AA 274A: Principles of Robot Autonomy I
Problem Set 1

Name: Pei-Chen Wu
SUID: pcwu1023

Problem 1

(i) Write a set of linear equations in the coefficients x_i, y_i for $i = 1, ..., 4$.
$x(0) = 0 = x_1$, $\dot{x} = 0 = x_2$, $x(t_f) = 5 = x_1 + 15x_2 + 225x_3 + 3375x_4$, $\dot{x}(t_f) = 0 = x_2 + 30x_3 + 675x_4$
$y(0) = 0 = y_1$, $\dot{y} = -0.5 = y_2$, $y(t_f) = 5 = y_1 + 15y_2 + 225y_3 + 3375y_4$, $\dot{y}(t_f) = -0.5 = y_2 + 30y_3 + 675y_4$

(ii) Why can we not set $V(t_f) = 0$?
If we set $V(t_f) = 0$, the unicycle will take infinite time in order to reach this final state condition at goal position. The J matrix will become non-invertible when $V = 0$.

(iii) CODE

(iv) CODE

(v) Please include differential_flatness.png in your write up.

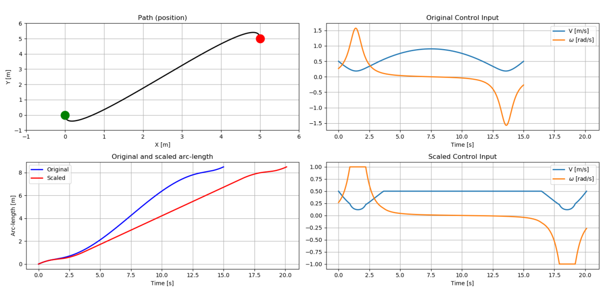

Figure 1: differential flatness

(vi) Simulation of open loop trajectory.

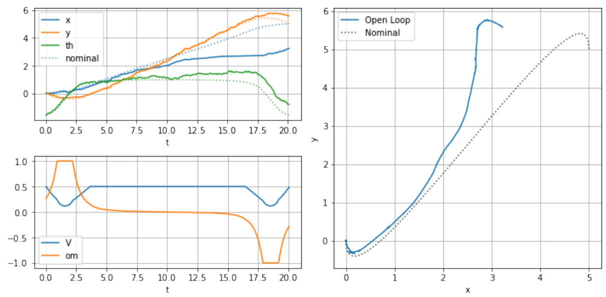

Figure 2: Simulation of the system with disturbances

Problem 2

(i) CODE

(ii) CODE

(iii) Please include all three plots (forward, reverse, and parallel) in your write up.

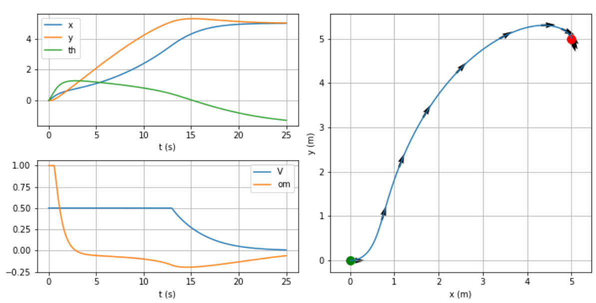

Figure 3: Forward

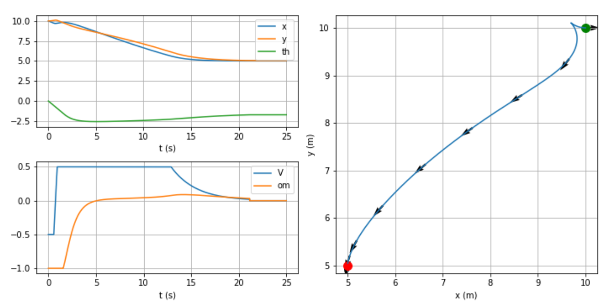

Figure 4: Reverse

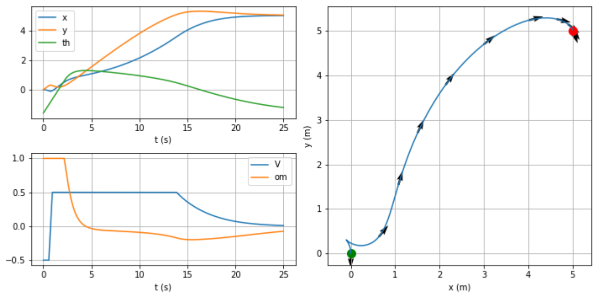

Figure 5: Parallel

Problem 3

(i) Write down a system of equations for computing the true control inputs (V, ω) in terms of the virtual controls $u_1, u_2 = (\ddot{x}, \ddot{y})$ and the vehicle state.

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \cos\theta & -V\sin\theta \\ \sin\theta & -V\cos\theta \end{bmatrix} \begin{bmatrix} \dot{V} \\ \omega \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

We can solve for $(\dot{V}, \omega)$ from the above equations with the desired control law (u_1, u_2).

(ii) CODE

(iii) Please include the resulting plot of trajectory tracking.

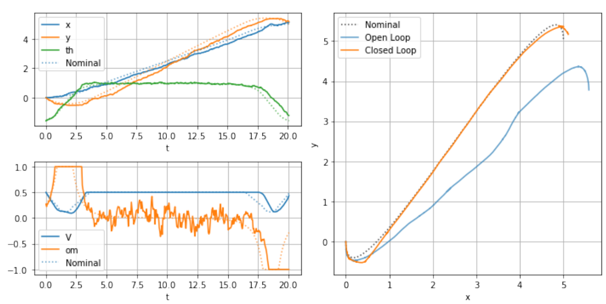

Figure 6: Trajectory Tracking w. closed-loop control

Problem 4

(i) Transcribe this optimal control problem into a finite dimensional constrained problem.

Cost:
$min_{(x_i,u_i)} \sum_{i=0}^{N-1} dt(V^2(t_i) + \omega^2(t_i)) + t_f\lambda$
Kinematic Model Constraint:
$x_{i+1} = x_i + h_i a(x_i, u_i, t_i)$

$$a(x_i, u_i, t_i) = \begin{bmatrix} \dot{x}(t_i) \\ \dot{y}(t_i) \\ \dot{\theta}(t_i) \end{bmatrix} = \begin{bmatrix} V(t_i)\cos(\theta(t_i)) \\ V(t_i)\sin(\theta(t_i)) \\ \omega(t_i) \end{bmatrix}$$

Initial and Final Constraints:
$x(0) = 0,\ y(0) = 0,\ \theta(0) = -\pi/2,\ x(t_f) = 5,\ y(t_f) = 5,\ \theta(0) = -\pi/2$
Control Input Limit:
$u \in U,\ |V(t)| \leq 0.5m/s,\ |\omega(t)| \leq 1rad/s$

(ii) CODE

(iii) Please include the optimal control resulting plots.

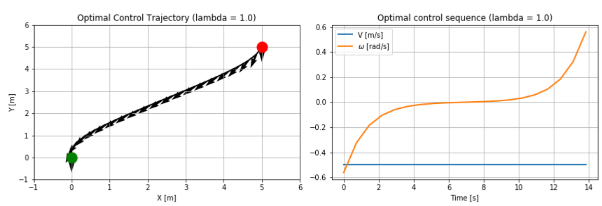

Figure 7: Optimal Control $\lambda = 1.0$

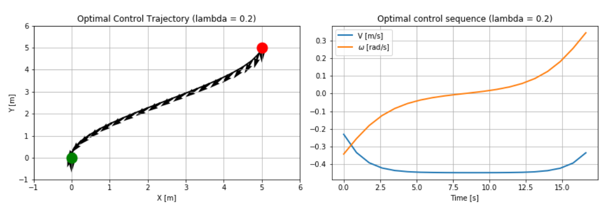

Figure 8: Optimal Control $\lambda = 0.2$

(iv) Explain the differences that you see with the different choices of λ.
Larger λ means higher cost and tf needs to be smaller in order to minimize the cost. We penalized the tf.