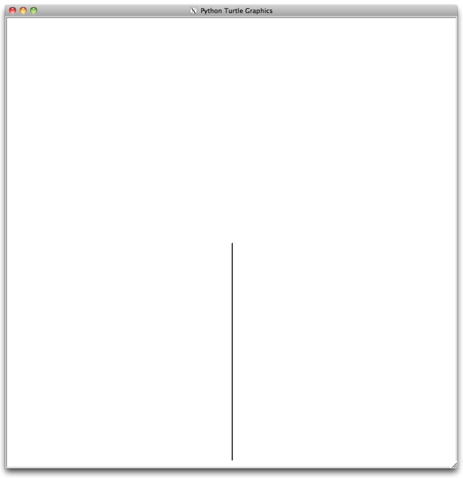
Computer Science I — CSCI141
Recursive Tree — Lecture (1/2)

08/26/2013

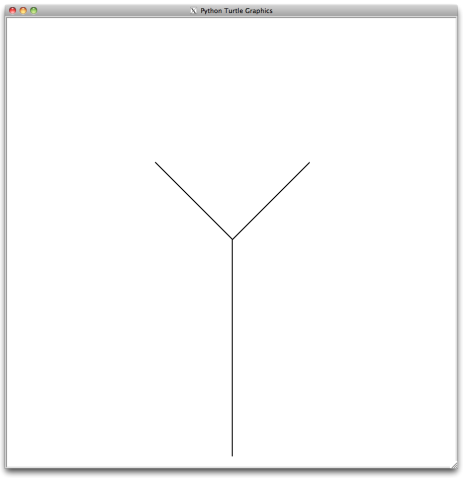
Problem

Write a program that prompts the user for a number of `segments` (a positive integer) and a `size` (also a non-zero positive integer) and then draws a *Y-tree* using the `segments` and the `size` as follows:

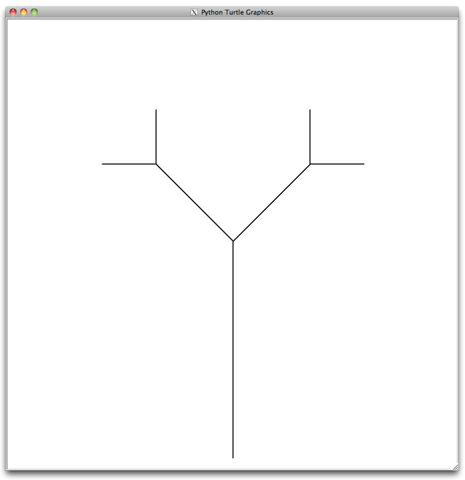
- If `segments == 0`, it draws nothing.
- If `segments == 1`, it draws a trunk (a line) of length `size` (a somewhat degenerate tree).
- If `segments == 2`, it draws a little tree, consisting of a trunk (a line) of length `size` that splits into two branches (two lines) of length `size/2`. The tree is symmetric and there is a right angle between the two branches.
- If `segments == 3`, it draws a tree with four more branches. The tree is the same as for `segments == 2`, but with additional splits at the ends of the branches of length `size/2`, where each of the new branches is of length `size/4`.
- For each greater value of `segments`, the program draws trees with yet more branches. Each tree is similar to the previous tree, but with additional splits, where each of the new branches is half the size of the branch from which it splits.

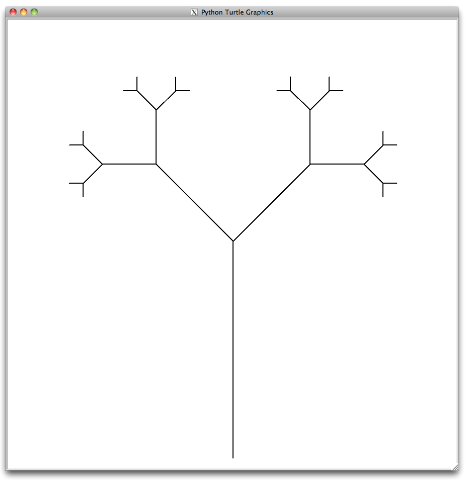


`segments == 1`



`segments == 2`





`segments == 3` `segments == 5`

Note that `segments` counts the number of line segments from the base of the tree to the end of any branch.

Problem Analysis and Solution Design

Development

How do we solve the problem for `segments == 1`?

```
def drawTree1(size):
    forward(size)
```

How do we solve the problem for `segments == 2`?

```
def drawTree2(size):
    forward(size)
    left(45)
    forward(size/2)
    forward(-size/2)
    right(45)
    right(45)
    forward(size/2)
```

These solutions work, but we've previously seen that it is good for figure-drawing functions to return to their initial position and orientation after drawing.

How do we solve the problem and return to the initial position and orientation for `segments == 1`?

```
def drawTree1(size):
    forward(size)
    forward(-size)
```

How do we solve the problem and return to the initial position and orientation for `segments == 2`?

```
def drawTree2(size):
    forward(size)
    left(45)
    forward(size/2)
    forward(-size/2)
    right(45)
    right(45)
    forward(size/2)
    forward(-size/2)
    left(45)
    forward(-size)
```

We can notice that there is some repetition in the solution for `segments == 2` and that the repeated code is similar to the solution for `segments == 1`.

```
def drawTree2(size):
    forward(size)
    left(45)
    drawTree1(size/2)
    right(45)
    right(45)
    drawTree1(size/2)
    left(45)
    forward(-size)
```

How do we solve the problem and return to the initial position and orientation for `segments == 3` (and reuse our solution for `segments == 2`)?

```
def drawTree3(size):
    forward(size)
    left(45)
    drawTree2(size/2)
    right(45)
    right(45)
    drawTree2(size/2)
    left(45)
    forward(-size)
```

How do we solve the problem and return to the initial position and orientation for `segments == 4` (and reuse our solution for `segments == 3`)?

```
def drawTree4(size):
    forward(size)
    left(45)
    drawTree3(size/2)
    right(45)
    right(45)
    drawTree3(size/2)
    left(45)
    forward(-size)
```

We observe much similarity between `drawTree2`, `drawTree3`, `drawTree4`, ...; we use the `segments` argument to distinguish the different `drawTree#` functions:

```
def drawTree(segments, size):
    if segments == 1:
        forward(size)
        forward(-size)
    else:
        forward(size)
        left(45)
        drawTree(segments-1, size/2)
        right(45)
        right(45)
        drawTree(segments-1, size/2)
        left(45)
        forward(-size)
```

How do we solve the problem for `segments == 0`?

```
def drawTree0(size):
    pass
```

Note that `pass` is the Python command to do nothing.

We can incorporate this into our `drawTree` function as follows:

```
def drawTree(segments, size):
    if segments == 0:
        pass
    elif segments == 1:
        forward(size)
        forward(-size)
    else:
        forward(size)
        left(45)
        drawTree(segments-1, size/2)
        right(45)
        right(45)
        drawTree(segments-1, size/2)
        left(45)
        forward(-size)
```

Finally, we can notice that the case `segments == 1` need not be treated specially — drawing a tree with `segments == 1` corresponds to drawing the trunk, then “drawing” two trees with `segments == 0`.

```
def drawTree1(size):
    forward(size)
    left(45)
    drawTree0(size/2)
    right(45)
    right(45)
    drawTree0(size/2)
    left(45)
    forward(-size)

def drawTree(segments, size):
    if segments == 0:
        pass
    else:
        forward(size)
        left(45)
        drawTree(segments-1, size/2)
        right(45)
        right(45)
        drawTree(segments-1, size/2)
        left(45)
        forward(-size)
```

Final Program

For the final program, in addition to the `drawTree` function developed in the previous section, we need an initialization function. Further, it is often worthwhile to have an entry-point function that does initialization and then calls the function of interest. All the functions are also carefully commented.

```
def drawTree( segments, size ):
    """
    drawTree recursively draws the tree.

    segments -- NonNegInteger;
                number of line segments from the base of the tree to
                the end of any branch should be integral and non-negative.
    size -- PosNumber;
            length of tree "trunk" to draw should be (strictly) positive.
    pre-conditions: segments >= 0, size > 0.
                    turtle is at base of tree,
                    turtle is facing along trunk of tree,
                    turtle is pen-down.
    post-conditions: a segments-level tree was drawn on the canvas,
                     turtle is at base of tree,
                     turtle is facing along trunk of tree,
                     turtle is pen-down.
    """
    if segments == 0:
        # base case: draw nothing
        pass
    else:
        # recursive case: draw trunk and two sub-trees
        forward( size )
        left( 45 )
        drawTree( segments - 1, size / 2 )
        right( 90 ) # turn 45 degrees twice
        drawTree( segments - 1, size / 2 )
        left( 45 )
        forward( -size )

def initWorld( size ):
    """
    initWorld initializes the drawing by establishing its pre-conditions.

    size -- PosNumber;
            length of tree trunk to draw should be positive.

    pre-conditions:
    post-conditions: coordinate system
                       is (-2*|size|,-2*|size|) at lower-left
                       to (2*|size|, 2*|size|) at upper-right.
                     turtle is at origin,
                     turtle is facing North,
                     turtle is pen-down.
    """
    margin = 2   # provides canvas boundary
    setup( 600, 600 )
    setworldcoordinates( -2*abs(size) - margin, -2*abs(size) - margin, \
                          2*abs(size) + margin, 2*abs(size) + margin )
    home()  # turtle is at origin, facing east, pen-down
    left( 90 )  # turtle is facing North
    down()  # turtle is pen-down
    pensize( 2 )


def initWorldAndDrawTree( segments, size ):
    """
    initWorldAndDrawTree prints a message, initializes the world,
    draws an instance of the recursive tree, and waits for ENTER.

    segments -- NonNegInteger;
                number of line segments from the base of the tree to
                the end of any branch should be integral and non-negative.
    size -- PosNumber;
            length of tree "trunk" to draw should be (strictly) positive.
    message -- String;
               message to display
    """
    print( "Drawing recursive tree with", (segments, size))
    initWorld( size )
    drawTree( segments, size )
    update()
    input( "Hit ENTER to quit." )
    bye()
```

Note that the `drawTree` expects the turtle's position and orientation to be at the base of the tree and facing along the trunk. This is called a **pre-condition** because a specific state is assumed to have been established *before* the function begins its execution.

Also, note that we are careful to return the turtle to its initial position and orientation (at the base of the tree and facing along the trunk) after drawing the tree. The turtle must be at exactly the same position and in exactly the same orientation as at the beginning of the drawing. Suppose otherwise: after drawing the left smaller tree, the turtle would not be at the crotch of the tree (the base of the smaller tree) and we would draw the right tree starting somewhere other than the crotch. (See what happens if you remove the "`move by length -size units`" from the pseudocode of `drawTree`.) This is called a **post-condition** because a specific state is guaranteed to have been established *after* the function finishes its execution.

We should always document pre- and post-conditions for functions. This allows other programmers to know what needs to be done before calling the function and what can be expected after calling the function. Beware: if you call a function without satisfying its pre-conditions, all bets are off!

Execution Diagram

We can visualize the execution of the `drawTree` function with an *execution diagram*.

```
drawTree( segments := 2, size := 100 )
    # segments <> 0
    forward( size )
    left( 45 )
    drawTree( segments := 1, size := 50 )
        # segments <> 0
        forward( size )
        left( 45 )
        drawTree( segments := 0, size := 25 )
            # segments == 0
            pass
        right( 90 )
        drawTree( segments := 0, size := 25 )
            # segments == 0
            pass
        left( 45 )
        forward( -size )
    right( 90 )
    drawTree( segments := 1, size := 50 )
        # segments <> 0
        forward( size )
        left( 45 )
        drawTree( segments := 0, size := 25 )
            # segments == 0
            pass
        right( 90 )
        drawTree( segments := 0, size := 25 )
            # segments == 0
            pass
        left( 45 )
        forward( -size )
    left( 45 )
    forward( -size )
```

Implementation

See `tree.py`.

Testing (Test Cases, Procedures, etc.)

We need to test the base case(s) and the recursion.

For the base case (that draws nothing), we should test `segments == 0` and a variety of values for `size` (e.g., 10, 100, 1, 0, possibly also -10).

For the simple recursive case (that draws a trunk and recurses to draw nothing), we should test `segments == 1` and a variety of values for `size` (e.g., 10, 100, 1, 0, possibly also -10).

For the complex recursive cases (that draws a trunk and recurses to draw something), we need to test `segments > 1` (e.g., 2, 3, 5) and a variety of values for `size`.

NOTE: The variety of values for `size` only matter if we do not adjust the world coordinates to the `size`.