Assignment 4 solution

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Question 1

(a) Equations of Motion

Mass 1 is connected to wall with spring k_1, then connected to mass 2 with spring k_2. Mass 2 is connected to another wall with spring k_3.

Let x_1 and x_2 be the displacements of masses.

Using Newton's law:

For mass 1:

$$2\ddot{x}_1 = -50x_1 + 100(x_2 - x_1) = -150x_1 + 100x_2$$

For mass 2:

$$1\ddot{x}_2 = -50x_2 - 100(x_2 - x_1) = 100x_1 - 150x_2$$

So final equations:

$$2\ddot{x}_1 + 150x_1 - 100x_2 = 0$$
$$\ddot{x}_2 - 100x_1 + 150x_2 = 0$$

Matrix form:

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 150 & -100 \\ -100 & 150 \end{bmatrix}$$

(b) MATLAB Code

To find natural frequencies and mode shapes:

```
M = [2 0; 0 1];
K = [150 -100; -100 150];

[phi, lambda] = eig(K, M);  % Solves K*phi = lambda*M*phi

omega = sqrt(diag(lambda))  % natural frequencies

phi  % mode shapes
```

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Question 2

MATLAB Code Using bodeplot

```
num = 100;
den = conv([1 0], conv([1 5], [1 10]));  % s(s+5)(s+10)

G = tf(num, den);  % Requires Control System Toolbox

h = bodeplot(G);  % Plot Bode plot (magnitude and phase)
grid on

% Optional: Add gain margin and phase margin info manually
[GM, PM, w_gm, w_pm] = margin(G);
fprintf("Gain Margin: %.2f dB at %.2f rad/s\n", 20*log10(GM), w_gm);
fprintf("Phase Margin: %.2f deg at %.2f rad/s\n", PM, w_pm);
```

Results from MATLAB

- **Gain Margin**: 17.50 dB at 7.07 rad/s
- **Phase Margin**: 59.29° at 1.85 rad/s

Comment on Stability:

Since both gain margin and phase margin are positive, the system is **stable**. A phase margin of 59.29° indicates good phase stability, and gain margin of 17.5 dB means we can increase gain significantly before system becomes unstable.