Millikan Oil Drop Experiment

Aritra Mukhopadhyay
(Roll. No.: 2011030)

1 Abstract

Millican's Oil Drop Experiment is used to determine the basic electric charge, or charge on an electron. We confirmed the quantization of charge in integer multiples of charge on an electron e in the experiment. The capacity to regulate, change, and balance extremely tiny forces of the order of 10-14 N is required for this experiment. In this experiment, small oil droplets from an atomizer are sprayed and allowed to fall between two plates, between which a potential difference may be used to provide force to the charged droplets owing to the electric field. The droplets become charged as they pass through the atomizer owing to frictional forces. Now, the droplet of appropriate size and charge is chosen and studied for forces under two conditions: The Dynamic method and Balancing method; to find their radii and charges.

2 Experimental Setup

The experimental apparatus, as shown in the picture, comprises of an oil drop chamber with parallel plate electrodes at a 5mm spacing utilising a thick ebonite ring with a hole near the centre for seeing the droplets. A gadget lights the compartment, and a hole on the top plate permits droplets from the atomizer to enter. A CCD camera attached to a monitor may be used to see the droplets. The upper plate voltage may be regulated and measured with a voltage supply, while the lower plate is grounded. The free fall and rising times are calculated using a time metre.

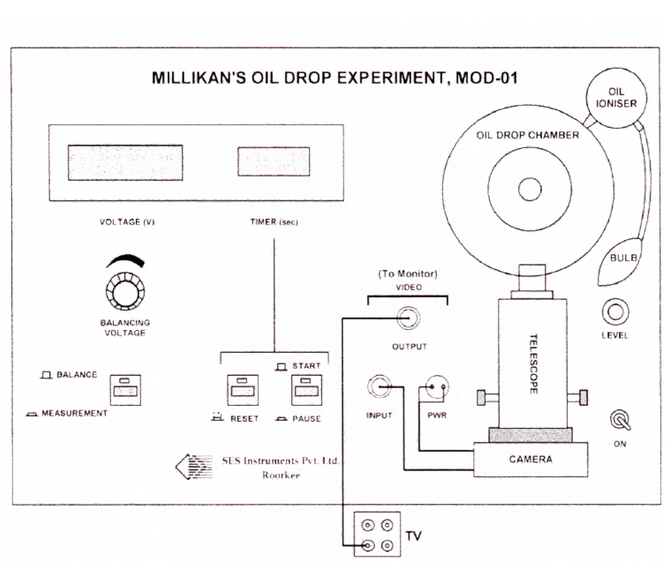

Figure 1: Top View of the Panel

3 Theory

To begin, the droplets are free to fall in the gravitational field. Because the droplet is also being forced upon by the viscous force caused by air particles, it will balance its weight if the droplet is tiny enough. The masses and radii of the droplets may be calculated by solving for the forces. The potential difference is now activated, and the negatively charged droplets begin to move upwards towards the positively charged top plate, while the positively charged droplets accelerate downwards. If the size and charge of the droplets allow, the negative droplet should reach its terminal velocity as the electric and viscous forces balance the droplet's weight. The charge on the droplet may now be calculated using the early discovered radii values. To reduce random errors, measurements are repeated using the same droplet. A droplet of small size and low charge should be found for the experiment.

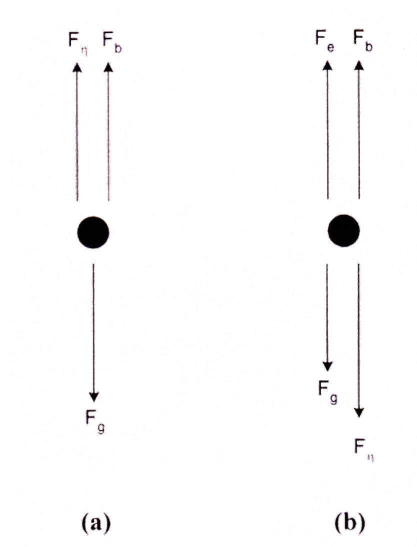

Figure 2: **(a) Dynamic Method, (b) Balancing Method**

Consider a droplet of radius r travelling with a terminal fall velocity of v in the absence of potential. The density of oil is ρ, while the density of air is ρ_a. As a result, the forces can be balanced as follows:

$$\frac{4\pi(\rho - \rho_a)r^3 g}{3} = 6\pi r\eta v_f$$

where the left-hand term equals the net downward force after removing the buoyant force due to air from the gravitational force, and the right-hand term equals the drag force by Stokes law and is the coefficient of viscosity of air.

$$\therefore v_f = \frac{2gr^3(\rho - \rho_a)}{9\eta}$$

Now, if the droplet is carrying charge ne and the potential difference between the plates at a distance d is V, then the force on the droplet due to the electric field is given by: $\frac{Vne}{d}$. Balancing the forces for a terminal rise velocity of v_r:

$$\frac{4\pi(\rho - \rho_a)r^3 g}{3} + 6\pi r\eta v_r = \frac{Vne}{d}$$

Using the equation for v_f, we get for Dynamic method:

$$ne = \frac{4d\pi(\rho - \rho_a)r^3 g}{3V}\left(1 + \frac{v_r}{v_f}\right) \tag{1}$$

The assumptions taken for the following calculations include:

- The droplets are falling slowly.
- There is no slipping between the droplet and the medium.
- The extent of the medium is large compared to the droplet size.
- The medium is homogeneous compared to the droplet size.

The final assumption does not apply in this situation because the mean free path of the air molecules in the medium is bigger than the size of the droplets, implying that the droplets would travel differently in that region than the rest, requiring an adjustment in the free fall velocity using kinetic theory.

$$v_f = \frac{2gr^2(\rho - \rho_a)}{9\eta}\left(1 + \frac{c}{Pr}\right)$$

where c ($= 6.17 \times 10^-8$m of Hg-m) is the correction factor and P is the atmospheric pressure in m of Hg. Now, replacing $\frac{2\eta v_f}{2g(\rho-\rho_a)}$ by ξ and $\frac{c}{2P}$ by ς:

$$r^2 + 2rc - \xi = 0$$

$$\therefore r = -\varsigma + \sqrt{\varsigma^2 + \xi} \tag{2}$$

For the Balancing method where potential (V_b) is controlled to reduce the rise velocity of the droplet to zero, the charge of the droplet is given as:

$$ne = \frac{4\pi dr^3 g(\rho - \rho_a)}{3V_b} \tag{3}$$

4 Observation

- Distance d between the plates = 5mm
- Distance L between the second line and second last line chosen on the monitor = 1mm
- Density of oil $\rho = 929Kgm^{-3}$
- Density of air $\rho_a = 1Kgm^-3$
- Room Temperature $T = 24°C$
- Atmospheric Pressure $P = 0.76Hg - m$
- Viscosity of air $\eta = 1.8432 \times 10^{-5} Kgm^{-1}s^{-1}$

Drop no	Sl no	Free Fall time (s)	Rise time (s)	Mean Free fall time	Mean Rise time	Mean free fall velocity ($v_f = {}^{L}/_{t_f} \times 10^5$) ($ms^{-1}$)	Voltage (V)
1	1	9.9	16.1	10.12	15.1	9.881	378
	2	10.4	14.6				
	3	10.3	14.8				
	4	10.3	14.2				
	5	9.7	15.8				
2	1	8.6	13.7	8.56	13.84	11.682	450
	2	8.1	13.9				
	3	8.7	14.9				
	4	9	13.3				
	5	8.4	13.4				
3	1	10.3	7	10.14	6.4	9.862	343
	2	10	6.4				
	3	10.1	6.2				
	4	10.1	6.2				
	5	10.2	6.2				
4	1	11.4	7	11.26	6.84	8.881	500
	2	11.2	6.6				
	3	11.3	6.8				
	4	11	7.1				
	5	11.4	6.7				
5	1	10.6	2.5	10.842	2.5	9.223	484
	2	11.2	2.3				
	3	10.9	2.4				
	4	11.11	2.6				
	5	10.4	2.7				
6	1	19.7	8.9	19.92	8.36	5.020	400
	2	20.4	7.7				
	3	18.9	8.3				
	4	20.4	8.7				
	5	20.2	8.2				
7	1	16.2	10.6	16.54	9.26	6.046	410
	2	16.8	9.9				
	3	15.8	9.6				
	4	16.9	8.1				
	5	17	8.1				

Table 1: Dynamic Method Data

Drop no	Sl no	Free Fall time (s)	Mean Free fall time (t_f) (s)	Mean free fall velocity ($v_f = {}^{L}/_{t_f}$ $\times 10^5$ (ms^{-1})	Balancing voltage(V)	Voltage (V)
1	1	4.9	4.64	0.0002		238
	2	4.6			238	
	3	4.5			238	
	4	4.4			238	
	5	4.8			238	
2	1	11.5	11.275	9E-05		306.3333
	2	11.2			306	
	3	11.3			306	
	4	11.1			307	
3	1	10.6	10.125	1E-04		230
	2	10.3			230	
	3	9.8			229	
	4	9.8			231	
4	1	15.7	15.15	7E-05		360
	2	14.7			360	
	3	15.3			359	
	4	14.9			361	
5	1	13.6	13.525	7E-05		264
	2	13.1			266	
	3	13.9			264	
	4	13.5			262	
6	1	13.8	13.95	7E-05		407.6667
	2	14.1			407	
	3	13.7			409	
	4	14.2			407	

Table 2: Balancing Method Data

5 Calculations

$C = \frac{4\pi dg(\rho-\rho_a)}{3} = 190.13$
$D = \frac{9\eta}{2g(\rho-\rho_a)} = 9.04 \times 10^{-9}$
$\varsigma = \frac{c}{2P} = 4.06 \times 10^{-8}$
$c = 6.17 \times 10^{-8}$

Droplet Number	Voltage (V)	$\eta(x10^14)$ (m^2)	r (10^{-8})	r^3 (10^{-19})	$T = 1 + \frac{t_f}{t_r}$	$ne \times 10^{-19}$
1	378	89.32	90.54	7.42	1.67	6.23
2	450	105.60	98.78	9.64	1.61	6.59
3	343	89.15	90.44	7.39	2.58	10.59
4	500	80.28	85.63	6.27	2.64	6.31
5	484	83.37	87.34	6.66	5.33	13.96
6	400	45.38	63.42	2.55	3.38	4.10
7	410	54.65	69.98	3.42	2.78	4.42

Table 3: Dynamic Method Calculations

Drop no	Voltage (V)	$\eta (x10^14)$ (m^2)	r (10^{-8})	r^3 (10^{-19})	$ne \times 10^{-19}$
1	238.00	194.82	135.57	24.92	19.90
2	306.33	80.17	85.57	6.26	3.88
3	230.00	89.28	90.51	7.41	6.13
4	360.00	59.66	73.29	3.93	2.07
5	264.00	66.83	77.79	4.70	3.39
6	407.67	64.80	76.54	4.48	2.09

Table 4: Balancing Method Calculation

5.1 Analysis and treatment of the charge ne on the droplets

$ne \times 10^{-19}$	ne divided by the lowest	n_{eff}	$e = \frac{ne}{n_{eff}}$ $(\times 10^{-19})$ (c)
6.24	1.52	2.00	3.12
6.59	1.61	2.00	3.30
10.60	2.58	3.00	3.53
6.32	1.54	2.00	3.16
13.97	3.40	3.00	4.66
4.10	1.00	1.00	4.10
4.43	1.08	1.00	4.43
Average value of e			3.76
δe (standard deviation)			0.63

Table 5: Dynamic Method find value of e

$ne \times 10^{-19}$	ne divided by the lowest	n_{eff}	$e = \frac{ne}{n_{eff}}$ $(\times 10^{-19})$ (c)
19.91	9.57	10.00	1.99
3.89	1.87	2.00	1.94
6.13	2.95	3.00	2.04
2.08	1.00	1.00	2.08
3.39	1.63	2.00	1.70
2.09	1.01	1.00	2.09
Average value of e			1.97
δe (standard deviation)			0.15

Table 6: Balancing Method find value of e

6 Results

1. **Dynamic Method:** Value of e was calculated to be:

$$e = (3.76 \pm 0.63) \times 10^{-19} C$$

Error when compared to the literature value of electron charge: 135%

2. **Balancing Method:** Value of e is:

$$e = (1.97 \pm 0.15) \times 10^{-19} C$$

Error when compared to the literature value of electron charge: 23%

7 Discussions

- The value for electron charge has a large error in the dynamic method (135%). The main reason for this is, the lowest value of ne calculated didn't have n=1, it must have been some multiple. Assuming n=2 for the least value will give us $e = 1.88 \times 10^{-19} C$. This result has an error of 17.5%. Which is quite small an error.
- Hence through the experiment we have proved the quantization of charge.
- The large error in the dynamic method is because it requires measurement of more quantities- the rise and fall time, so the errors due to both are factored in the total error, compared to the balancing method where only one measurement is to be made.
- Several droplets were observed changing trajectories or moving in all axes rather than in a straight line, which could not be accounted for in the measurements and must have contributed to the error. While the buoyant forces and weight are always in one direction for good approximations, the viscous force and electric force will depend on the horizontal levelling of the plates with the cylinder. As a result, the levelling should be corrected ahead of time to produce electric force in the same direction as mg.
- The droplet should not be lost sight of during measurements. The droplet may be observed as reducing in size or disappearing due to movement along other axes and can be controlled by refocusing on it with the telescope.

References

[SPS, 2022] SPS (2022). Lab manual. *Website*. `https://www.niser.ac.in/sps/sites/default/files/basic_page/Milican's%20Oil%20Drop%20Experiment.pdf`.