Expt3: Diffraction of light due to ultrasonic wave propagation in liquids

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1 Objectives

1. To study the diffraction of light due to propagation of ultrasonic wave in a liquids
2. To determine the speed of sound in various liquids at room temperature
3. To determine the compressibility of the given liquids

2 Apparatus

- Radio frequency oscillator fitted with a frequency meter
- Quartz crystal slab fitted with two leads
- Spectrometer
- Glass cell with Turpentine oil
- Sodium lamp
- Spirit level

3 Theory

Acoustic waves in **closed** liquids cause periodic high and low density regions in them. The spacing between high-density and low-density regions of ultrasonic waves in the MHz range is similar to the spacing in diffraction gratings. Since these density changes in liquids will cause changes in the index of refraction of the liquid, it can be shown that parallel light passed through the exciting liquid will be diffracted much as if it had passed through a grating. The experiment can be used as an indirect method to measure the velocity of sound in various liquids. The phenomenon of interaction between light and sound waves in a liquid is called the **DebyeSears effect**.

The successive separations between two compressions or rarefactions are equal to the wavelength of the ultrasonic wave, λ_u in the liquid. Due to reflections at the sides of the tank or the container, a stationary wave pattern is obtained with nodes and antinodes at regular intervals. In these periodic high and low density regions, the liquid acts as a diffraction grating.

If λ_u denotes the wavelength of sound in the liquid, λ the wavelength of incident light in air and θ_n is the angle of diffraction of n^{th} order, then we have:

$$d \sin\theta_n = n\lambda$$

Here, the liquid acts as the diffraction grating. So, here, the $d(= (e+b) = \frac{1}{N})$ becomes equal to λ_u. Thus:

$$\lambda_u \sin\theta_n = n\lambda$$

Now, if ν is the frequency of the crystal, the velocity V_u of the ultrasonic wave in the liquid will be:

$$V_u = \nu\lambda_u$$

Thus, the velocity of the ultrasonic wave in the liquid can be calculated from the diffraction pattern. The velocity of sound in the liquid can be calculated from the velocity of the ultrasonic wave in the liquid. The velocity of sound in the liquid is given by:

$$V_u = \frac{n\lambda\nu}{\sin\theta_n} \tag{1}$$

3.1 Compressibility of Liquid (K)

The speed of sound depends on both an inertial property of the medium (to store kinetic energy) and an elastic property (to store potential energy).

$$V_u = \sqrt{\frac{elastic\ property}{inertial\ property}}$$

For a liquid medium, the bulk modulus E accounts for the extent to which an element from the medium changes in volume when a pressure is applied.The bulk modulus is a measure of the compressibility of the liquid. The bulk modulus of a liquid is given by:

$$E = -\frac{\Delta p}{{}^{\Delta V}\!/_{V}}$$

Here, Δp is the change in pressure and $\Delta V/V$ is the fractional change in volume. The sign of Δp and $\Delta V/V$ is opposite. So, the bulk modulus is negative. The unit of E is Pascal (N/m^2) therefore the speed of sound in a liquid is given by:

$$V_u = \frac{n\lambda\nu}{\sin\theta} = \sqrt{\frac{E}{\rho}}$$

$$\therefore E = V_u^2\rho = \frac{1}{K} \tag{2}$$

Here, K = compressibility of the liquid,
ρ = density of the liquid,
E = bulk modulus of the liquid

4 Observations

Least Count of Spectrometer = $\frac{1}{60} = 1'$
Frequency of the Vibrating Crystal = 3.99 MHz
Density of the Liquid (turpentine oil) = 0.865 g/cm
Central Maxima position = 86.5° and 266.57° on two different scales

Order		Left (°)		Right (°)		a-b	$a' - b'$	mean 2θ	$\theta(°)$	V (m/s)
		a	a'	b	b'					
Set 1	2	86.33	266.40	86.85	266.70	0.52	0.30	0.41	0.21	1313.67
	1	86.60	266.50	86.70	266.60	0.10	0.10	0.10	0.05	2693.03
Set 2	2	86.30	266.40	86.85	266.70	0.55	0.30	0.43	0.21	1267.31
	1	86.60	266.50	86.70	266.60	0.10	0.10	0.10	0.05	2693.03
Set 3	2	86.20	266.40	86.85	266.70	0.65	0.30	0.48	0.24	1133.91
	1	86.60	266.50	86.70	266.60	0.10	0.10	0.10	0.05	2693.03

Table 1: Angular positions of the different fringes

Order	V_u (m/s)			Mean V_u (m/s)
	Set 1	Set 2	Set 3	
2nd order	1313.67	1267.31	1133.91	1238.30
1st order	2693.03	2693.03	2693.03	2693.03

Table 2: Velocity mean calculations

5 Calculations

The velocity of the sound waves in the liquid and the bulk modulus of the liquid are calculated from the 1st and 2nd order diffraction fringes formed due to the sound waves in the liquid.

5.1 Velocity of Wave:

The Mean velocity of waves through the liquid (turpentine oil) is calculated from the Equation 1:

For 1st order fringe:

$$V_u = \frac{n\lambda\nu}{\sin\theta_n} = \frac{1 \times 589\ nm \times 3.99\ MHz}{\sin(0.05°)} = 2693.03\ m/s$$

Similarly, For 2nd order fringe:

$$V_u\ (for\ \theta_n = 0.2050) = 1313.67\ m/s,$$

$$V_u\ (for\ \theta_n = 0.2125) = 1267.31\ m/s$$

and, $$V_u\ (for\ \theta_n = 0.2375) = 1133.91\ m/s.$$

5.2 Bulk Modulus of Elasticity (and Compressibility):

The literature value for the Bulk modulus of elasticity for turpentine oil is $1.28 \times 10^{9} N/m$ obtained from [LibreTexts, 2022]. The value is calculated from the Equation 2:

For 1st order fringe:

$$E = V_u^2 \rho = (2693.03)^2 \times 865 = 6.27 \times 10^{9} N/m$$

$$K = \frac{1}{6.27 \times 10^9} = 0.16\ nm/N$$

For 2nd order fringe:

$$E = V_u^2 \rho = (1238.3)^2 \times 865 = 1.32 \times 10^{9} N/m$$

$$K = \frac{1}{1.32 \times 10^9} = 0.75\ nm/N$$

6 Error Analysis

For error analysis, we use the following methods:

Least count of spectrometer $(\Delta\theta) = 10^{'} = 2.91 \times 10^{-}3\ rad$

6.1 Error in Velocity of Sound Waves:

The corresponding errors in the velocity for corresponding orders can be evaluated as follows:

We differentiate Eqn 1 with respect to θ_n. Thus we get:

$$\Delta V_u = V_u \cot\theta_n \ \ \Delta\theta_n \tag{3}$$

Using Eqn 3 for 1st order fringes we get:

$$\Delta V_u = -(2693.03 \times \cot 0.05^\circ \times 2.91 \times 10^{-}3)\ m/s = -8980.2\ m/s$$

And for 2nd order fringes:

$$\Delta V_u = -(1238.30 \times \cot 0.21^\circ \times 2.91 \times 10^{-}3)\ m/s = -983.15\ m/s$$

Thus we obtain the velocities to be:

$$V_u\ (for\ 1st\ order) = 2693.03 \mp 8980.2\ m/s$$

$$V_u\ (for\ 2nd\ order) = 1238.30 \mp 983.15\ m/s$$

6.2 Error in Calculating Bulk Modulus:

The corresponding errors in the bulk modulus for corresponding orders can be evaluated as follows:

We differentiate Eqn 1 with respect to V_u. Thus we get:

$$\Delta E = 2V_u \, \rho \, \Delta V_u \tag{4}$$

Using Eqn 4 for 1st order fringes we get:

$$\Delta E = 2 \times 2693.03 \times 865 \times 8980.2 = 41.8 \times 10^9 \, {}^N/_m$$

And for 2nd order fringes:

$$\Delta E = 2 \times 1238.3 \times 865 \times 983.15 = 2.1 \times 10^9 \, {}^N/_m$$

Thus we obtain the velocities to be:

$$E \; (for \; 2nd \; order) = 6.27 \pm 41.8 \times 10^9 \, {}^N/_m$$

$$E \; (for \; 1st \; order) = 1.32 \pm 2.1 \times 10^9 \, {}^N/_m$$

7 Conclusion

We see that our values for the 1st order fringes vary a lot from the literature values mentioned in the [SPS, 2022]. On the other hand, it is quite close to the literature values for the 2nd order fringes. This is because the 1st order fringes are much closer to the center of the diffraction pattern, and thus the error in the measurement of the angle is much more. The 2nd order fringes are much farther from the center, and thus the error in the measurement of the angle is relatively low. This is also the reason why the 1st order fringes are much more sensitive to the error in the measurement of the angle.

The 1st order fringes being close to the central maxima, are more affected by **backlash error** in the device. The 2nd order fringes being farther from the central maxima, are less affected by backlash error in the device. Hence those values are more accurate.

In terms of observation, I think we can improve the experiment by using digital meters for measuring the angles in spectrometer, that shall decrease much of the random error.

We should use devices with lesser values of least count, so that the error in the measurement of the angle is less. To do this, we can use spectrometers with larger diameters. We used a spectrometer with a larger diameter for **Experiment 2**, and the error in the measurement of the angle was much less.

References

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