Statistical Inference: Course Project

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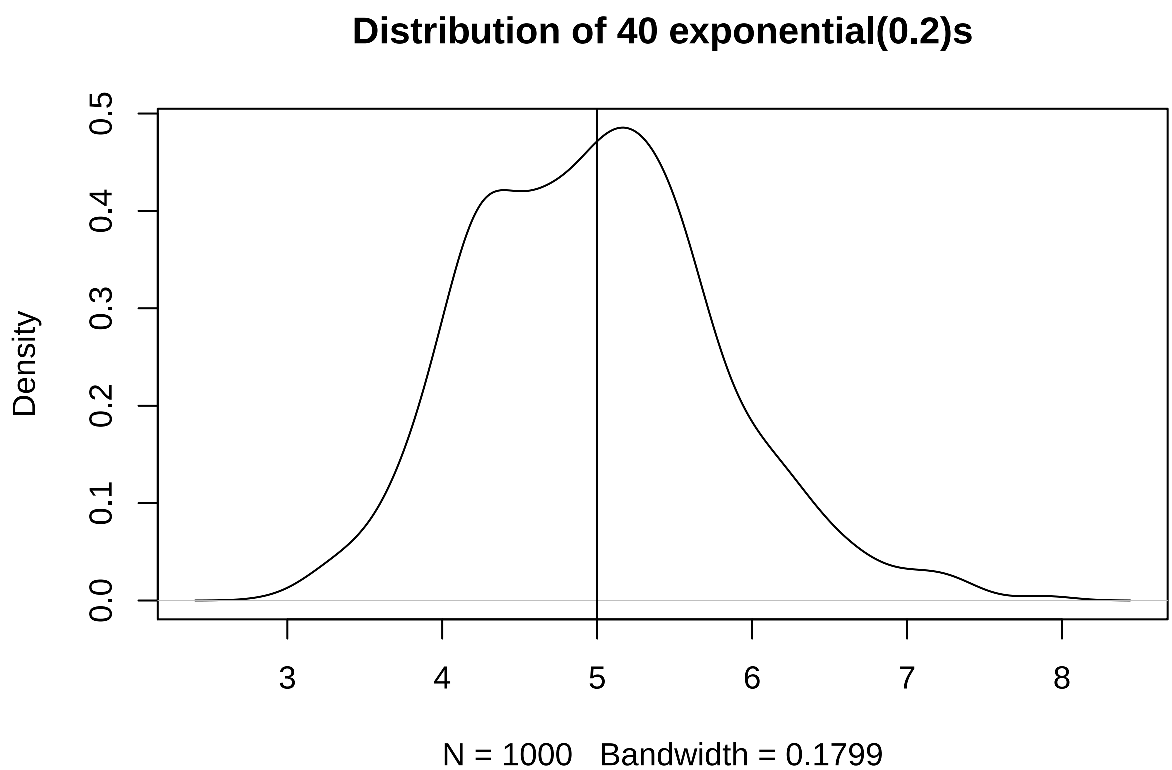
Part I.

The purpose of this report is to compare the distribution of sample means of an exponential distribution with 1000 simulations and lambda at 0.2.

1. Where the distribution is centered at comparing to the theoretical center of the distribution.

I plotted the data of sample means (mean of 40 exponentials). The vertical (x=5) is the theoretical center of the distribution. The plot shows that the peak of the distribution of the sample means is a little less than 5.



2. How variable it is comparing to the theoretical variance of the distribution. The standard deviation (SD) of each sample of 40 is centered around the true SD, 1/lambda. But the SD of the means of the 1000 40-exponentials samples is the standard error, which is S/sqrt(n). So the theoretical standard deviation of the 1000 means should be (1/lambda)/sqrt(40), and the variance should be

```
## [1] 0.625
```

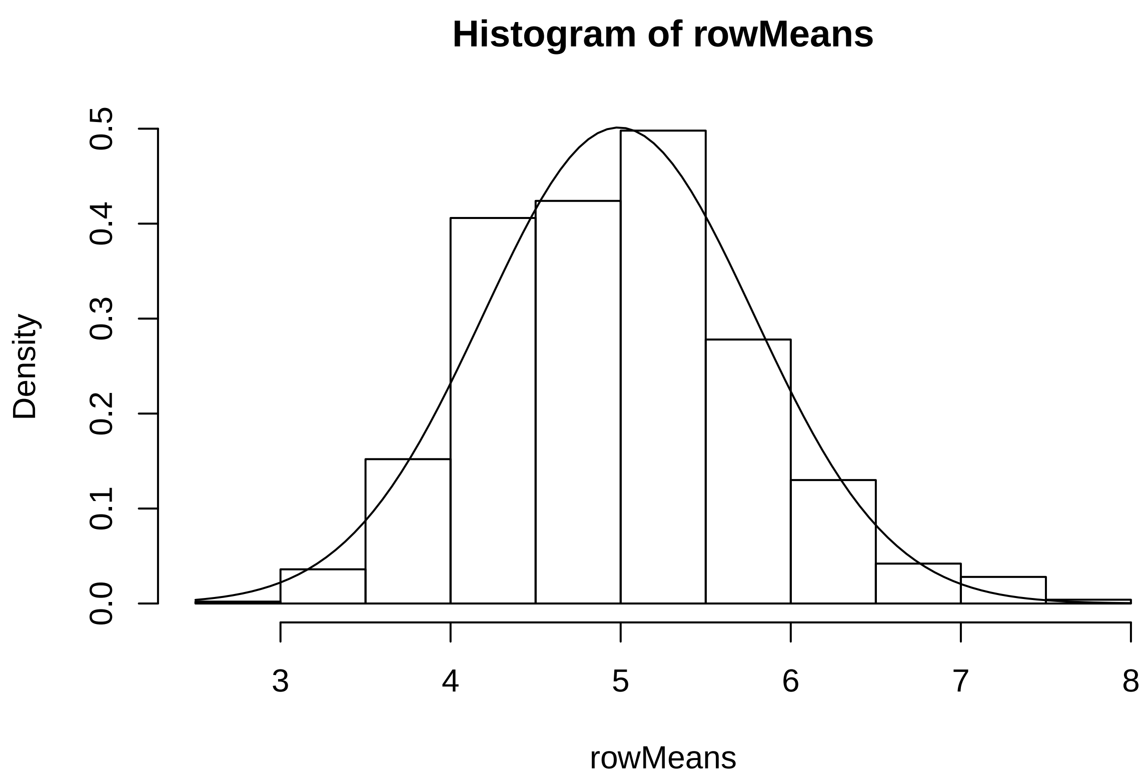
The variance of the dataset, by calculating its standard deviation then squaring it, gives

```
## [1] 0.6334
```

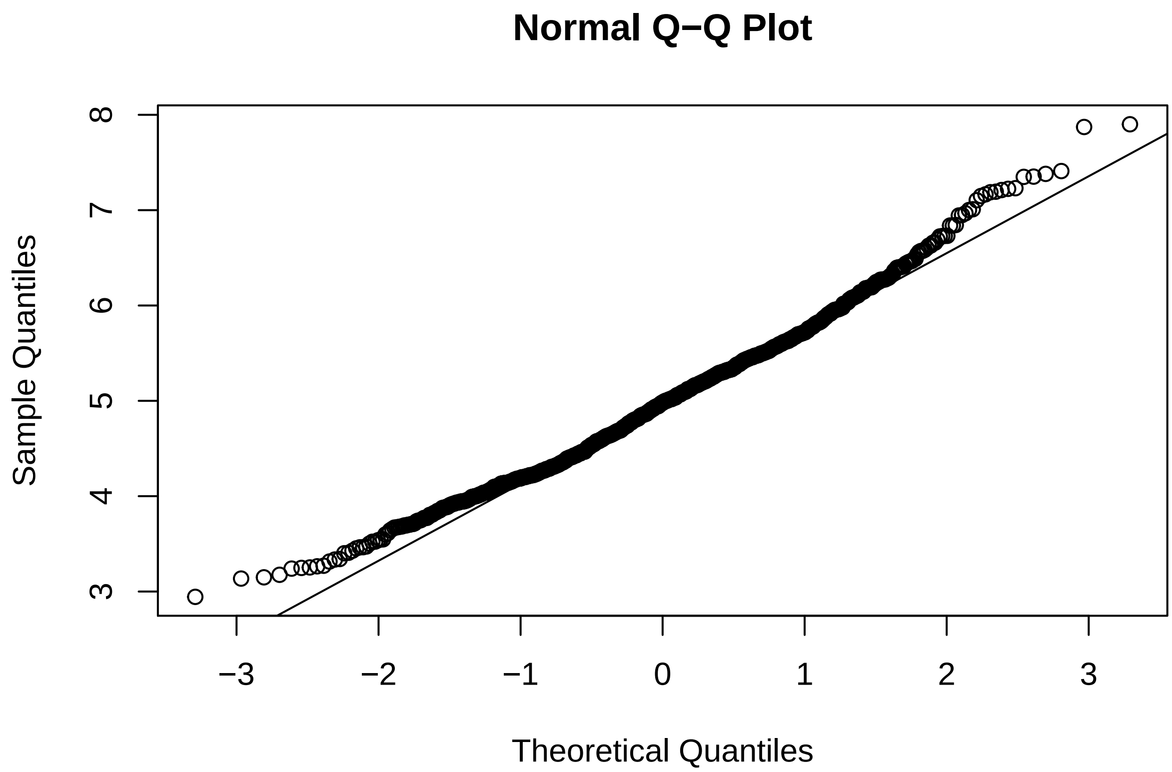
The theoretical variance is greater than the distribution's variance, so the distribution is less variable that its theoretical counterpart.

3. The distribution is approximately normal. The histogram of the distribution of sample means with a curve that describe the distribution as normal with the data's means and standard deviation.

Visually, the distribution seems normal.



To evaluate how close this distribution is to normal distribution, we can look at the sample means plotted in the quantile-quantile plot and against the line that represents normal distribution.



The samples means fall pretty much along the theoretical line, except at the quantiles farthest from the center on both sides, we see that the sample means deviate more from the line. Overall, we can say that the distribution is indeed approximately normal.

4. Evaluate the coverage of the confidence interval for 1/lambda: X±1.96*(S/sqrt(n)). I calculated the 95% confidence intervals for each sample of 40 exponentials (1000 confidence intervals in total). This is what this table of intervals looks like:

```
##   lower upper  mean
## 1 2.597 4.348 3.473
## 2 3.307 6.580 4.944
## 3 4.619 9.058 6.839
## 4 2.862 5.681 4.271
## 5 3.348 5.895 4.621
## 6 3.195 7.505 5.350
```

I calculated how many of those confidence intervals cover the theoretical mean of 1/lambda.

```
## [1] 0.927
```

93.7% of the time, the confidence intervals of the data contain the true mean. But it is lower than the 95% confidence level used in the equation.

Conclusion: As we can see, this dataset of sample means has a distribution slightly different from the theoretical distribution, on the mean, variance, and confidence level. One guess is that the data is not large enough. If we increase the simulations to a number higher than 1000, we might get closer to the parameters of the true exponential distribution.

Part II.

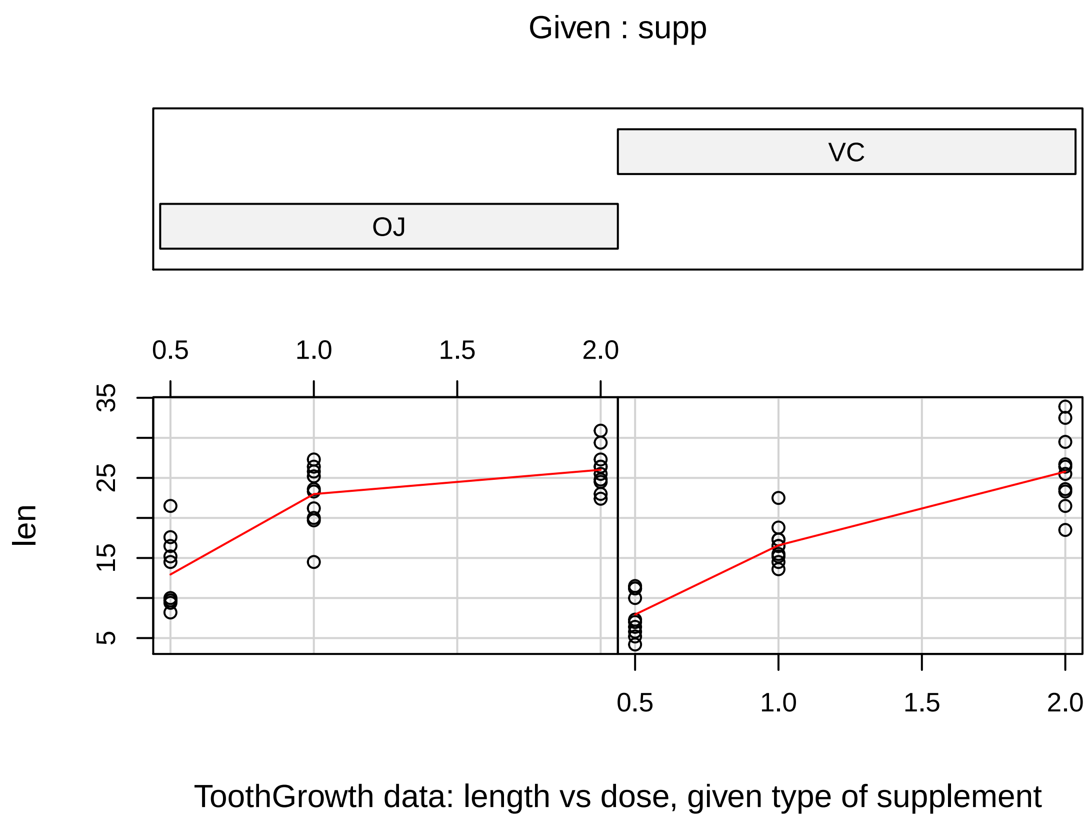
Analysis of the ToothGrowth Data

1. Provide a summary of the dataset.

	len	supp	dose
1	Min. : 4.2	OJ:30	Min. :0.50
2	1st Qu.:13.1	VC:30	1st Qu.:0.50
3	Median :19.2		Median :1.00
4	Mean :18.8		Mean :1.17
5	3rd Qu.:25.3		3rd Qu.:2.00
6	Max. :33.9		Max. :2.00

Table 1: Summary Statistics

Exploratory analysis of the data, examining the length of growth by supplement and dosage.



We observe several patterns. First, the tooth growths in the group receiving OJ are higher than those in the VC group. Second, in both OJ and VC groups, the higher dosage level, the more the tooth growth.

2. Use confidence intervals and hypothesis tests to compare tooth growth by supplement and dose. Tables 2 and 3 compare the standard deviations of the supplement type groups and dosage groups. We can see that the variances are indeed not equal across comparison groups.

	supp	mean	sd	N	se
1	OJ	20.663	6.606	30	1.206
2	VC	16.963	8.266	30	1.509

Table 2: Supplement data

	dose	mean	sd	N	se
1	0.500	10.605	4.500	20	1.006
2	1.000	19.735	4.415	20	0.987
3	2.000	26.100	3.774	20	0.844

Table 3: Dose data

Since this is not a pre/post test experiment where the same subjects receive different treatment, we cannot assume a matched t-test sample. Tables above also confirm the fact that the standard deviations (and hence variances) among the groups are not the same. So I assume the true variance is unequal and cannot be pooled and I use an independent t-test for comparing the supplement groups and dosage groups (basically all comparison groups).

I use a t-test to examine the null hypothesis that the tooth growths from the two supplement groups orange juice (OJ) v. ascorbic acid (VC) are the same.

```
    Welch Two Sample t-test

data:  len by supp
t = 1.915, df = 55.31, p-value = 0.06063
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.171  7.571
sample estimates:
mean in group OJ mean in group VC
           20.66            16.96
```

The mean tooth growth in the OJ group is higher than that of the VC group, but the test result is telling us that this difference is not significant at the 95% confidence level and p-values is greater than 0.05. We cannot reject the null hypothesis that there is no significant difference in tooth growth between the groups that received orange juice and ascorbic acid.

I use a pair-wise t-test to compare the tooth growths among the three groups receiving vitamin C dosage level of 0.5mg, 1mg, and 2mg. Again, the null hypothesis is that there is no significant difference in tooth growths in the comparison groups (for example, 0.5mg v. 1mg, or 1mg v. 2mg, etc).

```
    Pairwise comparisons using t tests with non-pooled SD

data:  ToothGrowth$len and ToothGrowth$dose

  0.5     1
1 2.5e-07 -
2 1.3e-13 1.9e-05

P value adjustment method: holm
```

The table shows comparisons among the three dosage levels. All of the p-values are less than 0.05, which means that tooth growth among all three groups with different dosage levels are significantly different.

Conclusion: The dosage levels of vitamic C, rather than the supplement type, make significant difference in the tooth growth of the subjects.