Emission spectra of Hydrogen and determination of Rydberg constant

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October 10, 2022

1 Abstract

An excited hydrogen electron emits energy when it returns to lower energy states. This energy is released in the form of light waves. From the wavelength of the light emitted, we can calculate the energy difference between it's energy levels.

1.1 Objective

1. To meaasure the wavelengths of visible spectral lines in Balmer series of atomic hydrogen.

2. To determine the value of Rydberg's constant from the wavelengths

1.2 Apparatus

- Spectrometer

- Diffraction grating

- Hg Lamp

- Hydrogen Lamp

- 60° Prism

2 Theory

Hydrogen is the simplest atom. It has only one electron which revolves around a single proton (the natural abundance of deuterium and tritium is very low). As a result, it is the best atom to study for beginners. When this electron gets some energy, it gets excited to higher energy states. Now, when this electron returns to lower energy states, it emits the energy back in the form of electromagnetic waves. The energy carried by this wave is equal to the energy difference between the two states. Different energies correspond to different wavelengths of electromagnetic waves. It has been experimentally observed that when the electron jumps from a higher energy state to the second energy state of the Hydrogen atom, the light emitted falls in the visible range of the human eye. These lights were first seen by **Swiss mathematician Johann Jakob Balmer**. That is why they are called Balmer lines of Hydrogen.

Now, according to Bohr's model of the Hydrogen atom, the wavelengths of spectral lines are given by

$$\frac{1}{\lambda} = R_y \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \tag{1}$$

Here, λ = wavelength released, R_y = Rydberg's constant, $n_1 = 2$ and $n_2 = 3, 4, 5...$

Also, we know that:

$$R_y = \frac{e^4 m_e}{8\epsilon_0^2 h^3 c} = 1.097 \times 10^7 m^{-1}$$

where e is the charge of 1 electron, m_e is mass of one electron, ϵ_0 is permittivity of air $= 8.85 \times 10^{-12}$, h is Planck's constant and c is velocity of light in vacuum.

The spectral beam is a collection of beams of different wavelengths. To study the spectrum, we must split the beam into components. We do that with the help of a diffraction grating. The principle is that if a monochromatic light of wavelength λ falls normally on an amplitude diffraction grating with a periodicity of lines given by g ($=1/N$, where N is the number of grating lines per unit length), the intensity peaks due to principal maxima occur under the condition:

$$g \sin \theta = p\lambda \tag{2}$$

Here θ is the diffraction angle and p is the order of diffraction.
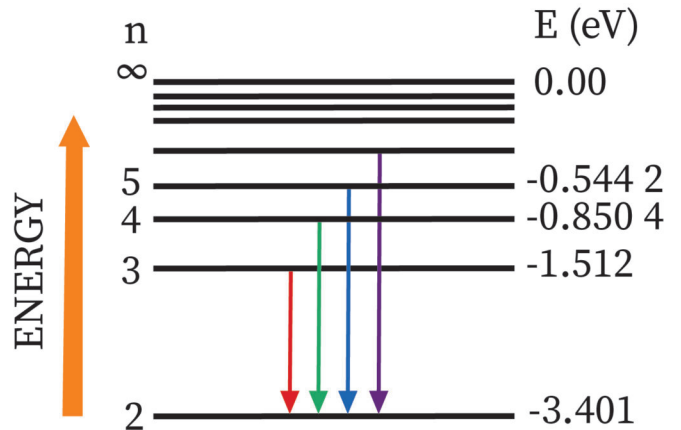

Figure 1: **Balmer series of Hydrogen**

3 Observation

Least Count of Spectrometer $= (1 - \frac{59}{60}) \times \frac{1}{2}^{\circ} = 30''$

Wavelength or Colour (λ nm)	Left Side (°)					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
413	335.5	45	335.875	156.0	0	156.000
488	334.5	4	334.533	154.5	25	154.708
501	332.5	5	332.542	152.5	17	152.642
560	331.0	53	331.442	151.5	20	151.667
588	329.0	48	329.400	149.5	5	149.542
614	328.5	23	328.692	148.5	57	148.975
627	328.0	0	328.000	148.0	2	148.017
637	327.5	15	327.625	147.5	42	147.850

Wavelength or Colour (λ nm)	Right Side (°)					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
413	3.5	41	3.842	183.5	52	183.933
488	4.5	25	4.708	185.0	20	185.167
501	6.5	10	6.583	187.0	0	187.000
560	8.5	27	8.725	188.5	35	188.792
588	9.5	23	9.692	189.5	56	189.967
614	10.5	0	10.500	190.5	2	190.517
627	11.0	22	11.183	191.0	37	191.308
637	11.0	25	11.208	191.0	39	191.325

2θ from V1 (deg)	2θ from V2 (deg)	Average θ (deg)	$\sin\theta$	g (nm)
20.283	20.067	20.175	0.345	1197.487
20.758	20.125	20.442	0.349	1397.265
20.875	20.358	20.617	0.352	1422.834
19.833	19.542	19.688	0.337	1662.264
20.908	20.492	20.700	0.353	1663.484
20.808	20.508	20.658	0.353	1740.390
20.817	20.675	20.746	0.354	1770.071
21.167	20.825	20.996	0.358	1777.839
			Average =	1578.954

Table 1: Determination of g for the grating

From the Table 1 we get **g = 1578.954 nm**.

With that value of g we calculate Table 2:

Colour	Left Side (°)					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
Red	326.0	10	326.083	146.0	30	146.250
Green	332.5	7	332.558	152.5	13	152.608
Violet	334.5	0	334.500	154.5	27	154.725

Colour	Right Side (°)					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
Red	12.5	0	12.500	192.5	13	192.608
Green	6.5	45	6.875	186.5	25	186.708
Violet	4.5	11	4.592	184.5	58	184.983

From Energy State (n_2)	2θ from V1 (deg)	2θ from V2 (deg)	Average θ (deg)	λ (nm)	Rydberg's Constant ($10^9 m^{-1}$)
3	21.417	21.142	21.279	573.039	0.012565
4	20.567	20.683	20.625	556.203	0.009589
5	20.908	20.292	20.600	555.558	0.008571
				Average =	0.010242

Table 2: Determination of spectral lines of hydrogen

From the Table 2 we got: the value of Rydberg's constant is:

$$R_y = 1.0242 \times 10^7 m^{-1}$$

4 Error Analysis

we know for Table 1 that:

$$g = \frac{p\lambda}{\sin\theta} \qquad (p = 1 \ and \ \Delta\lambda = 0)$$

$$\therefore \Delta g = -p\lambda \cot\theta \ cosec \ \theta \ \Delta\theta$$

$$\therefore \frac{\Delta g}{g} = -\cot\theta\Delta\theta$$

$\therefore \Delta g = g\cot\theta\Delta\theta$ [taking $\theta = 20.503°$ (average θ)]
$= 1.579 \times 10^{-6} \times 2.674 \times 1.45 \times 10^{-4} = 6.1 \times 10^{-10}$

$$\boxed{\therefore g = 1578.95 \pm 0.61 \ nm}$$

Differenciating Eqn 2 we get:

$$\Delta\lambda = g\cos\theta'\Delta\theta' + \sin\theta'\Delta g$$

$$\frac{\Delta\lambda}{\lambda} = \cot\theta'\Delta\theta' + \frac{\Delta g}{g}$$

$$\frac{\Delta\lambda}{\lambda} = (\cot\theta' - \cot\theta)\Delta\theta$$

taking $\theta' = 20.835°$ (average θ),
$\Delta\theta = \Delta\theta' = \frac{30}{60^2}^{\circ} = 1.45 \times 10^{-4} rad$
$g = 1.579 \times 10^{-6} \ m$

$$\frac{\Delta\lambda}{\lambda} = 6.74 \times 10^{-6}$$

Now differenciating Eqn 1 we get:

$$\frac{\Delta R_y}{R_y} = -\frac{\Delta\lambda}{\lambda} = 6.74 \times 10^{-6}$$

$$\boxed{\therefore R_y = (1.0242 \times 10^7 \pm 69) \ m^{-1}}$$

5 Discussion

- The value of the Rydberg constant came out to be pretty close to the literature value of $1.097 \times 10^7 \ m^{-1}$ (error = 6%). Value from from [SPS, 2022].

- The distinct lines in the spectrum were very close to each other. This was inside the backlash error of the spectrometer. This resulted in some odd values in the data (excluded in the report).

References

[SPS, 2022] SPS (2022). Lab manual. *Website.* https://www.niser.ac.in/sps/sites/default/files/basic_page/Balmer_series_manual.pdf.