D605 – Optimization

Performance Assessment #2 – Solve an Optimization Problem

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D605 – Optimization: Solve an Optimization Problem

A: Identify the Optimization Problem

Amazon Air seeks to minimize cargo shipment costs from two main hubs and three focus cities to 65 distribution centers worldwide. Each center has a monthly demand, and each hub and focus city have a maximum processing capacity. The goal is to determine the optimal flow of cargo from hubs to focus cities to centers, or directly from hubs to centers while satisfying demand and not exceeding capacity limits.

B. Create Mathematical Representations of the Components of the Optimization Problem

B1. Objective Function Expression

The goal of the objective function is to minimize the total cost of shipments across three pathways: from hubs to focus cities, from hubs to centers, and from focus cities to centers. This is represented mathematically as:

$$min\left(\sum_{i=1}^{2}\sum_{j=1}^{3} c_{ij}x_{ij} + \sum_{i=1}^{2}\sum_{k=1}^{65} c_{ik}y_{ik} + \sum_{j=1}^{3}\sum_{k=1}^{65} c_{jk}z_{jk}\right)$$

Where

- *xij*: Tons shipped from hub *i* to focus city *j*
- *yik*: Tons shipped from hub *i* into distribution center *k*
- *zjk:* Tons shipped from focus city *j* into distribution center *k*
- *cij, cik, cjk:* Unit shipping costs for each respective route

B2. Constraint Expressions

1. Hub Capacity Limits

Each hub (CVG and AFW) can only handle a limited amount of cargo per month. This includes both direct shipments to centers and indirect shipments via focus cities.

$$\sum_{j=1}^{3} X_{ij} + \sum_{k=1}^{65} y_{ik} \le HubCapacity_i, \qquad \text{for } i = 1, 2$$

The above means for each hub *i,* the total tons shipped to all focus cities *j* plus the total tons shipped directly to all centers *k* must not exceed that hub's monthly capacity.

2. Focus City Flow Balance

All tons that arrive at a focus city from hubs must be sent out to centers. Nothing gets stored or lost in the process.

$$\sum_{i=1}^{2} X_{ij} = \sum_{k=1}^{65} z_{jk}\,, \qquad \text{for } j = 1, 2, 3$$

The above means for each focus city *j,* the total cargo it receives from both hubs *i* must equal the cargo it sends out to all centers *k.* This maintains cargo balance and avoids buildup or shortage at the focus city.

3. Focus City Capacity Constraints

Focus cities also have a limit on how much cargo they can process per week or month.

$$\sum_{k=1}^{65} z_{jk} \le \mathrm{FocusCapacity_j}, \qquad \text{for } j = 1, 2, 3$$

Meaning, for each focus city *j* has a maximum monthly processing capacity. The total tons it sends to all centers *k* must not exceed this amount.

4. Center Demand Satisfaction

Each distribution center has a fixed amount of cargo demand per month. This must be met, either through direct shipments fro hubs or via focus cities.

$$\sum_{i=1}^{2} y_{ik} + \sum_{j=1}^{3} z_{jk} = \mathrm{Demand_k}, \qquad \text{for } k = 1, 2, \dots, 65$$

Which means each center *k* must receive exactly the number of tons it needs, whether directly from hubs *i* or via focus cities *j*. The model can choose the cheaper path, but the quantity received must match demand precisely – not more, not less.

5. Non-Negativity Constraints

   Shipping quantities cannot be negative. You cannot ship a negative ton of cargo. This is impled, but still worth noting.

$$X_{ij} \geq = \mathbf{0}, \quad \mathbf{y_{ik}} \geq 0, \quad \mathrm{z_{ik}} \geq 0$$

   The above means each shipment variable must be zero or more – you can't ship negative amounts of cargo between any nodes in the network.

B3. Decision Variables

These variables define how much cargo flows through each route and will be optimized to minimize total cost

- x_{ij}: Cargo shipped from hub *i* (CVG, AFW) to focus city *j* (Leipzig, Hyderabad, San Bernardino)
- y_{ik}: Cargo shipped from hub *i* to center *k*
- z_{jk}: Cargo shipped from focus city *j* to center *k*

C. Describe the Approach used to Solve the Optimization

This optimization problem is formulated as a linear program, which is appropriate because the objective function and constraints are all linear, and the decision variables are continuous shipment amounts. The model's objective is to minimize total shipment cost while satisfying customer demand and honoring all hub and focus city capacities.

C1.Optimzation Method or Algorithm Used to Solve

The chosen method is the Simplex algorithm, a standard approach for solving linear programming problems. It efficiently navigates feasible solutions to find the optimal one, given the cost structure, constraints, and valid routes.

C2. Tools and Technologies Used to Solve the Problem

The model will be implemented in Python using the PuLP library, which allows for clear definition of the objective function, decision variables, and constraints. PuLP integrates with solvers like CBC to produce optimal solutions. Supporting tools such as Excel or Google Sheets may be used for organizing cost and demand data before model execution.

D. Risks and Limitations In Approach

The primary risk lies in assuming that shipping costs, capacities, and demands are fixed and known. Real-world variables such as fluctuating fuel prices, weather delays, or labor shortages could affect the accuracy of the model. Another limitation is that the model assumes exact fulfillment of each center's demand, with no allowance for partial delivery or surplus, which is unrealistic in a real-world scenario. Furthermore, this linear model does not capture advanced logistics factors like aircraft routing schedules, priority freight handling, or volume discounts. Despite these limitations, the linear programming approach offers a strong baseline for cost-efficient logistics planning within the constraints provided.

References

No other sources were used outside of the WGU course materials provided