\to 0}\frac{(3+x)^2-9}{x}$.

A. 3 B. 0 C. undefined D. 6

29. If $f''(a)>0$ at $x=a$, then f has

A. Relative minimum B. Relative maximum C. No extremum D. Critical point

30. Evaluate $\int_0^1 xe^{-x}dx$

A. $1+2e^{-1}$ B. $1-2e^{-1}$ C. $2e^{-1}-1$ D. 0

31. A ball is thrown at the ground from the top of a tall building. The speed of the ball in meters per second is given by

$$v(t)=9.8t+v_0,$$

where t denotes the number of seconds since the ball has been thrown and v_0 is the initial speed of the ball (also in meters per second). If the ball travels 25 meters during the first 2 seconds after it is thrown, what was the initial speed of the ball?

A. $6m/s$ B. $3.5m/s$ C. $1.8m/s$ D. $2.7m/s$

Use the following information to answer question 32-33.

Air is blown into a balloon such the volume Vcm^3 increased at the rate of $2(100-t)cm^3/s$ after t seconds. If the volume V is $5cm^3$ at time $t=0$,

32. Find the volume of the balloon at t seconds.

A. $V=200t-t^2+c$ B. $V=200t+t^2+c$ C. $V=200t-t^2-c$ D. $V=t^2-200t+c$

33. Calculate the volume of the balloon at 3 seconds.

A. $600cm^3$ B. $550cm^3$ C. $596cm^3$ D. $610cm^3$

34. Find the integral $\int \frac{2\ln x}{x}dx$.

A. $\frac{1}{2}\ln x+c$ B. $(\ln x)^2+c$ C. $(\ln x)+c$ D. $(\ln x^2)+c$

35. Evaluate $\int 4\sin 8x dx$.

A. $\frac{1}{2}\cos 8x+c$ B. $\frac{1}{2}\cos x+c$ C. $-2\cos 8x+c$ D. $-\frac{1}{2}\cos 8x+c$

36. What is the equation of the tangent line to the curve $y=x^2\ln x$ at the point where $x=1$.

A. $y=x+1$ B. $y=1-x$ C. $y=x-1$ D. $y=x$

37. Find the second derivative of the function $f(x)=\frac{1}{\sqrt{x}}$.

A. $\frac{3}{4}x^{-\frac{5}{2}}$ B. $3x^{-\frac{5}{2}}$ C. $\frac{4}{3}x^{-\frac{3}{4}}$ D. $-\frac{3}{4}x^{-\frac{5}{2}}$

38. The length l meters of a certain metal rod at temperature θ^0C is given by $l=1+0.00005\theta+0.0000004\theta^2$. Determine the rate of change of length, in $m/^0C$, when the temperature is 100^0C.

A. $0.00015m/^0C$ B. $0.00011m/^0C$ C. $0.00008m/^0C$ D. $0.00013m/^0C$

39. An alternating voltage is given by $v=100\sin 200t$ volts, where t is the time in seconds. Calculate the rate of change of voltage when $t=0.005s$.

A. 110 volts per second B. 10506 volts per second C. 10806 volts per second D. 250 volts per second

40. Find the derivative of the function $y=3e^{4x^6}$.

A. $24x^5e^{4x^6}$ B. $72x^5e^{4x^6}$ C. $12x^6e^{4x^6}$ D. $3e^{4x^6}$

41. Differentiate the equation $y=5x^3\ln 2x$.

A. $5x^2(1+3\ln 2x)$ B. $5x^2(1-3\ln 2x)$ C. $5x^3(1+3\ln 2x)$ D. $5x^3(1-3\ln 2x)$

Use the following information to answer question 42-43.

Let $f(x)=x^3+3x^2-4$

42. What are the critical points?

A. $(-2,0)$ and $(0,0)$ B. $(1,0)$ and $(0,0)$ C. $(1,0)$ and $(0,-4)$ D. $(-2,0)$ and $(0,-4)$

43. Which point is the relative minimum?

A. $(0,-4)$ B. $(-2,0)$ C. $(1,0)$ D. $(2,0)$

44. Evaluate the integral $\int \cos^7 x \sin x dx$.

A. $\frac{1}{8}\cos^8 x + c$ B. $-\frac{1}{8}\cos^8 x + c$ C. $\frac{1}{8}\sin^8 x + c$ D. $frac18\sin^8 x + c$

45. Find f given that $f'' = 12x + 24x^2$.

A. $f(x) = 2x^3 + 2x^4 + cx + d$ B. $f(x) = 4x^3 + 8x^4 + cx + d$ C. $f(x) = 2x^3 + 4x^4 + cx + d$
D. $f(x) = 6x^3 + 4x^4 + cx + d$

46. Find the partial derivative $\frac{\partial z}{\partial x}$, if $z = x^2 + xy + y^2$.

A. $2x+2y$ B. $2x+y+2y$ C. $2x+y+y^2$ D. $2x+y$

47. Differentiate the function $f(x) = (3x^{-5} + \frac{1}{2}x - 2)^{-\frac{1}{2}}$

A. $-\frac{1}{2}(3x^{-5}+\frac{1}{2}x-2)^{-\frac{3}{2}}(-15x^{-6}+\frac{1}{2})$ B. $-\frac{1}{2}(3x^{-5}+\frac{1}{2}x-2)^{\frac{1}{2}}(-15x^{-6}+\frac{1}{2})$ C. $-\frac{1}{2}(3x^{-5}+\frac{1}{2}x-2)^{-\frac{3}{2}}(-15x^{-4}+\frac{1}{2})$ D. $-\frac{1}{2}(3x^{-5}+\frac{1}{2}x-2)^{-\frac{1}{2}}(-15x^{-4}+\frac{1}{2})$

Use the following information to answer question 48-50.

The slope of a secant is given as

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Let $y = -9x^2 + 2$ be a curve.

48. Find the slope of the secant that passes through the curve.

A. $-9x^2 - 18x - \Delta x$ B. $-18x - 9\Delta x$ C. $-18x - \Delta x$ D. $-18x - (\Delta x)^2$

49. If $\Delta x = 3$, find the slope of the secant at $x = -2$.

A. -35 B. -29 C. 11 D. 21

50. Find the slope of the tangent line to the curve at $x = -3$.

A. -54 B. -27 C. 54 D. 27

51. Find $\frac{dy}{dx}$ if $x^2y + 2y^3 = 3x + 2y$.

A. $\frac{3-2xy}{(x^2+6y^2-2)}$ B. $\frac{3+2xy}{(x^2+6y^2-2)}$ C. $\frac{3-2xy}{(x^2-6y^2+2)}$ D. $\frac{3-2xy}{(x^2+6y^3-2)}$

52. Find the integral $\int \sec x \tan x$.

A. $\tan x$ B. $\sec x$ C. $\sec^2 x$ D. $\tan^2 x$

53. An electrician estimated that if a particular revamping program for providing electricity to rural areas were initiated, then t years after its starts, n thousand rural communities will receive direct benefits, where,

$$n = \frac{t^3}{3} - 6t^2 + 32t \quad 0 \le t \le 12.$$

For what value of t does the maximum number receive benefits?

A. $t = 1$ B. $t = 2$ C. $t = 3$ D. $t = 4$

54. Find the integral $\int \csc^2 x dx$.

A. $\cot x + c$ B. $-\cot x + c$ C. $\sec x + c$ D. $-\sec x + c$

55. Evaluate $\int (3e^x - 2\sec^2 x) dx$.

A. $3e^x - 2\tan x + c$ B. $3e^x + 2\tan x + c$ C. $e^x - 2\tan x + c$ D. $e^x + 2\tan x + c$

56. Evaluate the definite integral $\int_0^1 (e^{-x} + \sqrt{x}) dx$.

A. 1.566 B. 1.877 C. 1.299 D. 0.556

57. A gas suspends to the law pv = constant. When the volume is $3m^3$ the pressure is 150 kPa. Given that work done = $\int_{v_1}^{v_2} pdv$ determine the work done as the gas expands from $2m^2$ to a volume of $6m^3$.

A. 394.4 kJ B. 594.4 kJ C. 694.4 kJ D. 494.4 kJ

58. Evaluate $\int_{y=1}^{2} \int_{x=0}^{3} (1 + 8xy) dxdy$.

A. 56 B. 57 C. 58 D. 59

59. Find the derivative of $f(x) = x^5 \cos x$.

A. $5x^4 \cos x - x^5 \sin x$ B. $5x^4 \cos x + x^5 \sin x$ C. $5x^3 \cos x - x^4 \sin x$ D. $5x^2 \cos x + x^2 \sin x$

60. Evaluate the following the limit,

$$\lim_{t \to \infty} \frac{2x^4 - x^2 + 8x}{-5x^4 + 7}.$$

A. $\frac{2}{5}$ B. $\frac{2}{3}$ C. $-\frac{2}{5}$ D. $-\frac{2}{7}$