Number Representations

123_{10} → One Hundred Twenty Three Base 10

$= 100_{10} + 20_{10} + 3_{10}$

$= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

Each place represents a power of 10. Hence, a Base 10 number

$1 \quad 2 \quad 3_{10}$

$10^2 \quad 10^1 \quad 10^0$

A Base 10 number has 10 digits

Base 10 = $\{0, 1, 2, 3, \ldots 9\}$

10 digits

The digits in Base 10 are are 0 to 9!

Computers do not work in Base 10. They work in Base 2. We prefer using Base 8 and Base 16 since it saves space and can easily be converted.

Base 2 → 2 digits = {0, 1}

Base 8 → 8 digits = {0, 1, 2, ... 7}

Base 16 → {0, 1, 2, ... 9, A, B, C, D, E, F}

Equivalence Number Representation

BASE 2	BASE 8	BASE 10	BASE 16	2	8	10	16
0	0	0	0	1000	10	8	8
1	1	1	1	1001	11	9	9
10	2	2	2	1010	12	10	A
11	3	3	3	1011	13	11	B
100	4	4	4	1100	14	12	C
101	5	5	5	1101	15	13	D
110	6	6	6	1110	16	14	E
111	7	7	7	1111	17	15	F

The ease of conversion

$8 = 2^3$ $16 = 2^4$

Base 2 can easily be converted into Base 8 and Base 16.

Base 8 is 3 places of Base 2
Base 16 is 4 places of Base 2

1010101101_2

Look at previous table to help

$\underline{1}\,\underline{010}\,\underline{101}\,\underline{101}_2$ $\underline{1010}\,\underline{1011}\,\underline{01}_2$

$2\ 5\ 3\ 5_8$ $5\ 5\ D_{,16}$

Every 3 Places Every 4 Plases

Which is equivalent in Base 10

$2\times8^3 + 5\times8^2 + 3\times8^1 + 5\times8^0 = 5\times16^2 + 5\times16^1 + D\times16^0$

$1024 + 320 + 24 + 5 = 1280 + 80 + 13$

$1373_{10} = 1373_{10}$

The conversion should be equivalent!

It Checks

Additional Examples

$1\ 2\ 3_8$ = ? in Base 2, 10, 16

$001\ 010\ 011_2$ Every 3 places from Base 2

0010100011_2

$0\ 5\ 3_{16}$ Every 4 places from Base 2

$123_8 = 1\times8^2 + 2\times8^1 + 3\times8^0$

$64 + 16 + 3 = \underline{\underline{83_{10}}}$

$1010011_2 = 1\times2^6 + 1\times2^4 + 1\times2^1 + 1\times2^0$

$64 + 16 + 2 + 1 = \underline{\underline{83_{10}}}$

$53_{16} = 5\times16^1 + 3\times16^0$

$80 + 3 = \underline{\underline{83_{10}}}$

The conversion back to Base 10 shows the equivalence of all 3 representations.

Another Example

$123_{16} = ?$ in Base 2, 8, 10

$1 \quad 2 \quad 3_{16}$

$0001 \quad 0010 \quad 0011_2$ Use 4 places for Base 2

$\underbrace{0001}\ \underbrace{0010}\ \underbrace{0011}_2$ From Base 2 use every 3 places

$4 \quad 4 \quad 3_8$

$443_8 = 100100011_2$

$4\times8^2+4\times8^1+3\times8^0 = 1\times2^8+1\times2^5+1\times2^1+1\times2^0$

$256+32+3 = 256+32+2+1$

$291_{10} = 291_{10}$

The conversion to Base 10 and its equivalence shows how correct the results are by agreement!

Test your knowledge. Complete the table by calculating and filling in the unknowns. Show the conversions!

Base 10	Base 2	Base 8	Base 16
127 =		=	=
=	10101	=	=
=		=	71 =
=		=	= AB

$127_{10} \longrightarrow 127 \% 16 = F$

$int(127/16) \% 16 = 7$

Just keep %ing and dividing by the base

$127_{10} = 7F_{16}$

$= 0111\ 1111_2$ every 4 places

$= 1\ 111\ 111_2$ every 3 places for Bac 8

$1\ 7\ 7_8$

$177_8 = 1 \times 8^2 + 7 \times 8^1 + 7 \times 8^0$

$= 64 + 56 + 7$

$= 127_{10}$

Base 10 → 16 → 2 → 8 → Back to 10

Same Answer it checks

$10101_2 = 25_8 = 15_{16}$

$1\times2^4 + 1\times2^2 + 1\times2^0 = 2\times8^1 + 5\times8^0 = 1\times16^1 + 5\times16^0$

$16 + 4 + 1 = 16 + 5 = 16 + 5$

$21_{10} = 21_{10} = 21_{10}$

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every 3 | even 4

$71_8 = 111\ 001_2 = 39_{16}$

$7\times8^1 + 1\times8^0 = 3\times16^1 + 9\times16^0$

$56 + 1 = 48 + 9$

$57_{10} = 57_{10}$

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4 | 3

$AB_{16} = 1010\ 1011_2 = 253_8$

$10\times16 + 11\times16^1 = 2\times8^2 + 5\times8^1 + 3\times8^0$

$160 + 11 = 128 + 40 + 3$

$171_{10} = 171_{10}$