

Conversions Float

$$\begin{aligned} .1_2 &= .100_2 = .4_8 \\ &= .1000_2 = .8_{16} \\ &= 1 \times 2^{-1} = .5_{10} \end{aligned}$$

$$\begin{aligned} .1_8 &= .001_2 = 1 \times 2^{-3} = 1/8 \\ .001_2 &= .0010_2 = .2_{16} = 2 \times 16^{-1} = 2/16 = 1/8 \\ &= 1/8 = .125_{10} \end{aligned}$$

$$\begin{aligned} .1_{16} &= .0001_2 = 1 \times 2^{-4} = 1/16 = .0625_{10} \\ &= .000100_2 = .04_8 = 4 \times 8^{-2} = 4/64 = 1/16 \end{aligned}$$

$$\begin{aligned} .1_{10} &\rightarrow .1 \times 16 = 1.6 \\ .6 \times 16 &= 9.6 \\ .6 \times 16 &= 9.6 \text{ repeats} \end{aligned}$$

$$= .1999\dots = .\underline{19}_{16} \leftarrow \text{repeat to infinity}$$

Note: $.19 < .\underline{19} < .19A$ Bounded

$$.\underline{19} = 1/16 + \sum_{i=2}^{\infty} 9/16^i = 1/16 + 9/16 \sum_{i=0}^{\infty} 16^{-i}$$

$$\begin{aligned} \text{Using } S &= \sum_{i=0}^N r^i = 1 + r + r^2 + \dots + r^N \\ - rS &= r + r^2 + \dots + r^N + r^{N+1} \\ \hline S(1-r) &= 1 - r^{N+1} \\ \lim_{N \rightarrow \infty} S &= \frac{1 - r^{N+1}}{1-r} = \frac{1}{1-r} \end{aligned}$$

mer 1/10

Then

$$\begin{aligned} .19_{16} &= 1/16 + 9/16^2 \left(\frac{1}{1 - 1/16} \right) = 1/16 + 9/16^2 \left(\frac{16}{15} \right) \\ &= 1/16 + 9/240 \\ &= \frac{15+9}{240} = 24/240 \\ &= \underline{\underline{.1_{10}}} \end{aligned}$$

Proof that $.19_{16} = .1_{10}$

. 1 9 9 9 9 9 9₁₆

Every
4 bits

. 0001 1001 1001 1001 1001 1001₂

can be written as

$. \underline{00011}_2$ where 0011 repeats forever

Now for base 8, group every 3 bits

. 000 110 011 001 100 110₂

. 0 6 3 1 4 6

$. \underline{06314}_8$

Note! The sequence repeats but looks very different depending on the base msh