DIFFRACTION OF LIGHT

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1 Abstract

When the slit width is on the order of the wavelength of light and the separation between the source, slit, and screen is practically infinite, Franhouffer diffraction occurs. By doing so, it is possible to determine the wavelength of the light used, the corresponding slit width, and the wire thickness by examining the minimas close to the central maxima. The phenomenon occurs when a single slit is replaced by a double slit, grating, or both. By differentiating the diffraction and interference minimas, we can determine the width of the slits and opaque region. Through the use of laser light passing through single, double, and thin wire in this experiment, we investigated the Fraunhofer diffraction phenomenon and discovered the aforementioned quantities.

2 Theory

When a coherent light beam is passed through a thin wire or slit with a thickness comparable to the wavelength of the light, a phenomenon known as diffraction occurs. This interference pattern on a distant screen is caused by secondary waves from the emerging beam's wavefront, which have a constant phase difference as they arrive at the screen by travelling different distances. Fraunhofer diffraction is the name given to this type of diffraction, in which the light source, the screen, and the object causing the diffraction are all practically at an infinite distance from one another. The slit width is on par with the light's wavelength. We will investigate Franhouffer diffraction in the experiment, which is caused by diffraction through single, double, and narrow wires.

2.1 Single Slit Diffraction

For single-slit diffraction, the intensity of the transmitted light on the screen is given by:

$$I = I_0 \frac{\sin \beta^2}{\beta^2}$$

Where $\beta = \frac{\pi b \sin\theta}{\lambda}$, b is the width of the slit. For minima $\beta = m\pi$, where m is set of integers. Therefore, the condition for minima is: $m\lambda = b \sin\theta$. For small angle of inclination (for larger distance between the source, slit and screen, $\theta \approx \sin\theta \approx \tan\theta = \frac{x}{2D}$ where x is distance between corresponding m-th minimas on both side of the principle maxima and D is distance between the slit and the screen).

Therefore we get:

$$x = \frac{2Dm\lambda}{b} \tag{1}$$

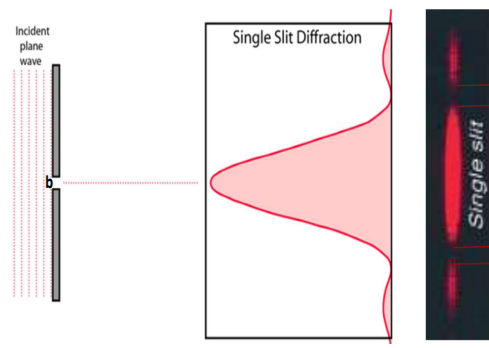


Figure 1: Schematics of Single Slit Diffraction

2.2 Diffraction of thin wire

A similar diffraction pattern to that of a single slit is obtained on the screen when the single-slit is replaced by a wire that blocks light with an equal amount of intensity as the single slit. Therefore, the following is the relationship between the minimas and inclination:

$$x = \frac{2Dm\lambda}{\pi d} \tag{2}$$

The general rule known as **Babinet's principle** is illustrated by the fact that the Fraunhofer diffraction pattern caused by an obstruction is almost identical to that of an opening of the same dimension. This idea can be tested by replacing the wire with a single slit once more and adjusting the slit width until the pattern is perfectly uniform. The slit width and wire thickness can then be compared.

2.3 Double slit Diffraction & Interference

The distance between the centres of the two slits is d(= c+b), and a double slit is made up of two openings of width b and an opaque region between them of width c. The diffraction patterns from the two slits interfere on the screen as the light beam travels through the slits. The following factors determine the intensity of the corresponding Franhouffer pattern:

$$I = I_0 \frac{\sin\beta^2}{\beta^2}\cos\gamma^2$$

$$\beta = \frac{\pi b \sin\theta}{\lambda}$$

$$\gamma = \frac{\pi d \sin\theta}{\lambda}$$

Where the $\sin\beta^2$ represents the diffraction pattern produced by the single-slit and the second term $\cos\gamma^2$ is the characteristic of interference produced by two beams of equal intensity and phase difference γ. The overall pattern, therefore, consists of single-slit diffraction fringes each broken into narrow maxima and minima of interference fringe. The conditions for minima are:

$$d\sin\theta = (p+0.5)\lambda$$

$$b\sin\theta = m\lambda$$

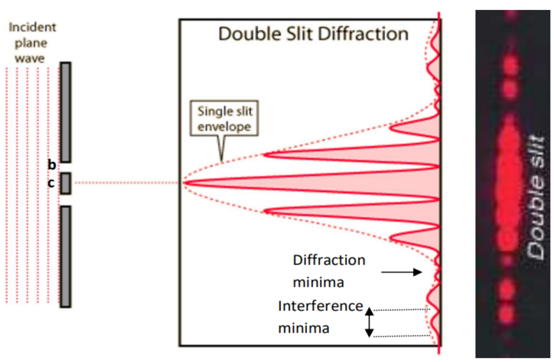
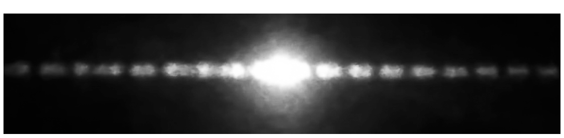


Figure 2: Schematics of Double Slit Diffraction



Instead of calculating the distance between the corresponding minimas on both sides, we will calculate the distance between n minimas on each side and average them because it is difficult to determine the order of the interference and diffraction pattern. Let the distance of $(m+n)$th diffraction minima from the central maxima be x_{m+n} and for m^th minima be x_m, then the distance between the minimas is: $\Delta x_m = x_{m+n} - x_m$. Similarly for the interference minima, the distance between n minimas is: $\Delta x_p = x_{p+n} - x_p$. For large distance the relation between the angle of inclination and the distance is given by:

$$\frac{n}{\Delta x_p} = \frac{d}{D\lambda} \tag{3}$$

$$\frac{n}{\Delta x_m} = \frac{b}{D\lambda} \tag{4}$$

3 Observation & Calculation

3.1 Determination of wavelength λ of the laser light using diffraction pattern

Slit width b = 0.2mm and D = 4m

Order (m)	Left Fringes			Right Fringes			x_m
	α_1^l	α_2^l	α^l	α_1^r	α_2^r	α^r	
1	11	22	16.5	11	22	16.5	16.5
2	22	31	26.5	22	33	27.5	27
3	31	42	36.5	33	44	38.5	37.5
4	42	54	48	44	54	49	48.5
5	54	66	60	54	66	60	60
6	66	78	72	66	74	70	71
7	78	89	83.5	74	86	80	81.75
8	89	99	94	86	97	91.5	92.75

Table 1: For Single Slit Diffraction

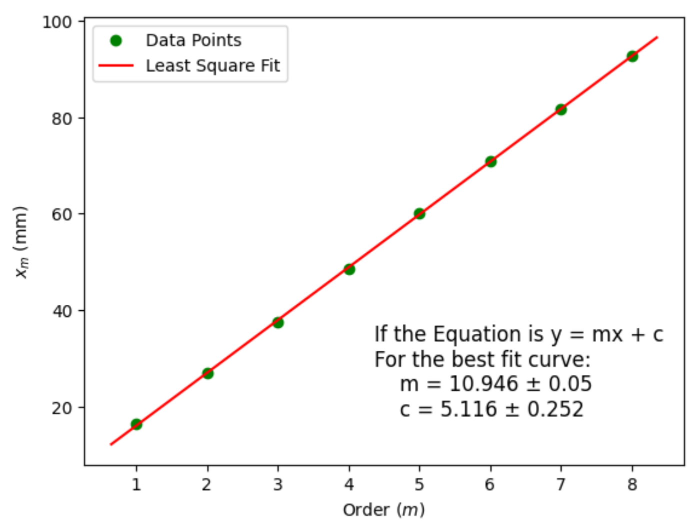


Figure 3: x_m vs m graph for Single Slit Diffraction

3.1.1 Calculation & error analysis

From Graph 1 we see that the slope:

$$m\ (slope) = 10.946mm$$

From Equation 1, we know:

$$m = \frac{2D\lambda}{b}$$

$$\therefore \lambda = \frac{mb}{2D} = 656.76nm$$

$$\delta\lambda = \lambda\sqrt{\left(\frac{\delta D}{D}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta m}{m}\right)^2}$$

$$\delta\lambda = 656.76 \times \sqrt{400^{-2} + 48^{-2} + 219^{-2}}$$

$$\delta\lambda = 14.1nm$$

$$\boxed{\lambda = 656.76 \pm 14.1\ nm}$$

3.2 Finding thickness of thin wire

Thickness b = 0.2mm (using travelling microscope). Distance between the wire and the screen D = 4m.

Order (m)	Left Fringes			Right Fringes			x_m
	α_1^l	α_2^l	α^l	α_1^r	α_2^r	α^r	
1	2	9	5.5	2	9	5.5	5.5
2	9	18	13.5	9	18	13.5	13.5
3	18	27	22.5	18	27	22.5	22.5
4	27	37	32	27	37	32	32
5	37	46	41.5	37	46	41.5	41.5
6	46	55	50.5	46	56	51	50.75
7	55	65	60	56	66	61	60.5
8	65	73	69	66	75	70.5	69.75
9	73	82	77.5	75	84	79.5	78.5

Table 2: Thin Wire Diffraction data

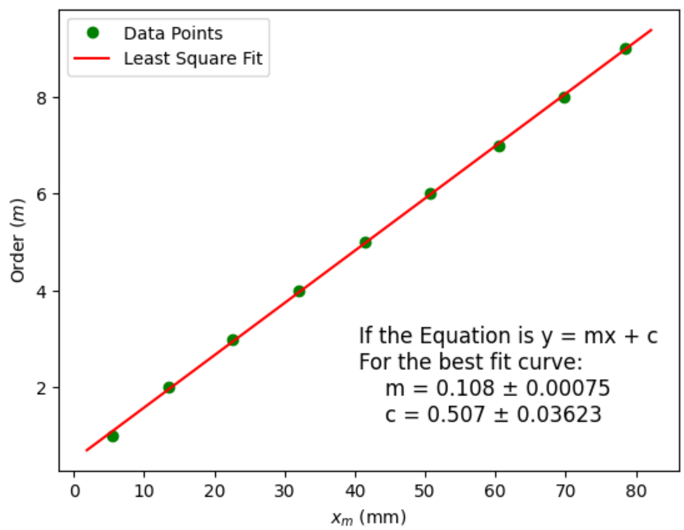


Figure 4: m vs x_m graph for Diffraction of thin wire

3.2.1 Calculation & error analysis

From Graph 2 we see that the slope:

$$m\ (slope) = 0.108mm^{-1} = 108m^{-1}$$

From Equation 2, we know:

$$s = \frac{b}{D\lambda}$$

For $\lambda = 656.76nm$ (found in the last part), we get:

$$b = D\lambda m = 0.283mm$$

$$\delta b = b\sqrt{\left(\frac{\delta D}{D}\right)^2 + \left(\frac{\delta\lambda}{\lambda}\right)^2 + \left(\frac{\delta m}{m}\right)^2}$$

$$\delta b = 0.0065mm\ (2.3\%)$$

$$\boxed{\lambda = 0.283 \pm 0065\ mm}$$

Error wrt to the value obtained from the travelling microscope is 29%.

3.3 Verifying Babinet's Principle

We tried to coincide the fringes with the single slit with that obtained from the wire. Then we measured the slit width with the help of travelling microscope. We got the slit width as 0.22mm.

We find that the single slit width is approximately equal to the wire thickness (0.2mm) for an approximate pattern. This verifies Babinet's principle. (Error = 10.7%)

3.4 Determination of the two slit widths in double slit experiment using diffraction pattern

From Travelling microscope:

Width of first slit (s_1) = $(8.85+0.020)-(8.80+0.04) = 8.870 - 8.840 = 0.030cm$

Width of Second slit (s_2) = $(8.90 + 0.008) - (8.85 + 0.035) = 8.908 - 8.885 = 0.023cm$

$$\therefore b = \frac{0.23 + 0.3}{2} = 0.265mm$$

$$\therefore d = 8.885 - 8.870cm = 0.15mm$$

$$\therefore c = 8.885 - 8.870cm = 0.28mm$$

Diffraction Order (m)	left fringes		Right fringes		x_m (mm)
	α^l	b	α^r	b	
1	14	0.188	14	0.188	14
2	27	0.195	26	0.202	26.5
3	40	0.197	38	0.207	39
4	55	0.191	49	0.214	52
5	70	0.188	65	0.202	67.5
6	82	0.192	78	0.202	80
7	95	0.194	88	0.209	91.5
8	109	0.193			109

Table 3: Double slit diffraction order

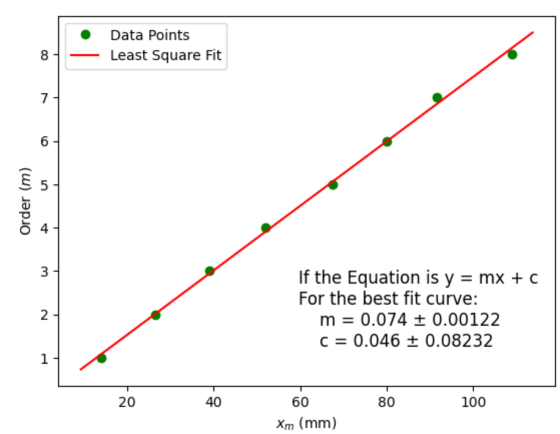


Figure 5: m vs x_m graph for double slit diffraction

From Graph 3 we see that the slope (m) is $0.074mm^{-1} = 74m^{-1}$.

Therefore, from Equation 3, we get:

$$m = \frac{b}{D\lambda}$$

$$b = mD\lambda = 0.19mm$$

Interference Order (p)	left fringes		Right fringes		x_p (mm)
	α^l	b	α^r	b	
1	5	0.525	6	0.438	5.5
2	9	0.584	11	0.478	10
3	14	0.563	14	0.563	14
4	16	0.657	16	0.657	16
5	22	0.597	20	0.657	21
6	27	0.584	26	0.606	26
7	30	0.613	28	0.657	29
8	36	0.584	34	0.618	35
9	40	0.591	38	0.622	39

Table 4: Double slit interference order

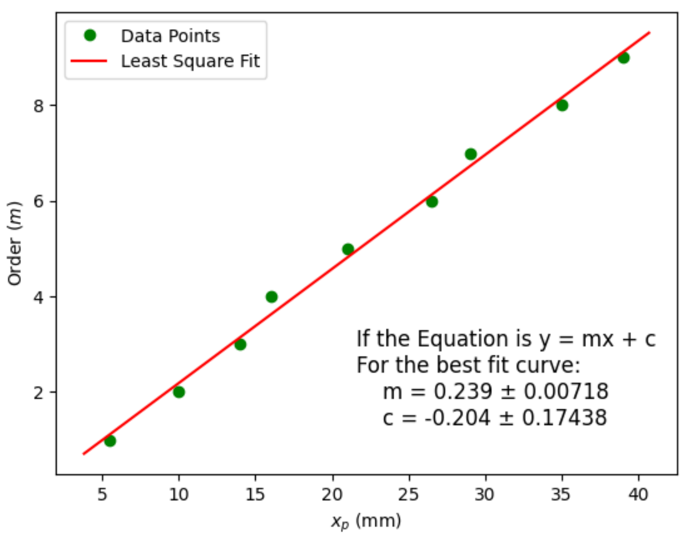


Figure 6: m vs x_p graph for double slit diffraction

From Graph 4 we see that the slope (m) is $0.239mm^{-1} = 239m^{-1}$.

Therefore, from Equation 4, we get:

$$m = \frac{d}{D\lambda}$$

$$d = mD\lambda = 0.63mm$$

from graph and table, $c = d - b = 0.44mm$.

The Error in the value of c when compared with the observed value with the help of travelling microscope (0.28mm) is 36.4%.

4 Discussion

- Compared to the literature value of 632.8 nm, the experimental value of wavelength 656.76 nm lands and error of $\approx 3.7\%$.
- The different errors include backlash error in the travelling microscope while taking measurements for the width of slits, which in the experiment was minimal and can be minised by tightening the screws. Also the microscope has to be levelled so that the slit edges are not tilted with respect to the marker on the lens.
- The other mistake includes parallax when seeing the pattern and writing their position on the graph since observing the pattern directly without interrupting it is not feasible. Also, the image may be unfocussed. The diffraction pattern may also not be perfectly horizontal, in which case the screen must be adjusted or the distance determined trigonometrically.
- Laser should not be viewed directly with the naked eye. The laser should be used with proper safety goggles.
- The pattern will be challenging to obtain if the laser is substituted with a monochromatic sodium lamp source. Because each filament of the sodium bulb acts as a separate source, the beam would be incoherent. Even though only a small piece of the wavefront passes through the slit, temporal coherence is established. Since light is monochromatic, spatial coherence will exist. However, the intensity may not be sufficient enough to generate a pattern at such a great distance. As a result, the coherence time and length are minimal, and obtaining the pattern will be tricky.
- We utilised the position of minimas for the calculation section since the formula simplifies to a simpler form in the case of minimas, but the form of the equation in the case of maximas involves tan theta, which is more difficult to calculate. Furthermore, the human eye is better able to distinguish between low and high intensity areas. As a result, the dark portions are more visible, whilst the brightest intensity zone is not. The larger bandwidth of the maximas prevents them from being used for other calculations.
- **Missing Order:** In the double slit experiment, when the diffraction pattern becomes zero, even if an interference bright fringe should have been observed in that region, its intensity remains zero. This phenomenon is called missing order.

References

[SPS, 2022] SPS (2022). Lab manual. `https://www.niser.ac.in/sps/sites/default/files/basic_page/Fabry%20Perot%20Interferometer.pdf`.

[Wikipedia, 2022] Wikipedia (2022). Helium-neon laser — Wikipedia, the free encyclopedia. `http://en.wikipedia.org/w/index.php?title=Helium%E2%80%93neon%20laser&oldid=1111514847`. [Online; accessed 21-November-2022].