How to represent negative numbers

P-Adic Numbers + 2's Compliment

We can create a measure

$$\left| Base^{d} \right| = \frac{1}{Base^{d}}$$

Let Base = 10

$$\left| (9 - -1) \right| = \left| 10^{1} \right| = \frac{1}{10}$$

$$\left| (99 - 1) \right| = \left| 10^{2} \right| = \frac{1}{10^{2}}$$

$$\left| (999 - -1) \right| = \left| 10^{3} \right| = \frac{1}{10^{3}}$$

$$\left| (\underline{9} - -1) \right| = \left| 10^{\infty} \right| = \frac{1}{10^{\infty}} = 0$$

Infinite 9's

So $\underline{9} = -1$

since the difference is 0

Now let's try for Base 2

$$|(1 \cdots 1)| = |2^1| = \frac{1}{2}$$

$$|11 \cdots 1| = |2^2| = \frac{1}{2^2}$$

$$|111 \cdots 1| = |2^3| = \frac{1}{2^3}$$

$$|(\underline{1} \cdots 1)| = 2^\infty = \frac{1}{2^\infty} = 0$$

So $\underline{1} = -\underline{1}$

Infinite $\underline{1}$'s

We have seen infinite digits to the right like $12/99 = .121212\ldots = .\underline{12}$

Now we have infinite digits to the left to represent negative values

$$\underline{1} = \ldots 111111 = -\underline{1}$$

So no matter how many bits or bytes

1 Byte → 11111111 = −1

2 Bytes → 1111111111111111 = −1

etc, ...

Now to formalize how to do the 2's compliment

a) Decide the data width

example 1 Byte, 8 bits

b) Take a number in Base 10 + Convert Base 2

$$25_{10} = 2^4 + 2^3 + 2^0$$

$$= 00011001$$

c) Form the 1's compliment, just flip bits

00011001

flip 11100110

0's → 1's
1's → 0's

Now add 1

	00011001
1's comp	11100110
	+1
2's comp	11100111 = -25

We know 25 - 25 = 0

So

$$\begin{array}{r l}
00011001 & = 25 \\
11100111 & = -25 \\
\hline
1\,|\,00000000 & = 0
\end{array}$$

off the end

We can find negative of any Base

$25_{10} = 19_{16}$ → 1 Byte representation

1's comp flip bit = $E6_{16}$

$$\begin{array}{r r}
15 & 15 \\
-1 & -9 \\
\hline
14 = E & 6
\end{array}$$

+1

2's comp = E7

$$\begin{array}{lrcl} \text{So} & {}^{1}\,19_{16} & = & 25_{10} \\ & E7_{16} & = & -25_{10} \\ \hline \text{off end of 1 Byte} & 100_{16} & = & 0 \end{array}$$

The idea is subtracting values is harder than adding.

So now we can easily take any number and use logic to make negative than add.

As a matter of fact adding is just a logic operation

Subtracting is a logic flip with addition

Multiplication is a shifted addition which again is just logic

Division is a shifted subtraction which is a logic flip add ultimately just a series of logic operations.