FABRY-PEROT INTERFEROMETER

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1 Abstract

A Fabry-Perot interferometer was used in the experiment as an etalon to measure the separation between the plates and as a stereoscopic tool to determine the diode laser's wavelength. It makes use of the phenomenon of multiple beam interference to create circular fringes with an equal inclination because the path differences of the interfering beams are the same for beams with an equal inclination. They have an interference pattern that is noticeably sharper than a Michelson interferometer due to multiple reflections and shared optical paths.

2 Theory

2.1 Fabry-Perot Interferometer

The Fabry-Perot Interferometer utilizes the phenomena of interference of multiple beams of light coming from a source that interferes with a screen. The wavelength of the light can be determined from the circular interference fringes of equal inclination formed on the screen. Other than that the interferometer is also used to find small differences in different wavelengths of light coming from a source, etc. The light coming from the source is bro- ken down into multiple beams as it continuously reflects between the high reflecting surface of two etalon plates while transmitting a portion of it through every reflection. The fringes are formed due to interference between these two beams when they are focused on a screen using a convex lens.
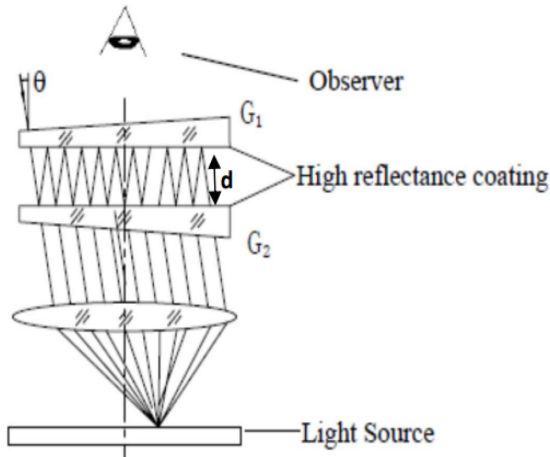

Figure 1: Schematics of a Fabry-Perot Interferometer

The components of the beam emerge parallel from G_2

at an angle of inclination theta with respect to the plate's surface, which is equal to the angle of inclination of the original beam striking the mirror G_1, as shown in the Figure 1. The convex lens's focal plane, where the parallel rays are now focused, is on the screen. Because the angle of the rays determines where the interference occurs. The corresponding interference regions from a circular are seen for the rays with the same inclination on the plates in all directions. These fringes are known as fringes of equal inclination because each circular figure corresponds to a specific inclination. The interference pattern exists due to the path difference (Δ) any two interfering rays of wavelength (λ), given by the equations:

- Constructive Interference: $\Delta = n\lambda$, $[n \in \mathbb{Z}]$

- Destructive Interference: $\Delta = \frac{(n+1)\lambda}{2}$, $[n \in \mathbb{Z}]$

Each interference maximum is caused by a corresponding ray path difference that depends on inclination and is therefore specific to each fringe. T the following path difference between two interfering beams (see Figure 2) can be calculated, one from direct transmission through G_1 and the other from two successive reflections in the air gap:
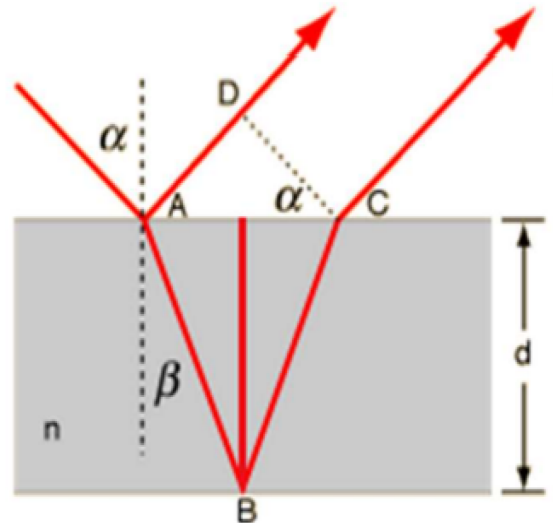

Figure 2: Calculation of Path Difference

In the Figure 2 the the reflected wavefront from C interferes with the wavefront from C. Hence the path difference between the two interfering beams is given by $\Delta = n(AB + BC) - AD$ [$n(\approx 1$ for air) is the refractive index of the medium]. d is the distance between the two etalons and β is the angle of incidence of the beam on the

second etalon. On putting the values of the variables in the equation AB, BC and DC in terms of β, we get:

$$\Delta = 2nd\cos\beta$$

2.2 Determination of Wavelength (λ):

For some light rays with the same inclination, the path difference changes as the distance between the plates changes. A maxima will be seen for a lower inclination for an increased d, while maxima for a decreasing d will vanish. The change in path difference will equal to an integer multiple of the wavelength of light for each recurrence of maxima at the same inclination. As a result, for small values of beta, the wavelength is given by (from the constructive interference formula):

$$\lambda = \frac{2(d_2 - d_1)}{m_2 - m_1} = 2 \times slope \tag{1}$$

Where d_2 and d_1 are the final and initial position of the fine micrometer respectively used for adjusting d; and $(m_2 - m_1)$ is the number of fringes appearing or disappearing at the centre, that is inclination of $0°$.

2.3 Determination of distance between the plates (d):

We know $\Delta = 2d\cos\theta_m$ for θ_m being the inclination of the m-th fringe. Therefore using small angle approximation:

$$\theta_m = \frac{r_m}{D}$$

here, r_m is radius of the m-th fringe and D is the distance between the etalon and the screen. For the m-th fringe, the path difference $\Delta = m\lambda$ implies:

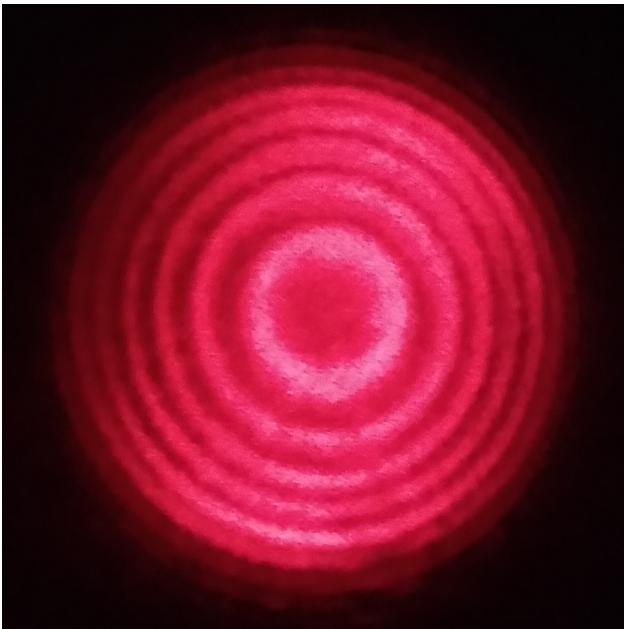$$d = \frac{m\lambda}{2\cos\theta_m} \tag{2}$$

Figure 3: Circular fringes

3 Observation, Calculations and Error Analysis

Least Count of Main Scale $= 0.5mm$
Number of divisions on circular scale $= 50$
Least Count of fine micrometer $= 0.01mm$

3.1 Determination of λ for diode laser:

Initial position of the micrometer screw $= 17.00mm$
here $D = (msd + 0.01 \times vsd)$ mm ($\because$ least count $= 0.01$mm)

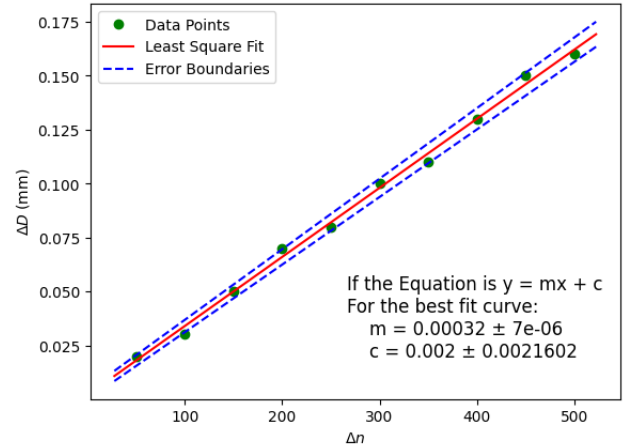| No. of fringes changed Δn	msd	vsd	D (mm)	ΔD (mm)
50	17	2	17.02	0.02
100	17	3	17.03	0.03
150	17	5	17.05	0.05
200	17	7	17.07	0.07
250	17	8	17.08	0.08
300	17	10	17.1	0.1
350	17	11	17.11	0.11
400	17	13	17.13	0.13
450	17	15	17.15	0.15
500	17	16	17.16	0.16

Table 1: For Laser

Figure 4: ΔD vs m graph for Laser Diode

From the graph, we can see that the slope of the graph is $(320 \pm 7)nm$, and the intercept is negligible under the error purview.

Thus, $\lambda = 2 \times slope = 2 \times 320nm$ [from Eqn 1]
Error Estimation:
From Slope we get: $\Delta\lambda = \pm2 \times 7nm = \pm7nm$
Calculating Standard Deviation of λ calculated with each data point of Table 1 we get: $\Delta\lambda = \pm54.24nm$

$$\boxed{\therefore \lambda = 640 \pm 54.24nm}$$

(**Note:** The uncertainty due to the least count of the micrometer screw gauge is taken care of in the calculation of the error in slope of the Graph 1. Although we did not

plot the error bars, we have taken that into account while calculating the error in the slope.)

3.2 Determination of separation between the etalon plates

$D = 34.5cm$

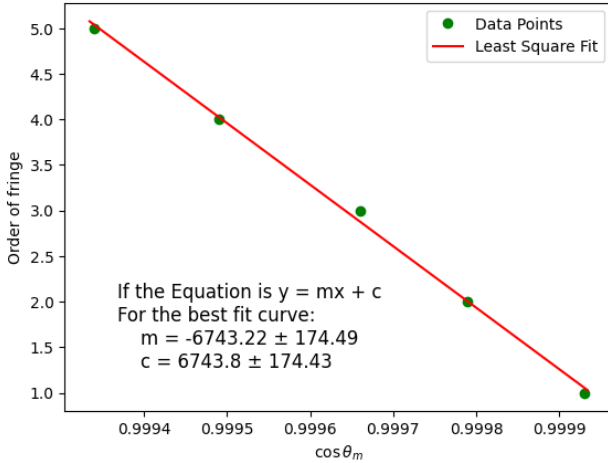| Order	a (cm)	b (cm)	R $\frac{b-a}{2}$ (cm)	θ_m	$\cos\theta_m$
5	8.5	11.0	1.3	0.036	0.99934
4	8.7	10.9	1.1	0.032	0.99949
3	8.9	10.7	0.9	0.026	0.99966
2	9.0	10.4	0.7	0.020	0.99979
1	9.4	10.2	0.4	0.012	0.99993

Table 2: For separation between etalon plates

Figure 5: $\cos\theta_m$ vs Order of fringe

from Graph 2, we can see that the slope of the graph is $m = 6743.22 \pm 174.5$ (unitless).

So, from Eqn 2, we get:

$$d = \frac{s\lambda}{2} = \frac{6743.22 \times 640}{2}nm = 2.16mm$$

Error Estimation:
Differentiating the above equation wrt d, we get:

$$\frac{\Delta d}{d} = \frac{\Delta s}{s} + \frac{\Delta\lambda}{\lambda}$$

$$\therefore \Delta d = 0.11 \times 2.16mm = 0.24mm$$

$$\boxed{\therefore d = 2.16 \pm 0.24mm}$$

4 Result Declaration

- Wavelength: $\lambda = 640 \pm 54.24nm$

- Distance between Etalon plates: $d = 2.16 \pm 0.24mm$

5 Discussion

- **Absence of Equipment:** We completed the whole experiment in a single day. We were not provided with a measuring tape that day. So to measure the value of D (during measurement of the distance between the etalon) we used a normal 30cm metal scale. We can see that the value of D is more than 30cm. So we had to put the scale twice which can lead to a lot of errors. On top of that, it was a metal scale in an AC room (contracts and expands with temperature change). Fortunately, this error leads to decreased accuracy but does not affect the result's precision. To support this claim, let's calculate the d for every data point wrt the intercept value obtained from Graph 2.

Order	$\cos\theta_m$	$\frac{6743-order}{\cos\theta_m}$
5	0.99934	6742.425
4	0.99949	6742.427
3	0.99965	6742.294
2	0.99979	6742.388
1	0.99993	6742.453
standard deviation =		0.0623

We can clearly see that the standard deviation of the slopes is pretty low ($\approx 0.0009\%$) wrt to the slope. Hence, this error just results in reduced accuracy but does not affect the precision of the result.

- **Error in Observations:** We assumed the value of m (number of fringes that appeared or disappeared) to be absolute while calculating the error. This is because uncertainty in calculating N can be solely due to random error. We cannot be sure about how much the value of Δm can be. **Reported Error consists of only Systematic Error.** Other Sources of the error have not been calculated or estimated.

References

[SPS, 2022] SPS (2022). Lab manual. *Website*. https://www.niser.ac.in/sps/sites/default/files/basic_page/Fabry%20Perot%20Interferometer.pdf.