

Age-structured compartmental model for COVID-19 using systems of ordinary differential equations.

Peilun (Tommy) Li

Vanderbilt University

- The SEIARHD model: Susceptible-Incubation-Symptomatically infectious-Asymptomatically infectious-Recovered-Hospitalized-Dead.
- Age-dependent parameters with groups of $[0, 9]$, $[10, 19]$, $[20, 29]$, $[30, 39]$, $[40, 59]$, $[60, 69]$, and $70+$.
- Compares simulations between controlled (raw), social distancing, and vaccination.

Model design—system of ODEs

$$\frac{dS_i}{dt} = -\beta \frac{S_i}{N} \sum_{j=1}^7 M_{ij}(I_j + A_j) + \gamma_i R_i$$

$$\frac{dE_i}{dt} = \beta \frac{S_i}{N} \sum_{j=1}^7 M_{ij}(I_j + A_j) - b_i E_i - n_i E_i$$

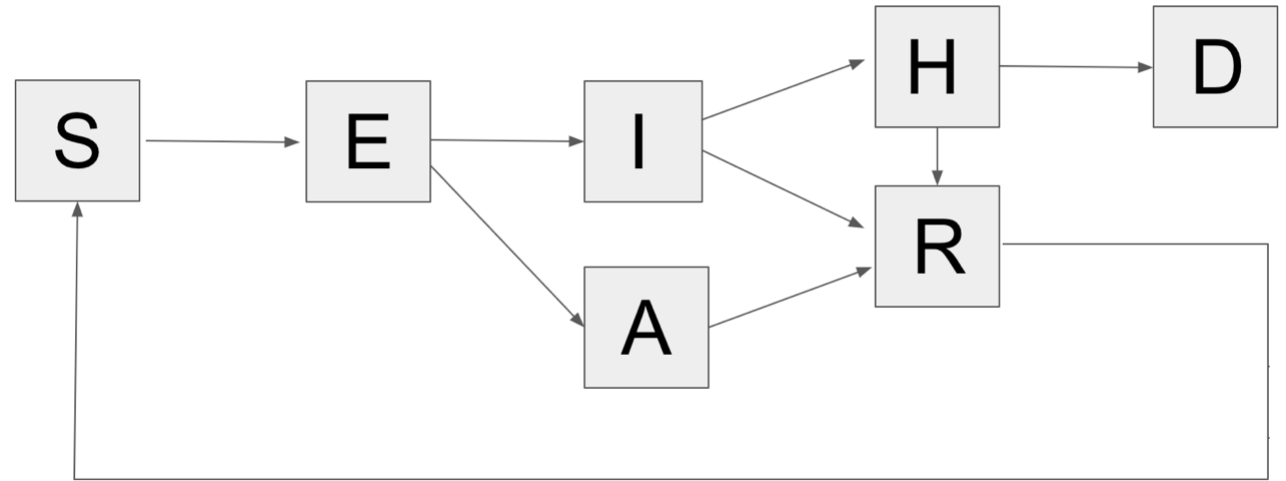
$$\frac{dI_i}{dt} = b_i E_i - c_i I_i - d_i I_i$$

$$\frac{dA_i}{dt} = n_i E_i - \alpha A_i$$

$$\frac{dR_i}{dt} = c_i I_i + \alpha A_i - \gamma_i R_i + r_i H_i$$

$$\frac{dH_i}{dt} = d_i I_i - \mu_i H_i - r_i H_i$$

$$\frac{dD_i}{dt} = \mu_i H_i$$



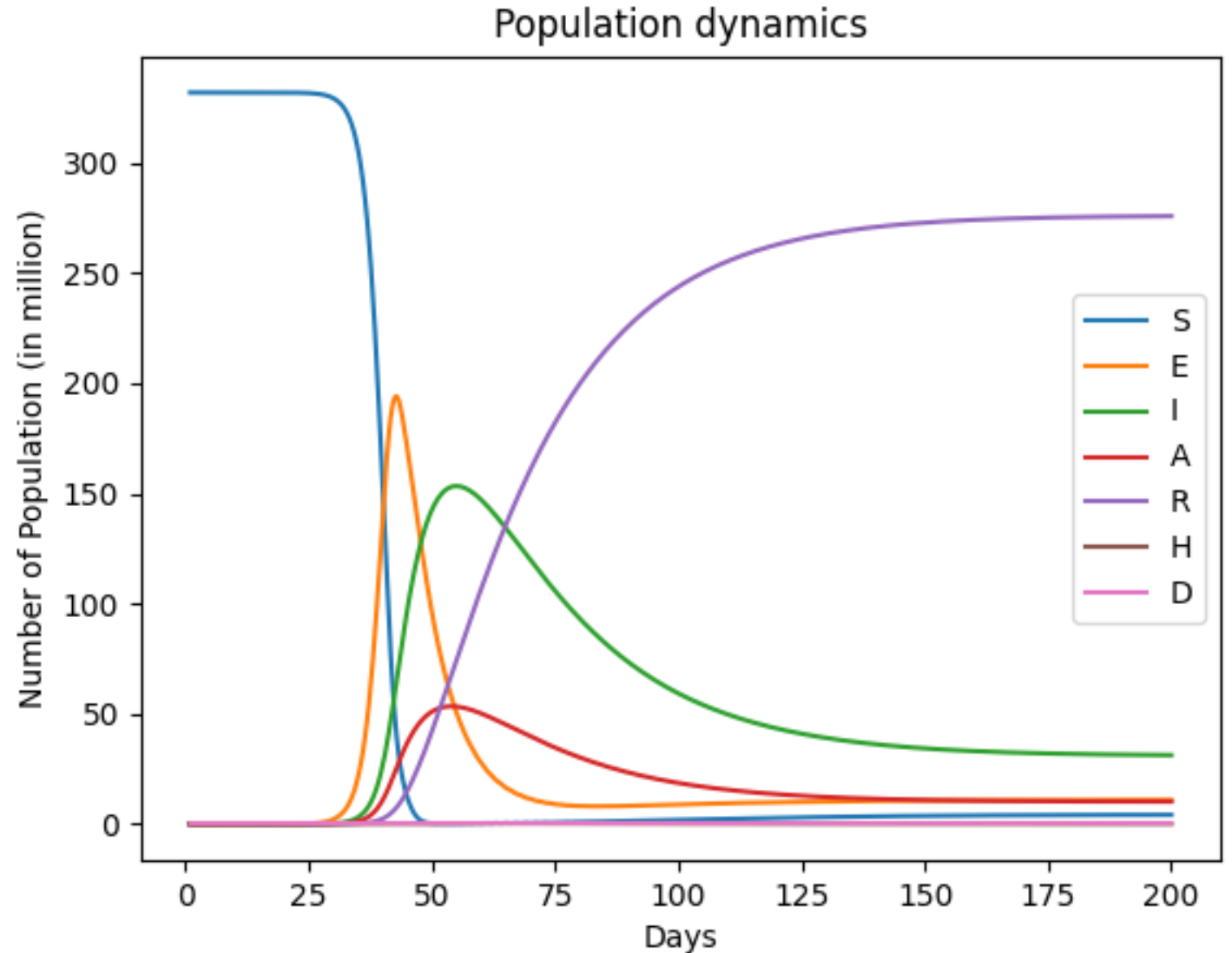
Determine infection—contact matrix \mathcal{M}

- \mathcal{M}_{ij} is proportional to number of contacts between age group i and age group j .
- The social conduct of people aged 70 to 79 is comparable to those aged 80 and above, while individuals between the ages of 40 to 49 and 50 to 59 exhibit mostly alike social behaviors.
- Social distancing: uniformly factor entries by a constant.

| Age | [0,9] | [10,19] | [20,29] | [30,39] | [40,59] | [60,69] | 70+ |
|---------|-------|---------|---------|---------|---------|---------|------|
| [0,9] | 19.2 | 4.8 | 3 | 7.1 | 3.39 | 2.3 | 1.4 |
| [10,19] | 4.8 | 42.4 | 6.4 | 5.4 | 6.23 | 1.8 | 1.7 |
| [20,29] | 3 | 6.4 | 20.7 | 9.2 | 6.69 | 2 | 0.9 |
| [30,39] | 7.1 | 5.4 | 9.2 | 16.9 | 8.42 | 3.4 | 1.5 |
| [40,59] | 3.39 | 6.23 | 6.69 | 8.42 | 9.55 | 3.06 | 1.95 |
| [60,69] | 2.3 | 1.8 | 2 | 3.4 | 3.06 | 7.5 | 3.2 |
| 70+ | 1.4 | 1.7 | 0.9 | 1.5 | 1.95 | 3.2 | 7.2 |

Raw simulation

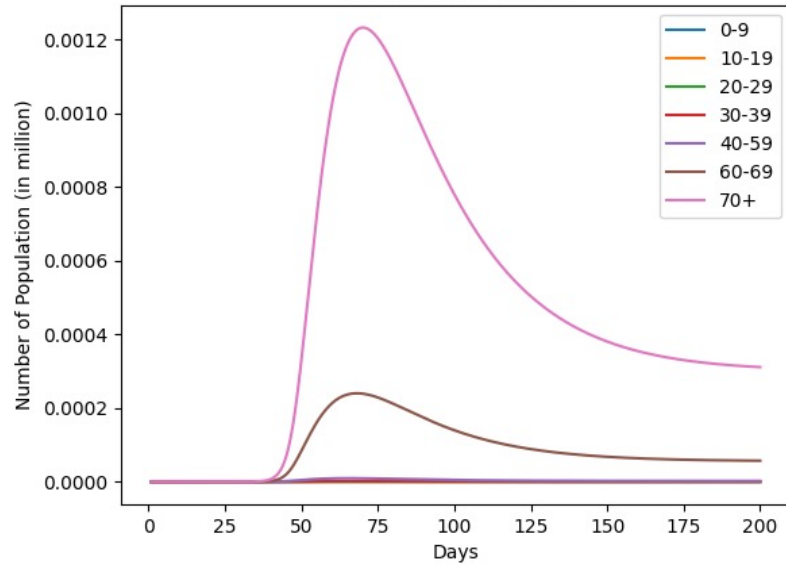
- Negligible effects before day 25
- Reach steady states around day 175
- Delays between curves (E, both I and A, R and H)



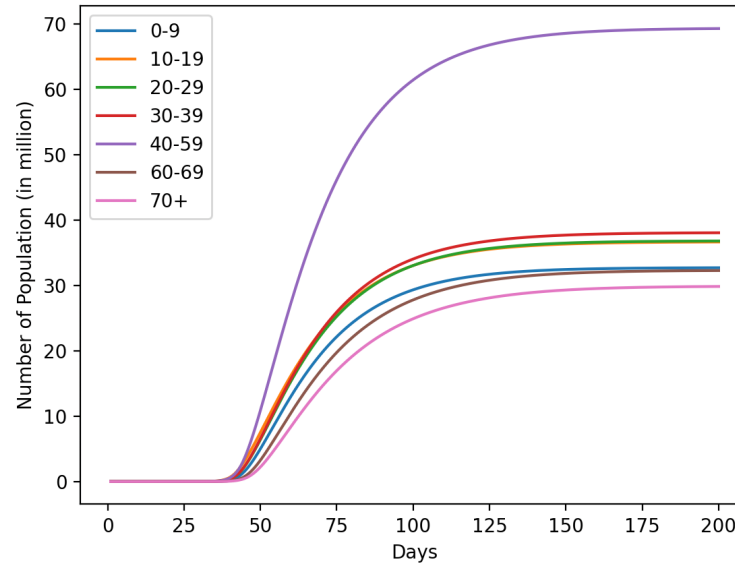
Raw simulation by age groups

- Asymptotically the same with small differences
- Will be used as controlled group for comparison

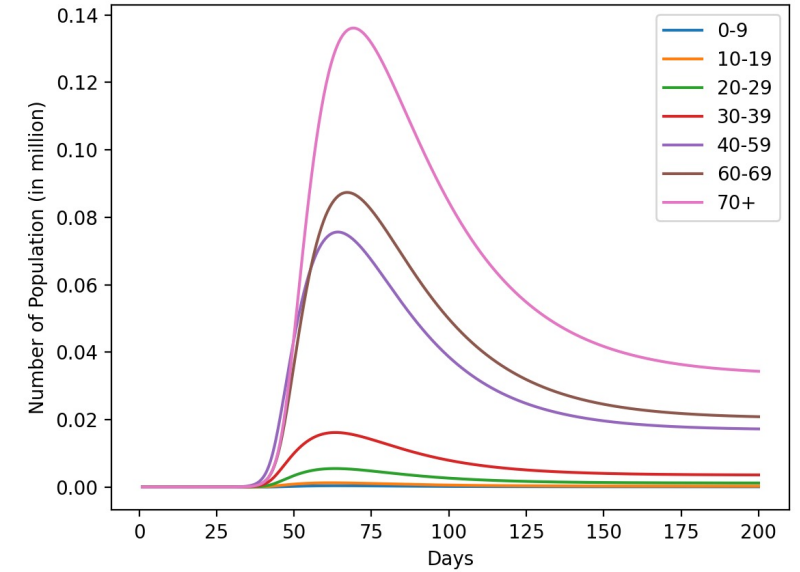
Death by age groups



Recovered by age groups

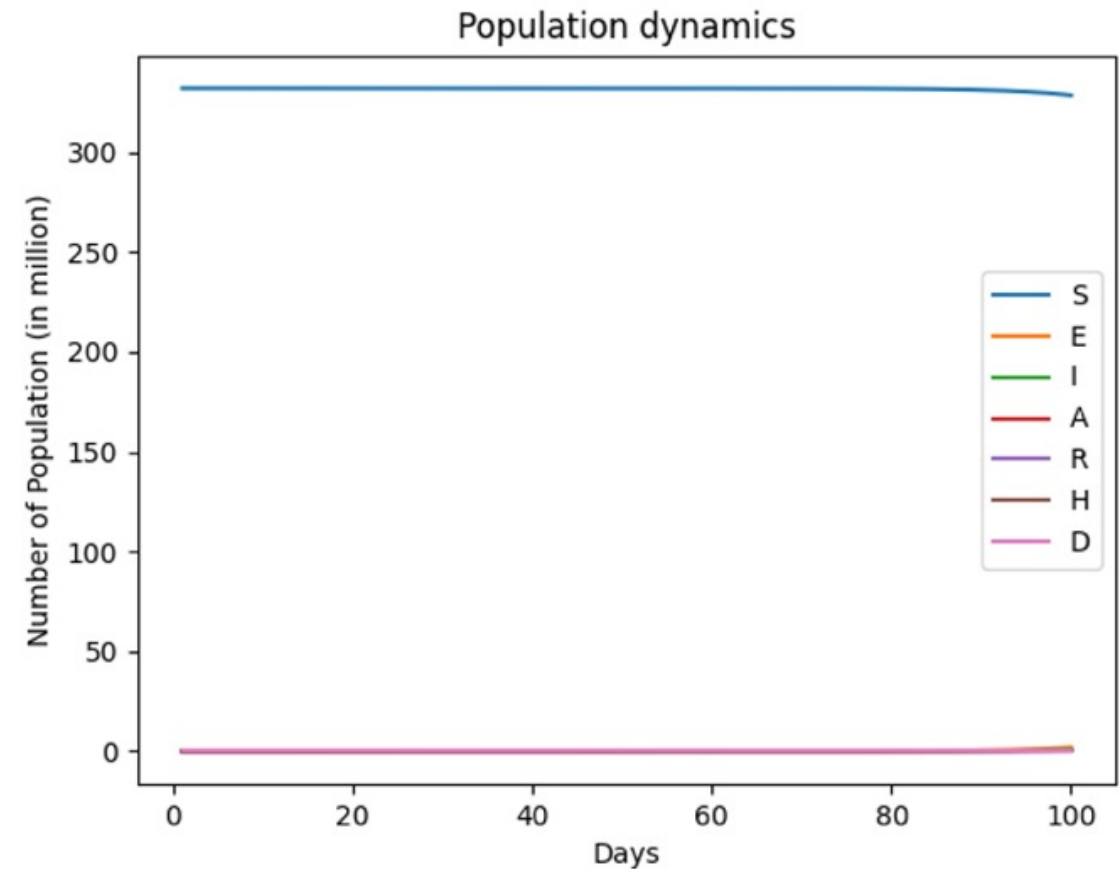
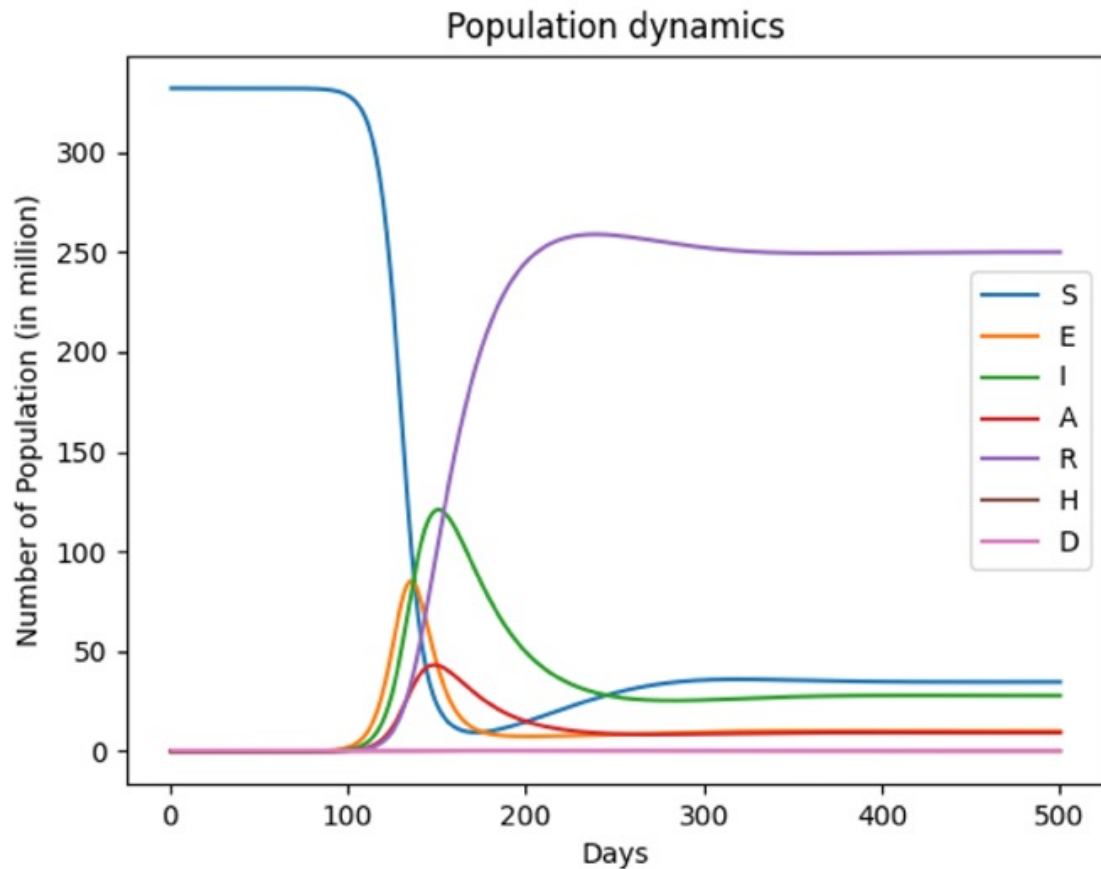


Hospitalized by age groups



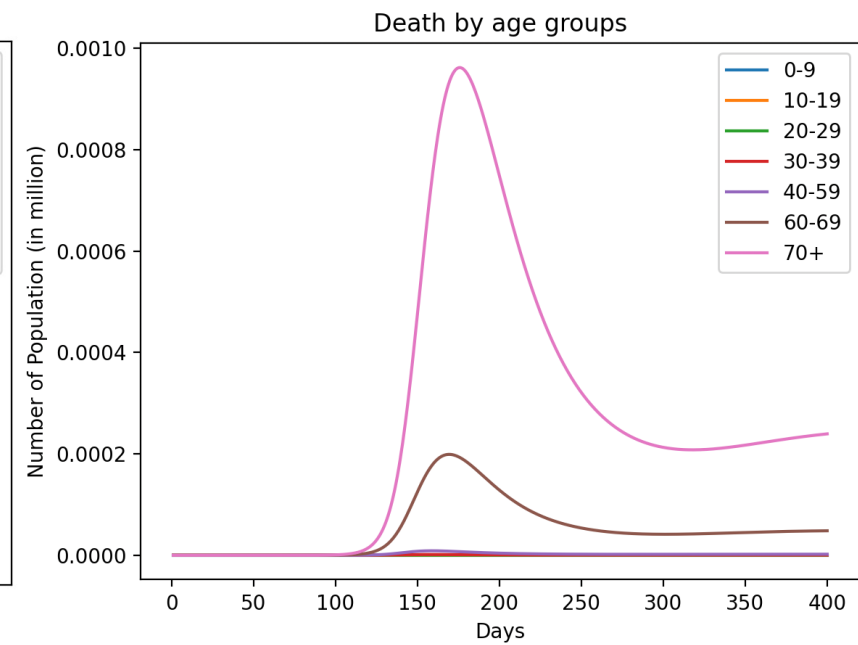
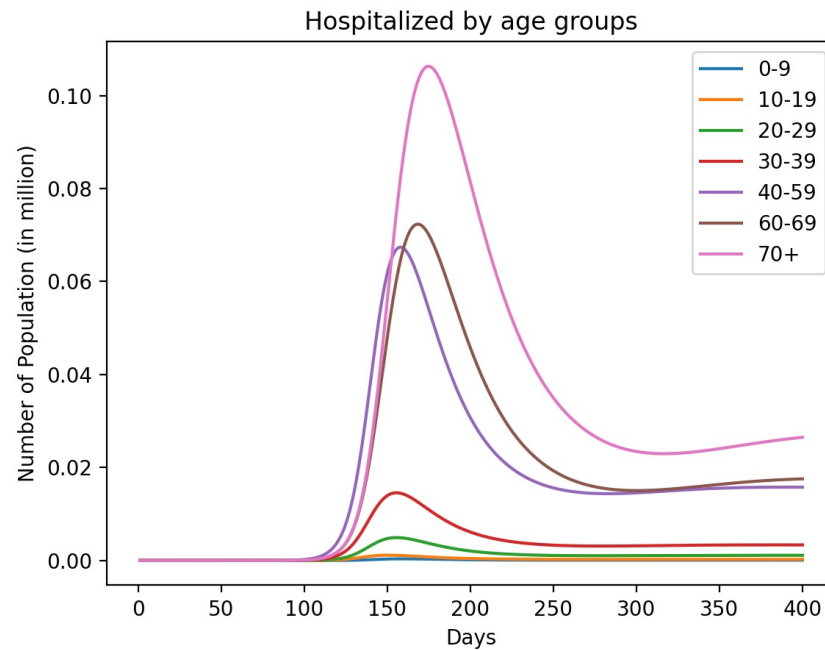
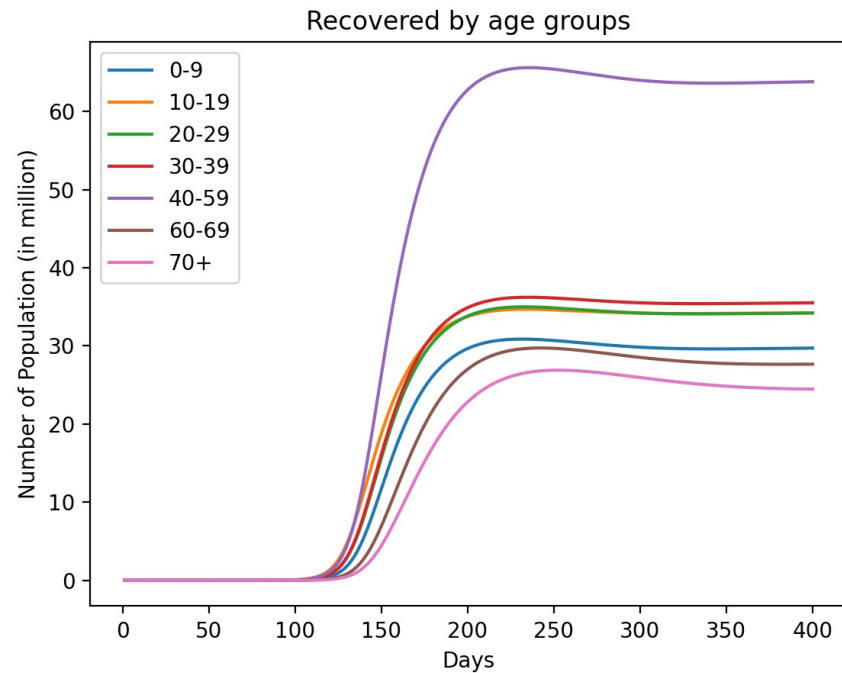
Simulation with social distancing

- More time to reach steady states
- Better protection of population



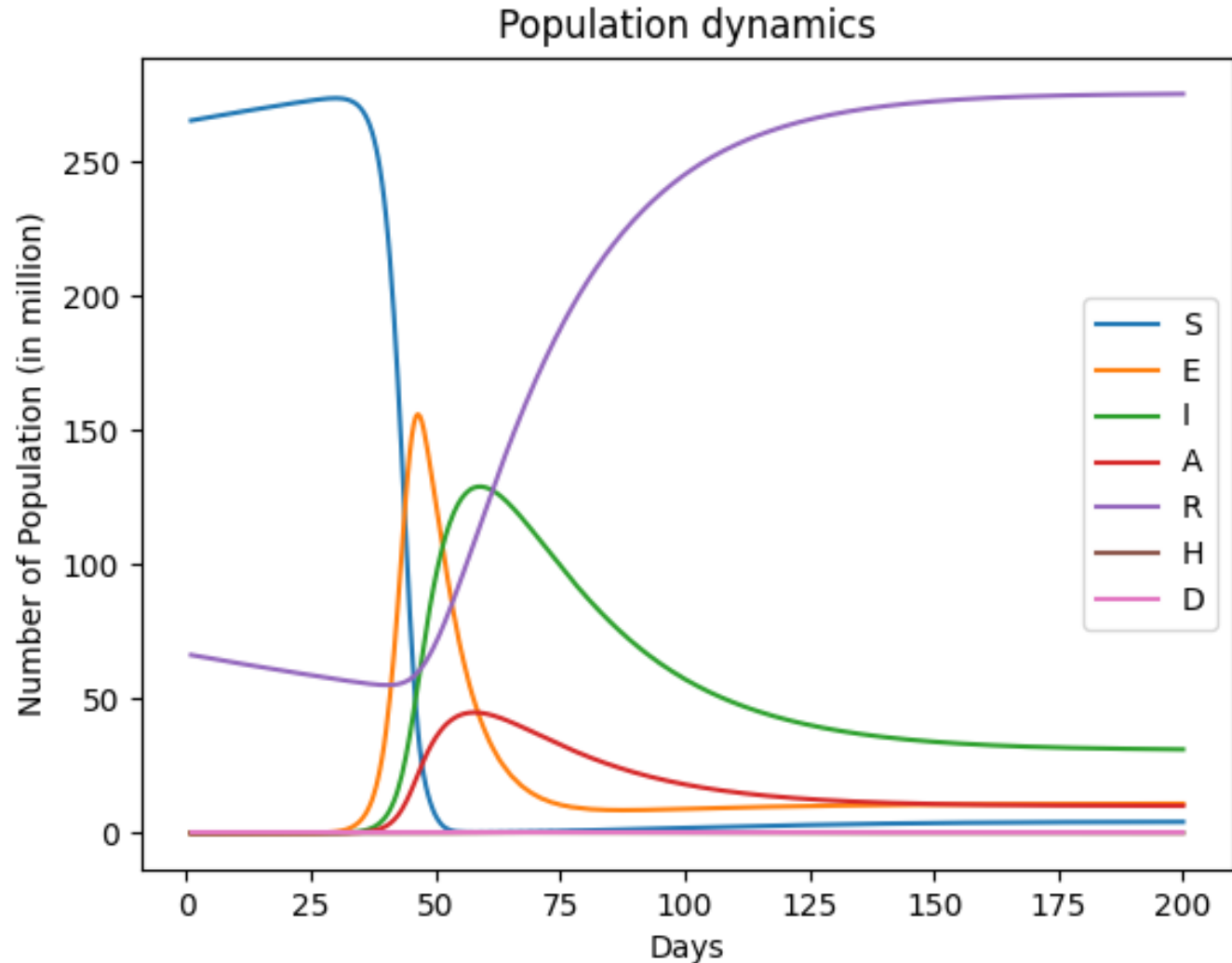
Social distancing by age groups

- Asymptotically the same with small differences
- R, H, and D has fewer cases than controlled simulation



Simulation with vaccination

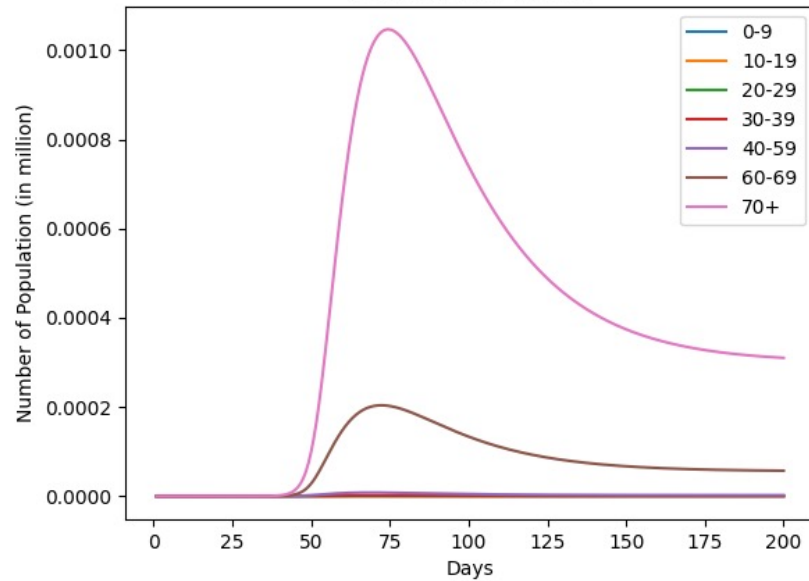
- Assume 20% vaccinated population
- Increase in S accompanied by decrease in R
- The behaviors of population is similar with those of raw simulation.



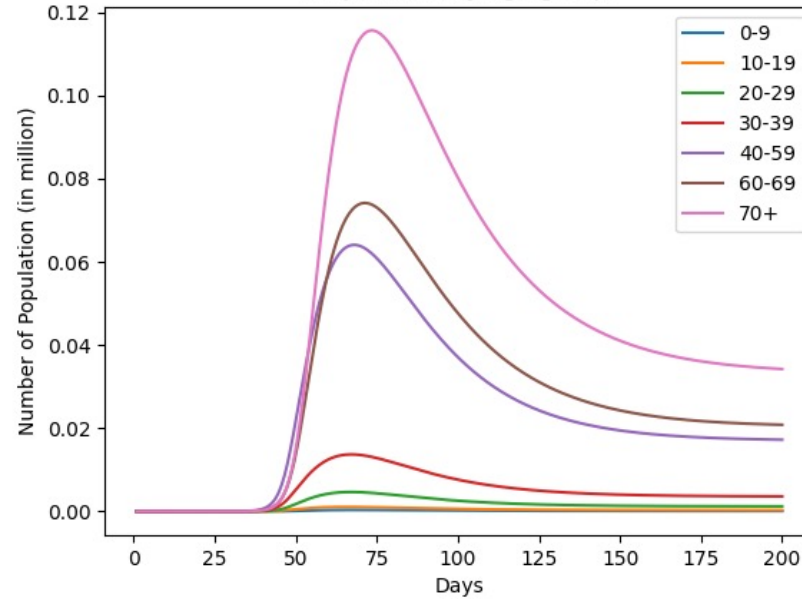
Vaccination by age groups

- Asymptotically the same with small differences
- Better than raw simulation
- Slightly worse than social distancing

Death by age groups



Hospitalized by age groups



Recovered by age groups

