Assignment 4. Posted on 02.04.2024 Due on 14.04.2024 Marks 30

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1. Generate your own random numbers in range [0,1] using *multiplicative linear congruential generator* defined by **[4]**

$$x_n = (a\, x_{n-1}) \bmod m$$

using (i) $a = 65$, $m = 1021$ and (ii) $a = 572$, $m = 16381$.

2. Use Monte Carlo and pRNG of Problem 1 to approximate the integral $\int_{-\pi/2}^{\pi/2} \cos x\, dx$. Consider $N \in [1000, 100000]$ in steps of 1000 and plot the convergence. **[4]**

3. Generate pseudo random numbers distributed as $\exp(-2x)$ for $0 \leq x \leq 3$ using inverse transform method and accept / reject method using sampling distribution $q(x) = 1 - x, \;\; 0 \leq x \leq 1$. Histogram the sampled RNG in both case (generate at least 2000 RNs for the purpose). **[5+5]**

4. Use Monte Carlo to evaluate the integral (taking $N = 10000$)

$$\int_0^2 \frac{e^{-2x}}{1+x^2}\, dx$$

with the following importance sampling functions and comment on the variance reduction that may be achieved. **[12]**

$$p_1(x) = 0.5 \qquad 0 < x < 2 \tag{1}$$

$$p_2(x) = e^{-x} \qquad x > 0 \tag{2}$$

$$p_3(x) = \frac{e^{-x/2}}{2(1 - e^{-1/2})} \qquad 0 < x < 2 \tag{3}$$