Expt 2: Splitting of sodium D-lines using diffraction grating

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1 Objectives

Measurement of the wavelength separation of sodium D-lines using a diffraction grating and to calculate the angular dispersive power of the grating.

2 Apparatus

- Spectrometer,
- prism,
- diffraction grating,
- sodium lamp with power supply.

3 Theory

3.1 Sodium Spectrum

The sodium spectrum consists mainly of two wavelengths **589.0 nm** and **589.6 nm**. Using an appropriate diffraction grating the wavelength of these two lines can be determined.

3.2 Diffraction Grating

A grating is an arrangement with many small slits of the same width separated by equal opaque spaces known as diffraction gratings. For N parallel slits, each with width e and separated by an opaque space of width b, the diffraction pattern consists of diffraction modulated interference fringes. The quantity $(e+b)$ is called the grating element and N $(=\frac{1}{e+b})$ is the number of slits per unit length, which could typically be 300 to 12000 lines per inch. For a large number of slits, the diffraction pattern consists of extremely sharp (practically narrow lines) principal maxima, together with weak secondary maxima in between the principal maxima. The various principal maxima are called orders.

For polychromatic incident light falling normally on a plane transmission grating, the principal maxima for each spectral wavelength are given by:

$$(e+b)\sin\theta = \pm m\lambda$$

$$\therefore \lambda = \left|\frac{\sin\theta}{m\times N}\right| \tag{1}$$

Where m is the order of principal maximum and θ is the angle of diffraction. Angular dispersive power:

The angular dispersive power of the grating is defined as the rate of change of angle of diffraction with the change in wavelength. It is obtained by differentiating Eqn. 1 and is given by

$$\frac{d\theta}{d\lambda} = \frac{m}{(e+b)\cos\theta} = \frac{m\times N}{\cos\theta} \tag{2}$$

4 Observations

Grating number (N) = 15000 lines per inch
Value of 1 Main Scale Division (MSD) = $20'$
Number of Vernier Scale Divisions (VSD) = 60
Least Count of the Vernier Scale = $20''$

The data readings have been included in the **Table 1** and **Table 2**.

Order	Left Side						
	Vernier 1			Vernier 2			Angle
	MSR	VSR	Total 1	MSR	VSR	Total 2	
1	59	20	59.011	239	14	239.008	20.291
	59	18	59.010	239	12	239.007	20.292
2	35	26	35.014	215	44	215.024	44.281
	35	37	35.021	215	50	215.028	44.276

Table 1: Left Side

Order	Rigth Side						
	Vernier 1			Vernier 2			Angle
	MSR	VSR	Total	MSR	VSR	Total	
1	99.6	50	99.628	279.6	4	279.602	20.315
	99.6	41	99.623	279.6	26	279.614	20.319
2	123.6	7	123.604	303.6	57	303.632	44.318
	123.6	0	123.600	303.6	17	303.609	44.305

Table 2: Right Side

5 Calculations

From the observation tables we can see that the grating number is

$$15000/in = \frac{15000}{0.0254}/m = 590551/m$$

5.1 Wavelength for 1st order fringe:

Average angle $(\theta_1) = (\frac{20.291+20.292+20.315+20.319}{4})^\circ = 20.304^\circ$

So, $\lambda_1 = \frac{\sin\theta_1}{mN} = \frac{\sin 20.304}{1\times 590551} = 5.876 \times 10^-7 = 587.6\ nm$

5.2 Wavelength for 2nd order fringe:

Average angle $(\theta_1) = (\frac{44.281+44.276+44.318+44.305}{4})^\circ = 44.295^\circ$

So, $\lambda_1 = \frac{\sin\theta_1}{mN} = \frac{\sin 44.295}{2\times 590551} = 5.912 \times 10^-7 = 591.2\ nm$

5.3 Difference in the two wavelengths

$$\Delta\lambda = \frac{591.2 - 587.6}{2}\, nm = 3.6\, nm$$

5.4 Angular Dispersive Power of the grating:

$$\left.\frac{d\theta}{d\lambda}\right|_{m=1} = \tfrac{1\times15000}{\cos 20.304} = 15.999 \times 10^3 rad/in$$

$$\left.\frac{d\theta}{d\lambda}\right|_{m=2} = \tfrac{1\times15000}{\cos 44.295} = 20.957 \times 10^3 rad/in$$

6 Error Analysis

For error analysis, we use the following methods:
Least count of spectrometer $(\Delta\theta) = 20^{''} = 9.69 \times 10^-5\ rad$

The corresponding errors in the wavelength for corresponding orders can be evaluated as follows:

We differentiate Eqn 1 with respect to θ and again divide the resultant equation by Eqn 1. Thus we get:

$$\Delta\lambda = \lambda \cot\theta \Delta\theta \tag{3}$$

Using Eqn 3 we get:

$$\Delta\lambda_1 = (587.6 \times \cot(20.304°) \times 9.69 \times 10^-5)\ nm = 0.154\ nm$$

$$\Delta\lambda_2 = (591.2 \times \cot(44.295°) \times 9.69 \times 10^-5)\ nm = 0.059\ nm$$

Thus we obtain the wavelengths to be:

$$\lambda_1 = 587.6 \pm 0.154 \text{ nm}$$
$$\lambda_2 = 591.2 \pm 0.059 \text{ nm}$$

7 Conclusion

We see that our values of the corresponding wavelength do not agree with the literature values, indicating significant amount of error during the measurement.

One reason of this error might be due to the rotation axis of the spectrometer table. It might not have been levelled properly with schuster's method.

The screws of the instruments have quite high backlash error, which might also have contributed to the error.

Also, we see that in this situation, the Fraunhoffer approximation for diffraction is quite good, and yeilds results that we expect.

In terms of observation, I think we can improve the experiment by using digital meters for measuring the angles in spectrometer, that shall decrease much of the random error.

Also, the readings tend to be erroneous over time of the experiment, because focusing on the small divisions of vernier scale in a dark room for long time, does affect the vision for short periods.

The Value of λ_1 came out to be quite accurate, but the value of λ_2 is not that good (according to literature values obtained from [**?**]). This is because I had to use a diffraction grating of 15000 per inch, which is quite large a value. This resulted in the 2nd order angle being quite large and thus the error is large there.