First name: ______________ Last name: _____________ Student ID: ______________

Algebra 1 Homework

1. Let A be the sum of ten positive real numbers and let B be the sum of the reciprocals of these ten numbers. What is the smallest possible value for AB?

2. For which real numbers a does the equation $|x-1|-2|x-2|+2|x-3|-|x-5|=a$ have a unique solution. Here $|x|$ denotes the absolute value of x, defined by $|x|=x$ if $x\ge 0$ and $|x|=-x$ if $x<0$.

3. Let $\square$ be a binary operation defined on the set of nonnegative integers. (This means that if x and y are any two nonnegative integers, then $x \square y$ is a nonnegative integer determined by x and y.) Now suppose that

(a) $(x+1) \square 0 = (0 \square x)+1$
(b) $0 \square (y+1) = (y \square 0)+1$, and
(c) $(x+1) \square (y+1) = (x \square y)+1$

are satisfied for all nonnegative integers x and y. If $1100 \square 450 = 2000$, find $1723 \square 3421$ and prove that your answer is correct.

4. Let $\square$ be a binary operation defined on the set of nonnegative integers. (This means that if x *and* y are any two nonnegative integers, then $x \square y$ is a nonnegative integer determined by x *and* y.) Now suppose that the formula $(x \square y)(y \square z) = x \square z$ holds for all nonnegative integers x, y and z. If $23 \square 47 \neq 0$, compute $61 \square 89$.

5. Find all positive real numbers x, y and z such that $x=\dfrac{1+z}{1+y}, \quad y=\dfrac{1+x}{1+z}, \quad z=\dfrac{1+y}{1+x}$.