Week 7 Project

Peilin Luo

Problem 1

Comparing the closed form greeks with the finite difference greeks using the general Black-Schole model (Table 2), we can conclude that these two methods got approximately the same results. Note that closed form rho cannot be calculated since the risk-free rate does not equal to the cost of carry in terms of problem 1.

Using binomial tree valuation for American options, I calculated the price for the call and put options with and without discrete dividends (Table 1). Also, I calculated the greeks of each option in Table 2. The results showed that the rho of options with discrete dividends differed largely from the rho calculated previously. Note that carry rho was not calculated for options with discrete dividends since the cost of carry is the same as the risk-free rate.

Table 1 bt American option prices (unit: $)

Binomial Tree American	Without discrete dividend	With discrete dividend
Call	4.2623	4.1258
Put	3.7497	4.1258

Table 2 option Greeks

Model	Type	Method	Delta	Gamma	Vega	Theta	Rho	Carry rho
GBSM	call	Closed form	0.5340	0.0400	19.7102	-24.8985	-	7.9662
		Finite difference	0.5339	0.0400	19.7101	-24.9322	-0.3827	7.9663
	put	Closed form	-0.4655	0.0400	19.7102	-18.7870	-	-6.9444
		Finite difference	-0.4656	0.0400	19.7101	-18.8207	-0.3326	-6.9444
BT American	call	Finite difference	0.5343	0.0000	19.8606	-25.0951	-0.3854	7.9600
	put	Finite difference	-0.4699	0.0032	19.8376	-19.4872	-0.2457	-5.7825
	Call with discrete dividend	Finite difference	0.5451	0.0316	19.8076	-25.1756	**6.8784**	-
	Put with discrete dividend	Finite difference	-0.4780	0.0314	20.0213	-18.7590	**-7.2741**	-

To illustrate the sensitivity of the put and call to a change in the dividend amount, I plotted the option prices for different dividend amounts from zero to $2. For the call option, the larger the dividend is, the lower the option price is (Figure 1). The sensitivity (slope) at $0.88 is -0.156. For the put option, the larger the dividend is, the higher the option price is (Figure 2). The sensitivity (slope) at $0.88 is 0.4818.

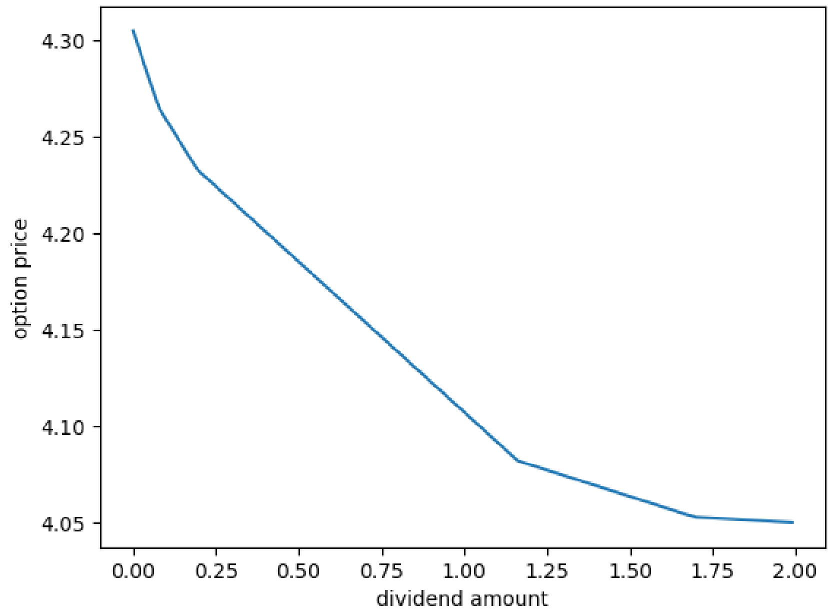

Figure 1 sensitivity of the call w.r.t. the dividend amount

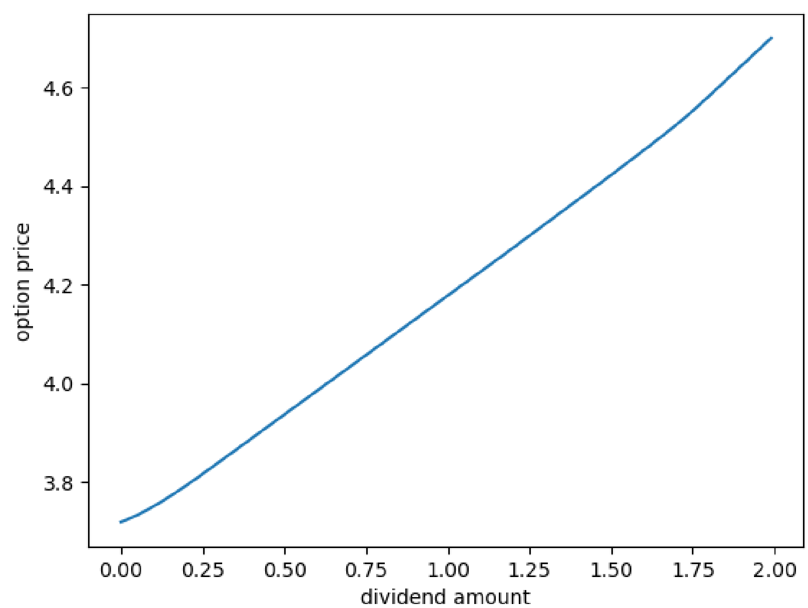

Figure 2 sensitivity of the put w.r.t. the dividend amount

Problem 2

To calculate the average PL, VaR, and ES, I fitted the AAPL returns with normal distribution and simulated the AAPL underlying prices 10 days ahead. And simulation results are shown in Table 4. Simulation results with AR(1) model from last week are shown in Table 3. Comparing Table 3 with Table 4, VaR and ES are approximately the same.

From Table 4, **VaR and ES using Delta-Normal are much smaller** than those calculated previously. To illustrate, according to Delta-Normal assumptions, asset prices have a linear relationship with the underlying prices. So, the portfolio value obeys normal distribution and is centered to mean value.

Table 3 PL mean, VaR, and ES for portfolios with GBSM (unit: $)

Portfolio	PL mean	VaR	ES
Straddle	1.5399	1.3797	1.3877
SynLong	-0.0292	16.1957	20.0312
CallSpread	-0.1029	3.8852	4.1873
PutSpread	0.3060	2.6282	2.8017
Stock	0.1742	15.9536	19.7661
Call	0.7554	6.0319	6.3690
Put	0.7845	4.3645	4.5891
CoveredCall	-0.7139	12.1403	15.8338
ProtectedPut	0.8338	8.0618	8.6943

Table 4 PL mean, VaR, and ES for portfolios with BT American (unit: $)

Portfolio	PL mean	VaR	ES	Delta VaR	Delta ES
Straddle	1.6758	1.3353	1.366	0.4053	0.5082
SynLong	0.4827	16.2335	19.3992	5.6162	6.9695
CallSpread	0.0232	3.7367	4.0183	1.5768	1.9981
PutSpread	0.2108	2.7786	2.8662	1.4852	1.8527
Stock	1.0272	15.1176	18.0711	5.6105	6.992
Call	1.0792	5.8636	6.1831	2.98	3.7085
Put	0.5965	4.5542	4.6663	2.6322	3.2616
CoveredCall	-0.1089	11.3826	14.2001	3.2782	4.1031
ProtectedPut	1.5485	7.2001	7.6135	3.6971	4.6728

Problem 3

Using the four-factor model to fit the stock returns and then simulate the returns for the past 10 years, I got the expected annual return of each stock and an annual covariance matrix. With the optimizer in SciPy, I plotted the efficient frontier for the 20 stocks portfolio (Figure 3). Then I found the super-efficient portfolio by comparing the shape ratio on the efficient frontier, shown in Table 5. **The super-efficient portfolio consists of four stocks, which are AAPL, UNH, NVDA, and XOM, with 9.82%, 25%, 19.54%, and 45.64% weight respectively**. The shape ratio for the super-efficient portfolio is 3.4594.

Table 5 super-efficient portfolio

Stock	AAPL	UNH	NVDA	XOM
Weight	0.0982	0.25	0.1954	0.4564
Risk: 0.0039 Expected return: 0.056 Shape ratio: 3.4594				

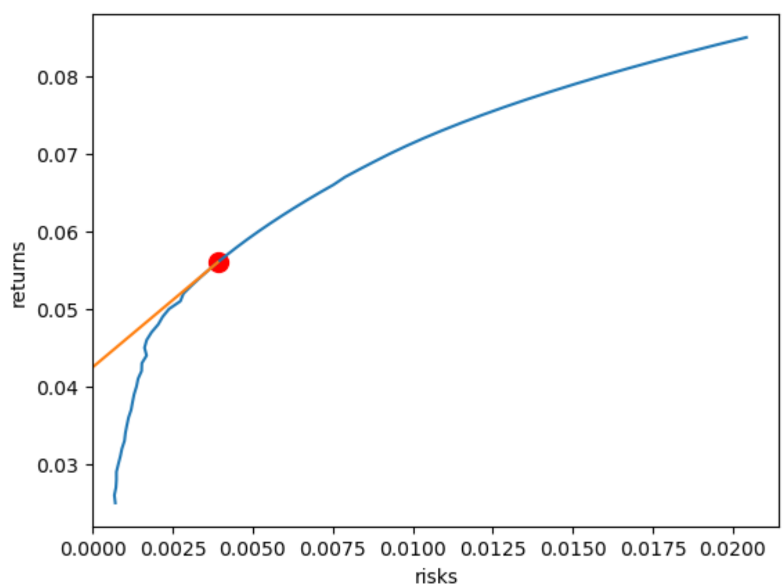

Figure 3 efficient frontier and super-efficient portfolio