

The Iwasawa λ invariant and Massey products

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Introduction

- ① In Iwasawa theory, the invariant λ controls the growth of the size of class group over a field extension.
- ② Kummer theory can relate class groups and cohomology groups
- ③ Massey product is a generalization of cup products and can be used to compute size of cohomology groups.

Application of Main theorem

Theorem

Let K be an imaginary quadratic field and p be an odd prime. Suppose p doesn't divide the size of class group $\text{Cl}(K)$ and p splits in K . Then the λ -invariant of cyclotomic \mathbb{Z}_p extension K_∞/K can be determined in terms of Massey products as follows:

Assume $\lambda \geq n - 1$. Then $\lambda \geq n \iff$ the n -fold Massey product $(\chi, \chi, \dots, \alpha) = 0$ relative to a proper defining system.

Remark

Let $n = 2$, we can recover the classical Gold's criterion and the theorem can be viewed as generalization of Gold's criterion.

Remark

We don't use "Iwasawa main conjecture" in the proof. The λ we computed is algebraic version of Iwasawa λ .