Assignment 6. ODE, PDE & Eigenvalue Marks 15

Posted on 13.11.2022 and due on 24.11.2022 midnight

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1. Use RK4 to solve the damped and forced harmonic oscillator satisfying

$$2\ddot{y} + \gamma\dot{y} + 2y = 2\cos(\omega t), \quad \text{where } y(0) = 2,\ \dot{y}(0) = -1$$

for $\gamma = 0.2$ and $\omega = 1.2$. **[4]**

2. Equation for heat conduction in a thin, un-insulated rod of length $L = 10$ m is

$$\frac{d^2T}{dx^2} + \alpha(T_a - T) = 0$$

where the heat transfer coefficient $\alpha = 0.01\,\mathrm{m}^{-2}$ parameterizes heat dissipated to the surrounding air and $T_a = 20^o\,\mathrm{C}$ is the ambient temperature. If $T(x = 0) = 40^o\,\mathrm{C}$ and $T(x = L) = 200^o\,\mathrm{C}$, solve the boundary value problem using *Shooting Method* with *RK4* integrator and determine at what x the temperature is $T = 100^o\,\mathrm{C}$. **[4]**

3. Solve the 1-dimensional heat equation $u_{xx} = u_t$ over a conducting bar, of length 2 units, kept at $0^o\,\mathrm{C}$ but is heated to $300^o\,\mathrm{C}$ at its center at time $t = 0$. Choose your Δx and Δt with care such that $\Delta t/(\Delta x)^2 \ll 0.5$. **[4]**

4. Find the dominant eigenvalue and its corresponding normalized eigenvector to a precision of 10^{-3} for the matrix given below. How many iterations does it take to achieve this precision? **[3]**

$$\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{pmatrix}$$