Public Economics

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Chapter 1

Introduction

1.1 What is Public Econometrics?

Subject: The role of the state in a ('mixed') economy

Method: Economic methodology

Methodological individualism: Rational Choice Theory, Self interest and rationality.

Normative individualism: Policy evaluation based on individual (citizen) preferences.

Formal Modeling: Neoclassical Theory (optimization, marginal conditions); Game Theory, Equilibrium analysis (effects of policy / rules on equilibrium).

Empirical Public economics (not in this course): Effects of policies, political institutions, estimated demands for public goods.

Chapter 2

General Equilibrium and Welfare Economics

2.1 Assumptions

There are four assumptions.

(a) There are only two consumers $h = 1, 2$ and two goods $i = 1, 2$.

(b) Preferences are describes by a utility function.

$$U^h = (x_1^h, x_2^h)$$

U^h represents **rational, continuous, locally unsaturated** preferences,a d indifference curves have the standard **convex** shape.

(c) Each consumer has an initial endowment.

$$\omega^h = (\omega_1^h, \omega_2^h) \in \mathbb{R}^2$$

(d) Consider a voluntary exchange economy and a pure exchange economy. More precisely, the market is competitive market and exchanges are mediated by a price system.

Note: Competitive Markets

(a) Goods are traded at common and **fixed** prices.

(b) **Decentralized:** Each person independently plans how much to buy/ sell at given prices. (No bargaining, no coordination)

(c) **Prices adjust** such that all plans are mutually compatible.

**But the adjust mechanism are exogenous given.*

Definition: Voluntary Exchange

We don't consider the production of the economy. All of the available products in the economy are given as initial endowments of consumers. Transfer of rights requires consent and exchange of goods also requires mutual consent.

Property rights (endowments) are protected (exogenous given status quo).

Note: Voluntary Exchange

1. Voluntary exchange is **one of many** imaginable (types of) allocation mechanisms.

2. There are infinitely many ways to organize a voluntary exchange.

3. The main substantive question is **how outcomes from voluntary exchange compare to those from other mechanisms which involve coercion**.

4. The model we are discussing is silent as to the precise rules governing exchange.

2.2 Constraints of the Optimal Problem

There are two aspects of constraint in this optimal problem.

(a) Feasibility: the total consumption of each good after exchange is equal to the total initial endowments.

(b) Utility: Each consumer's utility after the exchange is at least not less than the utility of initial endowments. Otherwise, the related will not consent the exchange.

2.2.1 The Edgeworth Box (Describe 'Feasibility' Constraint)

The Edgeworth box depicts the set of feasible consumption allocations. That is,

(a) Each point corresponds to one feasible allocation;

(b) The length and width of this box represents the total endowments. Consumption Plans are measured from the corners of the box.

Note: Edgeworth Box

(a) The set of feasible allocations does not depend on the initial distribution of the total endowment or on the allocation mechanism. It simply describe what is ' technologically ' possible.
(b) In competitive market, every price vector induces a budget line **through** the endowment point. In other words, the budget constraint line must cross the initial endowment point and rolls as the prices change.

Definition: Contract Curve

The contract curve or Pareto set is the set of points representing final allocations of two goods between two people that could occur as a result of mutually beneficial trading between those people given their initial allocations of the goods.

All the points on this locus are Pareto efficient allocations

In math, the feasibility constraint is

$$x_1^1 + x_1^2 = \omega_1^1 + \omega_1^2 \tag{2.1}$$

$$x_2^1 + x_2^2 = \omega_2^1 + \omega_2^2 \tag{2.2}$$

Note: x_1^2 represents the consumption amount of consumer 2 on good 1.

Definition: Excess Demand /Excess Supply

The **total amounts** that consumers plan to consume (demand) are larger or smaller than the total supply (endowments) available.

Note: Excess Demand /Excess Supply

(a) The excess demand is symmetric.

(b) At equilibrium, the **personal** excess demand may not be equal to 0. But the **total** excess demand should be zero.

2.2.2 The Utility Constraint

After the exchange , nobody's situation is worse than the initial endowments.

In math, it can be written as,

$$U^h(\omega_h^1, \omega_h^2) \leq U^h(x_h^1, x_h^2)$$

Definition: The Core

The core is a set of all points which satisfy the feasibility constraint and the utility constraint.

Note: The Core

1. The core is a part of the contract curve. It is a curve, and it contains infinitely many points.

2. All the points of the core is Pareto efficient.

3. Which point is realized may depend on specific rules and institutions (how trade is organized) and bargaining skills and strategies.

**One type of the institution is the price mechanism.*

2.3 Competitive Equilibrium

Definition: Competitive Equilibrium

A competitive equilibrium is a price vector $p^* = (p_1^*, p_2^*)$ are consumption plans for both consumers such that x^{h*} maximizes consumer h's utility given p^* and ω^h, and excess demand for both goods is zero at price p^*.

2.3.1 Walras' Law

Definition: Walras' Law

Walras' law states that the sum of the values of excess demands across all markets must equal zero, whether or not the economy is in a general equilibrium.

This implies that if positive excess demand exists in one market, negative excess demand must exist in some other market. **Thus, if all markets but one are in equilibrium, then that last market must also be in equilibrium.**

In math, the Walras' Law can be written as follows:

Define **market total excess demand** for good i as

$$Z_i(p,\omega) = x_i^1(p,\omega^1) + x_i^2(p,\omega^2) - \omega_i^1 - \omega_i^2$$

Where $x_1^h(p,\omega^h)$ and $x_2^h(p,\omega^h)$ solve utility maximization.

Then

$$p_1 x_1^h + p_2 x_2^h = p_1 \omega_1^h + p_2 \omega_2^h, h = 1,2 \tag{2.3}$$

Sum over $h = 1,2$

$$p_1(x_1^1 + x_1^2) + p_2(x_2^1 + x_2^2) = p_1(\omega_1^1 + \omega_1^2) + p_2(\omega_2^1 + \omega_2^2) \tag{2.4}$$

Rearrange and use the definition of $Z_i(p,\omega)$,

$$p_1 Z_1(p,\omega) + p_2 Z_2(p,\omega) = 0 \text{ [Walras' Law]} \tag{2.5}$$

Note: Walras Law

(a) The Walras' Law shows that the excess demand is zero **at any price, no matter the market is equilibrium or not**. *(If the market is equilibrium, then the excess demand of all products should be zero. At that time, Walras' Law is definitely true.)*

(b)Implications of Walras Law:

- If $Z_1(p,\omega) > 0$, then $Z_2(p,\omega) < 0$ (and vice versa), i.e. excess demand for one good implies excess supply of the other.
- If $Z_1(p,\omega) = 0$, then $Z_2(p,\omega) = 0$. That is to say, if one market clears, so does the other.

2.3.2 The Core and the Competitive Equilibrium

Many outcomes in the core are not part of a competitive equilibrium. The set of competitive equilibrium is a subset of the core.

2.4 Summary of the optimal problem

2.4.1 The Optimal Problem

For consumer a, his optimal problem can be written as

$$\max_{x_1^a, x_2^a, x_1^b, x_2^b} U^a(x_1^a, x_2^a) \tag{2.6}$$

subject to (s.t.)

(a) Feasibility Constraint

$$x_1^a + x_1^b = \omega_1^a + \omega_1^b \tag{2.7}$$

$$x_2^a + x_2^b = \omega_2^a + \omega_2^b$$

(b) Utility Constraint

$$U^b(x_1^b, x_2^b) \geq U^b(\omega_1^b, \omega_2^b) \tag{2.8}$$

Note: There, in the utility constraint, we only consider the other consumer, since in the optimal question of consumer 1, the utility of the final bundle is at least equal to the initial endowment. Otherwise, he will not exchange.

2.4.2 The Lagrange function and Lagrange multiplier

The related Lagrange function can be written as follows,

$$\begin{aligned} L = & U^a(x_1^a, x_2^a) \\ & - \mu_1(x_1^a + x_1^b - \omega_1^a - \omega_1^b) \\ & - \mu_2(x_2^a + x_2^b - \omega_2^a - \omega_2^b) \\ & - \mu_3[U^b(\omega_1^b, \omega_2^b) - U^b(x_1^b, x_2^b)] \end{aligned}$$

Then the F.O.C are

$$\frac{\partial L}{\partial x_1^a} = \frac{\partial U^a(x_1^a, x_2^a)}{\partial x_1^a} - \mu_1 = 0 \tag{2.9}$$

$$\frac{\partial L}{\partial x_2^a} = \frac{\partial U^a(x_1^a, x_2^a)}{\partial x_2^a} - \mu_2 = 0 \tag{2.10}$$

$$\frac{\partial L}{\partial x_1^b} = -\mu_1 + \mu_3 \frac{\partial U^b(x_1^b, x_2^b)}{\partial x_1^b} = 0 \tag{2.11}$$

$$\frac{\partial L}{\partial x_2^b} = -\mu_2 + \mu_3 \frac{\partial U^b(x_1^b, x_2^b)}{\partial x_2^b} = 0 \tag{2.12}$$

Hence, we have

$$\frac{\frac{\partial U^a(x_1^a,x_2^a)}{\partial x_2^a}}{\frac{\partial U^a(x_1^a,x_2^a)}{\partial x_2^a}} = \frac{\mu_1}{\mu_2} \tag{2.13}$$

$$\frac{\frac{\partial U^b(x_1^b,x_2^b)}{\partial x_2^b}}{\frac{\partial U^b(x_1^b,x_2^b)}{\partial x_2^b}} = \frac{\mu_1}{\mu_2} \tag{2.14}$$

Hence, we have the equilibrium condition – tangency.

Furthermore, Let's talk about the meaning of Lagrange multiplier.

We can found that

$$\mu_1 = \frac{\partial U^a(x_1^a, x_2^a)}{\partial x_1^a} \tag{2.15}$$

$$\mu_2 = \frac{\partial U^a(x_1^a, x_2^a)}{\partial x_2^a} \tag{2.16}$$

The meaning of Lagrange multiplier is shadow price, here, $\mu_1$and μ_2 are the marginal increase in the utility of consumer 1 if the total initial endowment of the two goods in society increases by one unit.

Since

$$\frac{\mu_1}{\mu_3} = \frac{\partial U^b(x_1^b, x_2^b)}{\partial x_1^b} \tag{2.17}$$

$$\frac{\mu_2}{\mu_3} = \frac{\partial U^b(x_1^b, x_2^b)}{\partial x_2^b} \tag{2.18}$$

$\frac{\mu_1}{\mu_3}$ and $\frac{\mu_2}{\mu_3}$ are the marginal utility increase of consumer 2 if we increase a unit of total endowments of good 1 and good 2 respectively.

2.5 Competitive Equilibrium and Pareto Optimal

2.5.1 The Arrow-Debreu Economy

The Arrow-Debreu Economy

In mathematical economics, the Arrow–Debreu model suggests that under certain economic assumptions (convex preferences, perfect competition, and demand independence) there must be a set of prices such that aggregate supplies will equal aggregate demands for every commodity in the economy.

Ingredients

(a) Goods $i = 1, ..., n$.
(b) Households $h = 1, ..., H$, with initial endowments $\omega^h = (\omega_1^h, \omega_2^h)$ and utility function $U^h(x^h)$ where $x^h = (x_1^h, ..., x_n^h)$ is a 'consumption plan'.
(c) Prices $p = (p_1, ...p_n)$ with each $p_i \geq 0$. These prices are fixed and common to all agents.

Note: Walras Law

Technically, $U^h(x^h)$ is strictly quasi-concave and satisfies local non-satiation.

Local non-satiation implies that consumers always plan to spend all of their wealth (Walras's Law).

2.5.2 Prices and aggregate demand at ***any given*** price vector

For any given price vector p, each household h has wealth

$$p\omega^h = p_1\omega_1^h + \cdots + p_n\omega_n^h \tag{2.19}$$

Given this, the household chooses $x^h = (x_1^h, \ldots, x_n^h)$ to solve

$$\max_{x^h} U^h(x^h)$$

$$\text{s.t. } px^h = p\omega^h$$

This yields $\Rightarrow x_i^h(p, \omega^h) =$ individual demand for each good i.

Aggregate (planned) demand

For each good i.

$$X_i(p, \omega) = \sum_{h=1}^{H} x_i^h(p, \omega^h) \tag{2.20}$$

Excess Demand as a function of p Then aggregate (planned) demand for good i is a function of p:

$$X_i(p) = X_i(p, \omega) \tag{2.21}$$

Excess (planned) demand for good i is defined as

$$Z_i(p) = X_i(p) - \sum_h \omega_i^h$$

Competitive Equilibrium.

Competitive Equilibrium

An array $\hat{p}, \{\hat{x}^h\}$ is a competitive equilibrium if

(a) For all consumers h, $\hat{x}^h = x^h(p)$.

(b) Markets clear: $Z_i(\hat{p}) = 0$ for all $i = 1, ..., n$.

Note: Necessary Conditions for competitive equilibrium (Edgeworth Box)

(a) Individual utility maximization

$\Rightarrow$ Indifference are tangent to the (same) budget line at the chosen points.

$$\frac{MU_1^h}{MU_2^h} = \frac{P_1}{P_2} \tag{2.22}$$

(b) Markets clear

$$\sum_{h=1}^{2} x_i^h = \sum_{h=1}^{2} \omega_i^h, \text{ i=1,2} \tag{2.23}$$

$\Rightarrow$ Consumption plans correspond to a single point in the box.

Definition: Normative Criterion: Pareto Optimal

A situation is Pareto optimal if the only way to make any person better off is to make at least one other person worse off.

Note: Pareto optimal and unanimous consent

1. If a situation is Pareto dominated, *unanimous voluntary agreement* on change is possible in principle.

2. If a situation is Pareto optimal, any change would be opposed by at least one person.

3. A Pareto optimal situation may not Pareto improve upon one that is not Pareto optimal.

4. Many government policies appear to be difficult to justify using this criterion (example, redistribution).

Definition:Pareto optimal allocations [feasibility]

An array of consumption vectors $\{x^1, \ldots, x^h, \ldots, x^H\}$ is feasible if

$$\sum_{h=1}^{H} x_i^h \leq \sum_{h=1}^{H} \omega_i^h \text{ for all i} \tag{2.24}$$

Definition : Pareto Optimal

(Pareto optimal) A feasible consumption array $\{\hat{x}^h\}$ is Pareto optimal if there does not exist another feasible array $\{\tilde{x}^h\}$ such that

$$U^h(\tilde{x}^h) \geq U^h(\hat{x}^h) \text{ for all h} \tag{2.25}$$

and

$$U^h(\tilde{x}^h) > U^h(\hat{x}^h) \text{ for at least one h} \tag{2.26}$$

2.5.3 Theorems of Welfare Economics

The 1st Theorem of Welfare EconomicsEfficient

Let $\{\hat{p}, \{\hat{x}^h\}\}$ be a competitive equilibrium, and assume that all households have locally non-satiated preferences. Then $\{\hat{x}^h\}$ is Pareto optimal.

General proof of 1st Theorem (H households, n goods)

Suppose $\{\hat{x}^h\}$ is a competitive equilibrium equilibrium allocation that is ***not*** Pareto optimal (seeking a contraction).

Then there exists a consumption array $\{\tilde{x}^h\}$ such that

1. $\tilde{X}_i \leq \sum_{h=1}^{H} \omega_i^h$ for all i (feasibility)

2. $U^h(\tilde{x}^h) \geq U^h(\hat{x}^h)$ for all h.

3. $U^h(\tilde{x}^h) > U^h(\hat{x}^h)$ for at least one h.

Local non-satiation, (2) and (3) together imply that $\hat{p}\tilde{x}^h \geq \hat{p}\hat{x}^h$ for all h, with a strict inequality for at least one h, and so

$$\hat{p}\tilde{X} > \hat{p}\hat{X} \tag{2.27}$$

According to the Walras' Law

$$\hat{p}\hat{X} = \hat{p}\omega \tag{2.28}$$

This is true ***for all prices***.

Combing the last two observations, we have

$$\hat{p}\tilde{X} > \hat{p}\hat{X} = \hat{p}\omega \tag{2.29}$$

Or

$$\hat{p}(\tilde{X} - \omega) > 0 \tag{2.30}$$

This contradicts feasibility, since there must then exist at least one good i for which $\tilde{X}_i - \sum_{h=1}^{H} \omega_i^h > 0$.

Note: The proof of 1st Welfare Theorem

Let's make a short review of the proof of the first theorem of welfare.

We want to prove that the equilibrium of competitive is Pareto efficient, i.e. no room for Pareto improvement. That is to prove, **under the price system resulting form the perfect competitive market**, no one is the Pareto improvement of the equilibrium allocation.

We use the rebuttal method, i.e. prove the contradict statement is not true. Hence, we assume that there is another consumption array $\tilde{x}$, which satisfies

1. $\tilde{x}$ is a feasible allocation among consumers;

2. $\tilde{x}$ is better than the allocation of competitive market $\hat{x}$ under the price system result from the competitive market.

If such $\tilde{x}$ doesn't exist, then we can prove that the first welfare theorem is true.

Form (2), we have

$$U^h(\tilde{x}^h) \geq U^h(\hat{x}^h) \text{ for all } h$$

and

$$U^h(\tilde{x}^h) > U^h(\hat{x}^h) \text{ for at least one } h$$

Since the utility function is monotonic, this condition implies that

$$\tilde{x}^h \geq \hat{x}^h \tag{2.31}$$

and

$$\tilde{x}^h > \hat{x}^h \text{ for at least one } h \tag{2.32}$$

According to Walras' Law

$$\hat{p}\hat{X} = \hat{p}\omega \tag{2.33}$$

Then

$$\hat{p}\tilde{X} > \hat{p}\hat{X} = \hat{p}\omega \tag{2.34}$$

That is not feasible.

Hence the first welfare theorem is proved.

Note:The interpretation of the first welfare theorem

- Adam Smith's 'invisible hand': Pursuit of self interest leads to a 'socially desirable' outcome.

- The decentralized decisions can result in a Pareto efficiency result.

- Conditions: Rationality, Perfect information, perfect competition. (*These are sufficient conditions, if these are violated, the result may be still Pareto efficient.*)

Decentralized Decision

- Individuals need to know only their own preferences and prices.
- Prices reflect the value of resources in alternative uses.
- Incentives to reveal preferences / costs through decisions to buy and sell.

The 2nd Theorem of Welfare Economics (Redistribution and the price mechanism)

Suppose that $\{\hat{x}^h\}$ is Pareto optimal, and let $\{\hat{y}^j\}$ be the corresponding production array. Assume that

(a) Preferences are locally non-satiated, convex, and continuous

(b) each $\{\hat{x}^h\}$ is interior to h's consumption set.

Then there exists $\hat{p}$ such that $\{\hat{p}, \{\hat{x}^h\}, \{\hat{y}^j\}\}$ is a competitive equilibrium after appropriate transfers of initial endowments.

Interpretation of 2nd Welfare Theorem

- Distributional objectives can be achieved by 'letting markets work'.
- Requires 'only' a redistribution of initial endowments.

The redistribution of endowments is unaffected by household consumption decisions - it is 'lump sum'.

Chapter 3

Commodity (and income) taxation

3.1 First Best

3.1.1 Assumption

- Government requires a certain amount of revenue (purpose unspecified). What is the "best way" to raise this revenue, from the point of view of taxpayers?
- The criteria for evaluation include two aspects: efficiency and fairness.
- Efficient taxation of commodities (consumption): What is the least painful way to take a given amount of money from a consumer?
  - a lump-sum tax is efficient.
  - Backwards of lump-sum tax: Unless all house holds are to pay the same tax, a true lump sum tax is impossible to implement in practice.

- The sources where the government can get tax: (a) consumption tax ; (2) income tax.
- Distortions resulting from commodity taxation.
- A commodity tax can be partially avoided by changing the consumption plan. (substitution effect).

3.1.2 Model

Assume the good is normal and have CRS(Constant Return to Scale) production technology.

Definition: Deadweight loss

The extent to which the consumer's welfare loss (monetary equivalent) exceeds the revence raised.

$$DWL \approx \frac{1}{2}t|dx| \tag{3.1}$$

The change in demand curve dx depends on the elasticity of demand curve.

$$\epsilon^d = -\frac{dx}{dp}\frac{p}{x}$$

$$|dx| = \frac{xt}{p}|\epsilon^d|$$

(Plug in to the definition of DWL)

$$DWL = \frac{1}{2}|\epsilon^d|\frac{x}{p}t^2$$

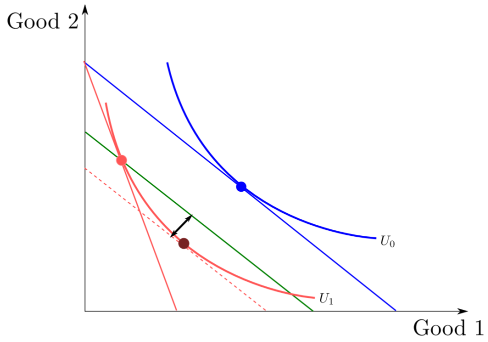


Figure 3.1: Deadweight Loss with 2 Goods

Note: Deadweight lost

(1)$DWL = \frac{1}{2}|\epsilon^d|\frac{x}{p}t^2$ implies that this function is convex. That suggests it is better to impose moderate tax on multiple goods than tax one good heavily.
(2) The DWL is increasing as $|\epsilon^d|$ increasing. That suggests that is better to tax inelastic demand goods.

However, these intuitions must be investigated in a model that includes multiple goods.

A tax will normally induce substitution and income effects.

Dead weight loss is associated only with **substitution effects**.

(i.e. if we don't change the relative prices between different goods or if the utility function is Leontief utility function (complement), there will be no substitution effects. Hence, there will also be no Deadweight loss.)

3.1.3 Model: How to allocate tax among different goods

3.1.4 Basic Setup

- One consumer with exogenous income y (numeraire good, p=1).
- 2 consumption goods produced by competitive firms (CRS technology)

at constant marginal cost c_i.

- Consumer's utility function $U(x_1, x_2)$.
- The government needs to secure $\hat{R}$ units of the numeraire.
- The government can tax both consumption goods.

3.1.5 Optimal (first best allocation)

$$\max_x U(x_1, x_2)$$
$$\text{s.t. } c_1x_1 + c_2x_2 = y - \hat{R}$$

The Lagrange function is

$$L = U(x_1, x_2) - \lambda(c_1x_1 + c_2x_2 - y + \hat{R}) \tag{3.2}$$
$$\Rightarrow \frac{\partial U}{\partial x_1} - \lambda C_1 = 0$$
$$\frac{\partial U}{\partial x_2} - \lambda C_2 = 0$$
$$\Rightarrow MU_i = \lambda c_i \tag{3.3}$$
$$\text{and } c_1x_1 + c_2x_2 = y - \hat{R}$$

3.1.6 Conclusion

Definition: First Best

When wealth is exogenous and all goods can be taxed, a uniform tax on all commodities achieves the first best.This is because a uniform consumption tax is equivalent to a lump sum tax on wealth.

3.2 Second Best

Questions

What if income is earned (i.e. endogenous)? More generally, what if some goods cannot be taxed?

Endogenous income: Income is the result of labor-leisure decisions.
The household chooses how much of his time (productive capability).
He will sell in the market as opposed to other ("homes") uses.

Definition: Leisure

Productive capacity not sold in the market. Since it is not traded, (we assume that) leisure cannot be taxed.

Distortion of taxing on other goods

A uniform tax on consumption goods make consuming 'leisure' relatively more attractive. The household may reduce labour supply, shifting productive capacity to other uses.

3.2.1 Simple Leisure- Consumption Model(Only one commodity)

Household is endowed with T units of (productive) time.

- He can sell his time at price w (wage).
- Thus, the value of his endowment is wT.

Feasible choices given pre-tax price p and wage w.

$$wL + p^{pre}c = wT \tag{3.4}$$

$$\Rightarrow p^{pre}c = w(T - L) \tag{3.5}$$

$$\Rightarrow c = \frac{w(T - L)}{p^{pre}} = \frac{wT}{p^{pre}} - \frac{w}{p^{pre}}L \tag{3.6}$$

**Another way to write the Budget constraint is that*

$$px = (T - L)w \tag{3.7}$$

where the LHS is consumption expenditure and the RHS is wage income.

Suppose that we want to raise w dollars (equivalent of one unit of time).

** The value of endowment:*wT

After the taxation, the value of endowment becomes $wT - w = w(T - 1)$.

Hence, the value of taxation (w) is equal to let individuals give up one unit of time (endowment).

Conclusion

The 1st best

1. The lump-sum tax is unrealistic, since the state would have to observe and tax earnings foregone as a result of time (productive productivity) not sold in the market.

2. Uniform tax is also unrealistic. The state would have to observe and tax earnings foregone as a result of time (productive capacity) not sold in the market.

In conclusion, both the lump sum tax and uniform tax are not realistic. A more realistic way is that the government can not tax leisure. It can only tax on consumption, which is traded in the markets.

Then, the price of leisure (opportunity cost) is fixed at ”1 productive hour” (worth w dollars), which means that the households can always choose the point $(T, 0)$.

Taxing consumption will change its price relative to leisure. Then at the chosen point

$$MRS = \frac{w}{p + t_c} \tag{3.8}$$

First best will not be chosen because there, $MRS^{FirstBest} = \frac{W}{P}$.

In conclusion, if one good (leisure) cannot be taxed, the best we can achieve by taxing the other is second best.

3.2.2 Model with 2 Commodities + Labor / Leisure

Assumption

1. 2 Consumption goods (x_1, x_2) produced by competitive firms using CRS technology (thus, no profits).

2. Labour is the only input in production, denoted by x_0 ($x_0 \geq 0$).

3. The price of labor (wage) is normalized to $p_0 = 1$.

4. Pre-tax prices for consumption goods $p_i^{pre} = c_i$ (marginal labour costs for good i).

5. The government taxes both consumption goods.

   (a) Labor is not taxed. This is a normalization.

   (b) Leisure is not taxed. This is a constraint.

6. After tax prices : $p_i^{post} = p_i^{pre} + t_i$.

7. Revenue constraint: $R = \sum_{i=1}^{2} t_i x_i$.

(1) Consumer's Decision Problem

The opyimal question of consuerms is

$$\max_{x_1, x_2, x_0} U(x_0, x_1, x_2) \tag{3.9}$$

$$\text{s.t } p_1^{post} x_1 + p_2^{post} x_2 = x_0 \tag{3.10}$$

The Lagrange function is that

$$L = U(x_0, x_1, x_2) - \lambda(p_1^{post} x_1 + p_2^{post} x_2 - x_0) \tag{3.11}$$

F.O.C

$$MU_0 = \lambda \tag{3.12}$$

$$MU_1 = \lambda p_1^{post} \tag{3.13}$$

$$MU_2 = \lambda p_2^{post} \tag{3.14}$$

where λ (Lagrange multiplier) is the marginal utility of lump sum income (i.e. income given exogeneously)

** F.O.C delivers consumption demands and labor supply as functions of p_1^{post} and p_2^{post}.*

(2) Government's maximizaion problem

By setting tax rates, the government directly determines post tax prices. Thus, we can equivalently think the government choosing the post tax prices $p^{post} = (1, p_1^{post}, p_2^{post})$, the consumer's indirectly utility is

$$V(p^{post}) = U(x_0(p^{post}), x_1(p^{post}), x_2(p^{post})) \tag{3.15}$$

The optimal question of the government: optimal commodity taxes maximizes this utility subject to the revenue constraint.

$$\max_{p_1^{post}, p_2^{post}} V(p^{post}) = U(x_0(p^{post}), x_1(p^{post}), x_2(p^{post}))$$
$$\text{s.t. } \sum_{i=1}^{2} t_i x_i(p^{post}) - R = 0$$

where $t_i = p^{post} - p_{pre}$.

The Lagrange function is that

$$L = V(p^{post}) - \mu(\sum_{i=1}^{2} t_i x_i(p^{post}) - R) \tag{3.16}$$
$$= V(p^{post}) - \mu[\sum_{i=1}^{2}(p^{post} - p^{pre}) x_i(p^{post}) - R] \tag{3.17}$$

F.O.C.

$$\frac{\partial v}{\partial p_1^{post}} = \mu(x_1^{post} + \sum_{i=1}^{2} t_1 \frac{\partial x_i}{\partial p_1^{post}}) \tag{3.18}$$
$$\frac{\partial v}{\partial p_2^{post}} = \mu(x_2^{post} + \sum_{i=1}^{2} t_1 \frac{\partial x_i}{\partial p_2^{post}}) \tag{3.19}$$

where

- $\frac{\partial v}{\partial p_1^{post}}$ and $\frac{\partial v}{\partial p_2^{post}}$ are the marginal utility loss due to taxation (the price of products is higher);
- μ is the marginal utility (to the consumer) of government revenue ($\mu < 0$);
- x_1^{post} and x_2^{post} (tax base) give direct effect of raising tax t on revenue;
  - Suppose x_i^{post} is fixed. Then increasing one unit of t, the increase of government's revenue is $x_i^{post} \times 1 = x_i^{post}$.

- Hence $\mu \times x_i^{post}$ measures how much utility will decrease due to higher government target revenue (direct effect; higher government target revenue means less personal disposable income).

- $\sum_{i=1}^{2} t_i \frac{\partial x_i}{p_i^{post}}$ represents indirect effect (via change in demands)
  - The change of utility is not only because of lower personal disposable income, but for the distortion of price mechanism.
  - $\frac{\partial x_i}{p_i^{post}}$ measures the distortion of consumption bundle due to taxation;
  - $t_i \frac{\partial x_i}{p_i^{post}}$ measures the taxation revenue loss due to imposing the policy (include both income effect and substitution effect).

Simplification: Independent Demands

To derive our first tax rule, suppose that demands are independent.

$$x_i = x_i(p_i^{post}) \tag{3.20}$$

** In the case of simplification, the demand only depends on its own price,i.e. there is no cross-price effect (substitution between the consumption goods).*

Then, the budget constraint is that

$$x_0(p_1^{post}, p_2^{post}) = p_1^{post} x_1(P_1^{post}) + p_2^{post} x_2(p_2^{post}) \tag{3.21}$$

According to the F.O.C. of the government.

$$\frac{\partial V}{\partial p_i^{post}} = -\mu(x_i^{post} + \sum_{j=1}^{2} t_i \frac{\partial x_j}{\partial p_i^{post}}) \tag{3.22}$$

According to Roy's identity $x_i(p,m) = -\frac{\partial V/\partial p_i}{\partial V/\partial m}$,

$$\frac{\partial V}{\partial p_i} = -x_i(p,m)\frac{\partial V}{\partial m} \tag{3.23}$$

Plug (3.21) into (3.20),

$$x_i(p, x_0)\frac{\partial V}{\partial x_0} = \mu(x_i^{post} + t_i \frac{\partial x_i}{\partial p_i^{post}}) \tag{3.24}$$

where x_0 is the lump-sum income and μ is the marginal utility of income. Therefore, under a (second best) optimal tax scheme, we have

$$x_i(p, x_0)(\frac{\partial V}{\partial x_0} - \mu) = \mu t_i \frac{\partial x_i}{\partial p_i^{post}}$$

$$x_i(\lambda - \mu) = \mu t_i \frac{\partial x_i}{\partial p_i^{post}}$$

$$\frac{t_i}{x_i}\frac{\partial x_i}{\partial p_i^{post}} = \frac{\lambda - \mu}{\mu} \tag{3.25}$$

where λ is the marginal utility of HHs' income and μ is the marginal utility of government's revenue.

* *Since* $\frac{\partial x_i}{\partial p_i^{post}} < 0$ *and* $t_i > 0$, $\lambda < \mu$.

The elasticity of demand is $\epsilon_i^d = -\frac{dx_i}{dp_i}\frac{p_i}{x_i}$, plug it into (3.23),

$$\epsilon_i^d \frac{p_i}{t_i} = \frac{\mu - \lambda}{\mu} \tag{3.26}$$

Rearrange the above equation, we can get **the inverse elasticity rule**:

$$\frac{t_i}{p_i} = \frac{1}{|\epsilon_i^d|}\frac{\mu}{\mu - \lambda} \tag{3.27}$$

Interpretation

1. The (proportional) rate of tax on good i should be inversely related to (the absolute value of) the elasticity of demand for good i.

2. Suggests that necessities should be high taxed.

   (a) From the equation of 'the inverse elasticity rule', we can know that the smaller $|\epsilon_i^d|$ is , the higher $\frac{t_i}{p_i}$ will be. And small $|\epsilon_i^d| \Leftrightarrow$ necessities.

   (b) Taxes on such necessities are difficult to avoid and therefore dead-weight loss is low.

3. This describes an efficient tax rule: equality concerns may suggest lower on necessities, as those are disproportionately consumed by lower income household.

4. Another interpretation of this rule is that leisure complements should be taxed more highly than leisure substitutes.

Definition: Leisure Substitutes and Leisure Complements

- **Leisure Substitutes:** If a tax causes the expenditure on x_i to fall, the household works less and therefore consumes more leisure.

- **Leisure Complements:** If a tax causes the expenditure on x_i to raise, the household works more and therefore consumes less leisure.

The expenditure of good i is $p_i x_i = e_i(p_i)$

Take the derivative w.r.t p_i, we have

$$\frac{de_i(p_i)}{dp_i} = x_i(p_i) + p_i \frac{dx_i(p_i)}{dp_i} \tag{3.28}$$

$$= x_i(p_i)(1 + \frac{p_i}{x_i}\frac{dx_i(p_i)}{dp_i}) \tag{3.29}$$

$$= x_i(1 - |\epsilon_i^d|) \tag{3.30}$$

- $|\epsilon_i^d| > 1 \Rightarrow$ elastic $\Rightarrow \frac{de_i(p_i)}{dp_i} < 0 \Rightarrow$ leisure substitute $\Rightarrow$ tax highly.

- $|\epsilon_i^d| < 1 \Rightarrow$ inelastic $\Rightarrow \frac{de_i(p_i)}{dp_i} > 0 \Rightarrow$ leisure complement $\Rightarrow$ tax lowly.

** For a linear demand function, the upper half of the image is the elastic region, while the bottom half of the image is the inelastic region. This is because for a linear demand function, the increase in the quantity consumed as*

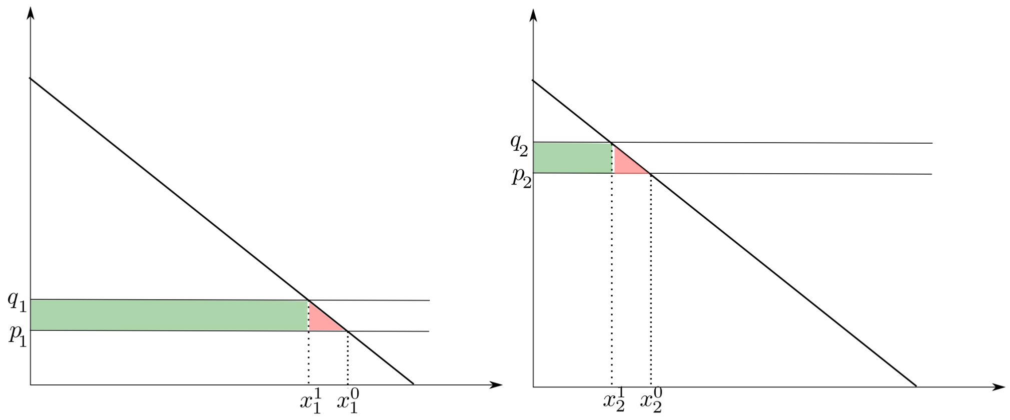


Figure 3.2: Second Best and Elasticity

a result of the same price decrease is fixed, the numerator remains constant, the denominator increases, and the absolute value of the fraction decreases.

Still, it is optimal to tax both goods such that the relative decline in demands is equalized.

General Analysis: Allowing for cross-price effects

$$x_i = x_i(p_1^{post}, p_2^{post}) \tag{3.31}$$

F.O.C. for the government:

$$\frac{\partial V}{\partial p_i^{post}} = -\mu(x_i + \sum_{j=1}^{2} t_j \frac{\partial x_j}{\partial p_i^{post}}) \tag{3.32}$$

According to Roy's identity, the LHS of the equation is

$$\frac{\partial V}{\partial p_i^{post}} = -x_i \frac{\partial V}{\partial x_0} = -\lambda x_i \tag{3.33}$$

* *Note that* $\frac{\partial V}{\partial x_0}$ *is the marginal utility of income, the meaning of Lagrange multiplier in household's utility optimal problem.*

Thus, we have

$$-\lambda x_i = -\mu(x_i + \sum_{j=1}^{2} t_j \frac{\partial x_j}{\partial p_i^{post}})$$

$$\Rightarrow (\mu - \lambda)x_i = -\mu(\sum_{j=1}^{2} t_j \frac{\partial x_j}{\partial p_i^{post}})$$

$$\Rightarrow -\frac{\mu - \lambda}{\mu} x_i = \sum_{j=1}^{2} t_j \frac{\partial x_j}{\partial p_i^{post}}$$

$$\Rightarrow -\frac{\mu - \lambda}{\mu} = \frac{\sum_{j=1}^{2} t_j \frac{\partial x_j}{\partial p_i^{post}}}{x_i} \tag{3.34}$$

According to Slutsky equation,

$$\frac{\partial x_j(p^{post}, x_0)}{\partial p_i^{post}} = \frac{\partial x_j^h(p^{post}, \bar{u})}{\partial p_i^{post}} - \frac{\partial x_j(p^{post}, x_0)}{\partial x_0} x_i(p^{post, x_0}) \tag{3.35}$$

Plug (3.35) into (3.34) ,

$$\frac{\sum_{j=1}^{2} t_j (\frac{\partial x_j^h}{\partial p_i^{post}} - \frac{\partial x_j}{\partial x_0} x_i)}{x_i} = -\frac{\mu - \alpha}{\mu} \tag{3.36}$$

Rearrange the equation,

$$\frac{\sum_{j=1}^{2} t_j \frac{\partial x_j^h}{\partial p_i^{post}}}{x_i} = -\frac{\mu - \alpha}{\mu} + \sum_{j=1}^{2} \frac{\partial x_j}{\partial x_0} t_j \tag{3.37}$$

Recall that $\frac{\partial x_j^h}{\partial p_i} = \frac{\partial x_i^h}{\partial p_j}$ (symmetry of substitution matrix). Thus,

$$\sum_{j=1}^{2} t_j \frac{\partial x_j^h}{\partial p_i^{post}} = \sum_{j=1}^{2} t_j \frac{\partial x_i^h}{\partial p_j^{post}} \tag{3.38}$$

Then (3.37) can be rewritten as

$$\frac{\sum_{j=1}^{2} t_j \frac{\partial x_i^h}{\partial p_j^{post}}}{x_i} = -\frac{\mu - \alpha}{\mu} + \sum_{j=1}^{2} \frac{\partial x_j}{\partial x_0} t_j \tag{3.39}$$

Then, define

$$\theta = -\frac{\mu - \alpha}{\mu} + \sum_{j=1}^{2} \frac{\partial x_j}{\partial x_0} t_j \tag{3.40}$$

Noting that θ does not depend on k, since for each kind of good's equation, the part "$\frac{\partial x_j}{\partial x_0} t_j$ "is always the same.

That is to say, for each kind of good,

$$\frac{\sum_{j=1}^{2} t_j \frac{\partial x_i^h}{\partial p_j^{post}}}{x_i} = \theta(\text{i.e. the same number}) \tag{3.41}$$

This is **Ramsey Rule**.

Interpretation of Ramsey Rule

An optional system of commodity taxation is such that the relative change in compensated demand is equalized across all goods.

Since $\frac{\partial x_i^h}{\partial p_j^{post}}$ means how does one unit of "good j's price change" affects the compensate demand of good i (total substitution effect for good i) , $t_j \frac{\partial x_i^h}{\partial p_j^{post}}$ measures the compensated demand for good k caused by the tax on good i.

- The effect of t_i itself is approximately $t_i \frac{\partial x_i^h}{\partial p_i^{post}}$.
- The effect of $t_j (i \neq j)$ itself is approximately $t_j \frac{\partial x_i^h}{\partial p_j^{post}}$

Extending this argument to the entire set of taxes, the LHS approximates the relative change in compensated demand for good k due to all the taxes in place.

That is: the relative change in demand for good k that would result if all taxes were reduced to zero and a utility-equivalent lump sum tax was imposed.

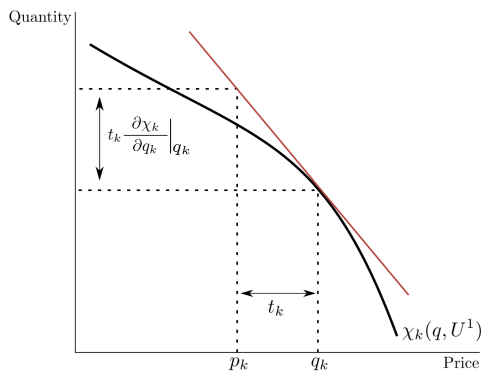


Figure 3.3: Approximation of Compensation Demand

Thus the Ramsey rule says the following: **An optimal system of commodity taxation is such that the relative change in compensated demand is equalized across all goods.**

Chapter 4

Public goods and Externalities

The chapter begins by defining a public good, and distinguishing between public goods and private goods.Doing so provides considerable insight into why market failure arises when there are public goods. **The Samuelson rule** characterizing the efficient provision of a public good.

Then focus of the chapter next terns to the consideration of methods through which efficiency can be reached. The first of this, **the Lindahl equilibrium**, is based on the observation that the price each consumer pays for the public good should reflect their valuation that the price each consumer pays for the public good should reflect their valuation of it. The Lindahl equilibrium achieves efficiency, but since the valuations are private information, it generates incentives for consumers to *provide false information.*

Mechanisms designed to elicit the correct statement of these valuation are then considered. The theoretical results are contrasted with the outcomes of experiments designed to test the extent of false statement of valuations and the use of market data to calculate valuations. These results are primary static in nature.

4.1 Public goods and private goods

Definition: Pure Public Good

A pure public good has the following 2 properties:

Nonexcludability: If the public good is supplied, no consumer can be excluded from consuming it.

Nonrivalty: Consumption of the public good by one consumer does not reduce the quantity avaliable for consumption by any other.

Note

Most public goods eventually suffer from congestion when too many consumers try to use them simultaneously.

For example, parks and roads are public goods that can become congested.

The effect of congestion is reduce the public good yields to each user.

Public goods that are excludable but at a cost, or suffer from congestion beyond some level of use are called **impure**.

	Rivalrous	**Non-Rivalrous**
Excludable	Private good	Club good
Non-Excludable	Common property resource	Public good

Figure 4.1: Pure goods and impure goods

In fact, it is helpful to envisage a continuum of goods that gradually vary in nature as the become more rivalrous or more easily excludable.

Note

(1) Common property resource (the commons): examples such as a lake that can be used to fishing by anyone who wishes or a field that can be used for grazing by any framer.

The problem with the common is the tendency of overusing then, and the usual solution is to establish property rights to govern access.
(2) Club goods are public goods for which exclusion is possible. The terminology is motivated by sport clubs whose facilities are public good for members but from which nonmembers can be excluded.

4.2 Public Provision and Private Provision

Public goods do not conform to assumptions required for a competitive economy to be benefit. Their characteristics of nonexcludability and nonrivalry lead to the wrong intensive to **rely on others to make purchases of the public good**. This reliance on others to purchase is called **free-riding**, and it is this that leads to inefficiency.

4.2.1 Assumptions

1. A simple model with quasi-linear preferences.
2. N consumers: $h=1, \ldots, H$
3. Private good (dollars): m^{h} (specific to Mr. h)
4. Public good: G (same for all h)
5. Cost of public good: one unit costs c dollars
6. Preferences: $U^{h}\left(m^{h}, G\right)=m^{h}+\phi^{h}(G)$,

7. Assume: $\phi^{h\prime} > 0, \phi^{h\prime\prime} < 0$

8. Heterogeneity: The functions ϕ^h differ, so $\phi^{h\prime}(G) \neq \phi^{k\prime}(G)$ for all G when $k \neq h$ (This simplifies part of the analysis.)

9. Endowments: Consumer h has ω_h dollars to start

10. Technical: allow $m^h < 0$ (to avoid corner solutions)

Feasibility

An **allocation** is a level of public good provision G and an amount of money m^h for each consumer. An allocation is **feasible** if

$$\sum_H m^h + c \times G \leq \sum_H \omega^h \tag{4.1}$$

Definition: Allocation

An allocation is a level of public good provision G and an amount of money m^h for each consumer.

**Allocation: to describe both consumption bundles and income.*

Definition: Pareto Efficient Allocation

A feasible allocation $(G, m^1, ..., m^H)$ is efficient if there is no other feasible allocation $(\tilde{G}, \tilde{m}^1, ..., \tilde{m}^H)$ such that:

$U^h\tilde{G}, \tilde{m}^1, ..., \tilde{m}^H) > U^h((G, m^1, ..., m^H))$for at least one h

and

$U^h\tilde{G}, \tilde{m}^1, ..., \tilde{m}^H) \geq U^h((G, m^1, ..., m^H))$ for all h

Fact: In this model, an allocation is Pareto Efficient if and only if the level of public good provision G maximizes "Marshallian Aggregate Surplus (MAS)".

$$S(G) = (\sum_h \phi^h(G)) - cG \tag{4.2}$$

Interpretation of Marchallian Aggregate Surplus(MAS)

This equation is true since the utility function is quasi-linear function. In fact, what MAS is actually trying to describe is the fact that the sum of the increase in utility of all individuals in society resulting from a one-unit increase in the supply of a public good is equal to the decrease in individual utility resulting from a decrease in the consumption of a private good by each individual as a result of having to pay for the public good.

Of course, different people do not necessarily pay equally for public goods.

4.2.2 Pareto Efficient Allocations (Public Provisions)

Any Pareto efficient allocation involves a level of G that maximizes MAS:

$$S(G) = (\sum_h \phi^h(G)) - cG \tag{4.3}$$

** In optimal problems, what parameters are adjusted to achieve optimality depends on who the actor is, i.e. "who is considering the problem".*

The problem he is considering will be the objective function, and the variables he is able to control determine which variables the Lagrangian function will be biased with respect to.

Take the derivative on both sides of the equation with respect to G at the same time (F.O.C). A pareto efficient level of the public good is therefore characterized by

$$\sum_h \phi'(G) \leq c \tag{4.4}$$

** The inequality sign here is very noteworthy. Whether or not public goods are provided, and if so how much should be provided, depends on the size relationship between $\sum_h MU(G)_h$ and $\sum_h MU(C)$. If $\sum_h MU(G)_h$ is very small, then the best option is not to provide the public good, in which case the inequality sign is taken.*

This is the **Samuelson condition**: the marginal cost of producing a public good should be equal to the sum of consumers' marginal utilities (i.e. the sum of everyone's willingness to pay for another unit).

4.2.3 Private provision using price mechanism

Assumption

- Public good sold on competitive market, at price $p = c$.
- HHs buy individually and independently.
- g^h is the amount which is purchased by Mr.h, so that $G = \sum_h g^h$.
- When deciding how much to buy, consumers take other's purchases as given.
- The standard approach to this type of problem is to look for a **Nash Equilibrium**.

**** Decentralized, individually decision: Nash Equilirbium.***

$\Rightarrow$ *Suppose the decision of the others doesn't change, will you change your current decision?*

If nobody say "yes", the situation can be a Nash Equilibrium. ***** Does there exist Pareto Improvement?***

When we consider questions related to "pareto improvement", we must recognize that we can change ***many*** *people's decisions to reach a better situation.*

Thus, Nash equilibrium is not necessary to be Pareto efficient. Also, a Pareto efficient equilibrium is also not necessary to be a Nash equilibrium. (e.g. prisoner's dilemma)

Individual's Optimal Problem in Decentralized Decision

Let $\hat{g}^h$ be h's purchase in equilibrium, and denote total contributions by

$$\hat{G} = \sum_h \hat{g}^h \tag{4.5}$$

Each $\hat{g}^h$ solves

$$\max_{g \geq 0} U^h(w^h - cg, \hat{G}^{-h} + g) \tag{4.6}$$

Suppose g^h, i.e. Mr. h purchases a positive amount.

Then Mr. h must value an extra unit at exactly c dollars:

$$\phi^{h'}(\hat{G}) = c \tag{4.7}$$

When marginal valuations differ, this condition cannot hold for several consumers at the same time.

Thus, *at most one consumer* can purchase a positive amount $g^h > 0$.

For all consumers who do not purchase positive amounts, must have $\phi^{k'}(\hat{G}) < c$.

- **Suppose** $\hat{G} = 0$, then this will occur whenever $\phi'(0) < c$ for all individuals.

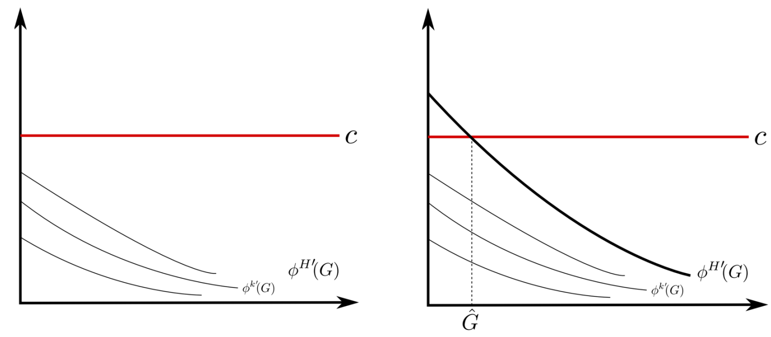


Figure 4.2: Left: $\hat{G} = 0$ Right: $G = \hat{G}$

- **Suppose** $\hat{G} > 0$, when marginal valuations differ, at most one person contributes. This must be the person with the highest marginal valuation (i.e. $\phi^{H'}(G)$).
  - The private provision will keep increasing until the person who have the highest marginal utility doesn't want to buy one more anymore.

4.2.4 Efficiency of Public Decision and Decentralized Decision

The public decision (the Samulson Condition)

$$\sum_h \phi^{h'}(\hat{G}) = c \tag{4.8}$$

The private decision

$$\phi^{H'}(\hat{G}) = c$$
$$\phi^{k'}(\hat{G}) > 0$$
$$\Rightarrow \sum_h \phi^{h'}(\hat{G}) > c \tag{4.9}$$

This is to say, the private decision provision is less than the public decision provision, since function $\phi^{h'}$ is decreasing.

4.2.5 Conclusion and interpretation

1. This analysis suggests that a fully decentralized price mechanism will result in inefficient under provision of public goods.

2. Many economists conclude from this that government intervention (forced contributions) is necessary.

3. Important substantive assumptions to highlight include:

   (a) Each consumer acts independently (no agreements between consumers possible)

   (b) Behavior of others is taken as given (no reciprocity, etc.)

   (c) Consumers are assumed to be 'selfish'

4. These assumptions might be regarded as somewhat 'pessimistic'.

   (a) Perhaps 'real people' contribute more than the analysis suggests?

   (b) Perhaps communication / private contracting can help even without government intervention?

4.3 Diagnosis: Market Failure

- When deciding independently, and using the price mechanism, individuals do not consider benefits bestowed on others.

- Only those who benefit **most** from a public good may provide a positive amount voluntarily.

- Nonexcludability means others can 'free ride'. The ability to free ride destroys incentive to pay.

⇒ Under provision relative to Pareto optimal benchmark.

4.4 Potential role for government intervention

4.4.1 'First best' policy

- Provide $G = G^0$ satisfying Samuelson condition.
- Use lump sum taxes (confiscation of endowments) to pay.
- Involves centralized decision making and coercion.

4.4.2 Problems

- Require knowledge of all individual valuations
- If preferences differ, consumers would have to pay different prices in order to implement a Pareto improvement.

4.5 Lindahl Pricing

If each consumer compares the full marginal cost of the public good (its price) to only her share of the marginal benefits, the provision of public good would be efficient. In this case, since marginal benefits differ, the 'price' (share) paid by different consumers would have to differ.

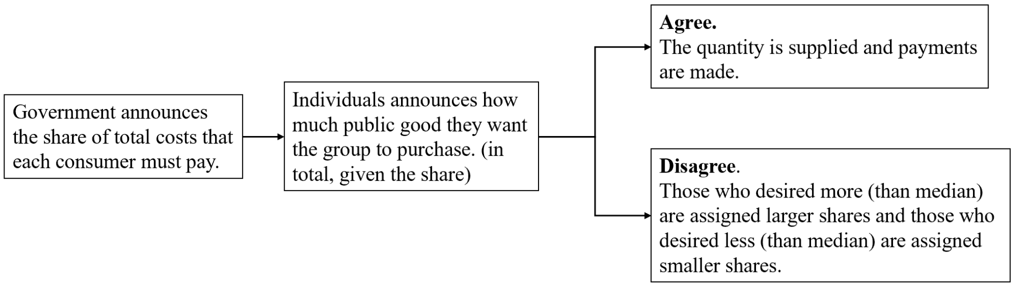


Figure 4.3: Lindahl Pricing Mechanism

Lindahl (1919) proposed a mechanism to find the appropriate prices (shares), and the optimal quantity to be provided.

Personalized pricing is aimed at two objectives

- Deal with the fact that consumers consider only their 'share' of the marginal benefits.
- 2 accommodate differences in consumers' marginal utilities.

4.5.1 The Process of Lindahl Pricing

Process repeats until demand match $\Rightarrow$ 'Lindahl Equilibrium'

4.5.2 Math model if Lindahl mechanism

Denote that

- τ^h = share of public good costs paid by household h
- $G^h\left(\tau^h\right)$ = quantity household h wants the community to demand, in total, given that he must pay a share τ^h

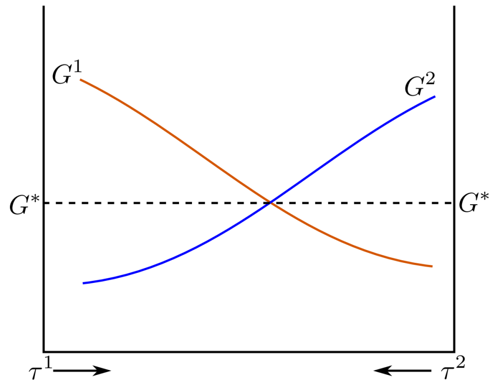


Figure 4.4: Pareto Optimality of Lindahl Equilibrium

Then, the Lindahl demand function of Mr.h is

$$U^h\left(\omega^h - \tau^h cG, G\right) = \omega^h - \tau^h cG + \phi^h(G) \tag{4.10}$$

** Note that in this decision-making process, the proportion of the public good that each individual has to pay (i.e. τ^h) is given. Mr.h is only able to choose the total amount of the public good that he wants to be provided given the proportion of the payment.*

F.O.C:

$$-\tau^h c + \phi^{h'}[G^h(\tau^h)] = 0$$

$$\Leftrightarrow \phi^{h'}(G^h) = c\tau^h \tag{4.11}$$

Taking first-order partial derivatives about τ^h on both sides of the equation

$$\phi^{h''}(G^h)\frac{dG^h}{d\tau^h} = c$$

$$\Rightarrow \frac{dG^h}{d\tau^h} = \frac{c}{\phi^{h''}(G^h)} < 0$$

since $\phi^{h''}(G^h) < 0$.

Hence, the function $G^h(\tau^h)$ is decreasing. Equilibrium is reached when $G^1 = G^2 = G^*$.

Then $\phi^{1'}(G^*) = \tau^h c$ for $h = 1, 2$ and so

$$\phi^{1'}(G^*) + \phi^{2'}(G^*) = \tau^1 c + (1 - \tau^1)c = c \tag{4.12}$$

So G^* satisfies the Samuelson condition.

4.5.3 Backwards of Lindahl Pricing

Construction of Lindahl Equilibrium assumed that households truthfully report their demands for each possible sharing rule.

But: If households expect this information to be used as it is, they may have an incentive to misreport their preferences.

To analyse this, assume that households seek to influence the amount purchased AND the share they pay.

The indifference curve of individuals is

$$w - \tau c G + \phi(G) = \bar{U} \tag{4.13}$$

where $\bar{U}$ is a constant.

Take derivative with respect if τ^h on both sides,

$$\begin{aligned} 0 &= -cG(\tau^h) - \tau^h c G'(\tau^h) + \phi^{h'} G'(\tau^h) \\ \Leftrightarrow \; cG(\tau^h) &= G'(\tau^h)(\phi^{h'} - \tau^h G) \end{aligned}$$

$$G'(\tau^h) = \frac{cG(\tau^h)}{\phi^{h'} - \tau^h G} \tag{4.14}$$

According to the first order condition if Lindahl equilibrium,

$$\phi^{h'} - \tau^h G = 0$$

Hence, at this point,

$$G'(\tau^h) \to \infty \tag{4.15}$$

i.e. For given τ^h, preferred G^h occurs where IC is vertical. The set of such points is Lindahl demand function.

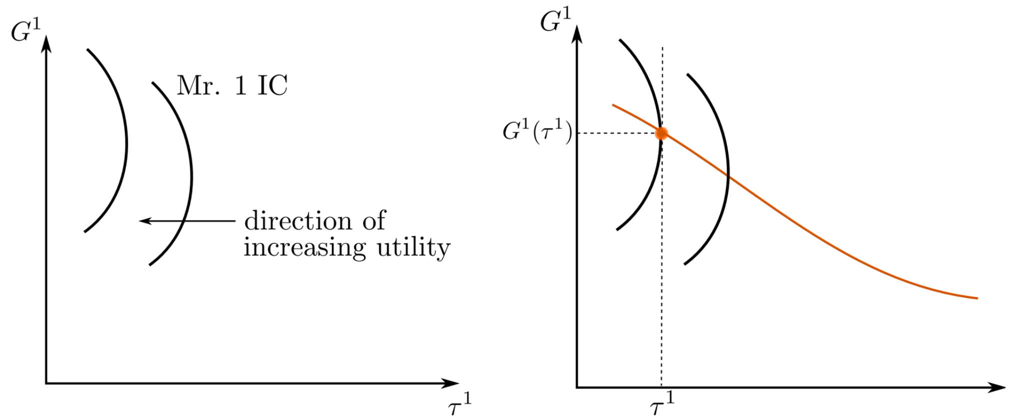


Figure 4.5: Preferences over G^h and τ^h and Lindahl Demand Curve

Incentives to be Dishonest

Mechanism motivating Lindahl Equilibrium implies incentives to be dishonest.

If household 1 always report $G^1(\tau^h)$ truthfully, household can gain more if he tells lie. (His share will be smaller and thus the indifference curve of Mr.2 will move rightwards. The new IC of Mr.2 represents higher utility.)

The reason why people have the incentive to be dishonest is that this mechanism lacks the means to test people for lying. That is, people can benefit from lying, since other people cannot realise that he is lying.

If people's lying behaviour is unavoidable, everyone will under-report their demand for public goods. As a consequence, society as a whole is under-provided with public goods.

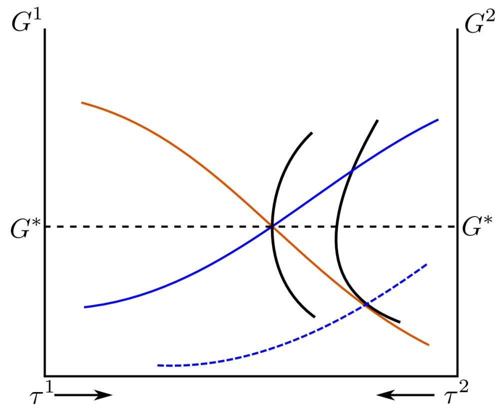


Figure 4.6: Incentives to be Dishonest

4.6 Clarke-Groves Mechanism

4.6.1 Simple Setting

- Decision whether or not to provide a public good ($G = 1$ or $G = 0$)
- Each household achieves a privately known net benefit v^h if the project is undertake.
    - Net benefits can be positive or negative.
        * Net Benefits= the utility gain from providing the public good - the utility loss due to the payment
    - The costs of providing the good (and how they are shared) are taken into account.
- Goal: Get households to reveal net benefits, and provide the good if the sum is positive.

Suppose there are 2 consumers, their net benefits are v^1 and v^2 respectively.

4.6.2 Game

- Each consumer submits a report r^h.
- The public good is provided if $r^1 + r^2 \geq 0$,i.e. $G = 1$.
- **If so, each consumer receives a side payment equal to the reported net benefit of *the other* consumer** (gross benefit=net benefit+ cost):
  - This is to say, if Mr.1 and Mr.2 report r^1 and r^2 respectively, then Mr.1 need to pay the amount reported by Mr.2, i.e. r^2. And similarly, Mr 2 needs to pay r^1.
    - Mr.1 's payoff (total gain from providing the public good) is $(v^1 + r^2)$.
    - Mr.2 's payoff is $(v^2 + r^1)$.
- If the public good is not provided, both receive a payoff of zero.

4.6.3 Theorem

Under the Clarke-Groves Mechanism, 'telling the truth' is a weakly dominant strategy.

* *It is important to note that the question we are considering here is simply whether the public good is provided, not how much of it is provided.*

In this context of considering only yes or no, an individual reporting a large enough but not too large value or a very, very large value actually leads to the same result, i.e., to provide the public good. Thus, individuals have no incentive to overstate their utility.

At the same time, since he pays the portion declared by others rather than the amount declared by himself, he also has no incentive to under-report the level of my utility.

4.6.4 Proof

We have already shown that $r^2 = v^2$ results in at least as great a payoff as any other report against any report r^1. So we still need to show that any other report $r^2 \neq v^2$ yields a lower payoff against some reports r^1.

Consider under-reporting, i.e. $r^r < v^2$ and take $r^1 \in (-v^2, -r^2)$.

Since $r^1 < -r^2$, the public good is not provided and thus Mr.2's payoff is 0.

But since $r^1 > -v^2$, declaring truthfully would have yield a positive payoff.

Thus, under-reporting never leads to a larger and sometimes leads to a smaller payoff than telling the truth.

A symmetric argument can be made about over-reporting, i.e. $r^2 > v^2$.

4.6.5 Interpretation of Clarke-Groves Mechanism

The Clarke-Groves mechanism induces truth telling and causes the public good to be provided whenever this optimal. *(i.e. in those situations where it would be provided in a Pareto optimal allocation.)*

The way this works is that people are paid exactly the social benefits they otherwise ignore.

4.6.6 Problem of Clark-Groves Mechanism

- Whenever provision is (strictly) efficient, side payments will exceed the social benefits!
- These payments have to come from outside the mechanism. The mechanism is not *revenue neutral.*
  - With two players, side payments total $(v^1 + v^2)$.
  - With three players, they total $2(v^1 + v^2 + v^3)$. *(Mr.1's cost is* $r^2 + r^3$*, Mr.2's cost is* $r^1 + r^3$ *and Mr.3's cost is* $r^1 + r^2$*.)*
- Obtaining truth is possible, but *information is costly.*

4.7 Clarke Mechanism

Clarke mechanism can reduce, but can not eliminate the costs of information.

4.7.1 Modified side payments

- For each player, check whether her report is pivotal: Would the sign of the sum have been different if she had said zero?
  - The **pivotal voter** is the person whose declaration can affect the provide of the public good, i.e., if drop the person, the positivity and negativity of the sum of payments will be changed.
- All pivotal players pay a tax equal to the absolute value of the sum of the others' announcements.
  - If the good is provided, that sum was negative.

  - If the good is not provided, that sum was positive.

- Any non-pivotal player does not pay a tax and receives no side payments.

4.7.2 Theorem

Under the Clarke Mechanism, 'telling the truth' is a weakly dominant strategy.

Some things to notice

- Any revenue collected must be disposed of 'outside' of the mechanism.
- It is possible for multiple or no players to be pivotal.
- If no player is pivotal, no payments are made. If multiple players are pivotal, all of them need to pay.

4.7.3 Discussion

1. The Clarke Mechanism induces truth telling (same reason as Clarke-Groves Mechanism)
2. By restricting payments to pivotal agents, truth comes cheaper - and is paid for by the agents. *(since only the pivotal voters rather than all voters need to pay)*
3. But: Any revenue must not be paid out to the agents! Doing so would distort the mechanism's incentives.
4. Both mechanisms suffer from two fundamental flaws:

(a) Although the public good is provided when Pareto optimal, the result is not necessarily a Pareto improvement over no provision. *(Except in special cases - which ones?)*

i. It is important to note that there is a duplicate payment issue here.

ii. If only one person is the pivotal voter in the current situation, then through a rational redistribution mechanism, his payment can be used to compensate for those voters who are not the same as the end result. Ultimately a Pareto efficient result is achieved.

iii. But if there is more than one pivotal voter, the first person's payment has already compensated the remaining people once. The second person's payment is equivalent to compensating those voters again. That is, those whose original decision was violated have been compensated more than once.

iv. That is, in the posterior analysis, even if offering the good is a Pareto-optimal outcome, this outcome may not be a Pareto-improvement compared to the original situation (not offering) if the Clark Mechanism is used to reach this outcome.

v. *The reason for this is that there is a cost to obtaining this information that everyone is telling the truth.*

(b) In fact, since taxes must 'disappear,' the result may not even be Pareto optimal (unless no agent is pivotal so that no taxes are paid).

5. No incentive compatible mechanism can always elicit the information necessary for optimal public goods provision at no cost (Gueth and

Hellwig 1986).

6. This highlights importance of collective decision making (politics) in the context of public goods.

Chapter 5

Externality

Definition: Externality

- "An externality is present whenever some economic agent's welfare (utility of profit) is directly affected by the action of another agent (consumer or producer) in the economy." (S. Coate)
- "An externality is the link between economic agents that lies outside the price system of the economy." (Hindriks Myles)
- "... an unintended impact... no price mechanism for coordinating individual actions" (D. Mueller)

Classic Examples

- Factory blowing smoke. (negative externality)
- Airplanes making noise. (negative externality)
- Bees pollinating fruit trees. (positive externality)

In each case, one party engages in an activity that affects another.

Negative effects are not paid for and positive effects are not rewarded.

5.1 Simple quasi-linear model

5.1.1 Assumption

Consumers are endowed with ω_h units of the numeraire good m. Good z produced by competitive firms as constant MC of c (units of m).

$$U^1(m_1, z_1) = m_1 + \phi(z_1) \tag{5.1}$$

$$U^2(m_2, z_2) = m_2 + \alpha z_1 \tag{5.2}$$

$$\phi' > 0 \text{ and } \phi'' < 0 \tag{5.3}$$

Technically, we allow $m_i < 0$ to avoid corner solutions.

Interpretation

Consumption of good z is associated with an externality.

α reflects household 2's marginal (dis)utility of household 1's consumption.

(positive externality: $\alpha > 0$; negative externality: $\alpha < 0$)

5.1.2 Pareto efficient allocations

Quasi-linear utility $\Rightarrow$ an allocation is Pareto efficient iff z_1 maximizes Marshallian aggregate surplus.

$$\max_{z_1} \phi(z_1) + \alpha z_1 - c z_1 \tag{5.4}$$

First order necessary condition

$$\phi'(z_1^0) + \alpha \leq c \ (= \text{if } z_1^0 > 0) \tag{5.5}$$

Remark

This characterizes a unique Pareto efficient level of z_1 only.

Any allocation of the remaining m is Pareto efficient.

5.1.3 Competitive equilibrium

We can normalize the price of good m to 1.

Equilibrium (producer and consumer) price of z:

$$p = c \tag{5.6}$$

Household 1 chooses z_1 to maximize

$$\max_{z_1} U^1(\omega^1 - cz_1, z_1) = (\omega^1 - cz_1) + \phi(z_1) \tag{5.7}$$

F.O.C.

$$\phi'(z_1^*) = c \tag{5.8}$$

5.1.4 Competitive Equilibrium v.s. Pareto Efficiency

Competitive equilibrium$\phi'(z_1^*) = c$.

Pareto efficiency: $\phi'(z_1^0) + \alpha \leq c$ (= if $z_1^0 > 0$)

Then, if $\alpha < 0$ (negative ecternality),

5.1.5 Diagnosis: Market Failure

The competitive equilibrium outcome (as modeled) will generally not be Pareto efficient.

Thus, the decentralized price mechanism does not coordinate behavior perfectly.

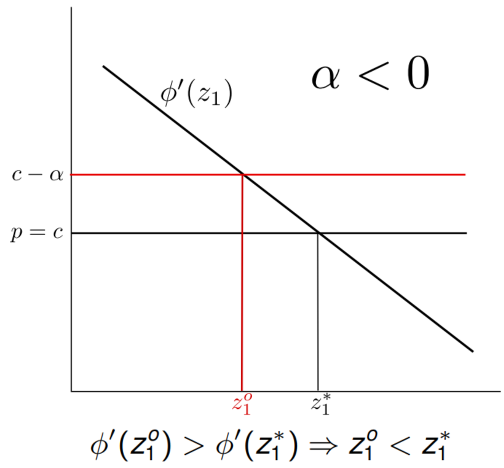


Figure 5.1: Competitive Equilibrium and Pareto Efficient Equilibrium with Negative Externality

All decisions are made by separate individuals comparing prices to their own marginal benefits.

These prices may not appropriately reflect (or communicate) all relevant costs and benefits. *(The (opportunity) costs from production and due to rivalry are included. But additional 'external' costs or benefits are not.)*

Traditional public finance offers two remedies:

- Direct (quantity) regulation
- Taxes and subsidies

5.2 Arthur Pigou (1920): The Economics of Welfare

The source of inefficiency is the divergence between social and private benefits.

* *Where 'Social benefits' = Sum of private benefits and costs.*

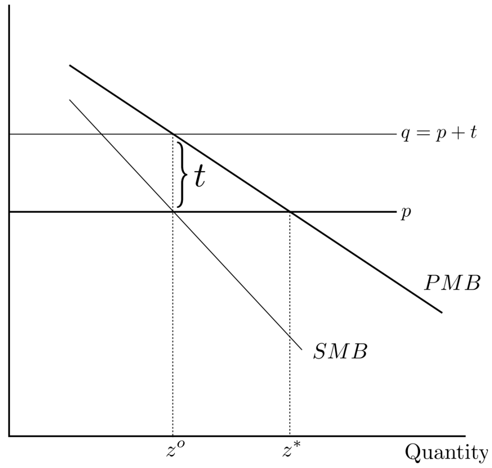


Figure 5.2: Pigouvian Taxation

Pigou's solution: adjust prices using appropriate taxes and subsidies.

Example : Negative Consumption Externality.

$$\text{(Private marginal benefit) } PMB = SMB \text{ (Social marginal benefit)} \quad (5.9)$$

$$\text{Appropriate tax} = \text{Marginal (monetary) damage caused by externality} \quad (5.10)$$

Implications for economic policy

1. Taxes and subsidies can force individuals to 'internalize' costs (benefits) imposed on others.

   (a) Change prices such that private cost=social cost

   (b) As a result, the deadweight loss will occur. (distortion of price mechanism).

2. Pigouvian taxation is very popular with economists, especially in the context of environmental policies. e.g.

(a) Eco taxes on fuel.

(b) Subsidies for solar panels.

3. An important alternative is quantity regulation, possibly combined with a market for licenses. e.g. EU carbon emissions market.

4. In addition to correcting the market failure, such policies can generate government revenue ('double dividend') .

5.2.1 Criticisms of Pigouvian Taxation

- Government (implicitly) modeled as omniscient and benevolent dictator.
  - omniscient: know everything it need.
  - benevolent: mercy
  - dictator: all decisions are made by one person. *(related to society's dictatorship preference)*
- Knowledge of costs and benefits is subjective, private, and dispersed. How can the policy maker obtain this information?
- Is it reasonable to assume that the policy maker's goal is to achieve efficiency?
  - Instruments may be used to raise revenue rather than control externalities.
  - Market participants will seek to influence these policies to their advantage.
- Pigouvian policies are typically not designed to achieve a Pareto improvement.

  - Imposing Pigouvian taxes harms those who are taxed.
  - Advovates rarely discuss the issue of compensation.

5.2.2 Implications for Law (Property Rights, liability Rules)

Individuals who are harmed by others' activities should be allowed to sue in court.

Courts should award damages corresponding to the (estimated) costs imposed.

This will cause individuals and firms to optimally trade off benefits and costs.

Examples

(1) Factory emitting smoke should pay for estimated costs (health, property value,...)

(2) Airline flying over neighborhood should pay for estimated value of sleep lost, etc.

5.3 Coase (1960) : The Problem of Social Cost

The Pigouvian analysis was long accepted by economists and legal scholars, until Coase(1960) argued that it is completely wrong!

Coase makes four important points

1. The problem considered is actually reciprocal in nature.

2. In an **ideal world**, private exchange can solve this problem.

3. In the **real world**, the problem we face is actually different.

4. Government intervention is only one of several ways that we might deal with the ***real problem***.

5.3.1 Step 1: the problem is reciprocal in nature

"It Takes Two"

- 'Externalities' result from the choices of multiple people.
  - Smoke from the factory would be okay if no one lived close to it.
  - Noise from airplanes would be okay if no one slept under them.
  - E.g. a doctor disturbed by noise from confectioner's machinery.
- But whichever way the issue is settled, someone will be harmed.
  - Either the doctor is distributed by noise.
  - Or the confectioner is harmed by stopping production.

Relevant Question (according to Coase)

Which harm should be avoided? Coase assumes that the goal is to maximize the total value of production. *(This is consistent with Pigouvian approach)*

Which is the lesser harm?

Suppose the confectioner's benefit from noise = 40;

Doctor's loss due to noise = 60.

Then indeed it appears to be efficient if the confectioner stops. But...

Suppose the doctor can move his office at a cost of 20.

Then it would be efficient if the confectioner continues. *(The confectioner compensate the office moving cost (20 dollars) and then the doctor's utility will be the same as before. But the confectioner can also gain 20 dollars.)*

Least Cost Avoider Principle

- Efficiency requires that the party that can avoid a conflict at lower cost should change their behavior.
- This will not always be the party that ostensibly 'causes' harm from an intuitive standpoint.

Implication

- Just because the confectioner causes the noise (in a physical sense), this does not mean that he (alone) is responsible for the externality.
- If the court rules in favor of the doctor, the result might be inefficient.
- According to Coase, the Court should recognize that the problem is **reciprocal in nature**.

Contrast with Standard Pigouvian Analysis

- The standard analysis (implicitly) assumes that only one party causes the externality.
  - Mr. 1 chooses how much z1 to consume.
  - Mr. 2 passively suffers external harm.

- The model we looked at disregards the possibility that Mr. 2 might have means to avoid the externality.

- To appreciate Coase's first point, we could try to modify the model to allow for this.

5.3.2 Step 2: : In a perfect world, the problem can be solved through private agreements.

Private Exchange (version 1)

Confectioner's benefit from making noise = 40, doctor's cost = 60.

Suppose the doctor has a property right to a quiet office, meaning

- Confectioner is prohibited from making noise.

- Doctor can allow noise in exchange for payment.

Assume that the parties can realize any mutually beneficial agreement at no cost.

Then, the doctor may ask for $40 (get total production surplus of the confectioner).

Efficiency of Costless Private Exchange

If the parties can exchange at no cost, they will decide on an arrangement that maximizes the value of production.

Different 'deals' are imaginable.

- 'Gains from trade' can be split in different ways.

- Any such deal will be Pareto efficient.
- Efficient deals may involve noise.

More General Point

- *Any* inefficiency due to 'market failure' (by definition) presents an *opportunity* for mutually beneficial exchange.
- In a *perfect world*, people would take advantage of such opportunities and negotiate an agreement.
- The Pigouvian analysis **doesn't explain** why private exchange doesn't solve the problem!

'Irrelevance' of legal regime in absence of transactions costs

- Irrespective of legal regime, costless exchange leads to efficient outcome in every case.
- But: Legal regime affects the distribution of costs and benefits

Coase Theorem

In a competitive economy with complete information and zero transactions costs, the allocation of resources will be efficient irrespective of the legal regime (assignment of liability, property rights).

Interpretation

- Affected parties can agree to solve externality problems efficiently.
- If private exchange is costless, legal regime would be 'irrelevant'.

- If the efficient solution is unique, outcome is invariant with respect to legal regime.
- No need for government intervention?
- Assignment of liability (by law, court rulings) irrelevant?
- $\Rightarrow$ The Pigouvian analysis
  - Doesn't justify government intervention or legal rules.
  - Doesn't explain the solutions employed in reality.

5.4 Step 3: The real world and the real problem

The real world

The realization of a mutually beneficial exchange is costly,i.e. **transactions costs**.

The real problem

- The standard analysis misdiagnoses the problem.
- The problem is not that decisions affect others.
- The problem is that transactions costs may prevent efficient exchanges.

Implication

- If there is no transaction cost,
  - the legal regime does matter.

  - Then the law can affect the efficiency of resulting allocation.
  - Then, government regulation may improve efficiency.

- If the cost of private negotiation are small,
  - Private exchange will lead to efficient outcomes.
  - The legal regime is 'irrelevant' (for efficiency)
  - Government intervention (taxes, regulation) is unnecessary (and possibly harmful)

- However, if the cost of private negotiation are large,
  - Efficient exchanges may not take place.
  - Legal regime makes a difference (for efficiency)
  - Interventions (e.g. Pigou style) may be beneficial.
  - But: other forms of private organizations may help as well.

- If alternative arrangements (public or private) are considered, these too will be associated with transactions costs.
  - Court proceedings, expert testimony, etc.
  - Public or private e bureaucracies estimating costs & benefits.
  - Democratic (non-unanimous) decision making procedures.

In conclusion, we need to compare advantages and disadvantages of alternative ways (private and public) to organize mutually beneficial exchanges.

What determines the costs of private transactions ?

- Number of persons involved.
- Communication technology / network structure.
- Ease of monitoring
- ......

Definition of property rights

Which (physical and 'abstract') resources are clearly 'owned' by private actors?

What exactly does 'ownership' of different resources entail?

Transactions costs and property rights

Property rights determine who may decide how certain resources are used, whether that person can prohibit others from interfering with that use, or whether he can sue for damages if they do so.

In many cases, 'owning' of a resource translates into a 'bundle' of rights concerning different aspects of their use.

e.g. Land Ownership

Transactions costs and property rights

According to Coase, efficiency requires that a given right (e.g. to build a tunnel or not) should be held by the party that values it most.

In the presence of transactions costs, it is therefore desirable to (initially) assign rights to those who value them most.

If so, the burden of avoiding conflict will fall on the party that can do so at lower cost. *('Least cost avoider principle')*

As a corollary, it is desirable to bundle different rights in such a way that they are complementary. (E.g. the right to build on a piece of land is bundled with the right to decide what happens immediately beneath and immediately above the surface, but not necessarily what happens at greater distances.)

Perhaps existing systems of property rights can be explained by assuming that they provide for efficient (default) solutions?

Starting point for the *Economic Analysis of Law* (R. Posner)

5.4.1 Property vs. liability rules

Legal scholars distinguish between property and liability rules

Under a property rule, the owner of an entitlement must voluntarily consent before another person can use it.

** Example: You can take my car only if I sell or lend it to you.*

Under a liability rule, the owner's consent is not required, but he must be compensated in accordance with relevant law.

** Example: If you scratch my car, you have to pay for the damage.*

In a zero transactions cost world, these rules don't matter. In reality, we may prefer one or the other type of rule, depending on the circumstances.

Property rules are likely to encourage private negotiations. The only way you can get my car is to bargain with me. This may prevent some cases where it's efficient for you to take it, but since it's just me you have to talk to, it should be easy (low transactions costs).

Liability rules may be better if private negotiations are too costly. If you had to ask for permission before walking close to someone's car, you'd have to negotiate with lots of people. This means that the 'price' you pay for scratching my car may be wrong in some cases, but it saves on transactions costs.

Transactions costs theory of (private) organizations

Pigouvian analysis portrays the market as a completely decentralized system of exchange. Decisions are coordinated only via the price mechanism.

In the absence of transactions costs, results are efficient. (All 'prices' include all costs and benefits.) If there are transactions costs, individuals may instead voluntarily agree to form organizations in which some decisions are centralized or hierarchically organized ('coercion' within a limited realm)

Fewer transactions are necessary ⇒ lower costs

Possible explanation for the existence of firms (Coase 1937)

Presumption: allocation within organization more efficient

Starting point for New Institutional Economics

See the overview article by Oliver Williamson (2000)

Political institutions (voting, delegation) constitute alternative ways of organizing transactions.

In contrast to private organizations, such institutions *do not necessarily require voluntary consent.*

Government intervention cannot be assumed to be costless and efficient.

- Dispersed information would have to be obtained.
- Politicians and bureaucrats would have to be properly incentivized.

Starting point for *Positive Political Theory and Public Choice Theory* (G. Tullock, J. Buchanan)

5.4.2 Summary and Conclusion

In presence of externalities, completely decentralized exchange may not produce a Pareto efficient result.

Traditional interpretation: Market failure Government intervention (Pigouvian taxation)

This view is vague about politics and pessimistic about private exchange.

Private exchange can lead to efficient outcomes if transactions costs are low.

If transactions costs are high

- Legal regime may affect efficiency, efficient regime will 'mimic' costless agreement.
- Government intervention (e.g. Pigouvian taxation) may be desirable.
- But: Private (hierarchical) organizations may also overcome transactions costs.

5.4.3 Discussion

Both public goods and externalities imply that voluntary agreement is possible *in principle.* The source of inefficiencies is due to the presence of transactions costs.

Government intervention is one way to reduce (but not eliminate) transactions costs

- Centralized decision making / coercion
- Don't need all affected parties to negotiate and agree

Government intervention need not produce full efficiency

- Would require benevolence, omnicience
- In reality, outcome of political processes / collective (e.g. democratic) decision making
- Political activity can also impose external costs

Relevant question: Which institutions (public or private) can be used to reduce transactions costs and realize Pareto improving arrangements in the presence of externalities?

Chapter 6

Club Goods

6.0.1 Definition: Club Goods

A club good is nonrivals or partly rivals and excludable. ** Clubs are goods that are exclusive but not competitive or imperfectly competitive. That is, after someone buys the good, others can benefit from it, but the purchaser of the good can exclude someone from benefiting from it.*

For such goods, an allocation consists of

- a set of consumer groups ('clubs')
- [**Membership**]a set of people included in each group ('members')
- [**Quantity**] a quantity supplied for each group

Private clubs will deal with trade-offs

1. Adding additional members
   (a) per capita cost of club good decreases (+)
   (b) reduce individual benefits in case of congestion (-)

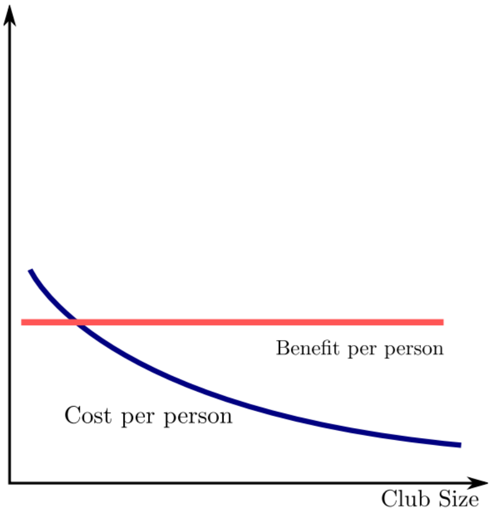


Figure 6.1: Pure Club Goods

2. Increasing quantity provided

   (a) increase per capita cost of club good. (-)

   (b) increase individual benefits (+)

- Per person cost falls as more people share.
- If purely non-rival, per person benefit is constant.
- Then the club size $n \to +\infty$.
  - Note that the further the distance between the red curve and the black curve, the better it will be.

In the case considering congestion, the marginal utility of members will decrease as the club size increase.

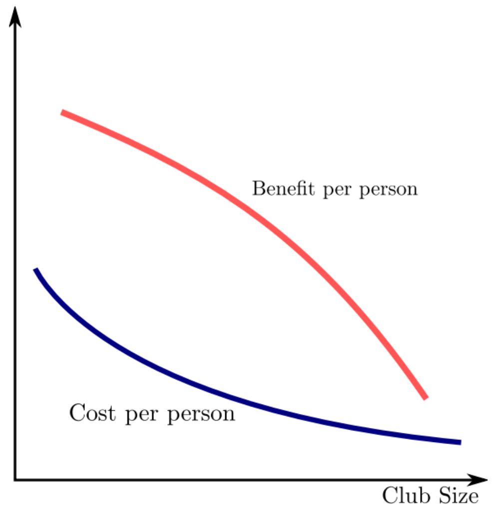


Figure 6.2: Partial Rivalry Club Good (Congestion)

6.1 Model (Buchnan, 1965)

6.1.1 Set Up

- n = the number of members.
- q = quantity of shared good.
- $B(q, n)$ = individual member's (gross) benefit from membership.
- $C(q, n)$ = per person cost
  - Special Case: $C(q, n) = \frac{F(q)}{n}$. ('fixed' cost of quantity q)

6.1.2 Step 1: Optimal n given q

Fix the quantity of the club good q^0,optimal the club size $n^0(q)$, which maximizes net benefits per person.

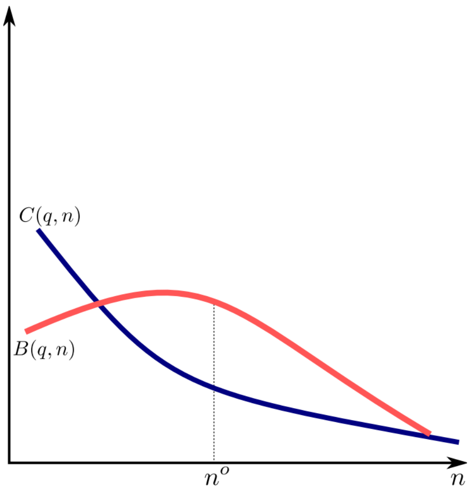


Figure 6.3: Optimal n given q

The net benefit is

$$U(q, n) = B(q, n) - C(q, n) \tag{6.1}$$

Take the first order partial derivative with the respect of n, we can get F.O.C.

$$\frac{\partial U}{\partial n} = \frac{\partial B(q, n)}{\partial n} - \frac{\partial C(q, n)}{\partial n} \tag{6.2}$$

Explanation of F.O.C.

Optimal membership size equalizes marginal loss in per capita benefits with marginal gain due to per capita cost savings.

** Note that in the above figure, when $n = n^0$, the slope of $C(q, n)$ and $B(q, n)$ are the same.*

6.1.3 Step 2: Optimal q given n

Fix the number of club members n, optimal the quantity of club goods $q^0(n)$ which maximizes net benefits per person.

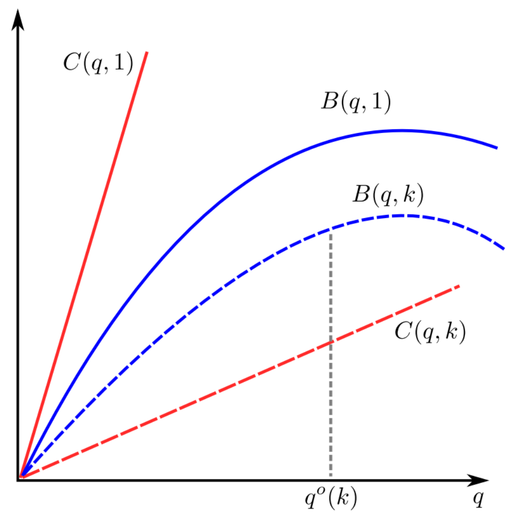


Figure 6.4: Optimal q given n

The net benefit is

$$U(q,n) = B(q,n) - C(q,n) \tag{6.3}$$

Take the first order partial derivative with the respect of q, we can get F.O.C.

$$\frac{\partial U}{\partial q} = \frac{\partial B(q,n)}{\partial q} - \frac{\partial C(q,n)}{\partial q} \tag{6.4}$$

In the special case, $C(q,n) = \frac{F(q)}{n} \Rightarrow \frac{\partial C(q^0,n)}{\partial q} = \frac{F'(q)}{n}$.

Then, the F.O.C. can be written as

$$\frac{\partial B(q,n)}{\partial q} = \frac{F'(q)}{n} \tag{6.5}$$

Rearrange the above equation, we have

$$n\frac{\partial B(q,n)}{\partial q} = F'(q) \tag{6.6}$$

* ***Technical Note***

(Corner Solutions): If $q^0(n) = 0$, we can have $LHS < RHS$. In our figure, this is for $n = 1$.

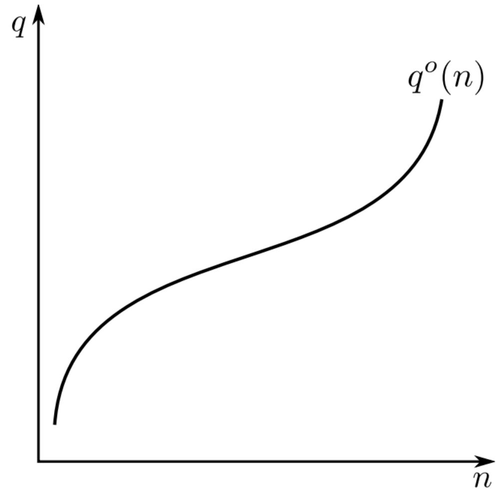


Figure 6.5: optimal q given n

k represents the number of people in the club, and as k goes up, the cost that each person has to pay goes down. Thus, $C(q,k)$ *is below* $C(q,1)$.

At the same time, the revenue per person per unit of club good will also decrease due to congestion, so $B(q,n)$ *is below* $B(q,1)$.

Objective: $B(q,n) - C(q,n)$ *is maximal, i.e., these two curves are furthest apart vertically.*

6.1.4 Combine these 2 Conditions Together

- The shape of $q^0(n)$ will depend on the type of good.
- It is reasonable to **assume** a (weakly) increasing relationship.
- The relationship between n^0 and q will depend on the type of good.

$$n^{0'}(q) = \frac{C_{qn} - B_{qn}}{B_{nn} - C_{nn}} \tag{6.7}$$

- It is reasonable to **assume** a (weakly) increasing relationship.

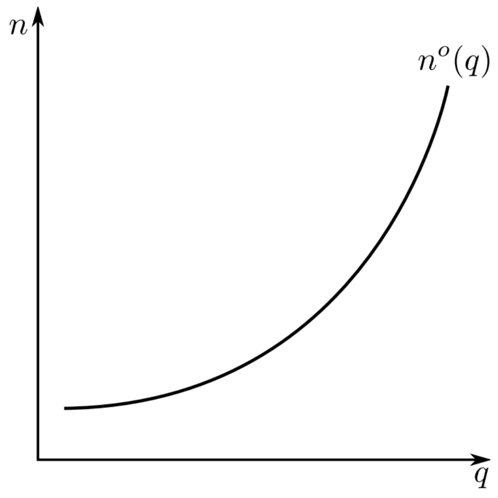


Figure 6.6: optimal n given q

*Note: calculation about $n^{0'}(q) = \frac{C_{qn}-B_{qn}}{B_{nn}-C_{nn}}$. The F.O.C of optimal n given q is that

$$\frac{\partial B(q,n^0)}{\partial q} = \frac{\partial C(q,n^0)}{\partial q} \tag{6.8}$$

We want to know the relationship between q and $n^0(q)$, i.e. $\frac{dn^0(q)}{dq}$.

Since $n^0 = n^0(q)$, the F.O.C. can be written as

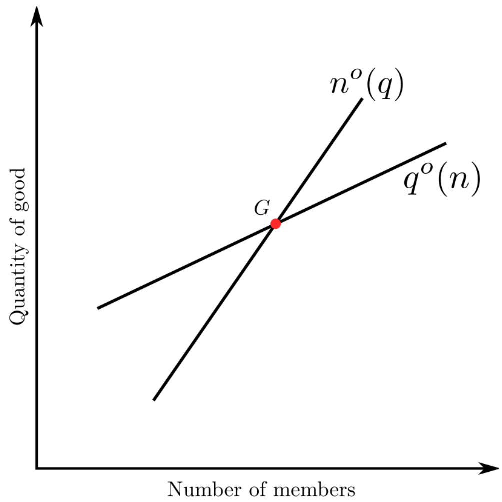
$$\frac{\partial B(q,n^0(q))}{\partial q} = \frac{\partial C(q,n^0(q))}{\partial q} \tag{6.9}$$

Take derivative w.r.t. q on both sides,

$$\frac{\partial^2 B}{\partial n \partial q} + \frac{\partial^2 B}{\partial n^2}\frac{dn^0}{dq} = \frac{\partial^2 C}{\partial n \partial q} + \frac{\partial^2 C}{\partial n^2}\frac{dn^0}{dq} \tag{6.10}$$

$$\Rightarrow (\frac{\partial^2 B}{\partial n^2} - \frac{\partial^2 C}{\partial n^2})\frac{dn^0(q)}{dq} = \frac{\partial^2 C}{\partial n \partial q} - \frac{\partial^2 B}{\partial n \partial q}$$

$$\begin{aligned}\Rightarrow \frac{dn^0(q)}{dq} &= \frac{\frac{\partial^2 C}{\partial n \partial q} - \frac{\partial^2 B}{\partial n \partial q}}{\frac{\partial^2 B}{\partial n^2} - \frac{\partial^2 C}{\partial n^2}} \\ &= \frac{C_{nq} - B_{nq}}{B_{nn} - C_{nn}} \ (B_{nn} \neq C_{nn})\end{aligned}$$



6.1.5 Optimal Combination of n and q

$n^0(q)$ and $q^0(n)$ can be derived as above. At the intersection of the two, both optimality conditions are satisfied:

$$\frac{\partial B(q, n^0(q))}{\partial q} = \frac{\partial C(q, n^0(q))}{\partial q} \text{ (A novel condition)} \tag{6.11}$$

$$\frac{\partial B(q^0(n), n)}{\partial q} = \frac{\partial C(q^0(n), n}{\partial q} \text{ (A modified Samuelson Condition)} \tag{6.12}$$

According to the rivality of the club good, we have 3 subcases.

Case 1: No Congestion ($\frac{\partial B(q,n)}{\partial q} = 0$)

For all q and n,

$$\frac{\partial B(q, n)}{\partial q} = 0 > \frac{\partial C(q, n)}{\partial q} \tag{6.13}$$

So that $n^0(q) = \bar{N}$ for all q.

The optimal club size is $\bar{N}$ and the optimal quantity is $q^0(\bar{N})$.

This is similar to a 'Pure Public Good' - except that this good is excludable.

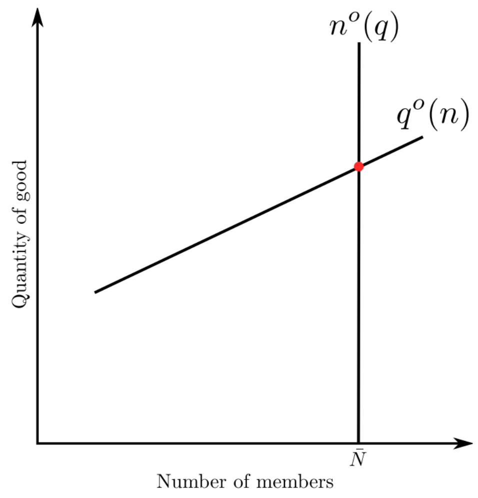


Figure 6.7: No Congestion

Case 2: Perfectly Rival

Small quantities can not be shared, so the optimal club size is 1.

Perhaps very large quantities would best be shared if they become available .$(n^0(q) > 1)$

However, even groups of people would not want to purchase such large quantities.

Then the optimal club size is 1 and the optimal quantity is $q^0(1)$.

This is a pure *private* good.

Case 3 : Partially Rival (Club) Good

At small quantities, optimal club size is larger than 1 and so people would benefit from sharing costs. (E.g. $n^0(q^0(1)) > 1$)

When costs are shared, the optimal quantity will be larger, and so perhaps it will be beneficial to share with yet a larger number.

...eventually, congestion sets in and even optimal cost sharing among more

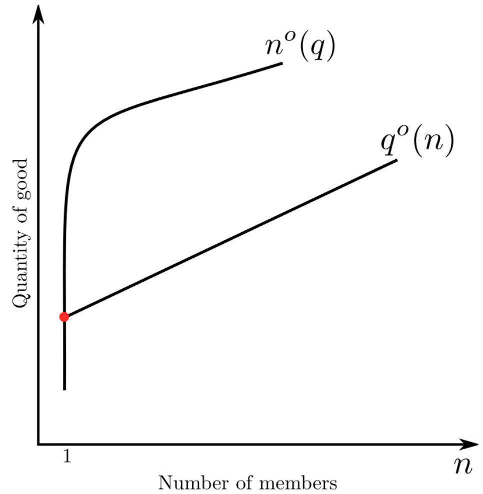


Figure 6.8: Perfectly Rival

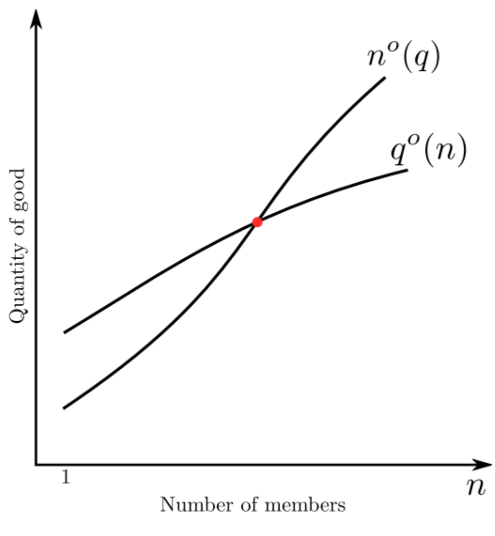


Figure 6.9: Partially Rival (Club) Good

members would lead to lower per capita benefits.

6.1.6 Efficiency of privately managed clubs

Buchanan assumed that private clubs will maximize net benefits per capita.

- Offer an optimal combination of q and n.

Buchanan argued that clubs must make zero profits in equilibrium, thus

- membership fee = per capita cost

The article's main contribution were

- Introduction of club good concept
- Conditions for optimal sharing arrangements (n and q)
- Prediction that price = average / per capita cost (not marginal cost)

Chapter 7

Local Public Good

7.1 Samuelson (1954) "The Pure Theory of Public Expenditure"

Decentralized market organization leads to an insufficient supply of public goods because individuals have no incentives to reveal their valuation ('demand') by making independent voluntary purchases.

Government intervention (taxation, centralized decision making) is an imperfect solution because the information necessary to identify and implement an efficient level of public good provision is difficult (perhaps impossible) to obtain.

Efficient provision would require that individuals truthfully reveal their preferences, but appropriate 'mechanisms' are lacking

- Lindahl pricing is not incentive compatible
- incentive compatible mechanisms are not revenue neutral
- democratic processes do not deal well with heterogeneity and may also

not be incentive compatible

7.2 Tiebout (1956) "A Pure Theory of Local Expenditures"

Many public goods are non-rivalrous only within a given geographic area.

Local public goods are (or could be) provided by local governments.

Citizens may reveal their valuation of such goods through their choice of residence. ("Voting with feet")

"Just as the consumer may be visualized as walking to a private market place to buy his goods, the prices of which are set, we place him in the position of walking to a community where the prices (taxes) of community services are set."

If many local governments compete, perhaps efficiency will be achieved, just as in markets for private goods.

Clearly, these ideas are closely related to Buchanan's Club Good concept.

7.2.1 Assumptions (Tiebout, 1956)

1. Large number of local communities
2. Public goods are purely local (no spillovers between jurisdictions)
3. Consumers are perfectly mobile (no moving costs)
4. Consumers are perfectly informed (know bundles of public goods and taxes being offered)
5. Employment and income do not depend on place of residence

6. The per capita cost of providing a given level of public good benefits is U-shaped as a function of population size.

7. Communities below optimal size grow, those above shrink

8. Public goods are financed by lump sum taxes equal to average (per person) costs.

7.2.2 Comparison Model

Consider the following 'Comparison Model'.

- Only private goods, available at exogenously given market prices.
- n consumer types (each with a certain preference and budget)
- Each associated with a utility maximizing bundle of private goods.

Then,

- Imagine a large number (¿ n) of communities, each forcing residents to purchase certain bundles.
- Each consumer chooses the community whose bundle of private goods he most prefers to purchase.
- Competition causes communities to mandate individually optimal bundles for each consumer type.

Alternatively,

- Communities could purchase goods and impose a tax equal to costs.
- Resulting allocation is the same $\Rightarrow$ hence Pareto efficient!

7.2.3 Bundles of Local Public Goods

Assume first: total cost of public goods bundle is proportional to community size (*average* cost constant).

Assume a large number of communities, each offering a different bundle of local public goods.

Then consumers sort into n (or more) communities, each of which provides exactly the right mix of public goods for one type of consumer.

For the same reason as with private goods, this outcome is *efficient.*

However,

- Proportional cost assumption means these are essentially private goods
- In the club good terminology, there is no benefit from sharing costs.
- In this case, it would (also) be optimal for each consumer to live alone!

Finally, drop proportional cost assumption.Instead, make assumption 6 and 7.

Assumption 6:

- Per capita cost of a given bundle is U-shaped in number of members.
- Then, the optimal community size is larger than one, but finite.

Assumption 7:

- Communities smaller than optimal size will grow.
- Communities above optimal size will shrink.

Then, Tiebout *conjectured* that:

Assuming large number of jurisdictions and large population divisible neatly into optimal groups, the competitive ('Tiebout') equilibrium is Pareto efficient.

7.2.4 Interpretation of Tiebout Model

1. The choice between (local) communities is comparable to a choice between different bundles of private goods.

2. Citizenship in a community is comparable to membership in a private club.

   (a) (Local) taxes resemble membership fees.

   (b) (Local) public goods are shared with other citizens.

3. Just as with private clubs, competition between jurisdictions may lead to an efficient allocation.

7.2.5 Policy Implications of Tiebout Model

1. Local public goods should be provided by small political units.

2. Citizens benefit from policies that increase mobility and information.

7.2.6 Criticisms of Tiebout Model

1. Tiebout's hypothesis depends on rather strong assumptions, notably

   (a) No spillovers between jurisdictions

   (b) Perfect information

   (c) No frictions (mobility, job, opportunities)

   (d) Large number of communities

2. Even if we accept these, Assumption 7 (on attaining optimal size) in particular would seem to require additional justification

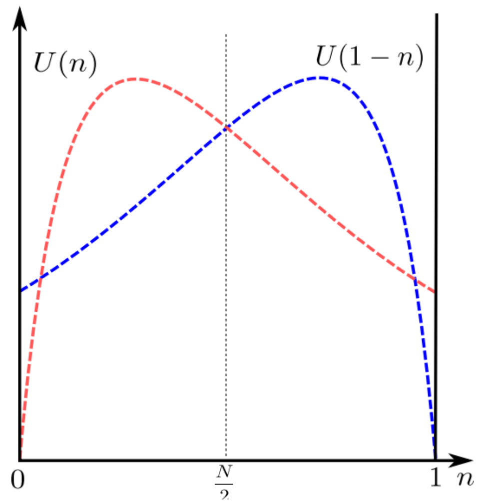


Figure 7.1: Hindriks and Myles's Model

(a) Individual citizens freely choose where to move to.

(b) No reason to expect population flows to achieve efficient separation and sorting.

(c) Possible problems: Failure to coordinate, failure to take into account external costs associated with movement.

7.2.7 Example (Hindriks and Myles)

Assumption

- Suppose all consumers are of the same type.
- Let there be 2 jurisdictions, normalize population size $N = 1$.
- $U(n)$ =utility of living in community of size n.
- Letting n =size of community 1, consider an individual's choice of residence.

Equilibrium requires either

- All citizens live in one district (preferred to living alone), or
- Utility is the same in both districts and noone wants to move

Points a, c, and e are equilibria.

Interpretation

- Benefits from sharing imply that choice of residence (club) may involve externalities.
  - Here: Economies of scale (other citizens / members pay less)
- Individuals do not take these into account when choosing place of residence.
- Other sources of inefficiency (Bewley 1981)
  - Failure to sort efficiently (coordination problem)
  - Inability to impose personalized tax rates
  - Possible nonexistence of an equilibrium

7.3 Bewley (1981) - Criticism of Tiebout

Bewley (1981) argues that Tiebout's Hypothesis depends on extremely restrictive assumptions.

Recall features of the Tiebout's 'comparison model':

- Cost of public goods proportional to population.
- Infinite number of communities

Bewley argues that these assumptions make local public goods **'essentially private,'** and that relaxing them causes problems in Tiebout's arguments.

Two Types of Public Goods

- **Pure public good:** Cost of provision are independent of population.
- **Pure public service:** Costs of provision are proportional to population.

Two Assumptions about Production

Free trade: Free trade between regions - firm location is irrelevant **Autarky:** Production takes place inside regions, no trade between them (We will ignore this - it creates additional problems not discussed below.)

Government objectives

Democratic governments: maximize welfare of own citizens / median voter. **Entrepreneurial governments:** maximize population or tax revenue.

Allocation

An allocation is

- A private consumption bundle for each consumer
- A production plan for each firm (location free)
- A public good bundle for each region
- An assignment of each consumer to a region

Definition: Tiebout Equilibrium

A Tiebout equilibrium consists of an allocation along with a price for each commodity and a head tax for each region such that

(1) Consumers choose optimal private consumption bundles

(2)Consumers choose optimal region of residence *(Taking tax rates and public goods as given.)*

(3)Markets clear

(4)All regions balance their budgets

(5)Each region's public goods and tax bundle maximizes *whatever objective they are pursuing*

- 'Democratic' governments take population as given
- 'Entrepreneurial' governments anticipate migration

7.3.1 Example 1: (Democratic Government, Pure public good

Set up

- 2 consumers, 2 regions
- 1 public good g and 1 private good l (leisure)
- Each consumer is endowed with 1 unit of leisure.
- Utility is $u(l, g_j) = g_j$, where g_j is the public good in the region he inhabits.
- Production possibilities are expressed by $g_j = L_j, j = 1, 2$ where L_j is the quantity of labor (leisure spent) for the production of the public goods for region j.

The following situation constitutes an equilibrium

- Price of labor = price of public good = 1
- Each region imposes a head tax of 1 and provides 1 unit of the public good.
- One consumer lives in each region, and sells all his labor to producers of the public good.

This equilibrium is not Pareto efficient

Both consumers would be better off if they lived together in one region and two units of public good were provided in that region.

Example 1 - What is the problem here?

Consumers don't take into account the economies of scale that result when they move into a region. (Here, the per capita cost of the public good is halved when a second person moves in.)

This problem would also arise if per capita cost were increasing but less than proportional to population.

An "opposite" problem would arise if per capita costs rose more than proportionally. Perhaps charging entry and exit fees could fix this - but this may violate freedom of movement.

7.3.2 Example 2 (Democratic Government, Pure public service)

Set Up

4 consumers (A, B, C, D), 2 regions. 4 types of public service (A, B, C, D) Each consumer is endowed with 1 unit of leisure. Consumers' utilities from living in region j are

$$U_A(g_{jA}, g_{jB}) = 2g_{jA} + g_{jB}$$
$$U_B(g_{jA}, g_{jB}) = g_{jA} + 2g_{jB}$$
$$U_C(g_{jC}, g_{jD}) = 2g_{jC} + g_{jD}$$
$$U_D(g_{jC}, g_{jD}) = g_{jC} + 2g_{jD}$$

Production possibilities are expressed by the equations

$$n_j(g_{jA} + g_{jB} + g_{jC} + g_{jD}) = 2L_j \tag{7.1}$$

for $j = 1, 2$, where n_j is the number of consumers in region j, L_j is the quantity of labor used to produce public services for region j, and g_{jk} is the quantity of public service k provided in region j, for $j = 1, 2$, and $k = A, B, C, D$.

*Note that we just assume that the RHS of the production function takes the above styles. One unit of labor can produce 2 units of goods,

This allocation is an equilibrium

Consumers A and C live in region 1, B and D live in region 2.

$$(g_{1A}, g_{1B}, g_{1C}, g_{1D}) = (1, 0, 1, 0)$$
$$(g_{2A}, g_{2B}, g_{2C}, g_{2D}) = (0, 1, 0, 1)$$

All prices are 1 in both regions, and the head tax is 1.

It is not Pareto optimal

Suppose B and C switch regions.

- Let $(g_{1A}, g_{1B}, g_{1C}, g_{1D}) = (1, 1, 0, 0)$;
- Let $(g_{2A}, g_{2B}, g_{2C}, g_{2D}) = (0, 0, 1, 1)$.

The new allocation is a Pareto improvement.

Example 2: What is the problem here?

- Consumers fail to sort correctly.
- No single consumer can improve the situation by moving.
- Coordination problem.

Possible solution?

The problems appearing in examples 1 and 2 might not arise if governments would initiate changes in public goods and tax bundles in anticipation of the migrations that would result.

Assume

Governments maximize profits (Head tax - cost of public goods).

Entrepreneurial governments establish regions and offer public goods / tax bundles anticipating migration into their region.

7.3.3 Example 3 (Entrepreneurial government, pure public good)

- 2 consumers, 2 regions
- 1 public good g and 1 private good I (leisure)
- Each consumer is endowed with 1 unit of leisure.

- Production possibilities are expressed by $g_j = L_j, j = 1, 2$ where L_j is the quantity of labor (leisure spent) for the production of the public goods for region j.
- Consumer 1's utility is $u(I, g_j) = g_j$
- Consumer 2's utility is $u_2(I, g_j) = 3I + g_j$ (where I is his leisure time)

This is an equilibrium

- In each region, the price of each good is 1.
- Region 1 provides 1 unit of g and imposes a head tax of 1.
- Region 2 provides 0 units of g and imposes no tax.
- Consumer i lives in region i, for $i = 1, 2$.
- Consumer 1 sells his labor, pays his tax, and consumes no leisure.
- Consumer 2 sells no leisure and consumes one unit of leisure.

This is not Pareto efficient

- Suppose Mr. 2 moves to region 1 without paying a tax.
- Then,Mr. 1 still gets $u_1 = 1$ but Mr. 2 gets $u_2 = 4$.

Why is this an equilibrium?

Each consumer is doing the best he can by living alone. Suppose a new region is formed to attract both consumers. Let the head tax in this region be τ units of labor. Then, at most 2τ units of the public good can be provided. Mr. 1 would get a utility of at most 2τ . Living alone he gets 1. Hence, for him to prefer the new region, we must have $\tau > \frac{1}{2}$. Mr.2 gets at most $3(1-\tau) + 2\tau = 3 - \tau$. So, he would prefer to live alone if $\tau > 0$.

What is the problem in this example?

Cannot charge different taxes from different consumer types.

In two-person case, Mr. 1 would voluntarily pay. But in a large population, type 1 consumers would not reveal their preferences.

7.3.4 Conclusion

- Examples 1-3 leave only one case to consider:
  - Pure public service, entrepreneurial government
- In this case, Tiebout equilibria are Pareto efficient.
- But then local public goods are 'essentially private'.

7.4 Empirical Evidence

7.4.1 Tiebout Hypotheses

The main empirical hypothesis attributed to Tiebout is that competition between local governments should increase efficiency of public services.

Areas in which the number of local communities is large and / or the costs of moving between them are small should have more efficient local governments than others. (Better benefit / tax ratio.)

Extensions of the Tiebout model predict that differences in the net-of-tax benefits provided by local communities should be 'capitalized' in housing prices.

E.g. houses located in good school districts should cost more than otherwise equivalent houses in bad school districts.

Another interesting prediction is that households will sort according to preferences over bundles of taxes and public goods.

Areas in which the number of local communities is large and / or the costs of moving between them are small should have more homogeneous populations within districts.

7.4.2 Results

- Hoxby (2000) uses a clever empirical technique to show that areas with a larger number of school districts enjoy better quality public schools. (Similar results have been found in a large number of studies.)

- Beginning with Oates (1969), a large number of studies have shown that property values are positively correlated with estimates of the net-of-tax benefits from local public services (e.g. quality of schooling).(These results have been confirmed in a large number of later studies.)

- Eberts and Gronberg (1981) test the sorting hypothesis looking at school districts. They find that the number of jurisdictions in an area is negatively correlated with a measure of heterogeneity based on income.

- Gramlich and Rubinfeld (1982) do a similar exercise regarding heterogeneity of preferences. (There is, however, conflicting evidence on this.)

- See Oates (2006) for further references.

Chapter 8

Social choice and voting theory

The problem of group choice

- Individual choice and market failure.
  - Individual optimum + decentralized decision making
- Possible 'functions' of / perspectives on collective choice.
  - Identification of group interests (Arrow, 1951)
  - Organization of 'non-market exchange' (Buchanan, 1954)
- Questions (related but different)
  - (How) should we define the 'preference of the group'?
  - Which rules should be used to make group decisions?
- This is the subject of voting theory and social choice theory.

8.1 Majority Rule

- The purpose of voting is to identify the objectively better alternative.

- Different of opinion reflect differences in information.
- Voting is a method of 'information aggregation'.

8.1.1 Model

Set Up

n individuals, 2 alternatives, a and b.

2 states of the world: S_a and S_b.

Each individual independently receives a signal t_i.

In state k, $t_i = s_k$ with the probability $p > \frac{1}{2}$.

Each individual votes according to his signal.

Fact

Jury Theorem: If the group uses simple majority rule, the probability that is makes the correct judgement approaches 1 as n becomes large.

Theorem (Nitzan and Paroush(1982); Sahpley and Grofrnman(1984)): For any n, simple majority rule maximizes the probability that the group judgement is correct.

Interpretation

1. In large groups, a majority is almost always 'correct'.
2. In small groups, a majority is more likely to be 'correct' than any smaller group.

Criticisms

1. The conclusion rests on the assumption that each voter receives and acts upon an **independently** drawn signal, such that **every vote**

contains additional information.

2. Especially in large groups, it is unlikely that each individual's opinion is based on an independent signal (source of information).

3. In addition, if individuals observes others' choices, they may be (rationally) influenced not to vote according to their signal.

8.1.2 Two Alternatives and Heterogeneous Preferences

Condorcet assumed that one option is truly best for the group. What if disagreement reflects genuinely different preferences?

Continue to consider two alternatives (e.g. 'status quo' v.s. 'reform')

Model

1. Set of alternatives : $x = \{x, y\}$.

2. The number of individuals is I and each with individual preference.

3. Let $\alpha_i = \{-1, 0, -1\}$, according to whether i prefers x to y, is different, or prefers y to x.

4. Profile of preferences : $\alpha = (\alpha_1, \ldots, \alpha_I) \in \{-1, 0, 1\}^I$.

Example

$I = May, Lily$

May prefers watching movie and Lily thinks this 2 choices are indifferent. Then $\alpha = \{1, 0\}$.

Social Welfare Functional (SWFL)

Definition

A social welfare functional (SWFL) is a rule $F(\ )$ that assigns a 'social preference' $F(\alpha_1, ..., \alpha_I) \in \{-1, 0, 1\}$ to any possible profile of individual preferences $(\alpha_1, ..., \alpha_I) \in \{-1, 0, 1\}$.

Note

A SWFL is not a 'social welfare function' !

It is a rule that translates any collection of individual preferences into a single 'social' preference.

Under certain conditions, such preferences could be represented by a social welfare function. Then the SWFL is an instrument that takes utility functions as an input and produces a social welfare function as an output.

Possible Axioms

- **Positive Responsiveness:** If $F(\alpha) \leq 0$ then for any $\alpha' \neq \alpha$ such that $\alpha' \geq \alpha$, we have $F(\alpha') = +1$.
  - The term '$\alpha' \geq \alpha$' means that each component of vector a is not smaller than the component of vector b in that direction.
- **Symmetry among agents:** If α' is a permutation of α, then $F(\alpha) = F(\alpha')$.
- **Neutrality between alternatives:** For any profile α , $F(-\alpha) = -F(\alpha)$.

The majority rule satisfies all three of these axioms.

May's Theorem

A social welfare functional $F(\alpha_1, \ldots, \alpha_I)$ is a majority voting social welfare functional if and only if it is symmetric among agents, neutral between alternatives, and positively reponsive.

Summary: 2 alternatives

Assuming that the group is choosing between 2 alternatives.

Majority voting may be a good rule for aggregating information when interests are perfectly aligned. (Condorcet Jury theorem)

8.1.3 Three or More Alternatives

So far, we have considered only choices between two alternatives.

In reality, groups face choices among many alternatives.

(1) Conduct majority vote between each pair of alternatives.

Problem: 'Condorcet's Paradox'. Collective preferences can be cycle, even if the preferences of individual voters are not cyclic. This is paradoxical, because it means that a majority wishes can be in conflict with each voter.

Possible solution: Plurality Rule

One vote among all alternatives. Alternative with most votes wins.

An Example of Plurality Rule

(2) Borda's Rule (the 'Borda Count')

- Each voter assigns points to every option.
  - Worst 0
  - Next Worst 1
  - Next Worst of Next Worst2

  - ......

- Borda Score: sum of points from all voters.
  - The rank of a, b, c is according to Borda score. The choice with the highest Borda score is the best one.

An Example of Borda's Rule

Condorcet's critique of Borda's Rule (3 alternatives)

Borda's rule (the points method) relies on irrelevant factors to form irrevant factors to form its judgements, it is bound to lead error, and that is the real reason why this method is defective. (Condorcet, 1788).

Note:

Borda's method is sensitive to an 'irrelevant' option being added or removed.

Borda's method may not choose an option that defeats all others in a pairwise majority vote. (A 'Condorcet Winner').

(3) Condorcet's Rule (for three or more alternatives)

As in the case of two options, Condorcet looked for a rational way to aggregate options.

Assumptions

- There exists a single correct ranking of the alternatives.
- Each voter' judgement about a given pair of alternatives is independent of other voters' judgements and also independent of her own judgement and also independent of her own judgement about other pairs.
- For any two alternatives, she ranks them correctly with probability $p > \frac{1}{2}$.

– Technical, with these assumptions, it is possible that a voter ranks $a \succ b$, $b \succ c$ and $c \succ a$.

An example of Condorcet Rule

Condorcet's Rule 'Social ranking' is the one with a maximum 'support'. We may call this the 'maximum likelihood ranking'.

Condorcet Winner A **Condorcet Winner** is an alternative that does not lose any pairwise majority vote against any other alternative. Condorcet argued that, when such an alternatives exists, it is the best choice for the group.

Condorcet v.s. Borda

1. Borda's method is sensitive to 'irrelevant alternatives', but condorcet's ranking not.
2. If a Condorcet winner exists, Condorcet ranks it first.
3. Borda's method may not rank a Condorcet winner first. If all ranking are equally likelu to be true.

- Condorcet's ranking is more likely to be true than Borda's.
- Option ranked highest by Borda is mist likely to be truly best.

8.2 Social Welfare Functional and Arrow impossible Axiom

8.2.1 The Problem of Social Choice (Arrow, 1951)

- $I = \{1, 2, \ldots, I\}$ = the set of individuals

- $X =$ the set of alternatives.
- $\succsim_i = Mr.i$'s rational (complete transitive) preference over X.
- $R =$ set of rational preference relations over X.
- $A = R^I =$ set of preference profiles. *(analogous to the domain of definition of a function)*

Definition: Social Welfare Functional(SWFL)

A social welfare functional is a rule that produces a 'social' preference relation $\succsim_s$ from a give profile of individual preference relations $(\succsim_1, \succsim_2, \ldots, \succsim_I)$.

We will write:

- $x \succsim_s y$ if the social preference ranks at least as good as y.
- $x \succ_s y$ if the social preference ranks strictly better than y.

8.2.2 Standards of Social Welfare Functional (Arrow, 1951)

1. Unrestricted domain: The rule must be applicable to all elements of A.
2. Rationality: the rule must be produce a complete and transitive ranking over all alternatives.
3. Pareto Principle: if $x \succ_i y$ for all i , then $x \succ_s y$.
4. Nondictatorship: there is no individual i such that for all x and y in X, $x \succsim_i y$ implies $x \succ_s y$ regardless of the preferences of other individuals.
5. Independence of irrelevant alternatives: the relative social ranking of any two options x and y depends only in the individual rankings of x and y , not on the relative ranking of any alternatives $z \neq x, y$.

None of the rules (pairwise, majority voting, Borda Count, Condorcet's rule, purality vote) have considered are 'acceptable' in Arrow sense.

8.2.3 Arrow's Impossibility Theorem

Suppose the number if alternatives is at least 3. Then, every social welfare functional that satisfies axioms I,P, R and U is dictatorial.

Proof of Arrow's impossible theorem

Interpretation (Positive)

- Group choices are not unambiguously determined by individual preferences alone.
- Collective decisions also depend on the rules and producers used.
- 'Democratic' producers may not produce rational preferences over all policies (or 'social states').
- Indecision and instability
  - Debates may cycle without agreement.
  - Status quo option winds up 'chosen'.

Perhaps decisions taken are later overturned.

8.3 One dimensional Policy Space

Many issues are such that options can be ordered in a natural way (small-big, left-right)

Often, each individual will have a favorite option , and other options are less preferred the further away they are.

Framework

N voters: $I = 1, 2, ..., N$ (N, odd). Set of alternatives $X \subseteq R$ Individual preferences $\succ_i$ (strict).

Definition: Single Peaked Preference and Ideal Point

Voter i has single peaked preferences if there exists an option x_i^* such that for any two options y and z,

If $x_i^* \geq z > y$, then $z \succ_i y$;

and

If $y > z \geq x_i^*$, then $z \succ_i y$.

We call x_i^* i's ***'ideal point'***.

Note: The further away from ideal point, the situation is worse.

Example

- $U_i = -|x - x_i^*|$
- $U_i(x) = -(x - x_i^*)^2$

Lemma Let $a < b < c$, if voter i has single peaked preferences, then either $b \succ_i a$ or $b \succ_i c$ (or both).

(i.e. the 'middle' option can not be the worst, for any voter who has single peaked preferences.)

Definition: Single Peaked Preference

Suppose that all N voters have single peaked preferences over $x \subseteq R$. Denote their ideal points by $(x_1^*, \ldots, x_N^*)$.

Mr. m is a median voter if

$\#\{i \in I : x_i^* \geq x_m^*\} \geq \frac{N}{2}$

and

$\#\{i \in I : x_i^* \leq x_m^*\} \geq \frac{N}{2}$

Black's Median Voter Theorem

Suppose that all voters have single peaked preferences over $x \subseteq R$.

Let m be a median voter.

Then, option x_m^* is not depend by any other option in a pairwise majority vote. That is, the median voter's ideal point is a Condorcet winner.

Then option x_m^* is not defeated by any other option in a pairwise majority vote. That is, the median voter's ideal points is a condorcet winner.

Example: Voting on redistribution

Set up

Consider majority voting on redistribution via income taxation.

Individuals choose how much income z they generate, and the tax system specifies how much consumption $c(z)$ they end up with.

For simplicity, restrict attention to linear tax system of the form

$$c(z) = b + (1 - t)z$$

where b is a fixed lump-sum benefit paid to every household.

Individual utility is given by

$$U(x, \frac{z}{s}) = x - \frac{1}{2}(\frac{z}{s})^2$$

Skill s is distributed according to the CDF $F(s)$.

Questions we need to answer now

What tax system (combination b and t) would be chosen using majority rule?

In particular: how does the chosen tax system depend on the distribution of skills ($F(s)$)?

Steps in analysis

(1) Derive labor supply for any given combination (t, b).

(2) Restrict attention to feasible systems, where b is fully funded. $\Rightarrow$ one dimentional space of alternatives.

(3) Derive a consumer's indirect utility from a given tax system.

(4) Check that preferences (indirect utility) are single peaked.

(5) Apply median voter theorem $\Rightarrow$ Median voter's preferred policy.

Calculation

The individual's utility function

$$U(c(z), \frac{z}{s}) = [b + (1-t)z] - \frac{1}{2}(\frac{z}{s})^2$$

where $b = t(1-t)E(s^2) = b(t)$.

Note:

(1) b is a parameter representing redistribution.

(2) s is the working efficiency of labors. Different individuals have different working efficiency. $s \sim N(\mu_s, \sigma_s^2)$.

(3) Consumers want to maximize their utility. Only the parameter z can be controlled by consumers.

$$\max_z b + (1-t)z - \frac{1}{2}(\frac{z}{s})^2$$

$$\text{F.O.C } \frac{dU}{dz} = (1-t) - \frac{z}{s^2} = 0$$

$$\Rightarrow z^* = (1-t)s^2$$

* *Since* $z^* = (1-t)s^2$ *is the personal optimal income(labor supply) for consumer* i*, and* s *is a random variable, the parameter* $b = t(1-t)E(s^2)$ *is can be seemed as "tax rate* $\times$ *expected income".*

For the group, they will decide the optimal tax rate to max individual's utility.

$$\begin{aligned}\max_t V(t) &= t(1-t)E(s^2) + (1-t^2)s^2 - \frac{1}{2}(1-t)^2 s^2 \\ &= t(1-t)E(s^2) + \frac{1}{2}(1-t)^2 s^2\end{aligned}$$

$$\begin{aligned}\text{Then F. O. C } \frac{dV(t)}{dt} &= (1-t)E(s^2) - tE(s^2) - \frac{1}{2} \times 2(1-t)s^2 \\ &= (s^2 - 2E(s^2))t + E(s^2) - s^2\end{aligned}$$

$$t = \frac{E(s^2) - s^2}{2E(s^2) - s^2}$$

And since $v_s'(t) = (1-t)(E(s^2)-s^2) - tE(s^2) = (E(s^2)-s^2) - (2E(s^2)-s^2)t$

$v_s''(t) = [s^2 - E(s^2)] - E(s^2) = -2E(s^2) + s^2$

$E(s^2)$ is the expectation of the random variable s^2, is only depends on the distribution. Here, we seem it as constant.

If a preference is a single peaked preference, then the utility function should be monotonic function or the function with only one local and global maximum point (i.e the function is increasing and then decreasing).

To satisfy these standard,

(a) If the slope of $v'(t)$ is negative, the intercept can be any value.

Then, if $E(s^2) - s^2 > 0$, $v(t)$ increasing firstly and then decreasing.

If $E(s^2) - s^2 < 0$, $v(t)$ is strictly decreasing.

(b) If the slope of $v'(t)$ is positive, the intercept must be positive. Then the function $V(t)$ can be strictly increasing.

This means that

$$E(s^2) - s^2 > 0 \Rightarrow s^2 < E(s^2)$$
$$E(2E(s^2) - s^2) > 0 \Rightarrow s^2 < 2E(s^2)$$

Hence combining these 2 situation, we can find that for any s, the function $V(t)$ can always satisfy these standards.

Thus, preferences over t are single peaked.

By the median voter theorem, the unique stable outcome of majority voting is the median voter's ideal t:

$$t^* = \frac{E(s^2) - s_m^2}{2E(s^2) - s_m^2} \tag{8.1}$$

8.3.1 Single Peaked Preferences in multiple dimensions

Black's theorem assumes that options can be aligned in a single dimension. Complications arise if we make the (more realistic) assumption that policies are multiple dimensional.

Assume that options are $x_j \in \mathbb{R}^L$ and voters have preferences $v_i(x) : \mathbb{R}^L \to \mathbb{R}$.

(e.g. $u(x_1, x_2, x_{1i}, x_{2i}) = -\sqrt{(x_1 - x_{1i})^2 + (x_2 - x_{2i}^2)}$).

Single peaked (distance) preferences in 2 dimensions

- Circular indifference curves.
- Preferred- to sets: areas inside indifference curve through given point.
- Intersection of preferred-to sets.

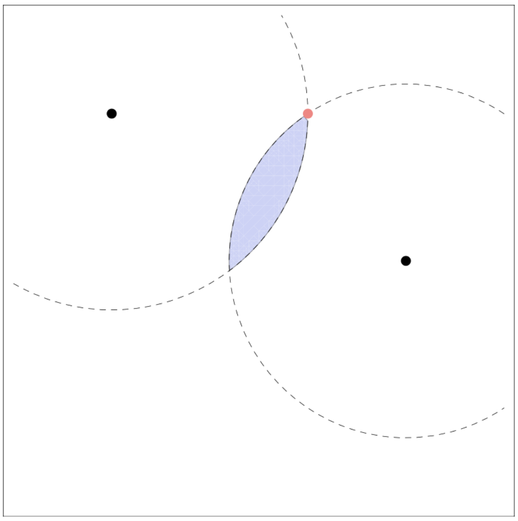


Figure 8.1: Pareto Improvements

- Mutually agreeable alternatives.

Pareto optimal Points (2 voters)

- Intersection of preferred-to sets is empty.
- Indifferent curves are tangent.

Pareto set in the case of 2 voters is a set that if points such that individuals cannot mutually agree to move. For any point not on this line. there exist other points that both prefer.

Three individuals

In the case of 3 individuals, the pairwise Pareto sets form a triangle. Any point inside of this triangle can be a Pareto Efficient point.

But the points on the line is not stable under majority rule byt stable under unanimity rule.

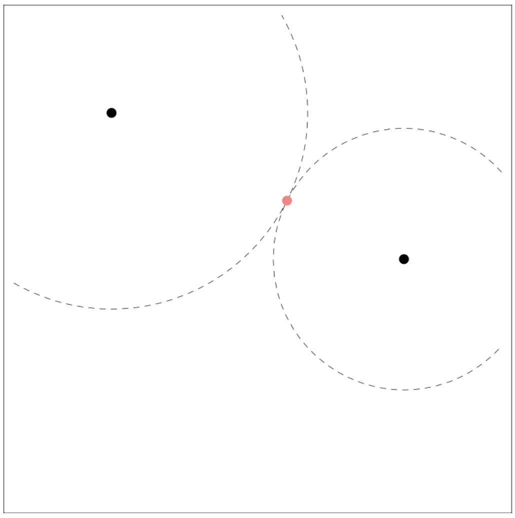


Figure 8.2: Pareto Optimal Points

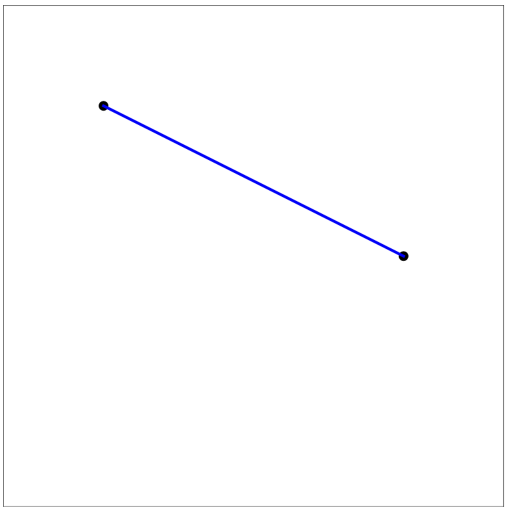


Figure 8.3: Pareto set (2 voters)

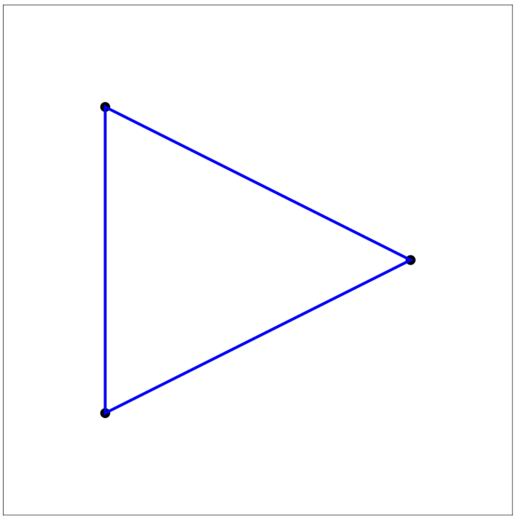


Figure 8.4: Pareto Set (3 voters)

Conclusion

1. Every point in the Pareto set is unstable under majority rule.
2. Every point outside Pareto set is unstable even under unanimity rule.
3. Thus, there are no stable points under majority rule (except under very implausible symmetry conditions).

Theorem: McKelvey 1979

- In two or more dimensions, the existence of Condorcet Winner is extremely unlikely.
- In general, any two points in the space are connected via some sequence of pairwise votes.

Interpretation

1. Illustrates relevance of Arrow's theorem and majority cycles.

2. Highlights the power of agenda setters:

    (a) Assuming that all participants vote sincerely at each stage ...

    (b) ... then an agenda setter can construct a sequence of pairwise votes such that any desired alternative wins.

Why so much stability?

In reality, committee voting outcomes do not cycle endlessly. The possible explanations are as follows.

1. Issue space may actually be one dimensional (Poole and Smith, 1994)

2. Committees often vote one dimension at a time.

    (a) 'Germanness Rule' prohibites unrelated amendments.

3. Legislative institutions involve (possibly implicit) supermajority requirements.

4. More generally: legislative rules may support 'structure induced equilibrium'. (Shepsle and Weingast, 1981)

Implication: Both outcomes and stability depend on institutional details beyond simple majority rule.

8.4 Summary

1. Social Choice Theory attempts to identify acceptable ways of attributing a social preference to a group.

2. There is no acceptable ('democratic') way to do this except in special cases.

(a) If all individuals have the same preference, unanimity rule would produce a consistent ranking and everyone would agree with all judgements.

(b) If individuals have single peaked preferences in a single dimension ,majority rule would produce consistent judgements (through not everyone will agree with each one).

(c) In all other cases (e.g. if alternatives are multi-dimensional), no ('acceptable') decision rule will produce anything resembling a rational group preference.

3. Many economists conclude from this that it is difficult to rationally justify collective decisions.

8.5 Critique (Buchanan, 1954)

Social Choice theorists take it for granted that **collective decisions** should be justifiable by reference to **collective preferences**.

This way of thinking is inherited from the way economists traditionally think about *individual choice*.

Buchanan argues that this is inconsistent with economists alleged commitment to methodological and normative individualism.

The idea of 'rationality (...) ' as an attribute of the social group in incompatible with the principle that "the individual is the only entity possessing ends or values".

In their analysis of market exchange, economists do not ask whether outcomes are 'socially preferred,' but only if they are efficient.

- The outcome (allocation) produced by market exchange is obviously not a ‘choice’ (social or otherwise).

- The emphasis is on investigating how markets can facilitate the realization of mutual advantages (Pareto improvements), not the maximization of ‘social utility’.

A consistent application of the ‘economic perspective’ to collective decisions should investigate how they can be organized in a way that facilitates additional Pareto improvements (beyond those achieved through voluntary private transactions).

8.5.1 Constitutional economics perspective

(How) can group members (as sovereign individuals) secure mutual gains by organizing some activities ’collectively’ ?

Which decision rules might group members mutually agree upon using for this purpose?

- Constitutional phase: Group members agree unanimously on
  - What types of collective actions they may undertake in the future.
  - Decision rules to be used to initiate action of a given type.
- Post-constitutional phase: Decision rules are used to make day-to-day decisions on certain previously defined matters.

Chapter 9

Electoral Competition

Why do people vote? An instrumentally rational citizen cast a vote if

$$p \times B > c$$

where

$p =$ probability that a single vote swings election (Voter is pivotal).

$B =$ individual benefit of getting the preferred candidate elected

$c =$ individual's voting cost

9.1 Pivotal Voter Model (Palfrey and Rosethal, 1985)

9.1.1 Set up

(1) Election between 2 parties $1, 2$.

(2) Proportion supporting each candidate known: σ_i, σ_2.

(3) Voting is costly and purely instrumental.

- **For (2) the proportion σ_1, σ_2:** suppose before voting, people know the proportion of people who supporting to candidate A and B respectively. But this is an overall proportion, not every one of the population will participate the vote. Hence, this proportion is usually not equal to the supporting proportion in the vote.

- **For (3), voting is purely instrumental:** This is to say that, people's utility is only affected by the result of voting (i.e.the policy the winner will impose) rather than the voting process itself.

9.1.2 Individual participation decision

The individual participation decision depends on possibility of being pivotal (P), which depends on

- Participation rates (ρ_1, ρ_2).

- Expected 'closeness' of the race $(\rho_1\sigma_1 - \rho_2\sigma_2)$.

$\sigma_1 -$ the proportion of the population who support party 1.

$\rho_1-$the proportion of people supporting party 1 who participate in the voting.

Hence, $\rho_1\sigma_1$ represents how many people support party 1 in the voting, represented by a proportion.

Interpretation

1. No participation $\rho_i = 0 (\forall i \in I = \{1, 2, ..., n\})$ is not an equilibrium. If no one votes, any one person can swing election. So some people have to be voting.

2. Palfrey and Rosenthal (1985) allow heterogeneous costs of voting and the participation depends on voting cost and party affiliation.

Main results:

- Higher participation when election is close. ('underdog' effect)
- Participation declines with population size N.

9.1.3 Objection

In fact, the probability of being pivotal is essentially zero in all major elections. It follows that

- Either people who vote are being irrational;
- Or they are doing so for 'non-instrumental' reasons.

Rational Ignorance (Down, 1957)

Given that the probability of being pivotal is effectively zero, voters have no (instrumental) benefit from getting informed.

Non- instrumental reason for voting (Riker and Ordeshook, 1970)

- Pleasure from compliance with an ethical norm/ guilt from violating it.
- Expressing support for political system.
- Expressing support for a political party or candidate.
- Interest in policies (enjoying the decision- making process itself).
- Feeling important / taking part in democratic process.

9.1.4 Implications for theory

- Public choice theorists (implicitly) acknowledge that voting is not instrumentally rational.
- Still, it is usually assumed that voters
  - carefully evaluate the alternatives being presented
  - support the alternative that they would prefer being selected.
- That is, although participation is not instrumentally rational, voters are assumed to behave as if they are being instrumental.
- Voting is treated exactly like other kinds of choice: the individual evaluates the option and 'chooses' the one that offers her greater utility.
- Brennan and Lomasky (1993) criticize this approach for being inconsistent ('pseudo-rational') and potentially misleading.

9.2 Expressive voting (Bernnan and Lomasky, 1993)

Voters do not decide which option will be chosen. Then decide which option to support.

People may enjoy supporting policies or candidates that they would not choose if it was up to them.

- 'Moral' considerations may become more important ? (Low cost to boost positive self image)
- Emotional appeal (charisma, image) may attract support (Everything is just a 'show'.)

9.3 Downsian Electoral Competition

Downs(1957): "Parties formulate policies in order to win elections rather than win elections in order to formulate policies in order to formulate policies."

9.3.1 Basic Model with 2 candidates

9.3.2 Set up

- Policies: $x \in [0, 1]$ (left- right)
- 2 Candidates A and B (exogenous)
- Office motivated.
- Can commit to any platform.
- voters vote sincerely.

Interpretation

(1) Office motivated: All they care about is making it into office. In reality, whether this policy will be implemented, its effects, and whether it aligns with the policy objectives of the political party are not important. The only thing that matters is to win as many votes as possible to achieve victory in the elections.

That is to say , the voters' preference will decide the outcome of the voting. The competition between parties is to search out the real preference of voters.

(2) Can commit to any platform. Voters believe that the party who wins the voting will impose the policy said before.

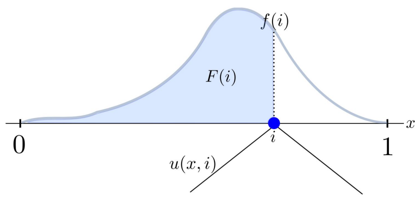


Figure 9.1: The PDF and the ideal point for voter i

(3)Voters vote sincerely: people will vote for the alternative they really prefer.

9.3.3 Voters

(1) Voters have single-peaked preferences over policy.

(2) Voter ideal points distributed according to $CDF\ F(i)$. Assume $f(i) > 0$ for all $i \in [0, 1]$ and $f(i) = 0$ for all other i. There is a unique i_M for which $F(i_M) = \frac{1}{2}$.This is the ideal point of the median voters.

9.3.4 Candidates

- 2 candidates: A, B.
- Office motivated: Payoff $R > 0$ if selected, 0 otherwise.
- $x_J \in [0, 1]$ candidate $J's$ platform.

9.3.5 Game

- Each candidate chooses a position $x_J, J \in \{A, B\}$.
- Each citizen votes for his/ her favourite candidate.
    - If $x_A < x_B$.

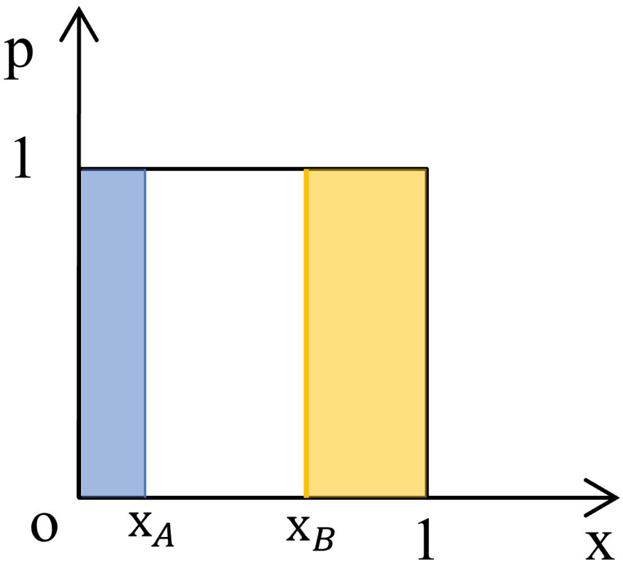


Figure 9.2: Uniform Distribution and 2 Candidates

- Indifferent voters $u(x_A, \hat{i}) = u(x_A, \hat{i})$
- All voters $i < \hat{i}$ vote for A.
- All voters $i > \hat{i}$ vote for B.

- Candidate with most votes is elected and implements x_J.

Note

People with their ideal points on the left side of x_A will always vote for candidate A.

Similarly, people with their ideal point on the right side of x_B will always vote for candidate B.

Half of people with ideal points $x_i \in [x_A, x_B]$ will vote for candidate A and half of them will vote for candidate B.

For example, in the given situation, $x_A = \frac{1}{8}$ and $x_B = \frac{3}{4}$. Then the support rate of candidate A is

$$R_A = \frac{1}{8} + \frac{1}{2}(\frac{3}{4} - \frac{1}{8}) = \frac{7}{16}$$

$$R_B = (1 - \frac{3}{4}) + \frac{1}{2}(\frac{3}{4} - \frac{1}{8}) = \frac{9}{16}$$

Since candidate B have higher support rate, he will win.

If candidate A want to win in the vote, he must move his policy point to the middle in order to improve his support rate.

Definition: Equilibrium of Electoral Competition

A pair of policy positions (x_A^*, x_B^*) is an equilibrium if x_J^* maximizes the probability that candidate J wins, given x_{-J}^*

** We can consider the process of seeking equilibrium as a game, with the equilibrium being the Nash equilibrium of the game. Therefore, we can understand equilibrium as a situation where, given the actions of other candidates, no one has an incentive to change their own strategy.*

Definition: Media Voter Theorem

Median Voter Theorem: There exists a unique equilibrium, with $x_A^* = x_B^* = x_M^*$.

Proof of Median Voter Theorem

(1) To show that $x_A^* = x_B^* = x_M^*$ is an equilibrium, consider candidate A, we need to show that choosing $x_A = x_M$ maximizes her probability of winning when B chooses $x_B = x_M^*$.

Given the action of other candidates, candidate A will not change her action since he has maximize his possibility to win.

Given these choices, he would receive half of the voters and win with probability $\frac{1}{2}$. Since x_M is a Condorcet winner (by Black's Theorem), no other position can get A more than half of the voters as long as $x_B = x_M$.

So there is no way to increase her probability of winning.

The same argument can be made for candidate B.

(2) To show that this equilibrium is unique, suppose there exist another equilibrium, and suppose (w.l.o.g) that $x_A^* \neq x_M$ in this equilibrium.

Then if candidate A gets fewer than half of the voters, he can improve his chances by moving to x_M.

Therefore, at least one candidate is not maximizing his probability of winning.

This contradicts the assumption that there is an equilibrium in which $x_A^* \neq x_M$, proving that no such equilibrium exists.

Again, the same argument can be repeated to show that we cannot have $x_B^* \neq x_M$.

9.3.6 Interpretation

- Two-party competition under majority rule favors moderate candidates and policies.
- Competing parties pursue similar policies, implying stability.

Note

Median Voter: no matter whose ideal point is compared to, the median voter's ideal point always has support from more than half of the people.

In other words, the middle voter benchmarks the concept of expectation of probability distribution, if we consider the ideal points of different voters as random variables that are not uniformly distributed, the middle voter's option represents the mathematical expectation of that probability distribution.

9.3.7 Modified model with three or more candidates

Set up

- Policies: $x \in [0, 1]$ (left-right).
- 3 candidates A, B, C. (exogenous)
- Candidates maximize their own vote share.
  - *Simpler than assuming they maximize the chance of winning.*
- Can commit to any platform.
- Voters vote sincerely.
  - *ideal points distributed according to CDF, F etc.*

The 'squeezing' effect

'Extreme' parties have incentive to move to center.

$\Rightarrow$ The moderate party get 'squeezed' !

Eventually, the moderate party wants to take a more extreme position.

Conclusions

It can be shown that **no equilibrium** exists if the number of parties is odd.

9.3.8 4 candidates, i uniform on $[0, 1]$

Unique equilibrium:

2 candidates at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.

Intuition: When two parties cater to the same ideology, there is less incentive to move towards the center because doing so will cede votes to the other (similar) party.

Interpretation

- competition for vote shares may not produce policy moderation when there are more than 2 candidates or parties.
- Moderate candidates or parties may fall prey to the 'squeezing effect'.
- The model predicts less policy moderation when more than 2 parties compete for vote shares.

9.4 Approval Voting

Each voter can vote for as many candidates as they like. Suppose voters cast one vote each for their top two choices.

$\Rightarrow$ Moderate party wins because it gets one vote from every voter, while the extreme parties share the remaining votes.

9.5 Some other alternatives to plurality voting

9.5.1 Majority rule, runoff election

Each voter votes for one candidate. If one has an absolute majority, he is chosen, If not, repeat procedure with the two that have the most first place votes.

9.5.2 Borda Count

9.5.3 Preferential Voting (or 'instant runoff')

Voters rank candidates. If one has a majority of first place votes, he is elected. If not, the candidate with the fewest first place votes is eliminated. This process is repeated.

9.5.4 Coombs System

Each voter votes against one candidate. If one has a majority of first place votes, he is elected. If not, the candidate with the most votes. Repeat until one is left stand in Mueller (Ch.7) evaluates these (and AV) rules on

- Informational requirements.(Partly implement)
- Condorcet efficiency: Does the rule choose a Condorcet winner when it exists?
- Utilitarian efficiency: Related to moderation.

9.5.5 Proportional requirements

Proportional requirements differs from all these rules in that it will usually not produce a single winner.

Under this rule, policy is determined through coalition bargaining.

There is a large literature on this, but we will not deal with it here.

Chapter 10

Legislative Bargaining

10.1 Distributive Politics

Many Political decisions involve the (redistribution) of resources.

- Public Projects conducted in different geometric regions.
- Allocation of ministries to members of different parties.
- Subsidies to different consituent groups.

Many of these decisions are made within legislatures or other relatively small decision making bodies.

Legislators usually have different properties / conflicting interests.

Budgets are crafted and agreed upon according to certain (formal/ informal) rules and procedures.

10.1.1 Legislative Bargaining Theories

The literature on legislative bargaining seeks to model this process.

Want to investigate how behavior and outcomes depend on rules.

10.1.2 Example: Cake distribution

Suppose Anna, Bonnie and Cindy must decide on how to distribute a cake. A feasible allocation is (x_A, x_B, x_C) such that $x_A + x_B + x_C \leq 1$. Suppose they use majority rule. How do you expect?

Expectation

Majority Rule: 3 voters, if a proposal wants to win, it need at least 2 voters' agree.

If the initial proposal is $(50\%, 50\%, 0)$, then voter 1 and voter 2 will agree.

$\Rightarrow$ If Voter 2 negotiated with voter 1 (or voter 2), to make sure voter 1(or voter 2) will agree with the new allocation, he may ask for less share. The new proposal may be $(60\%, 0, 40\%)$.

Then voter 2 may negotiate with voter 3, and come up with the allocation $(0, 40\%, 60\%)$.

$\cdots\cdots$

Conclusion: The outcome is not stable. Any feasible allocation can be defeated by another feasible allocation in pairwise vote.

Raise questions:

Why don't we see 'chaos' in democratic institutions such as legislatures?

Why are some outcomes stable? What kinds of outcomes are these?

10.2 Structure induced equilibrium (Shepsle and Weingast (1981))

Shepsle and Weingast (1981) suggest that democratic institutions are more than **pure majority rule (PMR)**.

Even all options can beat, an option can be stable if no option that would beat it can be proposed!

** That maybe why in the real world, most of proposals will be agreed although in analysis these proposals are not best (or the best proposal does not exist).*

All real-world Legislatures have procedural rules restricting the proposals that may offered.

* ¡Robert's Rules of Order¿ – A handbook about the US's decision mechanism.

10.3 Prototypical Amendment Rules

- **Closed Rule:** the legislature votes on a single proposal (usually proposed by a committee).
  - In case of failure, 'status quo' remains.
- **Open Rule:** before a vote is taken, (some)members may offer 'amendments'.

10.4 Baron and Ferehohn (1989) 'Bargaining in Legislatures'

Baron and Ferejohn (BF) proposed a formal model of legislative bargaining. In their model,

- Members must decide on how to divide a dollar (endogenous size of the overall budget).

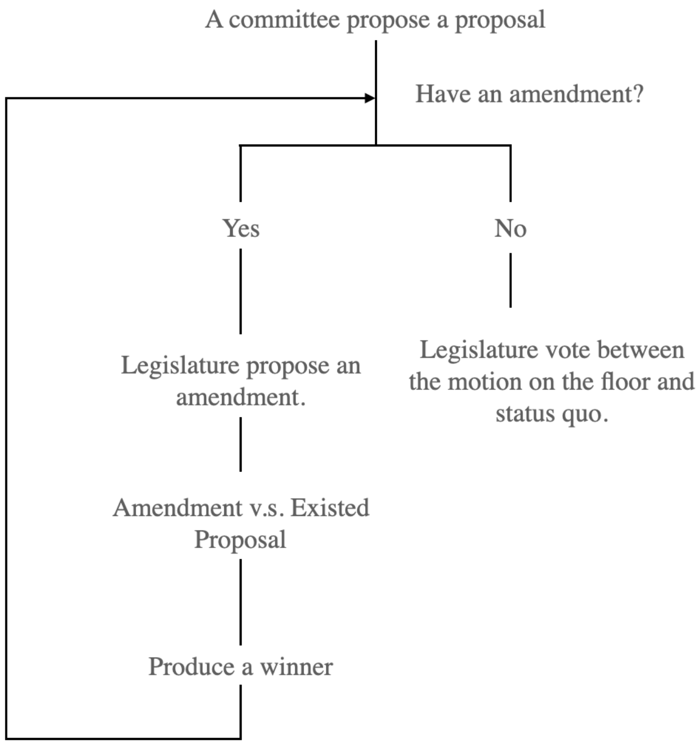


Figure 10.1: Decision Mechanism

- Process occurs in a sequence of 'sessions' (e.g. days on which the legislature meets)
- Members are impatient. They prefer to get a dollar today rather than tomorrow
- The legislature is assumed to operate using majority rule.

BF compare open and closed amendment rules. The model demonstrates how rules may influence the distribution of resources as well as the time required to reach agreement.

10.4.1 Simplified Model

Suppose the legislature meets for only two sessions ($t = 0, 1$).

At the beginning of each session, one member is 'recognized' at random. (Each member is recognized wiyh possibility $\frac{1}{n}$, selected by nature in game tree).

The recognized member makes a proposal $x \in R_+^n$ satisfying

$$\sum_{i=1}^{n} x_i \leq 1 \text{ (Feasibility)}$$

The proposal is immediately voted on using majority rule.

- If it passes, benefits are distributed and the legislature is dissolved.
- If not, the legislature adjourns until the second session.

If no agreement is reached in the second session, the legislature is dissolved and no benefits are distributed. If the distribution y is reached in session t, legislator i's utility is

$$u_i(y) = \delta^t y_i \tag{10.1}$$

where $\delta \in [0, 1]$ is a discount factor.

- $\delta \approx 0$ means legislators are very impatient.
- $\delta \approx 1$ means they are very patient.

Subgame Perfect Equilibrium: Proposals and voting decisions prescribed by the strategy must constitute best responses in every subgame.

Sincere Voting: In addition, assume that voters vote yes if they (weakly) prefer that a proposal passes, and 'no' otherwise.

10.5 Full model: Infinite Horizon,Closed Rule

- Potentially infinite number of rounds (sessions).
- Agreement requires $q \leq n$ 'yes' votes.
- Game continues until agreement is reached.

10.5.1 Equilibrium concept

- There are many subgame perfect equilibria!
- Symmetric, stationary Subgame Perfect Equilibrium (SSPE):
  - Strategies (proposals and votes) are stationary: The proposals a player makes as well as the way he votes on other proposals is the same in every session, irrespective of the date or the game's prior history.
  - All players use the same strategy (make the same (types of) proposals and vote the same as all other players).
- In addition: Assume that players vote yes if they (weakly) prefer that a proposal passes, and 'no' otherwise.

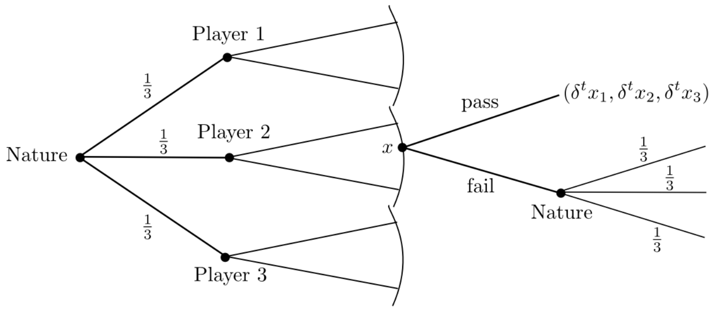


Figure 10.2: The Game Tree

10.5.2 Analysis (1): Voting decisions

Suppose a proposal $(x_1, ...x_n)$ has been made.

Let V_i be his expected utility (EU) in equilibrium.

If the proposal fails, the game continues to the next round.

* *Stationarity* implies that his *EU* from that point on is still V_i . Thus, his 'continuation value' is δV.

Thus Mr.i vote 'yes' if and only if

$$x_i \geq \delta V_i$$

By symmetry, $V_i = V$ (i.e. the same) for all i.

10.5.3 Analysis (2) : Proposals

What is the best proposal that a player can make?

Minimum winning coalitions

The proposer needs q votes. So, the best he can do is to 'buy' $(q-1)$ of the remaining players. He must offer each coalition member exactly δV.

**Note that the proposer is indifferent about whom to include, so he can choose randomly.*

10.5.4 Equilibrium Payoffs

If chosen to propose (probability $\frac{n-1}{n}$), a player gets $1-(q-1)\delta V$.

Otherwise (probability $\frac{n-1}{n}$), he gets δV if he is included in the winning coalition. (Probability μ_i, as yet unknown).

It follows that the expected utility for any player is given by

$$V_i = \frac{1}{n}(1-(q-1)\delta V) + \frac{n-1}{n}\mu_i \delta V \tag{10.2}$$

Since, $V_i = V$ for all i, it follows that μ_i must be the same for all i.

(By symmetry, this means that each proposer includes all other players with equal probability). Then we must have $\mu_i = \frac{q-1}{n-1}$, and the expected payoff simplifies to

$$V = \frac{1}{n} \tag{10.3}$$

This makes sense : given that the first proposal will pass, the entire pie will be distributed somehow,, and then symmetry implies that each player expects exactly $\frac{1}{n}$ of that pie.

10.5.5 Symmetric Stationary Subgame Perfect Equilibrium

Each player's vote has a 'price' equal to his continuation value.

$$\delta V = \frac{\delta}{n}$$

Proposers buy as many votes as are necessary $(q-1)$ and keeps

$$1-(q-1)\frac{\delta}{n} = \frac{\delta}{n} + (1-q\frac{\delta}{n})$$

The proposer receives a larger payoff than others.

This ' proposer advantage ' is decreasing in δ and q.

10.5.6 Appendix (1) : The Calculation Part

Assume that there are n voters. The possibility to be the proposer is equal to every members.

Then the possibility to be the proposer is $P_{proposer} = \frac{1}{n}$. And the possibility to be responsor is $P_{responsor} = 1 - P_{proposer} = \frac{n-1}{n}$.

Assume that to make sure that his proposal is approved, he need to gain at least q voter's agree. (including himself).

Note that the EU of every individual is V in the *next period.* Its present value is δV, which means if people are risk neutral and if they can get at least δV in this round, they will agree with this proposal.

This means that people need to pay at least $(q-1)\delta V$

Hence the revenue of the proposer is $V_{proposer} = 1 - (q-1)\delta V$. And the expected revenue of the responsor who is included in the minimum winning coalition is $V_{responsor} = \delta V$.

Hence, the expected revenue is

$$
\begin{aligned}
EU &= P_{proposer} \times V_{proposer} + P_{responsor} \times P_{included} \times V_{responsor} \\
&= \frac{1}{n} \times [1 - (q-1)\delta V] + \frac{n-1}{n} \times \frac{q-1}{n-1} \times \delta V \\
&= \frac{1}{n} \times [1 - (q-1)\delta V] + \frac{q-1}{n}\delta V \\
&= \frac{1}{n}
\end{aligned}
$$

Hence, $V = EU = \frac{1}{n}$.

10.6 Model: Open Rule, Infinite Horizon

Round 1:

One legislator is chosen at random to make proposal and this proposal becomes motion on the floor.

Immediately thereafter, another member is randomly chosen to

- Call for a vote on the proposal (”move the previous question”). **OR**
- Make an alternative proposal. (”offer an amendment”).

The winner becomes ”motion on the floor” in the next round. (*And the first step above is skipped*)

Another member can either ”move the previous question” or offer an amendment.

Discounting occurs whenever the ”motion on the floor” fails or an amendment is proposed.

10.6.1 Analysis (Intuition)

- Any member who is not offered a positive sharewould propose an amendment if recognized.
  - (i.e. or anybody who get less than δV will reject, but here the proposor want to maximize his own profit, which means that he will give people who are not included in the coalition 0)
- Proposers may buy more than a bare majority of votes in order to increase the chance that their proposal will not amended.

10.6.2 Equilibrium (Open Rule, Infinite Horizon)

Equilibrium properties depend on parameters δ and n.

Equilibrium coalitions may be larger than minimum winning.

The size of the proposed coalitions is

- Weakly decreasing in δ.

- Closer to minimum winning for larger n.

The proposer always gets ***less*** than under the closed rule.

Overall, benefits are distributed more equally.

Unless all members are included, delay may occur in equilibrium.

10.6.3 Experiments on BF Bargaining

McKelvey (1991), Frechette et.al (2003, 2005a, 2005b, 2005c), Diermeier and Morton (2005) and many others.

Focus is on closed rule and majority rule version of the game.

Authors want to test whether behavior corresponds to predicted equilibrium properties.

10.6.4 Main Findings

- (Most) proposers **do build** minimum winning coalitions.
- Responders are offered larger shares than predicted by theory.
- Equal splits within coalition are most common.
- Only a small number of proposals fall.
- Proposals more often fail under unanimity rule. (Buchanan and Tullock (1962))
- Individuals more often vote no under unanimity rule. (Buchanan and Tullock (1962))