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Quantum Tic-Tac-Toe: A Hybrid of Quantum and Classical Computing

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Tic-Tac-Toe is a well-known game that almost everyone has played at least in their childhood. It is generally played on paper or a classical computer. Recognizing the potential and power of quantum computers, we believe the future is quantum computing, where quantum computers might replace conventional computers due to their high efficiency at handling exceptional levels of complexity. We make our version of Quantum Tic-Tac-Toe by modifying some rules and adding some different types of moves. Through this, we want to show that a game as simple as Tic-Tac-Toe could be made much more exciting and fun by involving quantum circuits. The players have an option to choose from two different types of moves. The classical move which measures the box and results in collapse and the quantum move which entangles two separate boxes and favours the player if the control box collapses in his favour. We give a detailed explanation of the working of the quantum circuit along with the rules and strategies of the game. We have also described the algorithm in detail and provided the code for the game.

I. INTRODUCTION

Quantum physics is the study of energy and matter at its most fundamental level. It describes nature at the smallest scale, i.e., the atomic scale[1]. It provides the basis on which quantum computers are built. There are many differences between quantum and classical computers, but the main difference is the foundation on which they are built. In the case of quantum computers, qubits carry all the information which are analogous to the bits in classical computers. Unlike classical bits, quantum bits can be in a superposition state of 1 and 0 until they are measured. It is this probabilistic nature of qubits, which makes it uncertain to know the exact outcome of a measurement. This nature gives quantum computers an upper hand over classical computers. This probabilistic nature of quantum computers makes it very easy to generate random numbers whereas in classical computers we cannot generate actual random numbers. This virtue of quantum computer has profound applications in fields which require high levels of encryption. Though quantum physics is not a newly discovered branch under physical sciences, it is undoubtedly full of many mysteries yet to be uncovered. We tend to break things down to simplify and make them more understandable. Similarly, we breakdown quantum phenomena to get an in-depth understanding of them. We here, mainly focus on quantum game theory and its applications.

Quantum game theory is defined as the extension of classical game theory to the quantum domain[2]. It has remarkable applications in many fields such as social science, computer science, economy to name a few[3]. Many new research papers have been published recently on quantum versions of different games. Leaw and Cheong developed quantum algorithms to play quantum tic-tac-toe game[4], where players win the game depending upon the quantum advantage. Recently, a hybrid classical-quantum algorithm for solving Sudoku game has been developed by Pal *et al.*[5]. Paul *et al.*[6] have developed the quantum circuits for playing the quantum Monty Hall game on a quantum computer. A simple shooting game has been designed by Roy *et al.*[7], which can be played on a quantum computer with the proposed quantum circuits. Quantum Go game has also been introduced by Ranchin[8], where superposition and entanglement have been used to play the game more efficiently. A possible reason for the growing interest in this field could be the natural fascination towards games. Quantum game theory mainly focuses on the following phenomena[9,10].

Superposition: The ability of quantum objects to be in two places at once.

Entanglement: The phenomenon where distant components of a quantum system display strange correlations that cannot be explained.

Collapse: The phenomenon of the quantum states of a system getting reduced to classical states. Collapses occur when we measure the quantum circuit.

Quantum strategies involve these phenomenons; which are used in quantum gameplay.

History traces back Tic-Tac-Toe to ancient Egypt. The early variation of Tic-Tac-Toe was played in the Roman Empire, around the first century BC, and was called Terni lapilli[11]. Another closely related ancient game was Three Men's Morris which was also played on a simple grid and required three pieces in a row to finish[12]. The first print reference of a game called "Tick-Tack-Toe" occurred in 1884. "Tic-Tac-Toe" might have derived from the "tick-tack", the name of an old version of backgammon first described in 1558. The US renaming the "noughts and crosses" to "Tic-Tac-Toe" occurred in the 20th century[13].

Classical Tic-Tac-Toe is a relatively simple game with almost nil complexity. It is one of the first games to be played by children due to its fast setup and easy engagement[11]. Players soon discovered that the best play from both parties leads to a draw. Hence, Tic-Tac-Toe is mostly played by young children, who often have not yet found the optimal strategy. Classic Tic-Tac-Toe requires skills like strategy, tactics, observation, and hence is an effective way of improving these skills.

Tic-Tac-Toe is a two-player game, played on a 3×3 grid. Players choose a symbol usually either 'X' or 'O' to mark their boxes; hence the name "noughts and crosses". The game starts by a player marking one box with his specified mark and subsequently marking boxes on alternate turns with the same mark. The game continues until one of the players has three of his marks on a row, column, or diagonal of the grid. The player who satisfies this condition first is the winner. In case, none of them meets the winning condition, and all the boxes are marked, then it is considered a draw.

Unlike in the classical version, the initial state of all the boxes on the board is a superposition of the classical states[14]. Upon selecting a box, it is measured and is collapsed into one of the classical states i.e., $|0\rangle$ or $|1\rangle$. It has an equal probability of collapsing into either of the states. So, here, the player has no control over the collapsed state of the box. The players have two moves to choose from, namely classical move and quantum move, which are described in detail below. Though, the players have no direct control over the collapsed state of the box, which occurs when a classical move is applied on the box; However, due to the given option of a quantum move, they have a lot of chances to show off their intellect and forbid luck from taking the upper hand.

Our version of the quantum Tic-Tac-Toe can be thought of as an advancement to the classical version. Modifications have been made while still keeping the essence of the original game. Due to these modifications, it makes this version different but not entirely a new game. Quantum Tic-Tac-Toe could be used to get a brief understanding of the phenomena mentioned above.

II. RULES AND STRATEGY:

This section gives a detailed description of our version of the quantum Tic-Tac-Toe game. The game uses simple quantum computing principles, which makes it different and more interesting than the regular classical version of tic-tac-toe. Initially, the board is set up, and each box is in a superposition state of $|0\rangle$ and $|1\rangle$. We call this state quantum State. Quantum state has an equal probability of collapsing into a classical state, i.e., state $|0\rangle$ or state $|1\rangle$.

The game allows two legal moves:

1. **Classical Move:** This move when applied to a box, collapses it to either of the Classical states ($|0\rangle$ or $|1\rangle$) with equal probabilities.

2. **Quantum Move:** This move requires two boxes, one for being the control box other the target box. A quantum move just entangles the target box and the control box in such a way that when a classical move is applied to the control box, and if it collapses to the player's (*of the quantum move*) favored classical state it automatically reverses the state of the target box.

A classical move can only be applied to a box that is in a quantum state, i.e., the classical move cannot be applied to the same box twice. Quantum move's target box should be in a classical state, and the control box should be in a quantum state so that when the quantum state of the control box collapses to the favoured state of the player (*of the quantum move*), it reverses the classical state of the target box. Player X's favoured state is $|1\rangle$, and player O's favoured state is $|0\rangle$. A box is marked 'X' when it collapses to state $|1\rangle$ and 'O' when it collapses to state $|0\rangle$. The game begins with all the boxes being in a quantum state, player 'X' takes the first turn, the only movement allowed is the classical move as quantum move requires a box to be in the classical state for being its target. So player 'X' goes for a classical move on any one of the nine boxes, and the chosen box collapses into either state $|0\rangle$ or state $|1\rangle$. Suppose player 'X' gets lucky, and the box collapses to state $|1\rangle$, which is shown by marking the box 'X'. Now O's turn, now player 'O' could choose to go for a classical move and collapse any of the other boxes or could play a quantum move and entangle any other box with the box marked 'X', where the 'X' marked box becomes the target and any other box, the control of the entanglement. Now, whenever a classical move is applied onto the control box and if it collapses to classical state $|0\rangle$ (*player O's favored state*) it automatically reverses the state of the target, which in this case makes the previously 'X' marked the box 'O'. On the contrary, if the control would have collapsed into state $|1\rangle$ then the target state would have remained the same.

Let us take another case, players 'X' goes for a classical move to collapse any of the boxes, and it collapses to state $|0\rangle$. So the box turns 'O' now if in the next step player 'O' entangles this, and the control of this move in any subsequent step collapses to state $|0\rangle$ this makes the previously 'O' marked box turn into 'X'. So this shows that we do need to strategize our moves as there are few cases in which our own move could backfire us in any other subsequent step.

The condition for winning is the same as it is in the regular classical version of Tic-Tac-Toe. The first player to get three of his marks (*i.e. 'X' or 'O'*) in a single row, column or diagonal wins the game.

III. CIRCUIT EXPLANATION

The quantum circuit is set up with nine qubits, and nine classical bits and Hadamard gates are applied to each of the nine qubits. Hadamard gate creates the superposition state, which we call the quantum state.

Let us say player 'X' chooses to go for a classical move on box 5, then as soon as the input is received the qubit associated with box 5 (*i.e. qubit 4 as qubits count start*

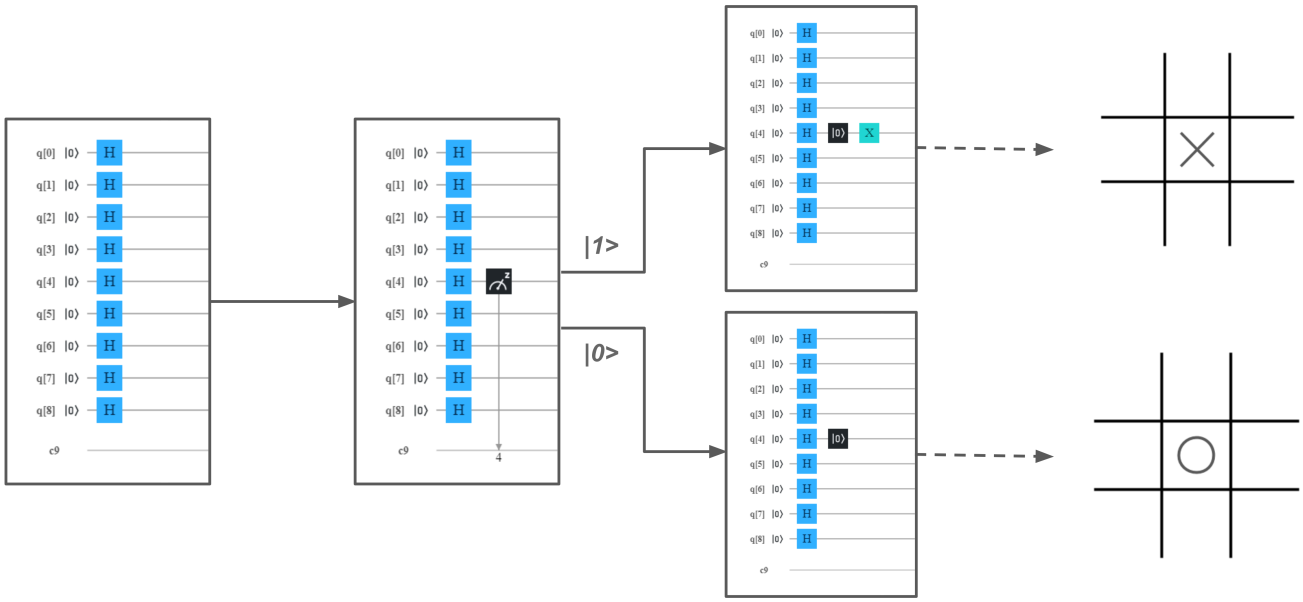

FIG. 1. Player 'X' goes for a classical move on box 5, so qubit 4 is measured and according to the output the qubit's state is fixed, and box 5 is marked.

from qubit 0) is measured and the result is analyzed. Box 5 is marked 'X' if the result is $|1\rangle$ and 'O' if the result is $|0\rangle$, on top of that the result of the measured qubit is fixed by adding a few gates, for example, a RESET gate is applied onto qubit 4 if it collapses to $|0\rangle$. Two gates, the RESET gate, and the NOT gate are applied onto qubit 4 if it collapses to $|1\rangle$. This step ensures that on subsequent turns whenever qubit 4 is measured even if the Hadamard gate collapses to a different state, the final result does not vary *Fig. 1*.

Moving on with the same example, now it is player O's turn and let us say he chooses to go for a quantum move. So player 'O' entangles box 5 with let us say box 2 (*i.e., qubit 4 with qubit 1*) with box 5 being the target and box 2 the control, so an anti-control NOT gate is applied, with qubit 1 being the control and qubit 4 the target. What this move does is, whenever qubit 1 (box 2) would be measured and if it collapses to state $|0\rangle$ then the anti-control NOT gate would reverse the state of qubit 4 and the previously marked 'X' would change to 'O', which would ultimately make both box 2 and box 5 marked as 'O'. On the contrary, if qubit 1 collapses to state $|1\rangle$ (*which would make box 2 'X' marked*), anti-control NOT gate would not be able to reverse the state of qubit 4, and both boxes would be left 'X' marked *Fig. 2*.

The game continues until the computer finds three symbols of the same type in a single row, column or diagonal. It checks if the symbol is 'X' then it prints "Player 'X' wins!" and if its 'O' then it prints "Player 'O' wins!".

An Anti-control NOT gate is added if player 'O' goes for a quantum move and a control NOT gate is applied if player 'X' goes for a quantum move as player X's favored state is $|1\rangle$ and player O's favoured state is $|0\rangle$.

In the quantum move, the adding of control NOT gate or the Anti-control NOT gate step is halted, it is added to the circuit only when the input to collapse the control of that gate (i.e., input for classical move on control box) is detected. This step makes sure the Hadamard gate's output on the control qubit does not keep affecting the target qubit's output even before measuring the control qubit (i.e., even before a classical move is played on the control qubit/box).

Protocol 1 Setting up of quantum circuit

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setup()

The Protocol:

- Creates the quantum circuit with nine quantum bits and nine classical bits
- Adds a Hadamard gate to each of the nine qubits
- Prints Legal Moves and Rules

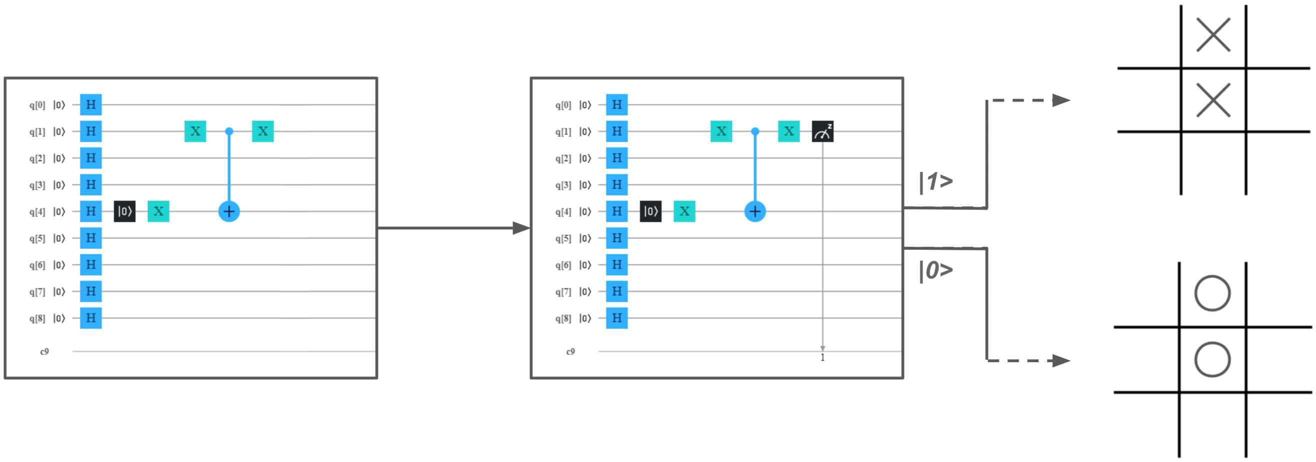

FIG. 2. Player 'O' goes for a quantum move with qubit 4 as target and qubit 1 as control, so a anti-control NOT gate is added to the circuit. According to the measurement of qubit 1 the boxes are modified.

Protocol 2 Player's Turn

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turn()

The Protocol:

- Checks and prints whether it is Player X's turn or Player O's turn
- Asks the user to choose between quantum move and classical move
- Calls function cmove() if user inputs 'c' and qmove if user inputs 'q'
- Asks the user to enter the position of the move

Protocol 3 Quantum Move

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qmove()

The Protocol:

- Stores target position and control position of the move in a variable so that cmove() could use it.
- Prints 'control move' and 'target move' linked

Protocol 4 Classical Move

---

cmove()

The Protocol:

- Calls the Aer() function in qiskit library
- Simulates the circuit on qasm simulator
- Checks if the given position is already a control position for any quantum move
  - Then adds a CNOT or Anti CNOT gate depending on the input from qmove()
- Measures the position entered by the user and stores in a variable called 'result'
- Fixes the qubit's state according to the result of the simulation
- Calls mark()

Protocol 5 Printing the Grid

mark()

The Protocol:

- Creates a 3×3 grid
- Re-simulates the circuit
- Checks the result and marks the box according to the result
- Prints the grid

IV. GAMEPLAY EXPLANATION

This section explains the gameplay by taking a few examples and suggesting some moves which would help the user to get used to the game. Initially, all the boxes are in a superposition state of 'X' and 'O', when a classical move is played on any box, it collapses it to either 'X' or 'O' with equal probability. For example, player X could go for a classical move on box 5, and then box 5 could collapse to either 'X' or 'O' with equal probability. We call the superposition state, the quantum state, and when it collapses to either 'X' or 'O', we say it has collapsed into a classical state. A quantum move requires two boxes to operate, the control needs to be a box in a quantum state, and the target needs to be in a classical state. If the control box of the quantum move collapses to the player's favour, it reverses the state of the target box. We will take an example to see how and when to apply these moves so that a player's chances of winning increases.

We start by running the code in jupyter notebook, the program prints out legal moves and basic rules, and it asks player X to choose a move between the classical move and the quantum move. Now as we know that quantum move requires a box to be in a classical state, so the only possible move is a classical move. So player X goes for a classical move on box 7, player X gets lucky, and the box collapses in his favour, i.e., 'X' *Fig. 3.a.* Now player O's turn, he could either go for a classical move and collapse any other box or could go for a quantum move. Player O sees that box 7 has collapsed into 'X' so for him the best move would be a quantum move in which he entangles box 7 with any other box, with box 7 being the target and box 5 (let us say) the control *Fig. 3.b.* Now if box 5 is measured and if it collapses to 'O' (Player O's favoured state), it would automatically reverse the state of box 7 and make it 'O'.

Now player X's turn, a quantum move would not benefit him in any way as box 7 is already 'X', hence no point in entangling it with any other box, he goes for a classical move. Now he could collapse box 5 (previously entangled by player O) or any other box, box 5 collapsing to 'O' would make the previously 'X' marked box 7, 'O'. Thus he chooses box 3, and he gets lucky again, and it collapses to 'X' *Fig. 3.c.* Now we have the board with box 7 and box 3 marked 'X'. Player O's turn, he goes with the same strategy and plays a quantum move and entangles box 3 with box 9 with box 3 as target and box 9 as control *Fig. 3.d.*

Now Player X's turn, if box 5 collapses to 'X' it would be a clear win for Player X, hence player X goes for a classical move on box 5, but this time it collapsed into 'O', this also made box 7 'O' as it was previously entangled by player O *Fig. 3.e.* It opens up a winning chance for player O, as previously Player O entangled box 3 and box 9, if box 9 collapses to 'O', it would reverse the state of box 3 and make it 'O' and player O would win the game. Thus player O goes for a classical move on box 9, and it collapsed into 'O', which made box 3 'O' and player O WINS the game *Fig. 3.f.*

This example shows how we could turn the game into our favour if we plan our moves properly, as we saw, player X started pretty well and got 2 boxes in his favour and even came close to winning the game. However, player O's properly planned quantum moves turned the game in his favour, and he ultimately won the game.

V. DISCUSSIONS

At first, this game might seem to be a game of chance or a game completely based on luck. However, as one gets used to the game and phenomena (mentioned above) associated with it, they start realizing that it is not only about chance or luck but also about strategy and tactics. The chance factor only comes into play when a measurement occurs (classical move). The entanglement through the quantum move should be well planned. This is well explained by the following example, for instance, if a player X entangles a box with 'X' mark then it would reverse it to 'O' if the control box collapses to 'X', this will result in player X losing one of his 'X' marked boxes. This example shows that one does need to plan their moves before applying. This is true not only for the quantum move but for the classical move as well since one needs to strategize when and which control box to collapse. While choosing the control box, one needs to keep in mind both the possibilities, i.e., if it collapses into the player's favour and if it does not. A player also needs to keep in mind that even if it does not collapse into his favour, this should not let the opponent directly receive a significant advantage like fulfilling the winning condition. Hence, we can say that, though this game shows a probabilistic nature, it is not a game completely based on luck, and it does involve strategies and demands proper thinking.

```
Player 'X' turn
Choose Classical Move(c) or Quantum Move(q)c
Enter Position(1-9): 7
[' ', ' ', ' ']
[' ', ' ', ' ']
['X', ' ', ' ']
```

FIG. 3.a

```
Player 'O' turn
Choose Classical Move(c) or Quantum Move(q)q
Enter Control Position(1-9): 5
Enter Target Position(1-9): 7
5 - 7   entangled
```

FIG. 3.b

```
Player 'X' turn
Choose Classical Move(c) or Quantum Move(q)c
Enter Position(1-9): 3
[' ', ' ', 'X']
[' ', ' ', ' ']
['X', ' ', ' ']
```

FIG. 3.c

```
Player 'O' turn
Choose Classical Move(c) or Quantum Move(q)q
Enter Control Position(1-9): 9
Enter Target Position(1-9): 3
9 - 3   entangled
```

FIG. 3.d

```
Player 'X' turn
Choose Classical Move(c) or Quantum Move(q)c
Enter Position(1-9): 5
[' ', ' ', 'X']
[' ', 'O', ' ']
['O', ' ', ' ']
```

FIG. 3.e

```
Player 'O' turn
Choose Classical Move(c) or Quantum Move(q)c
Enter Position(1-9): 9
[' ', ' ', 'O']
[' ', 'O', ' ']
['O', ' ', 'O']


Player O WINS!!!
```

FIG. 3.f

FIG. 3. Gameplay Explanation

VI. CONCLUSION

To conclude, we have analyzed and explained the formulation and working of our version of quantum Tic-Tac-Toe. We explained quantum phenomena which guard the basis of our game. The rules and strategies, along with the circuit, have been well elucidated with appropriate images. Algorithm of the code for the game is also well described. We have provided the code for the game, made by hybridizing quantum and classical computing, in the references[15]. The probabilistic nature of this game makes it eye-catching; this nature can be introduced into many games like Ludo, Snakes and Ladders, Bingo, Chess to make them more engaging. Hence, our work paves the path for developing other games that could be played using a quantum computer. This nature is useful in many fields other than just gaming. As stated in the above text, qubits can be in a superposition of the classical states; due to this property of qubits, they can be used to store relatively more information than classical bits. Hence, we believe quantum computers are capable of overcoming the limitations of classical computers and handling our ever-increasing levels of complexity.

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