Assignment 2. Posted on 04.03.2024 Due on 14.03.2024 Marks 30

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1. Solve the following linear equation by Cholesky decomposition (check for symmetric matrix) and Gauss-Seidel to a precision of 10^{-6}. **[5]**

$$\begin{pmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 2 \\ 1 \\ 2 \end{pmatrix}$$

2. Solve the following linear equation by Gauss-Jordon and LU factorization. **[5]**

$$\begin{pmatrix} 0 & 4 & 2 & 0 & 1 \\ 4 & 0 & 4 & 10 & 1 \\ 2 & 5 & 1 & 3 & 13 \\ 11 & 3 & 0 & 1 & 2 \\ 3 & 2 & 7 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 20 \\ 15 \\ 92 \\ 51 \\ 15 \end{pmatrix}$$

3. Solve the following *almost* sparse system $\mathbf{A}\,\mathbf{x} = \mathbf{b}$ using and Conjugate Gradient to find the inverse of the matrix $\mathbf{A}$ to a precision of 10^{-4}. Check the correctness of inverse. **[10]**

$$\begin{pmatrix} 2 & -3 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & -3 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} -5/3 \\ 2/3 \\ 3 \\ -4/3 \\ -1/3 \\ 5/3 \end{pmatrix}$$

4. Find the inverse of the following matrix using Conjugate Gradient by generating the matrix on fly *i.e.* without actually storing it. It is a two-dimensional 50×50 system with periodic boundary condition Plot the residue versus iteration steps. Use $m = 0.2$ and convergence criteria to be $\epsilon = 10^{-6}$. **[10]**

$$\mathbf{A} = \frac{1}{2}\Big(\delta_{x+\hat{\mu},y} + \delta_{x-\hat{\mu},y} - 2\delta_{x,y}\Big) + m^2\delta_{x,y}$$

where $\hat{\mu} \equiv \hat{x}$ or $\hat{y}$