Week 4 Project

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Problem 1

With the assumption: $r_t \sim N(0, \sigma^2)$, I calculated the expected value and standard deviation of the price of time t, given 3 types of returns.

For classical Brownian,

$$P_t = P_{t-1} + r_t$$

So we have

$$E(P_t) = E(P_{t-1} + r_t) = P_{t-1} + E(r_t) = P_{t-1}$$

$$\sigma(P_t) = \sigma(P_{t-1} + r_t) = \sigma(r_t) = \sigma$$

To demonstrate my expectation, I simulated P_t with $P_{t-1} = 100, \sigma = 0.1$. From table 1, the simulated results matched my calculation.

Table 1 classical Brownian

statistics	Expected value	Standard deviation
expected	100	0.1
simulated	100.0001	0.1000

For arithmetic return,

$$P_t = P_{t-1}(1 + r_t)$$

So we have

$$E(P_t) = E(P_{t-1}(1 + r_t)) = P_{t-1}E(1 + r_t) = P_{t-1}$$

$$\sigma(P_t) = \sigma(P_{t-1}(1 + r_t)) = P_{t-1}\sigma(1 + r_t) = P_{t-1}\sigma$$

To demonstrate my expectation, I simulated P_t with $P_{t-1} = 100, \sigma = 0.1$. From table 2, the simulated results matched my calculation.

Table 2 arithmetic return

statistics	Expected value	Standard deviation
expected	100	10
simulated	100.0090	10.0006

For geometric Brownian,

$$P_t = P_{t-1}e^{r_t}$$

So we have

$$E(P_t) = E(P_{t-1}e^{r_t}) = P_{t-1}E(e^{r_t}) = P_{t-1}e^{\mu+\frac{\sigma^2}{2}} = P_{t-1}e^{\frac{\sigma^2}{2}}$$

$$\sigma(P_t) = \sigma(P_{t-1}e^{r_t}) = P_{t-1}\sigma(e^{r_t}) = P_{t-1}\sqrt{(e^{\sigma^2}-1)e^{\sigma^2}}$$

To demonstrate my expectation, I simulated P_t with $P_{t-1} = 100, \sigma = 0.1$. From table 1, the simulated results matched my calculation.

Table 3 geometric Brownian

statistics	Expected value	Standard deviation
expected	100.5013	10.0753
simulated	100.5104	10.0780

Problem 2

To calculate the value at risk for META in different ways, I set α to be 5%. With the assumption of normal distribution, I extracted the 5% percentile of the normal distribution for meta returns as VaR1. To be specific, the normal distribution is

$$return \sim N(0, 0.0398^2)$$

Then I applied exponentially weights for λ=0.94, and calculated an exponentially weighted variance and VaR2. The new distribution is

$$return \sim N(0, 0.0556^2)$$

With the assumption of T distribution, I fitted the meta returns data in a T distribution with MLE method, and got VaR3. The T distribution is

$$return \sim t(3.923, (0, 0.0267))$$

To fit AR(1) model, I used AutoReg function in statsmodel package and set lags to be 1. My fitted results is (the intercept m is very close to 0)

$$Return_t = 0.0072Return_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, 0.0399^2)$$

With this AR(1) model, I simulated 1,000,000 draws of ϵ_t, used the last observed return as $Return_{t-1}$, and applied the formula to get 1,000,000 simulated values of $Return_t$. Then extracted the 5% percentile of these simulated values to get VaR4.

In histroric simulation, I randomly picked 1,000,000 returns with replacement and extracted the 5% percentile to get VaR5.

Table 4 VaR of META

Model	VaR	Rank
normal distribution	6.55%	3
normal distribution with ew variance	9.14%	1
MLE fitted T distribution	5.73%	4
fitted AR(1) model	6.58%	2
Historic simulation	5.59%	5

From table 4, the VaR in normal distribution with exponentially weighted variance has the largest loss, 9.14%. Look at the previous distribution formulas, I inferred the reason of this result is that the exponentially weighted variance (0.0556) is much larger than unweighted one, which makes the returns more dispersed. Hence, chances of extreme gains or losses are higher.

VaRs calculated from the other 4 models are very similar, ranging from 5.59% to 6.58%.

Problem 3

To calculate portfolios' VaR, I used both Monte Carlo model and historic simulation. In terms of Monte Carlo simulation, I chose PCA wit 100% explained as my method since it is both accurate and fast. In terms of historic simulation, I I randomly chose 25,000 draws from original returns and applied them to the last observed prices.

Table 5 VaR of portfolios

portfolio	A	B	C	Total
Monte Carlo with discrete return	5620.37$	4357.73$	3753.22$	13470.81$
Monte Carlo with log return	5539.85$	4382.22$	3736.56$	13450.62$
Historic sim with discrete return	3452.11$	3631.77$	2754.59$	11064.37$
Historic sim with log return	4558.32$	3631.77$	2754.59$	11064.37$

From the results, we can conclude that VaRs in same model but differenct return types are very similar. However, VaRs in different models but same return type are much different. In this case, total VaR in Monte Carlo model is 2000$ larger than that in historic simulation.