# Number Systems

| | | Unique Digits |
|---|---|---|
| Binary | {0,1} | 2 |
| Octal | {0,1,2,3,4,5,6,7} | 8 |
| Decimal | {0,1,2,3,4,5,6,7,8,9} | 10 |
| Hexadecimal | {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F} | 16 |

Are these the only representations? → No, Just those commonly related to Computer Science

**What does a number represent and how can it be written?**

| | | | | |
|---|---|---|---|---|
| $456_{10}$= | 456 Base 10 = | $4x10^2$ | + $5x10^1$ | + $6x10^0$ |
| $456_{10}$= | | 4x10^2 | + 5x10^1 | + 6x10^0 |
| $456_{10}$= | | 400 | 50 | 6 = 456 |

In Base 10 every position represents a power of 10.

**Scientific Notation**

$456_{10}$=
456x10^(0)
4,560x10^(-1)
45,600x10^(-2)
456,000x10^(-3)
45.6x10^(1)
4.56x10^(2)
0.456x10^(3)

Many different ways to represent the same thing

***The above definition provides the key to the representation of a number in another Base as well as it's equivalence!***

# Integer Representations

**Example converting base 10 to base 16 requires converting by powers to the Base 16**

| Positions → | 16^2 | 16^1 | 16^0 | |
|---|---|---|---|---|
| $456_{10}$= | 256 | 192 | 8 | |
| | 1x16^2 | 12x16^1 | 8x16^0 | |
| | 1x16^2 | Cx16^1 | 8x16^0 | = $1C8_{16}$ |
| | | | | = 1C8 Base 16 |

| | | | | |
|---|---|---|---|---|
| $1C8_{16}$ = | 1x16^2 | Cx16^1 | 8x16^0 | |
| | 1x16^2 | 12x16^1 | 8x16^0 | |
| | 256 | 192 | 8 | = $456_{10}$ |

**Example converting base 10 to base 2 requires converting by powers to the Base 2**

| Positions → | 2^8 | 2^7 | 2^6 | 2^5 | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 | |
|---|---|---|---|---|---|---|---|---|---|---|
| $456_{10}$= | 256 | 128 | 64 | 0 | 0 | 8 | 0 | 0 | 0 | |
| | 1x2^8 | 1x2^7 | 1x2^6 | 0x2^5 | 0x2^4 | 1x2^3 | 0x2^2 | 0x2^1 | 0x2^0 | = $111001000_2$ |

$456_{10}$= 256 + 128 + 64 + 8 = $111001000_2$

**Example converting base 10 to base 8 requires converting by powers to the Base 8**

| Positions → | 8^2 | 8^1 | 8^0 | |
|---|---|---|---|---|
| $456_{10}$= | 448 | 8 | 0 | |
| | 7*8^2 | 1x8^1 | 0x8^0 | = $710_8$ |

## Memorize

| Decimal | Binary | Octal | Hex |
|---|---|---|---|
| 0 | 0000 | 00 | 0 |
| 1 | 0001 | 01 | 1 |
| 2 | 0010 | 02 | 2 |
| 3 | 0011 | 03 | 3 |
| 4 | 0100 | 04 | 4 |
| 5 | 0101 | 05 | 5 |
| 6 | 0110 | 06 | 6 |
| 7 | 0111 | 07 | 7 |
| 8 | 1000 | 10 | 8 |
| 9 | 1001 | 11 | 9 |
| 10 | 1010 | 12 | A |
| 11 | 1011 | 13 | B |
| 12 | 1100 | 14 | C |
| 13 | 1101 | 15 | D |
| 14 | 1110 | 16 | E |
| 15 | 1111 | 17 | F |

**8 = 2^3** **Octal = 3 positions in Binary**
**16 = 2^4** **Hexadecimal = 4 positions in Binary**

| | | | | | |
|---|---|---|---|---|---|
| $456_{10}$= | 1 | C | 8 | Base 16 | 1C8 |
| | 0001 | 1100 | 1000 | Base 2 | 111001000 |
| | | | | | |
| | 111 | 001 | 000 | Base 2 | 111001000 |
| | 7 | 1 | 0 | Base 8 | 710 |

Easy to convert between the power of 2 Bases

**Procedures → Convert Base 10 to Base 16 then Base 16 to Base 2 then Base 2 to Base 8**