CHAPTER

2 Regular Expressions, Text Normalization, Edit Distance

```
User:   I am unhappy.
ELIZA:  DO YOU THINK COMING HERE WILL HELP YOU NOT TO BE UNHAPPY
User:   I need some help, that much seems certain.
ELIZA:  WHAT WOULD IT MEAN TO YOU IF YOU GOT SOME HELP
User:   Perhaps I could learn to get along with my mother.
ELIZA:  TELL ME MORE ABOUT YOUR FAMILY
User:   My mother takes care of me.
ELIZA:  WHO ELSE IN YOU FAMILY TAKES CARE OF YOU
User:   My father.
ELIZA:  YOUR FATHER
User:   You are like my father in some ways.
```

Weizenbaum (1966)

ELIZA

The dialogue above is from **ELIZA**, an early natural language processing system that could carry on a limited conversation with a user by imitating the responses of a Rogerian psychotherapist (Weizenbaum, 1966). ELIZA is a surprisingly simple program that uses pattern matching to recognize phrases like "I need X" and translate them into suitable outputs like "What would it mean to you if you got X?". This simple technique succeeds in this domain because ELIZA doesn't actually need to *know* anything to mimic a Rogerian psychotherapist. As Weizenbaum notes, this is one of the few dialogue genres where listeners can act as if they know nothing of the world. Eliza's mimicry of human conversation was remarkably successful: many people who interacted with ELIZA came to believe that it really *understood* them and their problems, many continued to believe in ELIZA's abilities even after the program's operation was explained to them (Weizenbaum, 1976), and even today such **chatbots** are a fun diversion.

chatbots

Of course modern conversational agents are much more than a diversion; they can answer questions, book flights, or find restaurants, functions for which they rely on a much more sophisticated understanding of the user's intent, as we will see in Chapter 26. Nonetheless, the simple pattern-based methods that powered ELIZA and other chatbots play a crucial role in natural language processing.

We'll begin with the most important tool for describing text patterns: the **regular expression**. Regular expressions can be used to specify strings we might want to extract from a document, from transforming "I need X" in Eliza above, to defining strings like *$199* or *$24.99* for extracting tables of prices from a document.

text normalization

We'll then turn to a set of tasks collectively called **text normalization**, in which regular expressions play an important part. Normalizing text means converting it to a more convenient, standard form. For example, most of what we are going to do with language relies on first separating out or **tokenizing** words from running text, the task of **tokenization**. English words are often separated from each other by whitespace, but whitespace is not always sufficient. *New York* and *rock 'n' roll* are sometimes treated as large words despite the fact that they contain spaces, while sometimes we'll need to separate *I'm* into the two words *I* and *am*. For processing tweets or texts we'll need to tokenize **emoticons** like `:)` or **hashtags** like `#nlproc`.

tokenization

Some languages, like Japanese, don't have spaces between words, so word tokenization becomes more difficult.

lemmatization

Another part of text normalization is **lemmatization**, the task of determining that two words have the same root, despite their surface differences. For example, the words *sang*, *sung*, and *sings* are forms of the verb *sing*. The word *sing* is the common *lemma* of these words, and a **lemmatizer** maps from all of these to *sing*. Lemmatization is essential for processing morphologically complex languages like Arabic. **Stemming** refers to a simpler version of lemmatization in which we mainly just strip suffixes from the end of the word. Text normalization also includes **sentence segmentation**: breaking up a text into individual sentences, using cues like periods or exclamation points.

stemming

sentence segmentation

Finally, we'll need to compare words and other strings. We'll introduce a metric called **edit distance** that measures how similar two strings are based on the number of edits (insertions, deletions, substitutions) it takes to change one string into the other. Edit distance is an algorithm with applications throughout language processing, from spelling correction to speech recognition to coreference resolution.

2.1 Regular Expressions

regular expression

One of the unsung successes in standardization in computer science has been the **regular expression** (**RE**), a language for specifying text search strings. This practical language is used in every computer language, word processor, and text processing tools like the Unix tools grep or Emacs. Formally, a regular expression is an algebraic notation for characterizing a set of strings. They are particularly useful for searching in texts, when we have a **pattern** to search for and a **corpus** of texts to search through. A regular expression search function will search through the corpus, returning all texts that match the pattern. The corpus can be a single document or a collection. For example, the Unix command-line tool `grep` takes a regular expression and returns every line of the input document that matches the expression.

corpus

A search can be designed to return every match on a line, if there are more than one, or just the first match. In the following examples we generally underline the exact part of the pattern that matches the regular expression and show only the first match. We'll show regular expressions delimited by slashes but note that slashes are *not* part of the regular expressions.

Regular expressions come in many variants. We'll be describing **extended regular expressions**; different regular expression parsers may only recognize subsets of these, or treat some expressions slightly differently. Using an online regular expression tester is a handy way to test out your expressions and explore these variations.

2.1.1 Basic Regular Expression Patterns

The simplest kind of regular expression is a sequence of simple characters. To search for *woodchuck*, we type `/woodchuck/`. The expression `/Buttercup/` matches any string containing the substring *Buttercup*; `grep` with that expression would return the line *I'm called little Buttercup*. The search string can consist of a single character (like `/!/`) or a sequence of characters (like `/urgl/`).

Regular expressions are **case sensitive**; lower case `/s/` is distinct from upper case `/S/` (`/s/` matches a lower case *s* but not an upper case *S*). This means that the pattern `/woodchucks/` will not match the string *Woodchucks*. We can solve this

RE	Example Patterns Matched
/woodchucks/	"interesting links to woodchucks and lemurs"
/a/	"Mary Ann stopped by Mona's"
/!/	"You've left the burglar behind again!" said Nori

Figure 2.1 Some simple regex searches.

problem with the use of the square braces [and]. The string of characters inside the braces specifies a **disjunction** of characters to match. For example, Fig. 2.2 shows that the pattern /[wW]/ matches patterns containing either *w* or *W*.

RE	Match	Example Patterns
/[wW]oodchuck/	Woodchuck or woodchuck	"Woodchuck"
/[abc]/	'a', 'b', *or* 'c'	"In uomini, in soldati"
/[1234567890]/	any digit	"plenty of 7 to 5"

Figure 2.2 The use of the brackets [] to specify a disjunction of characters.

The regular expression /[1234567890]/ specified any single digit. While such classes of characters as digits or letters are important building blocks in expressions, they can get awkward (e.g., it's inconvenient to specify

/[ABCDEFGHIJKLMNOPQRSTUVWXYZ]/

to mean "any capital letter"). In cases where there is a well-defined sequence associated with a set of characters, the brackets can be used with the dash (-) to specify any one character in a **range**. The pattern /[2-5]/ specifies any one of the characters *2*, *3*, *4*, or *5*. The pattern /[b-g]/ specifies one of the characters *b*, *c*, *d*, *e*, *f*, or *g*. Some other examples are shown in Fig. 2.3.

range

RE	Match	Example Patterns Matched
/[A-Z]/	an upper case letter	"we should call it 'Drenched Blossoms' "
/[a-z]/	a lower case letter	"my beans were impatient to be hoed!"
/[0-9]/	a single digit	"Chapter 1: Down the Rabbit Hole"

Figure 2.3 The use of the brackets [] plus the dash - to specify a range.

The square braces can also be used to specify what a single character *cannot* be, by use of the caret ^. If the caret ^ is the first symbol after the open square brace [, the resulting pattern is negated. For example, the pattern /[^a]/ matches any single character (including special characters) except *a*. This is only true when the caret is the first symbol after the open square brace. If it occurs anywhere else, it usually stands for a caret; Fig. 2.4 shows some examples.

RE	Match (single characters)	Example Patterns Matched
/[^A-Z]/	not an upper case letter	"Oyfn pripetchik"
/[^Ss]/	neither 'S' nor 's'	"I have no exquisite reason for't"
/[^.]/	not a period	"our resident Djinn"
/[e^]/	either 'e' or '^'	"look up ^ now"
/a^b/	the pattern 'a^b'	"look up a^b now"

Figure 2.4 The caret ^ for negation or just to mean ^. See below re: the backslash for escaping the period.

How can we talk about optional elements, like an optional *s* in *woodchuck* and *woodchucks*? We can't use the square brackets, because while they allow us to say "s or S", they don't allow us to say "s or nothing". For this we use the question mark /?/, which means "the preceding character or nothing", as shown in Fig. 2.5.

RE	Match	Example Patterns Matched
`/woodchucks?/`	woodchuck or woodchucks	"woodchuck"
`/colou?r/`	color or colour	"color"

Figure 2.5 The question mark ? marks optionality of the previous expression.

We can think of the question mark as meaning "zero or one instances of the previous character". That is, it's a way of specifying how many of something that we want, something that is very important in regular expressions. For example, consider the language of certain sheep, which consists of strings that look like the following:

baa!
baaa!
baaaa!
baaaaa!
...

This language consists of strings with a *b*, followed by at least two *a*'s, followed by an exclamation point. The set of operators that allows us to say things like "some number of *a*s" are based on the asterisk or *, commonly called the **Kleene *** (generally pronounced "cleany star"). The Kleene star means "zero or more occurrences of the immediately previous character or regular expression". So `/a*/` means "any string of zero or more *a*s". This will match *a* or *aaaaaa*, but it will also match *Off Minor* since the string *Off Minor* has zero *a*'s. So the regular expression for matching one or more *a* is `/aa*/`, meaning one *a* followed by zero or more *a*s. More complex patterns can also be repeated. So `/[ab]*/` means "zero or more *a*'s or *b*'s" (not "zero or more right square braces"). This will match strings like *aaaa* or *ababab* or *bbbb*.

Kleene *

For specifying multiple digits (useful for finding prices) we can extend `/[0-9]/`, the regular expression for a single digit. An integer (a string of digits) is thus `/[0-9][0-9]*/`. (Why isn't it just `/[0-9]*/`?)

Sometimes it's annoying to have to write the regular expression for digits twice, so there is a shorter way to specify "at least one" of some character. This is the **Kleene +**, which means "one or more occurrences of the immediately preceding character or regular expression". Thus, the expression `/[0-9]+/` is the normal way to specify "a sequence of digits". There are thus two ways to specify the sheep language: `/baaa*!/` or `/baa+!/`.

Kleene +

One very important special character is the period (`/./`), a **wildcard** expression that matches any single character (*except* a carriage return), as shown in Fig. 2.6.

RE	Match	Example Matches
`/beg.n/`	any character between *beg* and *n*	begin, beg'n, begun

Figure 2.6 The use of the period `.` to specify any character.

The wildcard is often used together with the Kleene star to mean "any string of characters". For example, suppose we want to find any line in which a particular word, for example, *aardvark*, appears twice. We can specify this with the regular expression `/aardvark.*aardvark/`.

Anchors

Anchors are special characters that anchor regular expressions to particular places in a string. The most common anchors are the caret ^ and the dollar sign $. The caret ^ matches the start of a line. The pattern `/^The/` matches the word *The* only at the

start of a line. Thus, the caret ^ has three uses: to match the start of a line, to indicate a negation inside of square brackets, and just to mean a caret. (What are the contexts that allow grep or Python to know which function a given caret is supposed to have?) The dollar sign $ matches the end of a line. So the pattern ␣$ is a useful pattern for matching a space at the end of a line, and /^The dog\.$/ matches a line that contains only the phrase *The dog.* (We have to use the backslash here since we want the . to mean "period" and not the wildcard.)

There are also two other anchors: \b matches a word boundary, and \B matches a non-boundary. Thus, /\bthe\b/ matches the word *the* but not the word *other*. More technically, a "word" for the purposes of a regular expression is defined as any sequence of digits, underscores, or letters; this is based on the definition of "words" in programming languages. For example, /\b99\b/ will match the string *99* in *There are 99 bottles of beer on the wall* (because 99 follows a space) but not *99* in *There are 299 bottles of beer on the wall* (since 99 follows a number). But it will match *99* in *$99* (since *99* follows a dollar sign ($), which is not a digit, underscore, or letter).

2.1.2 Disjunction, Grouping, and Precedence

Suppose we need to search for texts about pets; perhaps we are particularly interested in cats and dogs. In such a case, we might want to search for either the string *cat* or the string *dog*. Since we can't use the square brackets to search for "cat or dog" (why can't we say /[catdog]/?), we need a new operator, the **disjunction** operator, also called the **pipe** symbol |. The pattern /cat|dog/ matches either the string cat or the string dog.

disjunction

Sometimes we need to use this disjunction operator in the midst of a larger sequence. For example, suppose I want to search for information about pet fish for my cousin David. How can I specify both *guppy* and *guppies*? We cannot simply say /guppy|ies/, because that would match only the strings *guppy* and *ies*. This is because sequences like guppy take **precedence** over the disjunction operator |. To make the disjunction operator apply only to a specific pattern, we need to use the parenthesis operators (and). Enclosing a pattern in parentheses makes it act like a single character for the purposes of neighboring operators like the pipe | and the Kleene*. So the pattern /gupp(y|ies)/ would specify that we meant the disjunction only to apply to the suffixes y and ies.

Precedence

The parenthesis operator (is also useful when we are using counters like the Kleene*. Unlike the | operator, the Kleene* operator applies by default only to a single character, not to a whole sequence. Suppose we want to match repeated instances of a string. Perhaps we have a line that has column labels of the form *Column 1 Column 2 Column 3*. The expression /Column␣[0-9]+␣*/ will not match any number of columns; instead, it will match a single column followed by any number of spaces! The star here applies only to the space ␣ that precedes it, not to the whole sequence. With the parentheses, we could write the expression /(Column␣[0-9]+␣*)*/ to match the word *Column*, followed by a number and optional spaces, the whole pattern repeated zero or more times.

This idea that one operator may take precedence over another, requiring us to sometimes use parentheses to specify what we mean, is formalized by the **operator precedence hierarchy** for regular expressions. The following table gives the order of RE operator precedence, from highest precedence to lowest precedence.

operator precedence

Parenthesis	()
Counters	* + ? {}
Sequences and anchors	the ^my end$
Disjunction	|

Thus, because counters have a higher precedence than sequences, /the*/ matches *theeeee* but not *thethe*. Because sequences have a higher precedence than disjunction, /the|any/ matches *the* or *any* but not *thany* or *theny*.

Patterns can be ambiguous in another way. Consider the expression /[a-z]*/ when matching against the text *once upon a time*. Since /[a-z]*/ matches zero or more letters, this expression could match nothing, or just the first letter *o*, *on*, *onc*, or *once*. In these cases regular expressions always match the *largest* string they can; we say that patterns are **greedy**, expanding to cover as much of a string as they can.

greedy

non-greedy

*?

+?

There are, however, ways to enforce **non-greedy** matching, using another meaning of the ? qualifier. The operator ***?** is a Kleene star that matches as little text as possible. The operator **+?** is a Kleene plus that matches as little text as possible.

2.1.3 A Simple Example

Suppose we wanted to write a RE to find cases of the English article *the*. A simple (but incorrect) pattern might be:

```
/the/
```

One problem is that this pattern will miss the word when it begins a sentence and hence is capitalized (i.e., *The*). This might lead us to the following pattern:

```
/[tT]he/
```

But we will still incorrectly return texts with the embedded in other words (e.g., *other* or *theology*). So we need to specify that we want instances with a word boundary on both sides:

```
/\b[tT]he\b/
```

Suppose we wanted to do this without the use of /\b/. We might want this since /\b/ won't treat underscores and numbers as word boundaries; but we might want to find *the* in some context where it might also have underlines or numbers nearby (*the_* or *the25*). We need to specify that we want instances in which there are no alphabetic letters on either side of the *the*:

```
/[^a-zA-Z][tT]he[^a-zA-Z]/
```

But there is still one more problem with this pattern: it won't find the word *the* when it begins a line. This is because the regular expression [^a-zA-Z], which we used to avoid embedded instances of *the*, implies that there must be some single (although non-alphabetic) character before the *the*. We can avoid this by specifying that before the *the* we require *either* the beginning-of-line or a non-alphabetic character, and the same at the end of the line:

```
/(^|[^a-zA-Z])[tT]he([^a-zA-Z]|$)/
```

The process we just went through was based on fixing two kinds of errors: **false positives**, strings that we incorrectly matched like *other* or *there*, and **false negatives**, strings that we incorrectly missed, like *The*. Addressing these two kinds of

false positives

false negatives

errors comes up again and again in implementing speech and language processing systems. Reducing the overall error rate for an application thus involves two antagonistic efforts:

- Increasing **precision** (minimizing false positives)
- Increasing **recall** (minimizing false negatives)

2.1.4 A More Complex Example

Let's try out a more significant example of the power of REs. Suppose we want to build an application to help a user buy a computer on the Web. The user might want "any machine with at least 6 GHz and 500 GB of disk space for less than $1000". To do this kind of retrieval, we first need to be able to look for expressions like *6 GHz* or *500 GB* or *Mac* or *$999.99*. In the rest of this section we'll work out some simple regular expressions for this task.

First, let's complete our regular expression for prices. Here's a regular expression for a dollar sign followed by a string of digits:

```
/$[0-9]+/
```

Note that the $ character has a different function here than the end-of-line function we discussed earlier. Most regular expression parsers are smart enough to realize that $ here doesn't mean end-of-line. (As a thought experiment, think about how regex parsers might figure out the function of $ from the context.)

Now we just need to deal with fractions of dollars. We'll add a decimal point and two digits afterwards:

```
/$[0-9]+\.[0-9][0-9]/
```

This pattern only allows *$199.99* but not *$199*. We need to make the cents optional and to make sure we're at a word boundary:

```
/(^|\W)$[0-9]+(\.[0-9][0-9])?\b/
```

One last catch! This pattern allows prices like *$199999.99* which would be far too expensive! We need to limit the dollar

```
/(^|\W)$[0-9]{0,3}(\.[0-9][0-9])?\b/
```

How about disk space? We'll need to allow for optional fractions again (*5.5 GB*); note the use of ? for making the final s optional, and the of /␣*/ to mean "zero or more spaces" since there might always be extra spaces lying around:

```
/\b[0-9]+(\.[0-9]+)?␣*(GB|[Gg]igabytes?)\b/
```

Modifying this regular expression so that it only matches more than 500 GB is left as an exercise for the reader.

2.1.5 More Operators

Figure 2.7 shows some aliases for common ranges, which can be used mainly to save typing. Besides the Kleene * and Kleene + we can also use explicit numbers as counters, by enclosing them in curly brackets. The regular expression /{3}/ means "exactly 3 occurrences of the previous character or expression". So /a\.{24}z/ will match *a* followed by 24 dots followed by *z* (but not *a* followed by 23 or 25 dots followed by a *z*).

RE	Expansion	Match	First Matches
\d	[0-9]	any digit	Party␣of␣5
\D	[^0-9]	any non-digit	Blue␣moon
\w	[a-zA-Z0-9_]	any alphanumeric/underscore	Daiyu
\W	[^\w]	a non-alphanumeric	!!!!
\s	[␣\r\t\n\f]	whitespace (space, tab)	
\S	[^\s]	Non-whitespace	in␣Concord

Figure 2.7 Aliases for common sets of characters.

A range of numbers can also be specified. So /{n,m}/ specifies from *n* to *m* occurrences of the previous char or expression, and /{n,}/ means at least *n* occurrences of the previous expression. REs for counting are summarized in Fig. 2.8.

RE	Match
*	zero or more occurrences of the previous char or expression
+	one or more occurrences of the previous char or expression
?	exactly zero or one occurrence of the previous char or expression
{n}	*n* occurrences of the previous char or expression
{n,m}	from *n* to *m* occurrences of the previous char or expression
{n,}	at least *n* occurrences of the previous char or expression
{,m}	up to *m* occurrences of the previous char or expression

Figure 2.8 Regular expression operators for counting.

Newline

Finally, certain special characters are referred to by special notation based on the backslash (\) (see Fig. 2.9). The most common of these are the **newline** character \n and the **tab** character \t. To refer to characters that are special themselves (like ., *, [, and \), precede them with a backslash, (i.e., /\./, /*/, /\[/, and /\\/).

RE	Match	First Patterns Matched
*	an asterisk "*"	"K*A*P*L*A*N"
\.	a period "."	"Dr. Livingston, I presume"
\?	a question mark	"Why don't they come and lend a hand?"
\n	a newline	
\t	a tab	

Figure 2.9 Some characters that need to be backslashed.

2.1.6 Substitution, Capture Groups, and ELIZA

substitution

An important use of regular expressions is in **substitutions**. For example, the substitution operator s/regexp1/pattern/ used in Python and in Unix commands like vim or sed allows a string characterized by a regular expression to be replaced by another string:

```
s/colour/color/
```

It is often useful to be able to refer to a particular subpart of the string matching the first pattern. For example, suppose we wanted to put angle brackets around all integers in a text, for example, changing *the 35 boxes* to *the <35> boxes*. We'd like a way to refer to the integer we've found so that we can easily add the brackets. To do this, we put parentheses (and) around the first pattern and use the **number** operator \1 in the second pattern to refer back. Here's how it looks:

```
s/([0-9]+)/<\1>/
```

The parenthesis and number operators can also specify that a certain string or expression must occur twice in the text. For example, suppose we are looking for the pattern "the Xer they were, the Xer they will be", where we want to constrain the two X's to be the same string. We do this by surrounding the first X with the parenthesis operator, and replacing the second X with the number operator `\1`, as follows:

```
/the (.*)er they were, the \1er they will be/
```

Here the `\1` will be replaced by whatever string matched the first item in parentheses. So this will match *the bigger they were, the bigger they will be* but not *the bigger they were, the faster they will be*.

capture group

This use of parentheses to store a pattern in memory is called a **capture group**. Every time a capture group is used (i.e., parentheses surround a pattern), the resulting match is stored in a numbered **register**. If you match two different sets of parentheses, `\2` means whatever matched the *second* capture group. Thus

register

```
/the (.*)er they (.*), the \1er we \2/
```

will match *the faster they ran, the faster we ran* but not *the faster they ran, the faster we ate*. Similarly, the third capture group is stored in `\3`, the fourth is `\4`, and so on.

Parentheses thus have a double function in regular expressions; they are used to group terms for specifying the order in which operators should apply, and they are used to capture something in a register. Occasionally we might want to use parentheses for grouping, but don't want to capture the resulting pattern in a register. In that case we use a **non-capturing group**, which is specified by putting the commands `?:` after the open paren, in the form `(?: pattern )`.

non-capturing group

```
/(?:some|a few) (people|cats) like some \1/
```

will match *some cats like some cats* but not *some cats like some a few*.

Substitutions and capture groups are very useful in implementing simple chatbots like ELIZA (Weizenbaum, 1966). Recall that ELIZA simulates a Rogerian psychologist by carrying on conversations like the following:

User_1:	Men are all alike.
ELIZA_1:	IN WHAT WAY
User_2:	They're always bugging us about something or other.
ELIZA_2:	CAN YOU THINK OF A SPECIFIC EXAMPLE
User_3:	Well, my boyfriend made me come here.
ELIZA_3:	YOUR BOYFRIEND MADE YOU COME HERE
User_4:	He says I'm depressed much of the time.
ELIZA_4:	I AM SORRY TO HEAR YOU ARE DEPRESSED

ELIZA works by having a series or cascade of regular expression substitutions each of which matches and changes some part of the input lines. Input lines are first uppercased. The first substitutions then change all instances of *MY* to *YOUR*, and *I'M* to *YOU ARE*, and so on. The next set of substitutions matches and replaces other patterns in the input. Here are some examples:

```
s/.* I'M (depressed|sad) .*/I AM SORRY TO HEAR YOU ARE \1/
s/.* I AM (depressed|sad) .*/WHY DO YOU THINK YOU ARE \1/
s/.* all .*/IN WHAT WAY/
s/.* always .*/CAN YOU THINK OF A SPECIFIC EXAMPLE/
```

Since multiple substitutions can apply to a given input, substitutions are assigned a rank and applied in order. Creating patterns is the topic of Exercise 2.3, and we return to the details of the ELIZA architecture in Chapter 26.

2.1.7 Lookahead Assertions

Finally, there will be times when we need to predict the future: look ahead in the text to see if some pattern matches, but not advance the match cursor, so that we can then deal with the pattern if it occurs.

lookahead

These **lookahead** assertions make use of the `(?` syntax that we saw in the previous section for non-capture groups. The operator `(?= pattern)` is true if `pattern` occurs, but is **zero-width**, i.e. the match pointer doesn't advance. The operator `(?! pattern)` only returns true if a pattern does not match, but again is zero-width and doesn't advance the cursor. Negative lookahead is commonly used when we are parsing some complex pattern but want to rule out a special case. For example suppose we want to match, at the beginning of a line, any single word that doesn't start with "Volcano". We can use negative lookahead to do this:

zero-width

```
/^(?!Volcano)[A-Za-z]+/
```

2.2 Words

Before we talk about processing words, we need to decide what counts as a word. Let's start by looking at one particular **corpus** (plural **corpora**), a computer-readable collection of text or speech. For example the Brown corpus is a million-word collection of samples from 500 written English texts from different genres (newspaper, fiction, non-fiction, academic, etc.), assembled at Brown University in 1963–64 (Kučera and Francis, 1967). How many words are in the following Brown sentence?

corpus

corpora

He stepped out into the hall, was delighted to encounter a water brother.

This sentence has 13 words if we don't count punctuation marks as words, 15 if we count punctuation. Whether we treat period ("."), comma (","), and so on as words depends on the task. Punctuation is critical for finding boundaries of things (commas, periods, colons) and for identifying some aspects of meaning (question marks, exclamation marks, quotation marks). For some tasks, like part-of-speech tagging or parsing or speech synthesis, we sometimes treat punctuation marks as if they were separate words.

The Switchboard corpus of American English telephone conversations between strangers was collected in the early 1990s; it contains 2430 conversations averaging 6 minutes each, totaling 240 hours of speech and about 3 million words (Godfrey et al., 1992). Such corpora of spoken language don't have punctuation but do introduce other complications with regard to defining words. Let's look at one utterance from Switchboard; an **utterance** is the spoken correlate of a sentence:

utterance

I do uh main- mainly business data processing

This utterance has two kinds of **disfluencies**. The broken-off word *main-* is called a **fragment**. Words like *uh* and *um* are called **fillers** or **filled pauses**. Should we consider these to be words? Again, it depends on the application. If we are building a speech transcription system, we might want to eventually strip out the disfluencies.

disfluency

fragment

filled pause

But we also sometimes keep disfluencies around. Disfluencies like *uh* or *um* are actually helpful in speech recognition in predicting the upcoming word, because they may signal that the speaker is restarting the clause or idea, and so for speech recognition they are treated as regular words. Because people use different disfluencies they can also be a cue to speaker identification. In fact Clark and Fox Tree (2002) showed that *uh* and *um* have different meanings. What do you think they are?

Are capitalized tokens like *They* and uncapitalized tokens like *they* the same word? These are lumped together in some tasks (speech recognition), while for part-of-speech or named-entity tagging, capitalization is a useful feature and is retained.

How about inflected forms like *cats* versus *cat*? These two words have the same **lemma** *cat* but are different wordforms. A **lemma** is a set of lexical forms having the same stem, the same major part-of-speech, and the same word sense. The **wordform** is the full inflected or derived form of the word. For morphologically complex languages like Arabic, we often need to deal with lemmatization. For many tasks in English, however, wordforms are sufficient.

lemma

wordform

How many words are there in English? To answer this question we need to distinguish two ways of talking about words. **Types** are the number of distinct words in a corpus; if the set of words in the vocabulary is V, the number of types is the vocabulary size $|V|$. **Tokens** are the total number N of running words. If we ignore punctuation, the following Brown sentence has 16 tokens and 14 types:

word type

word token

They picnicked by the pool, then lay back on the grass and looked at the stars.

When we speak about the number of words in the language, we are generally referring to word types.

Corpus	Tokens = N	Types = $\|V\|$
Shakespeare	884 thousand	31 thousand
Brown corpus	1 million	38 thousand
Switchboard telephone conversations	2.4 million	20 thousand
COCA	440 million	2 million
Google N-grams	1 trillion	13 million

Figure 2.10 Rough numbers of types and tokens for some English language corpora. The largest, the Google N-grams corpus, contains 13 million types, but this count only includes types appearing 40 or more times, so the true number would be much larger.

Fig. 2.10 shows the rough numbers of types and tokens computed from some popular English corpora. The larger the corpora we look at, the more word types we find, and in fact this relationship between the number of types $|V|$ and number of tokens N is called **Herdan's Law** (Herdan, 1960) or **Heaps' Law** (Heaps, 1978) after its discoverers (in linguistics and information retrieval respectively). It is shown in Eq. 2.1, where k and β are positive constants, and $0 < \beta < 1$.

Herdan's Law

Heaps' Law

$$|V| = kN^{\beta} \tag{2.1}$$

The value of β depends on the corpus size and the genre, but at least for the large corpora in Fig. 2.10, β ranges from .67 to .75. Roughly then we can say that the vocabulary size for a text goes up significantly faster than the square root of its length in words.

Another measure of the number of words in the language is the number of lemmas instead of wordform types. Dictionaries can help in giving lemma counts; dictionary **entries** or **boldface forms** are a very rough upper bound on the number of

lemmas (since some lemmas have multiple boldface forms). The 1989 edition of the Oxford English Dictionary had 615,000 entries.

2.3 Corpora

Words don't appear out of nowhere. Any particular piece of text that we study is produced by one or more specific speakers or writers, in a specific dialect of a specific language, at a specific time, in a specific place, for a specific function.

Perhaps the most important dimension of variation is the language. NLP algorithms are most useful when they apply across many languages. The world has 7097 languages at the time of this writing, according to the online Ethnologue catalog (Simons and Fennig, 2018). Most NLP tools tend to be developed for the official languages of large industrialized nations (Chinese, English, Spanish, Arabic, etc.), but we don't want to limit tools to just these few languages. Furthermore, most languages also have multiple varieties, such as dialects spoken in different regions or by different social groups. Thus, for example, if we're processing text in African American Vernacular English (**AAVE**), a dialect spoken by millions of people in the United States, it's important to make use of NLP tools that function with that dialect. Twitter posts written in AAVE make use of constructions like *iont* (*I don't* in Standard American English (**SAE**)), or *talmbout* corresponding to SAE *talking about*, both examples that influence word segmentation (Blodgett et al. 2016, Jones 2015).

AAVE

SAE

It's also quite common for speakers or writers to use multiple languages in a single communicative act, a phenomenon called **code switching**. Code switching is enormously common across the world; here are examples showing Spanish and (transliterated) Hindi code switching with English (Solorio et al. 2014, Jurgens et al. 2017):

code switching

(2.2) Por primera vez veo a @username actually being hateful! it was beautiful:)
[For the first time I get to see @username actually being hateful! it was beautiful:)]

(2.3) dost tha or ra- hega ... dont wory ... but dherya rakhe
["he was and will remain a friend ... don't worry ... but have faith"]

Another dimension of variation is the genre. The text that our algorithms must process might come from newswire, fiction or non-fiction books, scientific articles, Wikipedia, or religious texts. It might come from spoken genres like telephone conversations, business meetings, police body-worn cameras, medical interviews, or transcripts of television shows or movies. It might come from work situations like doctors' notes, legal text, or parliamentary or congressional proceedings.

Text also reflects the demographic characteristics of the writer (or speaker): their age, gender, race, socioeconomic class can all influence the linguistic properties of the text we are processing.

And finally, time matters too. Language changes over time, and for some languages we have good corpora of texts from different historical periods.

Because language is so situated, when developing computational models for language processing, it's important to consider who produced the language, in what context, for what purpose, and make sure that the models are fit to the data.

2.4 Text Normalization

Before almost any natural language processing of a text, the text has to be normalized. At least three tasks are commonly applied as part of any normalization process:

1. Tokenizing (segmenting) words
2. Normalizing word formats
3. Segmenting sentences

In the next sections we walk through each of these tasks.

2.4.1 Unix Tools for Crude Tokenization and Normalization

Let's begin with an easy, if somewhat naive version of word tokenization and normalization (and frequency computation) that can be accomplished for English solely in a single UNIX command-line, inspired by Church (1994). We'll make use of some Unix commands: `tr`, used to systematically change particular characters in the input; `sort`, which sorts input lines in alphabetical order; and `uniq`, which collapses and counts adjacent identical lines.

For example let's begin with the 'complete words' of Shakespeare in one textfile, `sh.txt`. We can use `tr` to tokenize the words by changing every sequence of non-alphabetic characters to a newline ('A-Za-z' means alphabetic, the -c option complements to non-alphabet, and the -s option squeezes all sequences into a single character):

```
tr -sc 'A-Za-z' '\n' < sh.txt
```

The output of this command will be:

```
THE
SONNETS
by
William
Shakespeare
From
fairest
creatures
We
...
```

Now that there is one word per line, we can sort the lines, and pass them to `uniq -c` which will collapse and count them:

```
tr -sc 'A-Za-z' '\n' < sh.txt | sort | uniq -c
```

with the following output:

```
1945 A
72 AARON
19 ABBESS
25 Aaron
6 Abate
1 Abates
5 Abbess
6 Abbey
```

```
3 Abbot
...
```

Alternatively, we can collapse all the upper case to lower case:

```
tr -sc 'A-Za-z' '\n' < sh.txt | tr A-Z a-z | sort | uniq -c
```

whose output is

```
14725 a
  97 aaron
   1 abaissiez
  10 abandon
   2 abandoned
   2 abase
   1 abash
  14 abate
   3 abated
   3 abatement
   ...
```

Now we can sort again to find the frequent words. The `-n` option to `sort` means to sort numerically rather than alphabetically, and the `-r` option means to sort in reverse order (highest-to-lowest):

```
tr -sc 'A-Za-z' '\n' < sh.txt | tr A-Z a-z | sort | uniq -c | sort -n -r
```

The results show that the most frequent words in Shakespeare, as in any other corpus, are the short **function words** like articles, pronouns, prepositions:

```
27378 the
26084 and
22538 i
19771 to
17481 of
14725 a
13826 you
12489 my
11318 that
11112 in
   ...
```

Unix tools of this sort can be very handy in building quick word count statistics for any corpus.

2.4.2 Word Tokenization

tokenization

The simple UNIX tools above were fine for getting rough word statistics but more sophisticated algorithms are generally necessary for **tokenization**, the task of segmenting running text into words.

While the Unix command sequence just removed all the numbers and punctuation, for most NLP applications we'll need to keep these in our tokenization. We often want to break off punctuation as a separate token; commas are a useful piece of information for parsers, periods help indicate sentence boundaries. But we'll often want to keep the punctuation that occurs word internally, in examples like *m.p.h,*, *Ph.D.*, *AT&T*, *cap'n*. Special characters and numbers will need to be kept in prices

($45.55) and dates (01/02/06); we don't want to segment that price into separate tokens of "45" and "55". And there are URLs (http://www.stanford.edu), Twitter hashtags (#nlproc), or email addresses (someone@cs.colorado.edu).

Number expressions introduce other complications as well; while commas normally appear at word boundaries, commas are used inside numbers in English, every three digits: 555,500.50. Languages, and hence tokenization requirements, differ on this; many continental European languages like Spanish, French, and German, by contrast, use a comma to mark the decimal point, and spaces (or sometimes periods) where English puts commas, for example, 555 500,50.

clitic

A tokenizer can also be used to expand **clitic** contractions that are marked by apostrophes, for example, converting what're to the two tokens what are, and we're to we are. A clitic is a part of a word that can't stand on its own, and can only occur when it is attached to another word. Some such contractions occur in other alphabetic languages, including articles and pronouns in French (j'ai, l'homme).

Depending on the application, tokenization algorithms may also tokenize multiword expressions like New York or rock 'n' roll as a single token, which requires a multiword expression dictionary of some sort. Tokenization is thus intimately tied up with **named entity detection**, the task of detecting names, dates, and organizations (Chapter 18).

Penn Treebank tokenization

One commonly used tokenization standard is known as the **Penn Treebank tokenization** standard, used for the parsed corpora (treebanks) released by the Linguistic Data Consortium (LDC), the source of many useful datasets. This standard separates out clitics (*doesn't* becomes *does* plus *n't*), keeps hyphenated words together, and separates out all punctuation (to save space we're showing visible spaces '␣' between tokens, although newlines is a more common output):

Input: "The San Francisco-based restaurant," they said, "doesn't charge $10".

Output: "␣The␣San␣Francisco-based␣restaurant␣,␣"␣they␣said␣,␣ "␣does␣n't␣charge␣$␣10␣"␣.

In practice, since tokenization needs to be run before any other language processing, it needs to be very fast. The standard method for tokenization is therefore to use deterministic algorithms based on regular expressions compiled into very efficient finite state automata. For example, Fig. 2.11 shows an example of a basic regular expression that can be used to tokenize with the nltk.regexp_tokenize function of the Python-based Natural Language Toolkit (NLTK) (Bird et al. 2009; http://www.nltk.org).

Carefully designed deterministic algorithms can deal with the ambiguities that arise, such as the fact that the apostrophe needs to be tokenized differently when used as a genitive marker (as in *the book's cover*), a quotative as in *'The other class', she said*, or in clitics like *they're*.

Word tokenization is more complex in languages like written Chinese, Japanese, and Thai, which do not use spaces to mark potential word-boundaries.

hanzi

In Chinese, for example, words are composed of characters (called **hanzi** in Chinese). Each character generally represents a single unit of meaning (called a **morpheme**) and is pronounceable as a single syllable. Words are about 2.4 characters long on average. But deciding what counts as a word in Chinese is complex. For example, consider the following sentence:

(2.4) 姚明进入总决赛
"Yao Ming reaches the finals"

```
>>> text = 'That U.S.A. poster-print costs $12.40...'
>>> pattern = r'''(?x)    # set flag to allow verbose regexps
...     ([A-Z]\.)+        # abbreviations, e.g. U.S.A.
...   | \w+(-\w+)*        # words with optional internal hyphens
...   | \$?\d+(\.\d+)?%?  # currency and percentages, e.g. $12.40, 82%
...   | \.\.\.            # ellipsis
...   | [][.,;"'?():-_`]  # these are separate tokens; includes ], [
... '''
>>> nltk.regexp_tokenize(text, pattern)
['That', 'U.S.A.', 'poster-print', 'costs', '$12.40', '...']
```

Figure 2.11 A python trace of regular expression tokenization in the NLTK (Bird et al., 2009) Python-based natural language processing toolkit, commented for readability; the `(?x)` verbose flag tells Python to strip comments and whitespace. Figure from Chapter 3 of Bird et al. (2009).

As Chen et al. (2017) point out, this could be treated as 3 words ('Chinese Treebank' segmentation):

(2.5) 姚明 进入 总决赛
YaoMing reaches finals

or as 5 words ('Peking University' segmentation):

(2.6) 姚 明 进入 总 决赛
Yao Ming reaches overall finals

Finally, it is possible in Chinese simply to ignore words altogether and use characters as the basic elements, treating the sentence as a series of 7 characters:

(2.7) 姚 明 进 入 总 决 赛
Yao Ming enter enter overall decision game

In fact, for most Chinese NLP tasks it turns out to work better to take characters rather than words as input, since characters are at a reasonable semantic level for most applications, and since most word standards result in a huge vocabulary with large numbers of very rare words (Li et al., 2019).

word segmentation

However, for Japanese and Thai the character is too small a unit, and so algorithms for **word segmentation** are required. These can also be useful for Chinese in the rare situations where word rather than character boundaries are required. The standard segmentation algorithms for these languages use neural **sequence models** trained via supervised machine learning on hand-segmented training sets; we'll introduce sequence models in Chapter 8.

2.4.3 Byte-Pair Encoding for Tokenization

There is a third option to tokenizing text input. Instead of defining tokens as words (defined by spaces in orthographies that have spaces, or more complex algorithms), or as characters (as in Chinese), we can use our data to automatically tell us what size tokens should be. Perhaps sometimes we might want tokens that are space-delimited words (like *spinach*) other times it's useful to have tokens that are larger than words (like *New York Times*), and sometimes smaller than words (like the morphemes *-est* or *-er*. A morpheme is the smallest meaning-bearing unit of a language; for example the word *unlikeliest* has the morphemes *un-*, *likely*, and *-est*; we'll return to this on page 21.

subword

One reason it's helpful to have **subword** tokens is to deal with unknown words.

Unknown words are particularly relevant for machine learning systems. As we will see in the next chapter, machine learning systems often learn some facts about words in one corpus (a **training** corpus) and then use these facts to make decisions about a separate **test** corpus and its words. Thus if our training corpus contains, say the words *low*, and *lowest*, but not *lower*, but then the word *lower* appears in our test corpus, our system will not know what to do with it.

A solution to this problem is to use a kind of tokenization in which most tokens are words, but some tokens are frequent morphemes or other subwords like *-er*, so that an unseen word can be represented by combining the parts.

BPE The simplest such algorithm is **byte-pair encoding**, or **BPE** (Sennrich et al., 2016). Byte-pair encoding is based on a method for text compression (Gage, 1994), but here we use it for tokenization instead. The intuition of the algorithm is to iteratively merge frequent pairs of characters,

The algorithm begins with the set of symbols equal to the set of characters. Each word is represented as a sequence of characters plus a special end-of-word symbol _. At each step of the algorithm, we count the number of symbol pairs, find the most frequent pair ('A', 'B'), and replace it with the new merged symbol ('AB'). We continue to count and merge, creating new longer and longer character strings, until we've done k merges; k is a parameter of the algorithm. The resulting symbol set will consist of the original set of characters plus k new symbols.

The algorithm is run inside words (we don't merge across word boundaries). For this reason, the algorithm can take as input a dictionary of words together with counts. Consider the following tiny input dictionary with counts for each word, which would have the starting vocabulary of 11 letters:

	dictionary	**vocabulary**
5	l o w _	_, d, e, i, l, n, o, r, s, t, w
2	l o w e s t _	
6	n e w e r _	
3	w i d e r _	
2	n e w _	

We first count all pairs of symbols: the most frequent is the pair r _ because it occurs in *newer* (frequency of 6) and *wider* (frequency of 3) for a total of 9 occurrences. We then merge these symbols, treating r_ as one symbol, and count again:

	dictionary	**vocabulary**
5	l o w _	_, d, e, i, l, n, o, r, s, t, w, r_
2	l o w e s t _	
6	n e w e r_	
3	w i d e r_	
2	n e w _	

Now the most frequent pair is e r_, which we merge; our system has learned that there should be a token for word-final er, represented as er_:

	dictionary	**vocabulary**
5	l o w _	_, d, e, i, l, n, o, r, s, t, w, r_, er_
2	l o w e s t _	
6	n e w er_	
3	w i d er_	
2	n e w _	

Next e w (total count of 8) get merged to ew:

	dictionary	vocabulary
5	l o w _	_, d, e, i, l, n, o, r, s, t, w, r_, er_, ew
2	l o w e s t _	
6	n ew er_	
3	w i d er_	
2	n ew _	

If we continue, the next merges are:

Merge	Current Vocabulary
(n, ew)	_, d, e, i, l, n, o, r, s, t, w, r_, er_, ew, new
(l, o'	_, d, e, i, l, n, o, r, s, t, w, r_, er_, ew, new, lo
(lo, w)	_, d, e, i, l, n, o, r, s, t, w, r_, er_, ew, new, lo, low
(new, er_)	_, d, e, i, l, n, o, r, s, t, w, r_, er_, ew, new, lo, low, newer_
(low, _)	_, d, e, i, l, n, o, r, s, t, w, r_, er_, ew, new, lo, low, newer_, low_

When we need to tokenize a test sentence, we just run the merges we have learned, greedily, in the order we learned them, on the test data. (Thus the frequencies in the test data don't play a role, just the frequencies in the training data). So first we segment each test sentence word into characters. Then we apply the first rule: replace every instance of r _ in the test corpus with r_, and then the second rule: replace every instance of e r_ in the test corpus with er_, and so on. By the end, if the test corpus contained the word n e w e r _, it would be tokenized as a full word. But a new (unknown) word like l o w e r _ would be merged into the two tokens low er_.

Of course in real algorithms BPE is run with many thousands of merges on a very large input dictionary. The result is that most words will be represented as full symbols, and only the very rare words (and unknown words) will have to be represented by their parts. The full BPE learning algorithm is given in Fig. 2.12.

Wordpiece and Greedy Tokenization

wordpiece

There are some alternatives to byte pair encoding for inducing tokens. Like the BPE algorithm, the **wordpiece** algorithm starts with some simple tokenization (such as by whitespace) into rough words, and then breaks those rough word tokens into subword tokens. The **wordpiece** model differs from BPE only in that the special word-boundary token _ appears at the beginning of words rather than at the end, and in the way it merges pairs. Rather than merging the pairs that are most *frequent*, wordpiece instead merges the pairs that minimizes the language model likelihood of the training data. We'll introduce these concepts in the next chapter, but to simplify, the wordpiece model chooses the two tokens to combine that would give the training corpus the highest probability (Wu et al., 2016).

In the wordpiece segmenter used in BERT (Devlin et al., 2019), like other wordpiece variants, an input sentence or string is first split by some simple basic tokenizer (like whitespace) into a series of rough word tokens. But then instead of using a word boundary token, word-initial subwords are distinguished from those that do not start words by marking internal subwords with special symbols ##, so that we might split *unaffable* into ["un", "\#\#aff", "\#\#able"]. Then each word token string is tokenized using a greedy longest-match-first algorithm. This is different than the decoding algorithm we introduced for BPE, which runs the merges on the test sentence in the same order they were learned from the training set.

maximum matching

Greedy longest-match-first decoding is sometimes called **maximum matching** or **MaxMatch**. The maximum matching algorithm (Fig. 2.13) is given a vocabulary (a learned list of wordpiece tokens) and a string and starts by pointing at the

```
import re, collections

def get_stats(vocab):
    pairs = collections.defaultdict(int)
    for word, freq in vocab.items():
        symbols = word.split()
        for i in range(len(symbols)-1):
            pairs[symbols[i],symbols[i+1]] += freq
    return pairs

def merge_vocab(pair, v_in):
    v_out = {}
    bigram = re.escape(' '.join(pair))
    p = re.compile(r'(?<!\S)' + bigram + r'(?!\S)')
    for word in v_in:
        w_out = p.sub(''.join(pair), word)
        v_out[w_out] = v_in[word]
    return v_out

vocab = {'l o w </w>' : 5, 'l o w e s t </w>' : 2,
    'n e w e r </w>':6, 'w i d e r </w>':3, 'n e w </w>':2}
num_merges = 8

for i in range(num_merges):
    pairs = get_stats(vocab)
    best = max(pairs, key=pairs.get)
    vocab = merge_vocab(best, vocab)
    print(best)
```

Figure 2.12 Python code for BPE learning algorithm from Sennrich et al. (2016).

beginning of a string. It chooses the longest token in the wordpiece vocabulary that matches the input at the current position, and moves the pointer past that word in the string. The algorithm is then applied again starting from the new pointer position.

function MAXMATCH(string, dictionary) **returns** list of tokens T

if string is empty
return empty list
for *i* ← *length*(sentence) **downto** 1
firstword = first *i* chars of *sentence*
remainder = rest of *sentence*
if InDictionary(*firstword*, *dictionary*)
return list(firstword, MaxMatch(remainder,dictionary))

Figure 2.13 The MaxMatch (or 'greedy longest-first') algorithm for word tokenization using wordpiece or other vocabularies. Assumes that all strings can be successfully tokenized with the given dictionary.

Thus given the token `intention` and the dictionary:

`["in", "tent","intent","##tent", "##tention", "##tion", "#ion"]`

the BERT tokenizer would choose `intent` (because it is longer than `in`, and then `##ion` to complete the string, resulting in the tokenization `["intent" "##ion"]`. The BERT tokenizer applied to the string `unwanted running` will produce:

(2.8) `["un", "##want", "##ed", "runn", "##ing"]`

SentencePiece

Another tokenization algorithm is called **SentencePiece** (Kudo and Richardson,

2018). BPE and wordpiece both assume that we already have some initial tokenization of words (such as by spaces, or from some initial dictionary) and so we never tried to induce word parts across spaces. By contrast, the SentencePiece model works from raw text; even whitespace is handled as a normal symbol. Thus it doesn't need an initial tokenization or word-list, and can be used in languages like Chinese or Japanese that don't have spaces.

2.4.4 Word Normalization, Lemmatization and Stemming

normalization

Word **normalization** is the task of putting words/tokens in a standard format, choosing a single normal form for words with multiple forms like `USA` and `US` or `uh-huh` and `uhhuh`. This standardization may be valuable, despite the spelling information that is lost in the normalization process. For information retrieval or information extraction about the US, we might want see information from documents whether they mention the `US` or the `USA`.

case folding

Case folding is another kind of normalization. Mapping everything to lower case means that *Woodchuck* and *woodchuck* are represented identically, which is very helpful for generalization in many tasks, such as information retrieval or speech recognition. For sentiment analysis and other text classification tasks, information extraction, and machine translation, by contrast, case can be quite helpful and case folding is generally not done. This is because maintaining the difference between, for example, `US` the country and `us` the pronoun can outweigh the advantage in generalization that case folding would have provided for other words.

For many natural language processing situations we also want two morphologically different forms of a word to behave similarly. For example in web search, someone may type the string *woodchucks* but a useful system might want to also return pages that mention *woodchuck* with no *s*. This is especially common in morphologically complex languages like Russian, where for example the word *Moscow* has different endings in the phrases *Moscow*, *of Moscow*, *to Moscow*, and so on.

Lemmatization is the task of determining that two words have the same root, despite their surface differences. The words *am*, *are*, and *is* have the shared lemma *be*; the words *dinner* and *dinners* both have the lemma *dinner*. Lemmatizing each of these forms to the same lemma will let us find all mentions of words in Russian like Moscow. The lemmatized form of a sentence like *He is reading detective stories* would thus be *He be read detective story*.

How is lemmatization done? The most sophisticated methods for lemmatization involve complete **morphological parsing** of the word. **Morphology** is the study of the way words are built up from smaller meaning-bearing units called **morphemes**. Two broad classes of morphemes can be distinguished: **stems**—the central morpheme of the word, supplying the main meaning— and **affixes**—adding "additional" meanings of various kinds. So, for example, the word *fox* consists of one morpheme (the morpheme *fox*) and the word *cats* consists of two: the morpheme *cat* and the morpheme *-s*. A morphological parser takes a word like *cats* and parses it into the two morphemes *cat* and *s*, or a Spanish word like *amaren* ('if in the future they would love') into the morphemes *amar* 'to love', *3PL*, and *future subjunctive*.

morpheme
stem
affix

The Porter Stemmer

Lemmatization algorithms can be complex. For this reason we sometimes make use of a simpler but cruder method, which mainly consists of chopping off word-final affixes. This naive version of morphological analysis is called **stemming**. One of the most widely used stemming algorithms is the Porter (1980). The Porter stemmer

stemming
Porter stemmer

applied to the following paragraph:

```
This was not the map we found in Billy Bones's chest, but
an accurate copy, complete in all things-names and heights
and soundings-with the single exception of the red crosses
and the written notes.
```

produces the following stemmed output:

```
Thi wa not the map we found in Billi Bone s chest but an
accur copi complet in all thing name and height and sound
with the singl except of the red cross and the written note
```

cascade

The algorithm is based on series of rewrite rules run in series, as a **cascade**, in which the output of each pass is fed as input to the next pass; here is a sampling of the rules:

$$\begin{aligned} \text{ATIONAL} &\rightarrow \text{ATE} \quad \text{(e.g., relational} \rightarrow \text{relate)} \\ \text{ING} &\rightarrow \epsilon \quad \text{if stem contains vowel (e.g., motoring} \rightarrow \text{motor)} \\ \text{SSES} &\rightarrow \text{SS} \quad \text{(e.g., grasses} \rightarrow \text{grass)} \end{aligned}$$

Detailed rule lists for the Porter stemmer, as well as code (in Java, Python, etc.) can be found on Martin Porter's homepage; see also the original paper (Porter, 1980).

Simple stemmers can be useful in cases where we need to collapse across different variants of the same lemma. Nonetheless, they do tend to commit errors of both over- and under-generalizing, as shown in the table below (Krovetz, 1993):

Errors of Commission		Errors of Omission	
organization	organ	European	Europe
doing	doe	analysis	analyzes
numerical	numerous	noise	noisy
policy	police	sparse	sparsity

2.4.5 Sentence Segmentation

Sentence segmentation

Sentence segmentation is another important step in text processing. The most useful cues for segmenting a text into sentences are punctuation, like periods, question marks, and exclamation points. Question marks and exclamation points are relatively unambiguous markers of sentence boundaries. Periods, on the other hand, are more ambiguous. The period character "." is ambiguous between a sentence boundary marker and a marker of abbreviations like *Mr.* or *Inc.* The previous sentence that you just read showed an even more complex case of this ambiguity, in which the final period of *Inc.* marked both an abbreviation and the sentence boundary marker. For this reason, sentence tokenization and word tokenization may be addressed jointly.

In general, sentence tokenization methods work by first deciding (based on rules or machine learning) whether a period is part of the word or is a sentence-boundary marker. An abbreviation dictionary can help determine whether the period is part of a commonly used abbreviation; the dictionaries can be hand-built or machine-learned (Kiss and Strunk, 2006), as can the final sentence splitter. In the Stanford CoreNLP toolkit (Manning et al., 2014), for example sentence splitting is rule-based, a deterministic consequence of tokenization; a sentence ends when a sentence-ending punctuation (., !, or ?) is not already grouped with other characters into a token (such as for an abbreviation or number), optionally followed by additional final quotes or brackets.

2.5 Minimum Edit Distance

Much of natural language processing is concerned with measuring how similar two strings are. For example in spelling correction, the user typed some erroneous string—let's say `graffe`–and we want to know what the user meant. The user probably intended a word that is similar to `graffe`. Among candidate similar words, the word `giraffe`, which differs by only one letter from `graffe`, seems intuitively to be more similar than, say `grail` or `graf`, which differ in more letters. Another example comes from **coreference**, the task of deciding whether two strings such as the following refer to the same entity:

```
Stanford President John Hennessy
Stanford University President John Hennessy
```

Again, the fact that these two strings are very similar (differing by only one word) seems like useful evidence for deciding that they might be coreferent.

minimum edit distance

Edit distance gives us a way to quantify both of these intuitions about string similarity. More formally, the **minimum edit distance** between two strings is defined as the minimum number of editing operations (operations like insertion, deletion, substitution) needed to transform one string into another.

alignment

The gap between *intention* and *execution*, for example, is 5 (delete an `i`, substitute `e` for `n`, substitute `x` for `t`, insert `c`, substitute `u` for `n`). It's much easier to see this by looking at the most important visualization for string distances, an **alignment** between the two strings, shown in Fig. 2.14. Given two sequences, an **alignment** is a correspondence between substrings of the two sequences. Thus, we say `I` **aligns** with the empty string, `N` with `E`, and so on. Beneath the aligned strings is another representation; a series of symbols expressing an **operation list** for converting the top string into the bottom string: **d** for deletion, **s** for substitution, **i** for insertion.

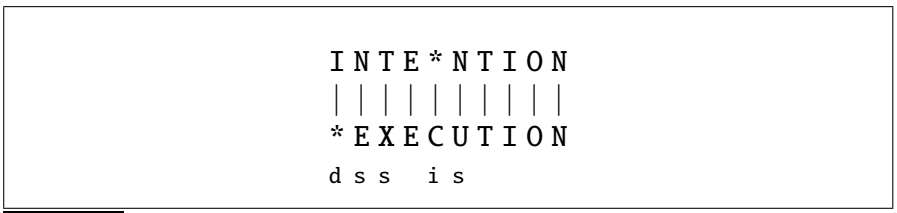

Figure 2.14 Representing the minimum edit distance between two strings as an **alignment**. The final row gives the operation list for converting the top string into the bottom string: d for deletion, s for substitution, i for insertion.

We can also assign a particular cost or weight to each of these operations. The **Levenshtein** distance between two sequences is the simplest weighting factor in which each of the three operations has a cost of 1 (Levenshtein, 1966)—we assume that the substitution of a letter for itself, for example, `t` for `t`, has zero cost. The Levenshtein distance between *intention* and *execution* is 5. Levenshtein also proposed an alternative version of his metric in which each insertion or deletion has a cost of 1 and substitutions are not allowed. (This is equivalent to allowing substitution, but giving each substitution a cost of 2 since any substitution can be represented by one insertion and one deletion). Using this version, the Levenshtein distance between *intention* and *execution* is 8.

2.5.1 The Minimum Edit Distance Algorithm

How do we find the minimum edit distance? We can think of this as a search task, in which we are searching for the shortest path—a sequence of edits—from one string to another.

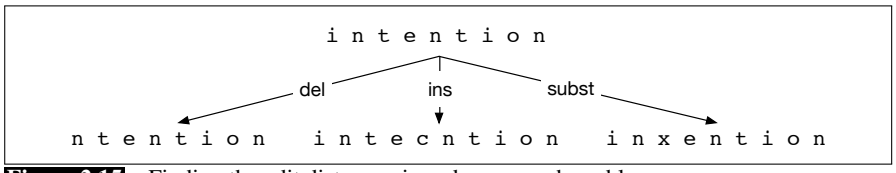

Figure 2.15 Finding the edit distance viewed as a search problem

The space of all possible edits is enormous, so we can't search naively. However, lots of distinct edit paths will end up in the same state (string), so rather than recomputing all those paths, we could just remember the shortest path to a state each time we saw it. We can do this by using **dynamic programming**. Dynamic programming is the name for a class of algorithms, first introduced by Bellman (1957), that apply a table-driven method to solve problems by combining solutions to sub-problems. Some of the most commonly used algorithms in natural language processing make use of dynamic programming, such as the **Viterbi** algorithm (Chapter 8) and the **CKY** algorithm for parsing (Chapter 13).

dynamic programming

The intuition of a dynamic programming problem is that a large problem can be solved by properly combining the solutions to various sub-problems. Consider the shortest path of transformed words that represents the minimum edit distance between the strings *intention* and *execution* shown in Fig. 2.16.

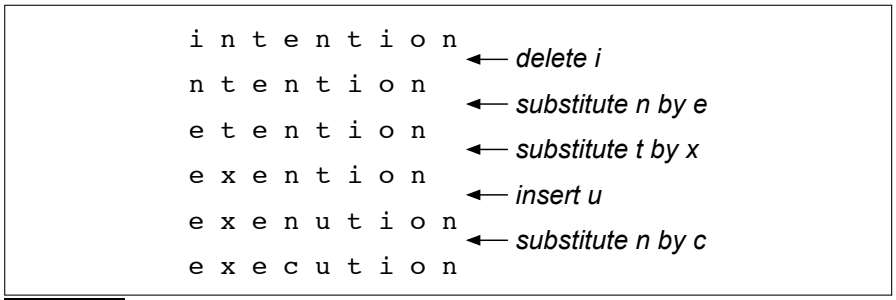

Figure 2.16 Path from *intention* to *execution*.

Imagine some string (perhaps it is *exention*) that is in this optimal path (whatever it is). The intuition of dynamic programming is that if *exention* is in the optimal operation list, then the optimal sequence must also include the optimal path from *intention* to *exention*. Why? If there were a shorter path from *intention* to *exention*, then we could use it instead, resulting in a shorter overall path, and the optimal sequence wouldn't be optimal, thus leading to a contradiction.

minimum edit distance

The **minimum edit distance** algorithm was named by Wagner and Fischer (1974) but independently discovered by many people (see the Historical Notes section of Chapter 8).

Let's first define the minimum edit distance between two strings. Given two strings, the source string X of length n, and target string Y of length m, we'll define $D[i,j]$ as the edit distance between $X[1..i]$ and $Y[1..j]$, i.e., the first i characters of X and the first j characters of Y. The edit distance between X and Y is thus $D[n,m]$.

We'll use dynamic programming to compute $D[n,m]$ bottom up, combining solutions to subproblems. In the base case, with a source substring of length i but an empty target string, going from i characters to 0 requires i deletes. With a target substring of length j but an empty source going from 0 characters to j characters requires j inserts. Having computed $D[i,j]$ for small i,j we then compute larger $D[i,j]$ based on previously computed smaller values. The value of $D[i,j]$ is computed by taking the minimum of the three possible paths through the matrix which arrive there:

$$D[i,j] = \min \begin{cases} D[i-1,j] + \text{del-cost}(source[i]) \\ D[i,j-1] + \text{ins-cost}(target[j]) \\ D[i-1,j-1] + \text{sub-cost}(source[i], target[j]) \end{cases}$$

If we assume the version of Levenshtein distance in which the insertions and deletions each have a cost of 1 (ins-cost(·) = del-cost(·) = 1), and substitutions have a cost of 2 (except substitution of identical letters have zero cost), the computation for $D[i,j]$ becomes:

$$D[i,j] = \min \begin{cases} D[i-1,j] + 1 \\ D[i,j-1] + 1 \\ D[i-1,j-1] + \begin{cases} 2; & \text{if } source[i] \neq target[j] \\ 0; & \text{if } source[i] = target[j] \end{cases} \end{cases} \tag{2.9}$$

The algorithm is summarized in Fig. 2.17; Fig. 2.18 shows the results of applying the algorithm to the distance between *intention* and *execution* with the version of Levenshtein in Eq. 2.9.

Knowing the minimum edit distance is useful for algorithms like finding potential spelling error corrections. But the edit distance algorithm is important in another way; with a small change, it can also provide the minimum cost **alignment** between two strings. Aligning two strings is useful throughout speech and language processing. In speech recognition, minimum edit distance alignment is used to compute the word error rate (Chapter 28). Alignment plays a role in machine translation, in which sentences in a parallel corpus (a corpus with a text in two languages) need to be matched to each other.

To extend the edit distance algorithm to produce an alignment, we can start by visualizing an alignment as a path through the edit distance matrix. Figure 2.19 shows this path with the boldfaced cell. Each boldfaced cell represents an alignment of a pair of letters in the two strings. If two boldfaced cells occur in the same row, there will be an insertion in going from the source to the target; two boldfaced cells in the same column indicate a deletion.

Figure 2.19 also shows the intuition of how to compute this alignment path. The computation proceeds in two steps. In the first step, we augment the minimum edit distance algorithm to store backpointers in each cell. The backpointer from a cell points to the previous cell (or cells) that we came from in entering the current cell. We've shown a schematic of these backpointers in Fig. 2.19. Some cells have multiple backpointers because the minimum extension could have come from multiple previous cells. In the second step, we perform a **backtrace**. In a backtrace, we start from the last cell (at the final row and column), and follow the pointers back through the dynamic programming matrix. Each complete path between the final cell and the initial cell is a minimum distance alignment. Exercise 2.7 asks you to modify the minimum edit distance algorithm to store the pointers and compute the backtrace to output an alignment.

backtrace

```
function MIN-EDIT-DISTANCE(source, target) returns min-distance

  n ← LENGTH(source)
  m ← LENGTH(target)
  Create a distance matrix distance[n+1,m+1]

  # Initialization: the zeroth row and column is the distance from the empty string
      D[0,0] = 0
      for each row i from 1 to n do
          D[i,0] ← D[i-1,0] + del-cost(source[i])
      for each column j from 1 to m do
          D[0,j] ← D[0,j-1] + ins-cost(target[j])

  # Recurrence relation:
  for each row i from 1 to n do
      for each column j from 1 to m do
          D[i,j] ← MIN( D[i−1,j] + del-cost(source[i]),
                        D[i−1,j−1] + sub-cost(source[i], target[j]),
                        D[i,j−1] + ins-cost(target[j]))
  # Termination
  return D[n,m]
```

Figure 2.17 The minimum edit distance algorithm, an example of the class of dynamic programming algorithms. The various costs can either be fixed (e.g., $\forall x, \text{ins-cost}(x) = 1$) or can be specific to the letter (to model the fact that some letters are more likely to be inserted than others). We assume that there is no cost for substituting a letter for itself (i.e., $\text{sub-cost}(x,x) = 0$).

Src\Tar	#	e	x	e	c	u	t	i	o	n
#	0	1	2	3	4	5	6	7	8	9
i	1	2	3	4	5	6	7	6	7	8
n	2	3	4	5	6	7	8	7	8	7
t	3	4	5	6	7	8	7	8	9	8
e	4	3	4	5	6	7	8	9	10	9
n	5	4	5	6	7	8	9	10	11	10
t	6	5	6	7	8	9	8	9	10	11
i	7	6	7	8	9	10	9	8	9	10
o	8	7	8	9	10	11	10	9	8	9
n	9	8	9	10	11	12	11	10	9	8

Figure 2.18 Computation of minimum edit distance between *intention* and *execution* with the algorithm of Fig. 2.17, using Levenshtein distance with cost of 1 for insertions or deletions, 2 for substitutions.

While we worked our example with simple Levenshtein distance, the algorithm in Fig. 2.17 allows arbitrary weights on the operations. For spelling correction, for example, substitutions are more likely to happen between letters that are next to each other on the keyboard. The **Viterbi** algorithm is a probabilistic extension of minimum edit distance. Instead of computing the "minimum edit distance" between two strings, Viterbi computes the "maximum probability alignment" of one string with another. We'll discuss this more in Chapter 8.

	#	e	x	e	c	u	t	i	o	n
#	0	← 1	← 2	← 3	← 4	← 5	← 6	← 7	← 8	← 9
i	↑ **1**	↖←↑ 2	↖←↑ 3	↖←↑ 4	↖←↑ 5	↖←↑ 6	↖←↑ 7	↖ 6	← 7	← 8
n	↑ 2	↖←↑ **3**	↖←↑ 4	↖←↑ 5	↖←↑ 6	↖←↑ 7	↖←↑ 8	↑ 7	↖←↑ 8	↖ 7
t	↑ 3	↖←↑ 4	↖←↑ **5**	↖←↑ 6	↖←↑ 7	↖←↑ 8	↖ 7	←↑ 8	↖←↑ 9	↑ 8
e	↑ 4	↖ 3	← 4	↖← **5**	← **6**	← 7	←↑ 8	↖←↑ 9	↖←↑ 10	↑ 9
n	↑ 5	↑ 4	↖←↑ 5	↖←↑ 6	↖←↑ 7	↖←↑ **8**	↖←↑ 9	↖←↑ 10	↖←↑ 11	↖↑ 10
t	↑ 6	↑ 5	↖←↑ 6	↖←↑ 7	↖←↑ 8	↖←↑ 9	↖ **8**	← 9	← 10	←↑ 11
i	↑ 7	↑ 6	↖←↑ 7	↖←↑ 8	↖←↑ 9	↖←↑ 10	↑ 9	↖ **8**	← 9	← 10
o	↑ 8	↑ 7	↖←↑ 8	↖←↑ 9	↖←↑ 10	↖←↑ 11	↑ 10	↑ 9	↖ **8**	← 9
n	↑ 9	↑ 8	↖←↑ 9	↖←↑ 10	↖←↑ 11	↖←↑ 12	↑ 11	↑ 10	↑ 9	↖ **8**

Figure 2.19 When entering a value in each cell, we mark which of the three neighboring cells we came from with up to three arrows. After the table is full we compute an **alignment** (minimum edit path) by using a **backtrace**, starting at the **8** in the lower-right corner and following the arrows back. The sequence of bold cells represents one possible minimum cost alignment between the two strings. Diagram design after Gusfield (1997).

2.6 Summary

This chapter introduced a fundamental tool in language processing, the **regular expression**, and showed how to perform basic **text normalization** tasks including **word segmentation** and **normalization**, **sentence segmentation**, and **stemming**. We also introduce the important **minimum edit distance** algorithm for comparing strings. Here's a summary of the main points we covered about these ideas:

- The **regular expression** language is a powerful tool for pattern-matching.
- Basic operations in regular expressions include **concatenation** of symbols, **disjunction** of symbols (`[]`, `|`, and `.`), **counters** (`*`, `+`, and `{n,m}`), **anchors** (`^`, `$`) and precedence operators (`(,)`).
- **Word tokenization and normalization** are generally done by cascades of simple regular expressions substitutions or finite automata.
- The **Porter algorithm** is a simple and efficient way to do **stemming**, stripping off affixes. It does not have high accuracy but may be useful for some tasks.
- The **minimum edit distance** between two strings is the minimum number of operations it takes to edit one into the other. Minimum edit distance can be computed by **dynamic programming**, which also results in an **alignment** of the two strings.

Bibliographical and Historical Notes

Kleene (1951) and (1956) first defined regular expressions and the finite automaton, based on the McCulloch-Pitts neuron. Ken Thompson was one of the first to build regular expressions compilers into editors for text searching (Thompson, 1968). His editor *ed* included a command "g/regular expression/p", or Global Regular Expression Print, which later became the Unix `grep` utility.

Text normalization algorithms have been applied since the beginning of the field. One of the earliest widely-used stemmers was Lovins (1968). Stemming was also applied early to the digital humanities, by Packard (1973), who built an affix-stripping morphological parser for Ancient Greek. Currently a wide vari-

ety of code for tokenization and normalization is available, such as the Stanford Tokenizer (http://nlp.stanford.edu/software/tokenizer.shtml) or specialized tokenizers for Twitter (O'Connor et al., 2010), or for sentiment (http://sentiment.christopherpotts.net/tokenizing.html). See Palmer (2012) for a survey of text preprocessing. NLTK is an essential tool that offers both useful Python libraries (http://www.nltk.org) and textbook descriptions (Bird et al., 2009) of many algorithms including text normalization and corpus interfaces.

For more on Herdan's law and Heaps' Law, see Herdan (1960, p. 28), Heaps (1978), Egghe (2007) and Baayen (2001); Yasseri et al. (2012) discuss the relationship with other measures of linguistic complexity. For more on edit distance, see the excellent Gusfield (1997). Our example measuring the edit distance from 'intention' to 'execution' was adapted from Kruskal (1983). There are various publicly available packages to compute edit distance, including Unix `diff` and the NIST `sclite` program (NIST, 2005).

In his autobiography Bellman (1984) explains how he originally came up with the term *dynamic programming*:

> "...The 1950s were not good years for mathematical research. [the] Secretary of Defense ...had a pathological fear and hatred of the word, research... I decided therefore to use the word, "programming". I wanted to get across the idea that this was dynamic, this was multistage... I thought, let's ... take a word that has an absolutely precise meaning, namely dynamic... it's impossible to use the word, dynamic, in a pejorative sense. Try thinking of some combination that will possibly give it a pejorative meaning. It's impossible. Thus, I thought dynamic programming was a good name. It was something not even a Congressman could object to."

Exercises

2.1 Write regular expressions for the following languages.

1. the set of all alphabetic strings;
2. the set of all lower case alphabetic strings ending in a *b*;
3. the set of all strings from the alphabet a, b such that each a is immediately preceded by and immediately followed by a b;

2.2 Write regular expressions for the following languages. By "word", we mean an alphabetic string separated from other words by whitespace, any relevant punctuation, line breaks, and so forth.

1. the set of all strings with two consecutive repeated words (e.g., "Humbert Humbert" and "the the" but not "the bug" or "the big bug");
2. all strings that start at the beginning of the line with an integer and that end at the end of the line with a word;
3. all strings that have both the word *grotto* and the word *raven* in them (but not, e.g., words like *grottos* that merely *contain* the word *grotto*);
4. write a pattern that places the first word of an English sentence in a register. Deal with punctuation.

2.3 Implement an ELIZA-like program, using substitutions such as those described on page 10. You might want to choose a different domain than a Rogerian psychologist, although keep in mind that you would need a domain in which your program can legitimately engage in a lot of simple repetition.

2.4 Compute the edit distance (using insertion cost 1, deletion cost 1, substitution cost 1) of "leda" to "deal". Show your work (using the edit distance grid).

2.5 Figure out whether *drive* is closer to *brief* or to *divers* and what the edit distance is to each. You may use any version of *distance* that you like.

2.6 Now implement a minimum edit distance algorithm and use your hand-computed results to check your code.

2.7 Augment the minimum edit distance algorithm to output an alignment; you will need to store pointers and add a stage to compute the backtrace.

CHAPTER

3 N-gram Language Models

"You are uniformly charming!" cried he, with a smile of associating and now and then I bowed and they perceived a chaise and four to wish for.

Random sentence generated from a Jane Austen trigram model

Predicting is difficult—especially about the future, as the old quip goes. But how about predicting something that seems much easier, like the next few words someone is going to say? What word, for example, is likely to follow

```
Please turn your homework ...
```

Hopefully, most of you concluded that a very likely word is *in*, or possibly *over*, but probably not *refrigerator* or *the*. In the following sections we will formalize this intuition by introducing models that assign a **probability** to each possible next word. The same models will also serve to assign a probability to an entire sentence. Such a model, for example, could predict that the following sequence has a much higher probability of appearing in a text:

```
all of a sudden I notice three guys standing on the sidewalk
```

than does this same set of words in a different order:

```
on guys all I of notice sidewalk three a sudden standing the
```

Why would you want to predict upcoming words, or assign probabilities to sentences? Probabilities are essential in any task in which we have to identify words in noisy, ambiguous input, like **speech recognition**. For a speech recognizer to realize that you said *I will be back soonish* and not *I will be bassoon dish*, it helps to know that *back soonish* is a much more probable sequence than *bassoon dish*. For writing tools like **spelling correction** or **grammatical error correction**, we need to find and correct errors in writing like *Their are two midterms*, in which *There* was mistyped as *Their*, or *Everything has improve*, in which *improve* should have been *improved*. The phrase *There are* will be much more probable than *Their are*, and *has improved* than *has improve*, allowing us to help users by detecting and correcting these errors.

Assigning probabilities to sequences of words is also essential in **machine translation**. Suppose we are translating a Chinese source sentence:

他 向 记者 介绍了 主要 内容
He to reporters introduced main content

As part of the process we might have built the following set of potential rough English translations:

he introduced reporters to the main contents of the statement
he briefed to reporters the main contents of the statement
he briefed reporters on the main contents of the statement

A probabilistic model of word sequences could suggest that *briefed reporters on* is a more probable English phrase than *briefed to reporters* (which has an awkward *to* after *briefed*) or *introduced reporters to* (which uses a verb that is less fluent English in this context), allowing us to correctly select the boldfaced sentence above.

Probabilities are also important for **augmentative and alternative communication** systems (Trnka et al. 2007, Kane et al. 2017). People often use such **AAC** devices if they are physically unable or sign but can instead using eye gaze or other specific movements to select words from a menu to be spoken by the system. Word prediction can be used to suggest likely words for the menu.

AAC

Models that assign probabilities to sequences of words are called **language models** or **LMs**. In this chapter we introduce the simplest model that assigns probabilities to sentences and sequences of words, the **n-gram**. An n-gram is a sequence of N words: a 2-gram (or **bigram**) is a two-word sequence of words like "please turn", "turn your", or "your homework", and a 3-gram (or **trigram**) is a three-word sequence of words like "please turn your", or "turn your homework". We'll see how to use n-gram models to estimate the probability of the last word of an n-gram given the previous words, and also to assign probabilities to entire sequences. In a bit of terminological ambiguity, we usually drop the word "model", and thus the term **n-gram** is used to mean either the word sequence itself or the predictive model that assigns it a probability. In later chapters we'll introduce more sophisticated language models like the RNN LMs of Chapter 9.

language model
LM
n-gram

3.1 N-Grams

Let's begin with the task of computing $P(w|h)$, the probability of a word w given some history h. Suppose the history h is "*its water is so transparent that*" and we want to know the probability that the next word is *the*:

$$P(\textit{the}|\textit{its water is so transparent that}). \tag{3.1}$$

One way to estimate this probability is from relative frequency counts: take a very large corpus, count the number of times we see *its water is so transparent that*, and count the number of times this is followed by *the*. This would be answering the question "Out of the times we saw the history h, how many times was it followed by the word w", as follows:

$$P(\textit{the}|\textit{its water is so transparent that}) = \frac{C(\textit{its water is so transparent that the})}{C(\textit{its water is so transparent that})} \tag{3.2}$$

With a large enough corpus, such as the web, we can compute these counts and estimate the probability from Eq. 3.2. You should pause now, go to the web, and compute this estimate for yourself.

While this method of estimating probabilities directly from counts works fine in many cases, it turns out that even the web isn't big enough to give us good estimates in most cases. This is because language is creative; new sentences are created all the time, and we won't always be able to count entire sentences. Even simple extensions of the example sentence may have counts of zero on the web (such as "*Walden Pond's water is so transparent that the*"; well, *used to* have counts of zero).

Similarly, if we wanted to know the joint probability of an entire sequence of words like *its water is so transparent*, we could do it by asking "out of all possible sequences of five words, how many of them are *its water is so transparent*?" We would have to get the count of *its water is so transparent* and divide by the sum of the counts of all possible five word sequences. That seems rather a lot to estimate!

For this reason, we'll need to introduce cleverer ways of estimating the probability of a word w given a history h, or the probability of an entire word sequence W. Let's start with a little formalizing of notation. To represent the probability of a particular random variable X_i taking on the value "the", or $P(X_i = \text{"the"})$, we will use the simplification $P(the)$. We'll represent a sequence of N words either as $w_1 \dots w_n$ or w_1^n (so the expression w_1^{n-1} means the string $w_1, w_2, ..., w_{n-1}$). For the joint probability of each word in a sequence having a particular value $P(X = w_1, Y = w_2, Z = w_3, ..., W = w_n)$ we'll use $P(w_1, w_2, ..., w_n)$.

Now how can we compute probabilities of entire sequences like $P(w_1, w_2, ..., w_n)$? One thing we can do is decompose this probability using the **chain rule of probability**:

$$\begin{aligned} P(X_1 ... X_n) &= P(X_1)P(X_2|X_1)P(X_3|X_1^2)\dots P(X_n|X_1^{n-1}) \\ &= \prod_{k=1}^{n} P(X_k|X_1^{k-1}) \end{aligned} \tag{3.3}$$

Applying the chain rule to words, we get

$$\begin{aligned} P(w_1^n) &= P(w_1)P(w_2|w_1)P(w_3|w_1^2)\dots P(w_n|w_1^{n-1}) \\ &= \prod_{k=1}^{n} P(w_k|w_1^{k-1}) \end{aligned} \tag{3.4}$$

The chain rule shows the link between computing the joint probability of a sequence and computing the conditional probability of a word given previous words. Equation 3.4 suggests that we could estimate the joint probability of an entire sequence of words by multiplying together a number of conditional probabilities. But using the chain rule doesn't really seem to help us! We don't know any way to compute the exact probability of a word given a long sequence of preceding words, $P(w_n|w_1^{n-1})$. As we said above, we can't just estimate by counting the number of times every word occurs following every long string, because language is creative and any particular context might have never occurred before!

The intuition of the n-gram model is that instead of computing the probability of a word given its entire history, we can **approximate** the history by just the last few words.

bigram

The **bigram** model, for example, approximates the probability of a word given all the previous words $P(w_n|w_1^{n-1})$ by using only the conditional probability of the preceding word $P(w_n|w_{n-1})$. In other words, instead of computing the probability

$$P(\text{the}|\text{Walden Pond's water is so transparent that}) \tag{3.5}$$

we approximate it with the probability

$$P(\text{the}|\text{that}) \tag{3.6}$$

When we use a bigram model to predict the conditional probability of the next word, we are thus making the following approximation:

$$P(w_n|w_1^{n-1}) \approx P(w_n|w_{n-1}) \tag{3.7}$$

Markov

The assumption that the probability of a word depends only on the previous word is called a **Markov** assumption. Markov models are the class of probabilistic models that assume we can predict the probability of some future unit without looking too far into the past. We can generalize the bigram (which looks one word into the past) to the trigram (which looks two words into the past) and thus to the **n-gram** (which looks $n-1$ words into the past).

n-gram

Thus, the general equation for this n-gram approximation to the conditional probability of the next word in a sequence is

$$P(w_n|w_1^{n-1}) \approx P(w_n|w_{n-N+1}^{n-1}) \tag{3.8}$$

Given the bigram assumption for the probability of an individual word, we can compute the probability of a complete word sequence by substituting Eq. 3.7 into Eq. 3.4:

$$P(w_1^n) \approx \prod_{k=1}^{n} P(w_k|w_{k-1}) \tag{3.9}$$

maximum likelihood estimation

normalize

How do we estimate these bigram or n-gram probabilities? An intuitive way to estimate probabilities is called **maximum likelihood estimation** or **MLE**. We get the MLE estimate for the parameters of an n-gram model by getting counts from a corpus, and **normalizing** the counts so that they lie between 0 and 1.[1]

For example, to compute a particular bigram probability of a word y given a previous word x, we'll compute the count of the bigram $C(xy)$ and normalize by the sum of all the bigrams that share the same first word x:

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)}{\sum_w C(w_{n-1}w)} \tag{3.10}$$

We can simplify this equation, since the sum of all bigram counts that start with a given word w_{n-1} must be equal to the unigram count for that word w_{n-1} (the reader should take a moment to be convinced of this):

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)}{C(w_{n-1})} \tag{3.11}$$

Let's work through an example using a mini-corpus of three sentences. We'll first need to augment each sentence with a special symbol <s> at the beginning of the sentence, to give us the bigram context of the first word. We'll also need a special end-symbol. </s>[2]

```
<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>
```

[1] For probabilistic models, normalizing means dividing by some total count so that the resulting probabilities fall legally between 0 and 1.

[2] We need the end-symbol to make the bigram grammar a true probability distribution. Without an end-symbol, the sentence probabilities for all sentences of a given length would sum to one. This model would define an infinite set of probability distributions, with one distribution per sentence length. See Exercise 3.5.

Here are the calculations for some of the bigram probabilities from this corpus

$P(\texttt{I}|\texttt{<s>}) = \frac{2}{3} = .67$ $P(\texttt{Sam}|\texttt{<s>}) = \frac{1}{3} = .33$ $P(\texttt{am}|\texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>}|\texttt{Sam}) = \frac{1}{2} = 0.5$ $P(\texttt{Sam}|\texttt{am}) = \frac{1}{2} = .5$ $P(\texttt{do}|\texttt{I}) = \frac{1}{3} = .33$

For the general case of MLE n-gram parameter estimation:

$$P(w_n|w_{n-N+1}^{n-1}) = \frac{C(w_{n-N+1}^{n-1}w_n)}{C(w_{n-N+1}^{n-1})} \tag{3.12}$$

relative frequency

Equation 3.12 (like Eq. 3.11) estimates the n-gram probability by dividing the observed frequency of a particular sequence by the observed frequency of a prefix. This ratio is called a **relative frequency**. We said above that this use of relative frequencies as a way to estimate probabilities is an example of maximum likelihood estimation or MLE. In MLE, the resulting parameter set maximizes the likelihood of the training set T given the model M (i.e., $P(T|M)$). For example, suppose the word *Chinese* occurs 400 times in a corpus of a million words like the Brown corpus. What is the probability that a random word selected from some other text of, say, a million words will be the word *Chinese*? The MLE of its probability is $\frac{400}{1000000}$ or .0004. Now .0004 is not the best possible estimate of the probability of *Chinese* occurring in all situations; it might turn out that in some other corpus or context *Chinese* is a very unlikely word. But it is the probability that makes it *most likely* that Chinese will occur 400 times in a million-word corpus. We present ways to modify the MLE estimates slightly to get better probability estimates in Section 3.4.

Let's move on to some examples from a slightly larger corpus than our 14-word example above. We'll use data from the now-defunct Berkeley Restaurant Project, a dialogue system from the last century that answered questions about a database of restaurants in Berkeley, California (Jurafsky et al., 1994). Here are some text-normalized sample user queries (a sample of 9332 sentences is on the website):

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

Figure 3.1 shows the bigram counts from a piece of a bigram grammar from the Berkeley Restaurant Project. Note that the majority of the values are zero. In fact, we have chosen the sample words to cohere with each other; a matrix selected from a random set of seven words would be even more sparse.

Figure 3.2 shows the bigram probabilities after normalization (dividing each cell in Fig. 3.1 by the appropriate unigram for its row, taken from the following set of unigram probabilities):

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Here are a few other useful probabilities:

$P(\texttt{i}|\texttt{<s>}) = 0.25$ $P(\texttt{english}|\texttt{want}) = 0.0011$

$P(\texttt{food}|\texttt{english}) = 0.5$ $P(\texttt{</s>}|\texttt{food}) = 0.68$

Now we can compute the probability of sentences like *I want English food* or *I want Chinese food* by simply multiplying the appropriate bigram probabilities together, as follows:

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Figure 3.1 Bigram counts for eight of the words (out of $V = 1446$) in the Berkeley Restaurant Project corpus of 9332 sentences. Zero counts are in gray.

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Figure 3.2 Bigram probabilities for eight words in the Berkeley Restaurant Project corpus of 9332 sentences. Zero probabilities are in gray.

$$\begin{aligned} P(\texttt{<s> i want english food </s>}) &= P(\texttt{i}|\texttt{<s>})P(\texttt{want}|\texttt{i})P(\texttt{english}|\texttt{want}) \\ &\quad P(\texttt{food}|\texttt{english})P(\texttt{</s>}|\texttt{food}) \\ &= .25 \times .33 \times .0011 \times 0.5 \times 0.68 \\ &= .000031 \end{aligned}$$

We leave it as Exercise 3.2 to compute the probability of *i want chinese food*.

What kinds of linguistic phenomena are captured in these bigram statistics? Some of the bigram probabilities above encode some facts that we think of as strictly **syntactic** in nature, like the fact that what comes after *eat* is usually a noun or an adjective, or that what comes after *to* is usually a verb. Others might be a fact about the personal assistant task, like the high probability of sentences beginning with the words *I*. And some might even be cultural rather than linguistic, like the higher probability that people are looking for Chinese versus English food.

Some practical issues: Although for pedagogical purposes we have only described bigram models, in practice it's more common to use **trigram** models, which condition on the previous two words rather than the previous word, or **4-gram** or even **5-gram** models, when there is sufficient training data. Note that for these larger n-grams, we'll need to assume extra context for the contexts to the left and right of the sentence end. For example, to compute trigram probabilities at the very beginning of the sentence, we can use two pseudo-words for the first trigram (i.e., $P(\texttt{I}|\texttt{<s><s>})$.

trigram
4-gram
5-gram

We always represent and compute language model probabilities in log format as **log probabilities**. Since probabilities are (by definition) less than or equal to 1, the more probabilities we multiply together, the smaller the product becomes. Multiplying enough n-grams together would result in numerical underflow. By using log probabilities instead of raw probabilities, we get numbers that are not as small.

log probabilities

Adding in log space is equivalent to multiplying in linear space, so we combine log probabilities by adding them. The result of doing all computation and storage in log space is that we only need to convert back into probabilities if we need to report them at the end; then we can just take the exp of the logprob:

$$p_1 \times p_2 \times p_3 \times p_4 = \exp(\log p_1 + \log p_2 + \log p_3 + \log p_4) \tag{3.13}$$

3.2 Evaluating Language Models

extrinsic evaluation

The best way to evaluate the performance of a language model is to embed it in an application and measure how much the application improves. Such end-to-end evaluation is called **extrinsic evaluation**. Extrinsic evaluation is the only way to know if a particular improvement in a component is really going to help the task at hand. Thus, for speech recognition, we can compare the performance of two language models by running the speech recognizer twice, once with each language model, and seeing which gives the more accurate transcription.

intrinsic evaluation

Unfortunately, running big NLP systems end-to-end is often very expensive. Instead, it would be nice to have a metric that can be used to quickly evaluate potential improvements in a language model. An **intrinsic evaluation** metric is one that measures the quality of a model independent of any application.

training set
test set
held out

For an intrinsic evaluation of a language model we need a **test set**. As with many of the statistical models in our field, the probabilities of an n-gram model come from the corpus it is trained on, the **training set** or **training corpus**. We can then measure the quality of an n-gram model by its performance on some unseen data called the **test set** or test corpus. We will also sometimes call test sets and other datasets that are not in our training sets **held out** corpora because we hold them out from the training data.

So if we are given a corpus of text and want to compare two different n-gram models, we divide the data into training and test sets, train the parameters of both models on the training set, and then compare how well the two trained models fit the test set.

But what does it mean to "fit the test set"? The answer is simple: whichever model assigns a **higher probability** to the test set—meaning it more accurately predicts the test set—is a better model. Given two probabilistic models, the better model is the one that has a tighter fit to the test data or that better predicts the details of the test data, and hence will assign a higher probability to the test data.

Since our evaluation metric is based on test set probability, it's important not to let the test sentences into the training set. Suppose we are trying to compute the probability of a particular "test" sentence. If our test sentence is part of the training corpus, we will mistakenly assign it an artificially high probability when it occurs in the test set. We call this situation **training on the test set**. Training on the test set introduces a bias that makes the probabilities all look too high, and causes huge inaccuracies in **perplexity**, the probability-based metric we introduce below.

development test

Sometimes we use a particular test set so often that we implicitly tune to its characteristics. We then need a fresh test set that is truly unseen. In such cases, we call the initial test set the **development** test set or, **devset**. How do we divide our data into training, development, and test sets? We want our test set to be as large as possible, since a small test set may be accidentally unrepresentative, but we also want as much training data as possible. At the minimum, we would want to pick

the smallest test set that gives us enough statistical power to measure a statistically significant difference between two potential models. In practice, we often just divide our data into 80% training, 10% development, and 10% test. Given a large corpus that we want to divide into training and test, test data can either be taken from some continuous sequence of text inside the corpus, or we can remove smaller "stripes" of text from randomly selected parts of our corpus and combine them into a test set.

3.2.1 Perplexity

perplexity

In practice we don't use raw probability as our metric for evaluating language models, but a variant called **perplexity**. The **perplexity** (sometimes called *PP* for short) of a language model on a test set is the inverse probability of the test set, normalized by the number of words. For a test set $W = w_1 w_2 \dots w_N$,:

$$\begin{aligned} \text{PP}(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}} \end{aligned} \tag{3.14}$$

We can use the chain rule to expand the probability of W:

$$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_1 \dots w_{i-1})}} \tag{3.15}$$

Thus, if we are computing the perplexity of W with a bigram language model, we get:

$$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_{i-1})}} \tag{3.16}$$

Note that because of the inverse in Eq. 3.15, the higher the conditional probability of the word sequence, the lower the perplexity. Thus, minimizing perplexity is equivalent to maximizing the test set probability according to the language model. What we generally use for word sequence in Eq. 3.15 or Eq. 3.16 is the entire sequence of words in some test set. Since this sequence will cross many sentence boundaries, we need to include the begin- and end-sentence markers <s> and </s> in the probability computation. We also need to include the end-of-sentence marker </s> (but not the beginning-of-sentence marker <s>) in the total count of word tokens N.

There is another way to think about perplexity: as the **weighted average branching factor** of a language. The branching factor of a language is the number of possible next words that can follow any word. Consider the task of recognizing the digits in English (zero, one, two,..., nine), given that (both in some training set and in some test set) each of the 10 digits occurs with equal probability $P = \frac{1}{10}$. The perplexity of this mini-language is in fact 10. To see that, imagine a test string of digits of length N, and assume that in the training set all the digits occurred with equal probability. By Eq. 3.15, the perplexity will be

$$\begin{aligned} \text{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\ &= \left(\frac{1}{10}^{N}\right)^{-\frac{1}{N}} \\ &= \frac{1}{10}^{-1} \\ &= 10 \end{aligned} \tag{3.17}$$

But suppose that the number zero is really frequent and occurs far more often than other numbers. Let's say that 0 occur 91 times in the training set, and each of the other digits occurred 1 time each. Now we see the following test set: 0 0 0 0 0 3 0 0 0 0. We should expect the perplexity of this test set to be lower since most of the time the next number will be zero, which is very predictable, i.e. has a high probability. Thus, although the branching factor is still 10, the perplexity or *weighted* branching factor is smaller. We leave this exact calculation as exercise 12.

We see in Section 3.7 that perplexity is also closely related to the information-theoretic notion of entropy.

Finally, let's look at an example of how perplexity can be used to compare different n-gram models. We trained unigram, bigram, and trigram grammars on 38 million words (including start-of-sentence tokens) from the *Wall Street Journal*, using a 19,979 word vocabulary. We then computed the perplexity of each of these models on a test set of 1.5 million words with Eq. 3.16. The table below shows the perplexity of a 1.5 million word WSJ test set according to each of these grammars.

	Unigram	**Bigram**	**Trigram**
Perplexity	962	170	109

As we see above, the more information the n-gram gives us about the word sequence, the lower the perplexity (since as Eq. 3.15 showed, perplexity is related inversely to the likelihood of the test sequence according to the model).

Note that in computing perplexities, the n-gram model P must be constructed without any knowledge of the test set or any prior knowledge of the vocabulary of the test set. Any kind of knowledge of the test set can cause the perplexity to be artificially low. The perplexity of two language models is only comparable if they use identical vocabularies.

An (intrinsic) improvement in perplexity does not guarantee an (extrinsic) improvement in the performance of a language processing task like speech recognition or machine translation. Nonetheless, because perplexity often correlates with such improvements, it is commonly used as a quick check on an algorithm. But a model's improvement in perplexity should always be confirmed by an end-to-end evaluation of a real task before concluding the evaluation of the model.

3.3 Generalization and Zeros

The n-gram model, like many statistical models, is dependent on the training corpus. One implication of this is that the probabilities often encode specific facts about a given training corpus. Another implication is that n-grams do a better and better job of modeling the training corpus as we increase the value of N.

CHAPTER

6 Vector Semantics and Embeddings

The asphalt that Los Angeles is famous for occurs mainly on its freeways. But in the middle of the city is another patch of asphalt, the La Brea tar pits, and this asphalt preserves millions of fossil bones from the last of the Ice Ages of the Pleistocene Epoch. One of these fossils is the *Smilodon*, or sabre-toothed tiger, instantly recognizable by its long canines. Five million years ago or so, a completely different sabre-tooth tiger called *Thylacosmilus* lived in Argentina and other parts of South America. Thylacosmilus was a marsupial whereas Smilodon was a placental mammal, but Thylacosmilus had the same long upper canines and, like Smilodon, had a protective bone flange on the lower jaw. The similarity of these two mammals is one of many examples of parallel or convergent evolution, in which particular contexts or environments lead to the evolution of very similar structures in different species (Gould, 1980).

The role of context is also important in the similarity of a less biological kind of organism: the word. Words that occur in *similar contexts* tend to have *similar meanings*. This link between similarity in how words are distributed and similarity in what they mean is called the **distributional hypothesis**. The hypothesis was first formulated in the 1950s by linguists like Joos (1950), Harris (1954), and Firth (1957), who noticed that words which are synonyms (like *oculist* and *eye-doctor*) tended to occur in the same environment (e.g., near words like *eye* or *examined*) with the amount of meaning difference between two words "corresponding roughly to the amount of difference in their environments" (Harris, 1954, 157).

distributional hypothesis

In this chapter we introduce **vector semantics**, which instantiates this linguistic hypothesis by learning representations of the meaning of words, called **embeddings**, directly from their distributions in texts. These representations are used in every natural language processing application that makes use of meaning, and underlie the more powerful **contextualized word representations** like **ELMo** and **BERT** that we will introduce in Chapter 10.

vector semantics
embeddings

These word representations are also the first example in this book of **representation learning**, automatically learning useful representations of the input text. Finding such **self-supervised** ways to learn representations of the input, instead of creating representations by hand via **feature engineering**, is an important focus of NLP research (Bengio et al., 2013).

representation learning

We'll begin, however, by introducing some basic principles of word meaning, which will motivate the vector semantic models of this chapter as well as extensions that we'll return to in Chapter 19, Chapter 20, and Chapter 21.

6.1 Lexical Semantics

How should we represent the meaning of a word? In the N-gram models we saw in Chapter 3, and in many traditional NLP applications, our only representation of a word is as a string of letters, or perhaps as an index in a vocabulary list. This representation is not that different from a tradition in philosophy, perhaps you've seen it in introductory logic classes, in which the meaning of words is represented by just spelling the word with small capital letters; representing the meaning of "dog" as DOG, and "cat" as CAT).

Representing the meaning of a word by capitalizing it is a pretty unsatisfactory model. You might have seen the old philosophy joke:

> Q: What's the meaning of life?
> A: LIFE

Surely we can do better than this! After all, we'll want a model of word meaning to do all sorts of things for us. It should tell us that some words have similar meanings (*cat* is similar to *dog*), other words are antonyms (*cold* is the opposite of *hot*). It should know that some words have positive connotations (*happy*) while others have negative connotations (*sad*). It should represent the fact that the meanings of *buy*, *sell*, and *pay* offer differing perspectives on the same underlying purchasing event (If I buy something from you, you've probably sold it to me, and I likely paid you).

More generally, a model of word meaning should allow us to draw useful inferences that will help us solve meaning-related tasks like question-answering, summarization, detecting paraphrases or plagiarism, and dialogue.

lexical semantics

In this section we summarize some of these desiderata, drawing on results in the linguistic study of word meaning, which is called **lexical semantics**; we'll return to and expand on this list in Chapter 19.

Lemmas and Senses Let's start by looking at how one word (we'll choose *mouse*) might be defined in a dictionary: [1]

```
mouse (N)
1.  any of numerous small rodents...
2.  a hand-operated device that controls a cursor...
```

lemma **citation form**

Here the form *mouse* is the **lemma**, also called the **citation form**. The form *mouse* would also be the lemma for the word *mice*; dictionaries don't have separate definitions for inflected forms like *mice*. Similarly *sing* is the lemma for *sing*, *sang*, *sung*. In many languages the infinitive form is used as the lemma for the verb, so Spanish *dormir* "to sleep" is the lemma for *duermes* "you sleep". The specific forms *sung* or *carpets* or *sing* or *duermes* are called **wordforms**.

wordform

As the example above shows, each lemma can have multiple meanings; the lemma *mouse* can refer to the rodent or the cursor control device. We call each of these aspects of the meaning of *mouse* a **word sense**. The fact that lemmas can be **polysemous** (have multiple senses) can make interpretation difficult (is someone who types "mouse info" into a search engine looking for a pet or a tool?). Chapter 19 will discuss the problem of polysemy, and introduce **word sense disambiguation**, the task of determining which sense of a word is being used in a particular context.

Synonymy One important component of word meaning is the relationship between word senses. For example when one word has a sense whose meaning is

[1] This example shortened from the online dictionary WordNet, discussed in Chapter 19.

identical to a sense of another word, or nearly identical, we say the two senses of those two words are **synonyms**. Synonyms include such pairs as

synonym

couch/sofa vomit/throw up filbert/hazelnut car/automobile

A more formal definition of synonymy (between words rather than senses) is that two words are synonymous if they are substitutable one for the other in any sentence without changing the *truth conditions* of the sentence, the situations in which the sentence would be true. We often say in this case that the two words have the same **propositional meaning**.

propositional meaning

While substitutions between some pairs of words like *car* / *automobile* or *water* / H_2O are truth preserving, the words are still not identical in meaning. Indeed, probably no two words are absolutely identical in meaning. One of the fundamental tenets of semantics, called the **principle of contrast** (Girard 1718, Bréal 1897, Clark 1987), is the assumption that a difference in linguistic form is always associated with at least some difference in meaning. For example, the word H_2O is used in scientific contexts and would be inappropriate in a hiking guide—*water* would be more appropriate— and this difference in genre is part of the meaning of the word. In practice, the word *synonym* is therefore commonly used to describe a relationship of approximate or rough synonymy.

principle of contrast

Word Similarity While words don't have many synonyms, most words do have lots of *similar* words. *Cat* is not a synonym of *dog*, but *cats* and *dogs* are certainly similar words. In moving from synonymy to similarity, it will be useful to shift from talking about relations between word senses (like synonymy) to relations between words (like similarity). Dealing with words avoids having to commit to a particular representation of word senses, which will turn out to simplify our task.

similarity

The notion of word **similarity** is very useful in larger semantic tasks. Knowing how similar two words are can help in computing how similar the meaning of two phrases or sentences are, a very important component of natural language understanding tasks like question answering, paraphrasing, and summarization. One way of getting values for word similarity is to ask humans to judge how similar one word is to another. A number of datasets have resulted from such experiments. For example the SimLex-999 dataset (Hill et al., 2015) gives values on a scale from 0 to 10, like the examples below, which range from near-synonyms (*vanish*, *disappear*) to pairs that scarcely seem to have anything in common (*hole*, *agreement*):

vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

Word Relatedness The meaning of two words can be related in ways other than similarity. One such class of connections is called word **relatedness** (Budanitsky and Hirst, 2006), also traditionally called word **association** in psychology.

relatedness

association

Consider the meanings of the words *coffee* and *cup*. Coffee is not similar to cup; they share practically no features (coffee is a plant or a beverage, while a cup is a manufactured object with a particular shape).

But coffee and cup are clearly related; they are associated by co-participating in an everyday event (the event of drinking coffee out of a cup). Similarly the nouns

scalpel and *surgeon* are not similar but are related eventively (a surgeon tends to make use of a scalpel).

semantic field

One common kind of relatedness between words is if they belong to the same **semantic field**. A semantic field is a set of words which cover a particular semantic domain and bear structured relations with each other.

For example, words might be related by being in the semantic field of hospitals (*surgeon*, *scalpel*, *nurse*, *anesthetic*, *hospital*), restaurants (*waiter*, *menu*, *plate*, *food*, *chef*), or houses (*door*, *roof*, *kitchen*, *family*, *bed*).

topic models

Semantic fields are also related to **topic models**, like **Latent Dirichlet Allocation**, **LDA**, which apply unsupervised learning on large sets of texts to induce sets of associated words from text. Semantic fields and topic models are very useful tools for discovering topical structure in documents.

In Chapter 19 we'll introduce even more relations between senses, including **hypernymy** or **IS-A**, **antonymy** (opposite meaning) and **meronymy**) (part-whole relations).

Semantic Frames and Roles Closely related to semantic fields is the idea of a

semantic frame

semantic frame. A semantic frame is a set of words that denote perspectives or participants in a particular type of event. A commercial transaction, for example, is a kind of event in which one entity trades money to another entity in return for some good or service, after which the good changes hands or perhaps the service is performed. This event can be encoded lexically by using verbs like *buy* (the event from the perspective of the buyer), *sell* (from the perspective of the seller), *pay* (focusing on the monetary aspect), or nouns like *buyer*. Frames have semantic roles (like *buyer*, *seller*, *goods*, *money*), and words in a sentence can take on these roles.

Knowing that *buy* and *sell* have this relation makes it possible for a system to know that a sentence like *Sam bought the book from Ling* could be paraphrased as *Ling sold the book to Sam*, and that Sam has the role of the *buyer* in the frame and Ling the *seller*. Being able to recognize such paraphrases is important for question answering, and can help in shifting perspective for machine translation.

connotations

Connotation Finally, words have *affective meanings* or **connotations**. The word *connotation* has different meanings in different fields, but here we use it to mean the aspects of a word's meaning that are related to a writer or reader's emotions, sentiment, opinions, or evaluations. For example some words have positive connotations (*happy*) while others have negative connotations (*sad*). Some words describe positive evaluation (*great*, *love*) and others negative evaluation (*terrible*, *hate*). Positive or negative evaluation expressed through language is called **sentiment**, as we saw in Chapter 4, and word sentiment plays a role in important tasks like sentiment analysis, stance detection, and many applications of natural language processing to the language of politics and consumer reviews.

sentiment

Early work on affective meaning (Osgood et al., 1957) found that words varied along three important dimensions of affective meaning. These are now generally called *valence*, *arousal*, and *dominance*, defined as follows:

valence: the pleasantness of the stimulus
arousal: the intensity of emotion provoked by the stimulus
dominance: the degree of control exerted by the stimulus

Thus words like *happy* or *satisfied* are high on valence, while *unhappy* or *annoyed* are low on valence. *Excited* or *frenzied* are high on arousal, while *relaxed* or *calm* are low on arousal. *Important* or *controlling* are high on dominance, while *awed* or *influenced* are low on dominance. Each word is thus represented by three

numbers, corresponding to its value on each of the three dimensions, like the examples below:

	Valence	Arousal	Dominance
courageous	8.05	5.5	7.38
music	7.67	5.57	6.5
heartbreak	2.45	5.65	3.58
cub	6.71	3.95	4.24
life	6.68	5.59	5.89

Osgood et al. (1957) noticed that in using these 3 numbers to represent the meaning of a word, the model was representing each word as a point in a three-dimensional space, a vector whose three dimensions corresponded to the word's rating on the three scales. This revolutionary idea that word meaning word could be represented as a point in space (e.g., that part of the meaning of *heartbreak* can be represented as the point $[2.45, 5.65, 3.58]$) was the first expression of the vector semantics models that we introduce next.

6.2 Vector Semantics

How can we build a computational model that successfully deals with the different aspects of word meaning we saw in the previous section (word senses, word similarity and relatedness, lexical fields and frames, connotation)?

vector semantics

A perfect model that completely deals with each of these aspects of word meaning turns out to be elusive. But the current best model, called **vector semantics**, draws its inspiration from linguistic and philosophical work of the 1950's.

During that period, the philosopher Ludwig Wittgenstein, skeptical of the possibility of building a completely formal theory of meaning definitions for each word, suggested instead that "the meaning of a word is its use in the language" (Wittgenstein, 1953, PI 43). That is, instead of using some logical language to define each word, we should define words by some representation of how the word was used by actual people in speaking and understanding.

Linguists of the period like Joos (1950), Harris (1954), and Firth (1957) (the linguistic distributionalists), came up with a specific idea for realizing Wittgenstein's intuition: define a word by its environment or distribution in language use. A word's distribution is the set of contexts in which it occurs, the neighboring words or grammatical environments. The idea is that two words that occur in very similar distributions (that occur together with very similar words) are likely to have the same meaning.

Let's see an example illustrating this distributionalist approach. Suppose you didn't know what the Cantonese word *ongchoi* meant, but you do see it in the following sentences or contexts:

(6.1) Ongchoi is delicious sauteed with garlic.

(6.2) Ongchoi is superb over rice.

(6.3) ...ongchoi leaves with salty sauces...

And furthermore let's suppose that you had seen many of these context words occurring in contexts like:

(6.4) ...spinach sauteed with garlic over rice...

(6.5) ...chard stems and leaves are delicious...

(6.6) ...collard greens and other salty leafy greens

The fact that *ongchoi* occurs with words like *rice* and *garlic* and *delicious* and *salty*, as do words like *spinach*, *chard*, and *collard greens* might suggest to the reader that ongchoi is a leafy green similar to these other leafy greens.[2]

We can do the same thing computationally by just counting words in the context of *ongchoi*; we'll tend to see words like *sauteed* and *eaten* and *garlic*. The fact that these words and other similar context words also occur around the word *spinach* or *collard greens* can help us discover the similarity between these words and *ongchoi*.

Vector semantics thus combines two intuitions: the distributionalist intuition (defining a word by counting what other words occur in its environment), and the vector intuition of Osgood et al. (1957) we saw in the last section on connotation: defining the meaning of a word *w* as a vector, a list of numbers, a point in N-dimensional space. There are various versions of vector semantics, each defining the numbers in the vector somewhat differently, but in each case the numbers are based in some way on counts of neighboring words.

Figure 6.1 A two-dimensional (t-SNE) projection of embeddings for some words and phrases, showing that words with similar meanings are nearby in space. The original 60-dimensional embeddings were trained for sentiment analysis. Simplified from Li et al. (2015).

The idea of vector semantics is thus to represent a word as a point in some multi-dimensional semantic space. Vectors for representing words are generally called **embeddings**, because the word is embedded in a particular vector space. Fig. 6.1 displays a visualization of embeddings that were learned for a sentiment analysis task, showing the location of some selected words projected down from the original 60-dimensional space into a two dimensional space.

embeddings

Notice that positive and negative words seem to be located in distinct portions of the space (and different also from the neutral function words). This suggests one of the great advantages of vector semantics: it offers a fine-grained model of meaning that lets us also implement word similarity (and phrase similarity). For example, the sentiment analysis classifier we saw in Chapter 4 only works if enough of the important sentimental words that appear in the test set also appeared in the training set. But if words were represented as embeddings, we could assign sentiment as long as words with *similar meanings* as the test set words occurred in the training

[2] It's in fact *Ipomoea aquatica*, a relative of morning glory sometimes called *water spinach* in English.

set. Vector semantic models are also extremely practical because they can be learned automatically from text without any complex labeling or supervision.

As a result of these advantages, vector models of meaning are now the standard way to represent the meaning of words in NLP. In this chapter we'll introduce the two most commonly used models. First is the **tf-idf** model, often used as a baseline, in which the meaning of a word is defined by a simple function of the counts of nearby words. We will see that this method results in very long vectors that are **sparse**, i.e. contain mostly zeros (since most words simply never occur in the context of others).

Then we'll introduce the **word2vec** model, one of a family of models that are ways of constructing short, **dense** vectors that have useful semantic properties.

We'll also introduce the **cosine**, the standard way to use embeddings (vectors) to compute functions like *semantic similarity*, the similarity between two words, two sentences, or two documents, an important tool in practical applications like question answering, summarization, or automatic essay grading.

6.3 Words and Vectors

Vector or distributional models of meaning are generally based on a **co-occurrence matrix**, a way of representing how often words co-occur. This matrix can be constructed in various ways; let's begin by looking at one such co-occurrence matrix, a term-document matrix.

6.3.1 Vectors and documents

term-document matrix

In a **term-document matrix**, each row represents a word in the vocabulary and each column represents a document from some collection of documents. Fig. 6.2 shows a small selection from a term-document matrix showing the occurrence of four words in four plays by Shakespeare. Each cell in this matrix represents the number of times a particular word (defined by the row) occurs in a particular document (defined by the column). Thus *fool* appeared 58 times in *Twelfth Night*.

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Figure 6.2 The term-document matrix for four words in four Shakespeare plays. Each cell contains the number of times the (row) word occurs in the (column) document.

vector space model

The term-document matrix of Fig. 6.2 was first defined as part of the **vector space model** of information retrieval (Salton, 1971). In this model, a document is represented as a count vector, a column in Fig. 6.3.

vector

vector space

dimension

To review some basic linear algebra, a **vector** is, at heart, just a list or array of numbers. So *As You Like It* is represented as the list [1,114,36,20] and *Julius Caesar* is represented as the list [7,62,1,2]. A **vector space** is a collection of vectors, characterized by their **dimension**. In the example in Fig. 6.3, the vectors are of dimension 4, just so they fit on the page; in real term-document matrices, the vectors representing each document would have dimensionality $|V|$, the vocabulary size.

The ordering of the numbers in a vector space is not arbitrary; each position indicates a meaningful dimension on which the documents can vary. Thus the first dimension for both these vectors corresponds to the number of times the word *battle* occurs, and we can compare each dimension, noting for example that the vectors for *As You Like It* and *Twelfth Night* have similar values (1 and 0, respectively) for the first dimension.

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Figure 6.3 The term-document matrix for four words in four Shakespeare plays. The red boxes show that each document is represented as a column vector of length four.

We can think of the vector for a document as identifying a point in $|V|$-dimensional space; thus the documents in Fig. 6.3 are points in 4-dimensional space. Since 4-dimensional spaces are hard to draw in textbooks, Fig. 6.4 shows a visualization in two dimensions; we've arbitrarily chosen the dimensions corresponding to the words *battle* and *fool*.

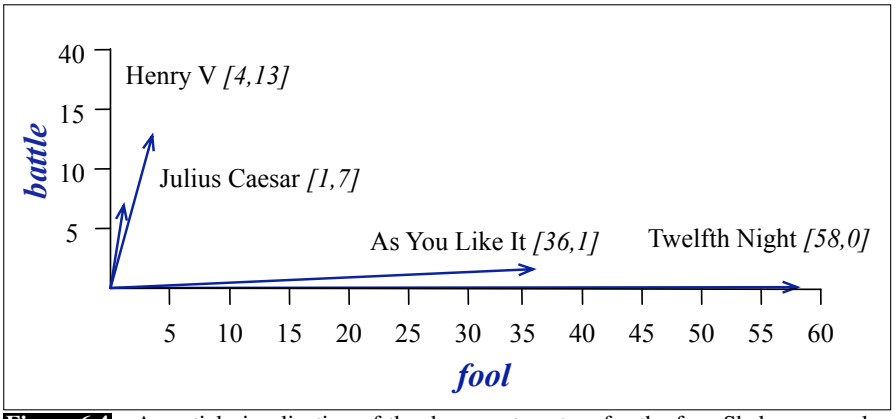

Figure 6.4 A spatial visualization of the document vectors for the four Shakespeare play documents, showing just two of the dimensions, corresponding to the words *battle* and *fool*. The comedies have high values for the *fool* dimension and low values for the *battle* dimension.

Term-document matrices were originally defined as a means of finding similar documents for the task of document **information retrieval**. Two documents that are similar will tend to have similar words, and if two documents have similar words their column vectors will tend to be similar. The vectors for the comedies *As You Like It* [1,114,36,20] and *Twelfth Night* [0,80,58,15] look a lot more like each other (more fools and wit than battles) than they look like *Julius Caesar* [7,62,1,2] or *Henry V* [13,89,4,3]. This is clear with the raw numbers; in the first dimension (battle) the comedies have low numbers and the others have high numbers, and we can see it visually in Fig. 6.4; we'll see very shortly how to quantify this intuition more formally.

A real term-document matrix, of course, wouldn't just have 4 rows and columns, let alone 2. More generally, the term-document matrix has $|V|$ rows (one for each word type in the vocabulary) and D columns (one for each document in the collection); as we'll see, vocabulary sizes are generally in the tens of thousands, and the number of documents can be enormous (think about all the pages on the web).

information retrieval

Information retrieval (IR) is the task of finding the document d from the D documents in some collection that best matches a query q. For IR we'll therefore also represent a query by a vector, also of length $|V|$, and we'll need a way to compare two vectors to find how similar they are. (Doing IR will also require efficient ways to store and manipulate these vectors by making use of the convenient fact that these vectors are sparse, i.e., mostly zeros).

Later in the chapter we'll introduce some of the components of this vector comparison process: the tf-idf term weighting, and the cosine similarity metric.

6.3.2 Words as vectors

We've seen that documents can be represented as vectors in a vector space. But vector semantics can also be used to represent the meaning of *words*, by associating each word with a vector.

row vector

The word vector is now a **row vector** rather than a column vector, and hence the dimensions of the vector are different. The four dimensions of the vector for *fool*, [36,58,1,4], correspond to the four Shakespeare plays. The same four dimensions are used to form the vectors for the other 3 words: *wit*, [20,15,2,3]; *battle*, [1,0,7,13]; and *good* [114,80,62,89]. Each entry in the vector thus represents the counts of the word's occurrence in the document corresponding to that dimension.

For documents, we saw that similar documents had similar vectors, because similar documents tend to have similar words. This same principle applies to words: similar words have similar vectors because they tend to occur in similar documents. The term-document matrix thus lets us represent the meaning of a word by the documents it tends to occur in.

However, it is most common to use a different kind of context for the dimensions of a word's vector representation. Rather than the term-document matrix we use the

word-word matrix

term-term matrix, more commonly called the **word-word matrix** or the **term-context matrix**, in which the columns are labeled by words rather than documents. This matrix is thus of dimensionality $|V| \times |V|$ and each cell records the number of times the row (target) word and the column (context) word co-occur in some context in some training corpus. The context could be the document, in which case the cell represents the number of times the two words appear in the same document. It is most common, however, to use smaller contexts, generally a window around the word, for example of 4 words to the left and 4 words to the right, in which case the cell represents the number of times (in some training corpus) the column word occurs in such a ± 4 word window around the row word. For example here is one example each of some words in their windows:

is traditionally followed by	**cherry**	pie, a traditional dessert
often mixed, such as	**strawberry**	rhubarb pie. Apple pie
computer peripherals and personal	**digital**	assistants. These devices usually
a computer. This includes	**information**	available on the internet

If we then take every occurrence of each word (say **strawberry**) and count the context words around it, we get a word-word co-occurrence matrix. Fig. 6.5 shows a simplified subset of the word-word co-occurrence matrix for these four words computed from the Wikipedia corpus (Davies, 2015).

Note in Fig. 6.5 that the two words *cherry* and *strawberry* are more similar to each other (both *pie* and *sugar* tend to occur in their window) than they are to other words like *digital*; conversely, *digital* and *information* are more similar to each other than, say, to *strawberry*. Fig. 6.6 shows a spatial visualization.

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	
strawberry	0	...	0	0	1	60	19	
digital	0	...	1670	1683	85	5	4	
information	0	...	3325	3982	378	5	13	

Figure 6.5 Co-occurrence vectors for four words in the Wikipedia corpus, showing six of the dimensions (hand-picked for pedagogical purposes). The vector for *digital* is outlined in red. Note that a real vector would have vastly more dimensions and thus be much sparser.

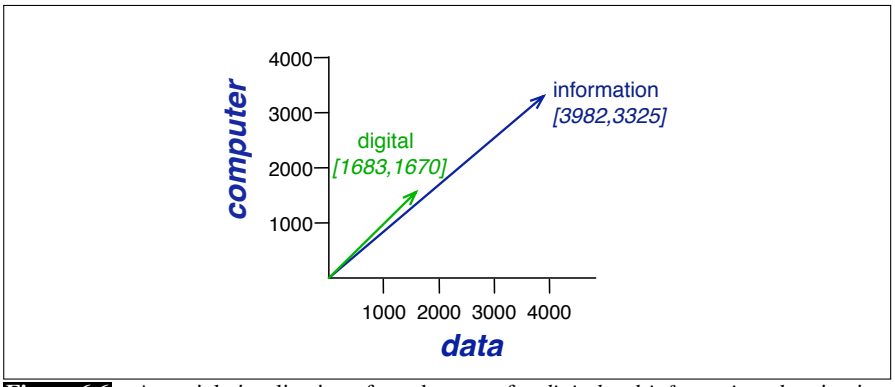

Figure 6.6 A spatial visualization of word vectors for *digital* and *information*, showing just two of the dimensions, corresponding to the words *data* and *computer*.

Note that $|V|$, the length of the vector, is generally the size of the vocabulary, usually between 10,000 and 50,000 words (using the most frequent words in the training corpus; keeping words after about the most frequent 50,000 or so is generally not helpful). But of course since most of these numbers are zero these are **sparse** vector representations, and there are efficient algorithms for storing and computing with sparse matrices.

Now that we have some intuitions, let's move on to examine the details of computing word similarity. Afterwards we'll discuss the tf-idf method of weighting cells.

6.4 Cosine for measuring similarity

To define similarity between two target words v and w, we need a measure for taking two such vectors and giving a measure of vector similarity. By far the most common similarity metric is the **cosine** of the angle between the vectors.

dot product
inner product

The cosine—like most measures for vector similarity used in NLP—is based on the **dot product** operator from linear algebra, also called the **inner product**:

$$\text{dot product}(\mathbf{v},\mathbf{w}) = \mathbf{v}\cdot\mathbf{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N \tag{6.7}$$

As we will see, most metrics for similarity between vectors are based on the dot product. The dot product acts as a similarity metric because it will tend to be high just when the two vectors have large values in the same dimensions. Alternatively, vectors that have zeros in different dimensions—orthogonal vectors—will have a dot product of 0, representing their strong dissimilarity.

vector length

This raw dot product, however, has a problem as a similarity metric: it favors **long** vectors. The **vector length** is defined as

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^{N} v_i^2} \tag{6.8}$$

The dot product is higher if a vector is longer, with higher values in each dimension. More frequent words have longer vectors, since they tend to co-occur with more words and have higher co-occurrence values with each of them. The raw dot product thus will be higher for frequent words. But this is a problem; we'd like a similarity metric that tells us how similar two words are regardless of their frequency.

The simplest way to modify the dot product to normalize for the vector length is to divide the dot product by the lengths of each of the two vectors. This **normalized dot product** turns out to be the same as the cosine of the angle between the two vectors, following from the definition of the dot product between two vectors $\mathbf{a}$ and $\mathbf{b}$:

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= |\mathbf{a}||\mathbf{b}| \cos\theta \\ \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} &= \cos\theta \end{aligned} \tag{6.9}$$

cosine

The **cosine** similarity metric between two vectors $\mathbf{v}$ and $\mathbf{w}$ thus can be computed as:

$$\text{cosine}(\mathbf{v},\mathbf{w}) = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}||\mathbf{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}} \tag{6.10}$$

unit vector

For some applications we pre-normalize each vector, by dividing it by its length, creating a **unit vector** of length 1. Thus we could compute a unit vector from $\mathbf{a}$ by dividing it by $|\mathbf{a}|$. For unit vectors, the dot product is the same as the cosine.

The cosine value ranges from 1 for vectors pointing in the same direction, through 0 for vectors that are orthogonal, to -1 for vectors pointing in opposite directions. But raw frequency values are non-negative, so the cosine for these vectors ranges from 0–1.

Let's see how the cosine computes which of the words *cherry* or *digital* is closer in meaning to *information*, just using raw counts from the following shortened table:

	pie	**data**	**computer**
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

$$\begin{aligned} \cos(\text{cherry},\text{information}) &= \frac{442*5+8*3982+2*3325}{\sqrt{442^2+8^2+2^2}\sqrt{5^2+3982^2+3325^2}} = .017 \\ \cos(\text{digital},\text{information}) &= \frac{5*5+1683*3982+1670*3325}{\sqrt{5^2+1683^2+1670^2}\sqrt{5^2+3982^2+3325^2}} = .996 \end{aligned}$$

The model decides that *information* is way closer to *digital* than it is to *cherry*, a result that seems sensible. Fig. 6.7 shows a visualization.

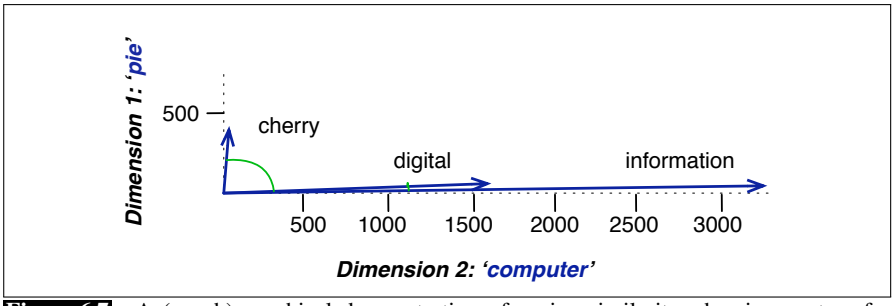

Figure 6.7 A (rough) graphical demonstration of cosine similarity, showing vectors for three words (*cherry*, *digital*, and *information*) in the two dimensional space defined by counts of the words *computer* and *pie* nearby. Note that the angle between *digital* and *information* is smaller than the angle between *cherry* and *information*. When two vectors are more similar, the cosine is larger but the angle is smaller; the cosine has its maximum (1) when the angle between two vectors is smallest (0°); the cosine of all other angles is less than 1.

6.5 TF-IDF: Weighing terms in the vector

The co-occurrence matrix in Fig. 6.5 represented each cell by the raw frequency of the co-occurrence of two words.

It turns out, however, that simple frequency isn't the best measure of association between words. One problem is that raw frequency is very skewed and not very discriminative. If we want to know what kinds of contexts are shared by *cherry* and *strawberry* but not by *digital* and *information*, we're not going to get good discrimination from words like *the*, *it*, or *they*, which occur frequently with all sorts of words and aren't informative about any particular word. We saw this also in Fig. 6.3 for the Shakespeare corpus; the dimension for the word *good* is not very discriminative between plays; *good* is simply a frequent word and has roughly equivalent high frequencies in each of the plays.

It's a bit of a paradox. Words that occur nearby frequently (maybe *pie* nearby *cherry*) are more important than words that only appear once or twice. Yet words that are too frequent—ubiquitous, like *the* or *good*— are unimportant. How can we balance these two conflicting constraints?

The **tf-idf algorithm** (the '-' here is a hyphen, not a minus sign) is the product of two terms, each term capturing one of these two intuitions:

term frequency

The first is the **term frequency** (Luhn, 1957): the frequency of the word t in the document d. We can just use the raw count as the term frequency:

$$\text{tf}_{t,d} = \text{count}(t,d) \tag{6.11}$$

Alternatively we can squash the raw frequency a bit, by using the $\log_{10}$ of the frequency instead. The intuition is that a word appearing 100 times in a document doesn't make that word 100 times more likely to be relevant to the meaning of the document. Because we can't take the log of 0, we normally add 1 to the count:[3]

$$\text{tf}_{t,d} = \log_{10}(\text{count}(t,d)+1) \tag{6.12}$$

If we use log weighting, terms which occur 10 times in a document would have a tf=2, 100 times in a document tf=3, 1000 times tf=4, and so on.

[3] Or we can use this alternative: $\text{tf}_{t,d} = \begin{cases} 1+\log_{10}\text{count}(t,d) & \text{if } \text{count}(t,d)>0 \\ 0 & \text{otherwise} \end{cases}$

The second factor is used to give a higher weight to words that occur only in a few documents. Terms that are limited to a few documents are useful for discriminating those documents from the rest of the collection; terms that occur frequently across the entire collection aren't as helpful. The **document frequency** df_t of a term t is the number of documents it occurs in. Document frequency is not the same as the **collection frequency** of a term, which is the total number of times the word appears in the whole collection in any document. Consider in the collection of Shakespeare's 37 plays the two words *Romeo* and *action*. The words have identical collection frequencies (they both occur 113 times in all the plays) but very different document frequencies, since Romeo only occurs in a single play. If our goal is find documents about the romantic tribulations of Romeo, the word *Romeo* should be highly weighted, but not *action*:

document frequency

	Collection Frequency	Document Frequency
Romeo	113	1
action	113	31

We emphasize discriminative words like *Romeo* via the **inverse document frequency** or **idf** term weight (Sparck Jones, 1972). The idf is defined using the fraction N/df_t, where N is the total number of documents in the collection, and df_t is the number of documents in which term t occurs. The fewer documents in which a term occurs, the higher this weight. The lowest weight of 1 is assigned to terms that occur in all the documents. It's usually clear what counts as a document: in Shakespeare we would use a play; when processing a collection of encyclopedia articles like Wikipedia, the document is a Wikipedia page; in processing newspaper articles, the document is a single article. Occasionally your corpus might not have appropriate document divisions and you might need to break up the corpus into documents yourself for the purposes of computing idf.

idf

Because of the large number of documents in many collections, this measure too is usually squashed with a log function. The resulting definition for inverse document frequency (idf) is thus

$$\text{idf}_t = \log_{10}\left(\frac{N}{\text{df}_t}\right) \tag{6.13}$$

Here are some idf values for some words in the Shakespeare corpus, ranging from extremely informative words which occur in only one play like *Romeo*, to those that occur in a few like *salad* or *Falstaff*, to those which are very common like *fool* or so common as to be completely non-discriminative since they occur in all 37 plays like *good* or *sweet*.[4]

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

[4] *Sweet* was one of Shakespeare's favorite adjectives, a fact probably related to the increased use of sugar in European recipes around the turn of the 16th century (Jurafsky, 2014, p. 175).

tf-idf

The **tf-idf** weighted value $w_{t,d}$ for word t in document d thus combines term frequency $\text{tf}_{t,d}$ (defined either by Eq. 6.11 or by Eq. 6.12) with idf from Eq. 6.13:

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t \tag{6.14}$$

Fig. 6.8 applies tf-idf weighting to the Shakespeare term-document matrix in Fig. 6.2, using the tf equation Eq. 6.12. Note that the tf-idf values for the dimension corresponding to the word *good* have now all become 0; since this word appears in every document, the tf-idf algorithm leads it to be ignored in any comparison of the plays. Similarly, the word *fool*, which appears in 36 out of the 37 plays, has a much lower weight.

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Figure 6.8 A tf-idf weighted term-document matrix for four words in four Shakespeare plays, using the counts in Fig. 6.2. For example the 0.049 value for *wit* in *As You Like It* is the product of $\text{tf} = \log_{10}(20+1) = 1.322$ and $\text{idf} = .037$. Note that the idf weighting has eliminated the importance of the ubiquitous word *good* and vastly reduced the impact of the almost-ubiquitous word *fool*.

The tf-idf weighting is the way for weighting co-occurrence matrices in information retrieval, but also plays a role in many other aspects of natural language processing. It's also a great baseline, the simple thing to try first. We'll look at other weightings like PPMI (Positive Pointwise Mutual Information) in Section 6.7.

6.6 Applications of the tf-idf vector model

In summary, the vector semantics model we've described so far represents a target word as a vector with dimensions corresponding to all the words in the vocabulary (length $|V|$, with vocabularies of 20,000 to 50,000), which is also sparse (most values are zero). The values in each dimension are the frequency with which the target word co-occurs with each neighboring context word, weighted by tf-idf. The model computes the similarity between two words x and y by taking the cosine of their tf-idf vectors; high cosine, high similarity. This entire model is sometimes referred to for short as the **tf-idf** model, after the weighting function.

One common use for a tf-idf model is to compute word similarity, a useful tool for tasks like finding word paraphrases, tracking changes in word meaning, or automatically discovering meanings of words in different corpora. For example, we can find the 10 most similar words to any target word w by computing the cosines between w and each of the $V-1$ other words, sorting, and looking at the top 10.

The tf-idf vector model can also be used to decide if two documents are similar. We represent a document by taking the vectors of all the words in the document, and computing the **centroid** of all those vectors. The centroid is the multidimensional version of the mean; the centroid of a set of vectors is a single vector that has the minimum sum of squared distances to each of the vectors in the set. Given k word vectors $w_1, w_2, ..., w_k$, the centroid **document vector** d is:

centroid

document vector

$$d = \frac{w_1 + w_2 + ... + w_k}{k} \tag{6.15}$$

Given two documents, we can then compute their document vectors d_1 and d_2, and estimate the similarity between the two documents by $\cos(d_1, d_2)$.

Document similarity is also useful for all sorts of applications; information retrieval, plagiarism detection, news recommender systems, and even for digital humanities tasks like comparing different versions of a text to see which are similar to each other.

6.7 Optional: Pointwise Mutual Information (PMI)

An alternative weighting function to tf-idf is called PPMI (positive pointwise mutual information). PPMI draws on the intuition that the best way to weigh the association between two words is to ask how much **more** the two words co-occur in our corpus than we would have a priori expected them to appear by chance.

pointwise mutual information

Pointwise mutual information (Fano, 1961)[5] is one of the most important concepts in NLP. It is a measure of how often two events x and y occur, compared with what we would expect if they were independent:

$$I(x,y) = \log_2 \frac{P(x,y)}{P(x)P(y)} \tag{6.17}$$

The pointwise mutual information between a target word w and a context word c (Church and Hanks 1989, Church and Hanks 1990) is then defined as:

$$\text{PMI}(w,c) = \log_2 \frac{P(w,c)}{P(w)P(c)} \tag{6.18}$$

The numerator tells us how often we observed the two words together (assuming we compute probability by using the MLE). The denominator tells us how often we would **expect** the two words to co-occur assuming they each occurred independently; recall that the probability of two independent events both occurring is just the product of the probabilities of the two events. Thus, the ratio gives us an estimate of how much more the two words co-occur than we expect by chance. PMI is a useful tool whenever we need to find words that are strongly associated.

PMI values range from negative to positive infinity. But negative PMI values (which imply things are co-occurring *less often* than we would expect by chance) tend to be unreliable unless our corpora are enormous. To distinguish whether two words whose individual probability is each 10^{-6} occur together less often than chance, we would need to be certain that the probability of the two occurring together is significantly different than 10^{-12}, and this kind of granularity would require an enormous corpus. Furthermore it's not clear whether it's even possible to evaluate such scores of 'unrelatedness' with human judgments. For this reason it is more common to use Positive PMI (called **PPMI**) which replaces all negative PMI values

PPMI

[5] Pointwise mutual information is based on the **mutual information** between two random variables X and Y, which is defined as:

$$I(X,Y) = \sum_x \sum_y P(x,y) \log_2 \frac{P(x,y)}{P(x)P(y)} \tag{6.16}$$

In a confusion of terminology, Fano used the phrase *mutual information* to refer to what we now call *pointwise mutual information* and the phrase *expectation of the mutual information* for what we now call *mutual information*