

Number Systems

Number Systems (cont)

Scientific Notation (cont)

$$0.06640625_{10} = 0.6640625 \times 10^{-1}$$

Convert to Binary

Positions →	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	
$0.06640625_{10} =$	0.5	0.25	0.125	0.0625	0.03125	0.015625	0.0078125	0.00390625	
				1×2^{-4}				1×2^{-8}	$= 0.00010001_2$

$$0.00010001_2 = 0.10001 \times 2^{-3}$$

Once you have a number in Base 2, 8, or 16 then the conversion to any of the other is direct

0.00010001_2	$=$.0001	0001	Base 2 use every 4 digits to convert to Base 16
	$=$	0.1	1	Base 16 $= 0.11_{16}$

0.00010001_2	$=$.000	100	010	Base 2 use every 3 digits to convert to Base 8
	$=$.0	4	2	$= 0.042_8$

As a check convert Base 8 result back to Base 10 and see if same value

4×8^{-2}	2×8^{-3}	
0.0625	0.00390625	$= 0.06640625$

Therefore $0.6650625 \times 10^{-1} = 0.10001 \times 2^{-3} = 0.42 \times 8^{-1} = 0.11 \times 16^0$
 where the base is represented by the power exponent

Quickest Conversion for Integer

Example converting base 10 to base 16, then base 2, and finally base 8

1023.60546875₁₀

The approach should be to split the number between the integer and float.
2 parts, convert the integer first, then the floating part

Here is how I quickly convert the integer, I mod with the base, divide by the base and repeat for each digit till less than 1
All the following are integer operations, no floats/fractions involved

				Value	Digit
1023	→	1023 % 16	=	F	1
		(1023/16 = 63) % 16	=	F	2
		(63/16 = 3) % 16	=	3	3

So, 1023 Base 10 is equivalent to 3FF Base 16

Convert to Base 2

3	F	F	→ Base 16
0011	1111	1111	→ Base 2

Convert to Base 8

001	111	111	111	→ Base 2
1	7	7	7	→ Base 8

As a check, convert the Base 8 solution back to Base 10, if agree then all intermediate steps check. Is 1777 Base 8 = 1023 Base 10
Add

1x8 ³	7x8 ²	7x8 ¹	7x8 ⁰	
512	448	56	7	= 1023

Quickest Conversion for Float

Example converting base 10 to base 16, then base 2, and finally base 8

$$1023.60546875_{10}$$

Now for the quick conversion of the fraction. Simply multiply by the base and keep the integer, repeat with remainder

0.60546875 Base 10					Value	Digit
0.60546875	x	16	=	9.6875	9	1
0.6875	x	16	=	11	B	2

There is no fraction/decimal to continue therefore 0.60546875 Base 10 = 0.9B Base 16

Converting Base 16 to Base 2

.9	B	→ Base 16
0.1001	1011	→ Base 2
0.100	110	→ Base 2
0.4	6	→ Base 8

Checking Base 8 final value and comparing to Base 10 starting point will assure all intermediate steps were correct.

$$\begin{array}{r} 4 \times 8^{(-1)} \\ 0.5 \end{array} \quad \begin{array}{r} 6 \times 8^{(-2)} \\ 0.09375 \end{array} \quad \begin{array}{r} 6 \times 8^{(-3)} \\ 0.01171875 \end{array} = 0.60546875 \rightarrow \text{Base 10}$$

It checks, so the conversion equivalence is

$$1023.60546875_{10} = 3FF.9B_{16} = 1111111111.10011011_2 = 1777.466_8$$

$$0.102360546875 \times 10^{(4)} = 0.3FF9B \times 16^{(3)} = 0.111111111110011011 \times 2^{(10)} = 0.1777466 \times 8^{(4)}$$