

Functional Autoencoder

Plan for Discussion Sessions

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Motivation

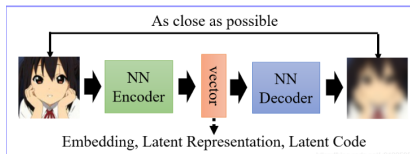
- Autoencoders (AE) are powerful unsupervised learning methods for representation learning, dimensionality reduction, and data compression.
- Functional data (curves, time series, continuous signals) frequently appear in finance, medicine, and physical experiments.
- Most existing AEs handle discrete vectors; extending them to functional spaces is still an open question.

Key Questions:

- ① How to generalize AE theory to functional spaces (e.g. L^2)?
- ② Can we outperform classical methods such as Karhunen–Loève expansion / PCA?
- ③ If it can outperform in downstream task compared with linear model?

Research Approach

- 1 **Mathematical review:** revisit the AE objective and its equivalence to PCA in the linear case.
- 2 **Functional modeling:** represent functions using basis expansions (splines, polynomials, Fourier) to embed them into a neural network.
- 3 **Practical exploration and potential extensions:** implement a functional AE and compare with KL expansion for curve reconstruction.
Explore VAE formulation, regularization terms, and interpretability of the latent space.



Reading List & Plan

- Umberto Michelucci(2022): *An introduction to autoencoders*
- Kingma & Welling (2013): *Auto-Encoding Variational Bayes*
- Ramsay & Silverman (2005): *Functional Data Analysis*
- Sidi Wu & Cédric Beaulac(2024): *Functional Autoencoder for Smoothing and Representation Learning*

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|-----------|-------------------------------------------------------|
| Session 1 | Introduction to autoencoder + Some samples |
| Session 2 | Functional data introduction + Functional AE modeling |
| Session 3 | Extensions: VAE / Downstream task |

Thank You!