1. Consider a 4×4 matrix given below.

    (a) Find its largest eigenvalue using Power method.   [**5**]

    (b) Find all the eigenvalues using QR factorization, using Gram-Schmidt orthogonalization scheme.   [**5**]

    $$\mathbf{A} = \begin{pmatrix} 4 & 2/3 & -4/3 & 4/3 \\ 2/3 & 4 & 0 & 0 \\ -4/3 & 0 & 6 & 2 \\ 4/3 & 0 & 2 & 6 \end{pmatrix}$$

2. The data from an experiment is given in the file `assign2fit.txt`. Try a cubic least square fit using

    $$y = f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

    Try fitting the data set with a modified basis

    $$y = f(x) \quad = \quad a_0 \phi_0(x) + a_1 \phi_1(x) + a_2 \phi_2(x) + a_3 \phi_3(x)$$
    $$\text{where,} \quad \phi_0(x) = 1, \quad \phi_2(x) = 2x - 1, \quad \phi_3(x) = 8x^2 - 8x + 1$$
    $$\phi_3(x) = 32x^3 - 48x^2 + 18x - 1$$

    where the above functions ϕ_α are modified Chebyshev functions. Can you explain the difference by comparing the relative merit of using different functions calculating the condition number? (You may use online condition number or norm calculator.)   [**10**]

3. Letter grades of 600 students in Physics over couple of years are observed to be

Letter Grade	Observed distribution of students
A	77
B	150
C	210
D	125
F	38

The expected distribution is assumed to *standard normal*. Perform χ^2-test at 5% and 10% level of significance to determine whether the distribution is *normal*. Prepare a pdf file (for uploading) to show necessary calculations and your conclusion.   [**5**]

4. Lab tests for focal length of two independent shipment of convex lenses yield the following result

Shipment A f (cm)	Shipment B f (cm)
4.65	4.75
4.84	4.79
4.59	4.74
4.75	4.74
4.63	4.77
4.75	4.58
4.58	4.81
4.82	
4.86	
4.60	
4.77	
4.65	
4.80	

Determine whether the lenses in shipment A and B are from the same population using F-test and t-test. Prepare a pdf file (for uploading) to show necessary calculations and your conclusion.   [**5**]