Rutherford scattering: Measuring the scattering rate as a function of the scattering angle and measuring the atomic number of aluminium

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By blasting alpha particles in a vacuum chamber via a thin gold leaf, we have researched Rutherford scattering and determined the relationship for the scattering rate at various angles using a detector. We have also utilised the relationship to determine the charge number of aluminium by utilising an identical-sized piece of aluminium foil. The experiment counts the number of detections at various angles using electrical components like a counter and discriminator.

I. THEORY

A. Scattering Principle

When alpha-particles are allowed to collide with a thin gold foil in a process known as Rutherford Scattering, the pertubative potential of the colliding nucleus causes the particles to scatter or be deflected in various ways. The graphic below illustrates how most alpha-particle scattering occurs at angles smaller than one degree. However, a small number of particles exhibit significantly large scattering angles (θ), and in rare situations, they may even reverse their original direction at 180° (back scattering). The only way to explain these early qualitative findings is to assume that the gold atoms have a very tiny nucleus that virtually contains the entire atomic mass and is positively charged. The nucleus was found in this manner.

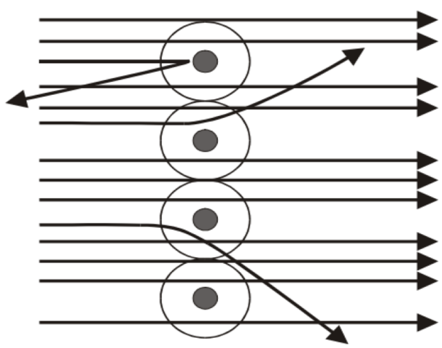

FIG. 1: Scattering of alpha-particles by the nucleus

B. Scattering Rate

The angular distribution of scattering rate is given by $N(\theta)$, and is equal to the number of particles scattered per unit time per unit solid angle. For a material with nuclear charge Z, the scattering rate for alpha particles ($Z = 2$) emitted at a rate N_0 with energy E_α is given by:

$$N(\theta) = \frac{N_0 c_F Z^2 e^4 d_F}{(8\pi\epsilon_0 E_\alpha)^2 \sin^4 \frac{\theta}{2}} \tag{1}$$

here c_F is the atomic concentration in the foil and d_F the thickness of the foil.

The relevant shape of the scattering rate vs angle graph is defined by the sine function, and is given by:

$$f(\theta) = \frac{1}{\sin^4 \frac{\theta}{2}}$$

With increasing scattering angle theta, the values of $f(theta)$ rapidly decline, and there is a singularity at $\theta = 0°$. Higher scattering angles result in very tiny counting rates, hence in order to achieve an acceptable level of precision, the gate times $t(\theta)$ for calculating the counting rate $N(\theta)$ must be raised with rising angle theta. As a result, in the initial portion of the experiment, we begin at 5° and gradually increase the gate duration as we climb higher until 30°.

1. *Space Correction for Scattering Rate*

The scattering rates $N_d(\theta)$ are determined by recording the pulse counts $n(\theta)$ for a given angle θ over a gate time t.

$$N_d(\theta) = \frac{n(\theta)}{t}$$

However, because to the clear design of the chamber utilised in this experiment, this $N_d(\theta)$ is for a flat scattering geometry. However, Rutherford's formula indicates that the theoretical function is connected to a three-dimensional geometry. The relationship between them is illustrated in the Figure 2.

Therefore, the plane angular fifferential $d\theta$ corresponds in three dimentions to a spatial angular differential $d\Omega$:

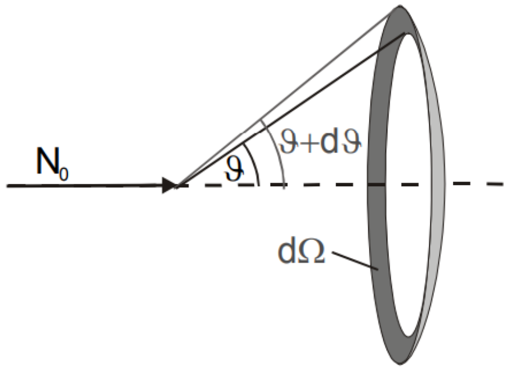

FIG. 2: Angular deflection of alpha-particles

$$d\Omega = 2\pi \sin\theta d\theta$$

Hence the space correction to $N_d(\theta)$ and the spatial scattering rate $N(\theta)$ is:

$$N(\theta) = \frac{N_d(\theta)}{2\pi \sin\theta}$$

2. Determination of Atomic Number (Z) of Aluminium

This experiment can be used to find the atomic number (Z) of Aluminium by replacing the gold foil with Aluminium foil keeping all other quantities (like dimentions of the foil, angle, radioactive source etc.) unchanged. From Equation 1 we can conclude:

$$N(\theta) \propto d \times Z^2$$

where N is the count per unit time, d is the thickness of the foil and Z is the atomic number of the foil.

$$\therefore \frac{N_{Au}}{N_{Al}} = \frac{d_{Au} Z_{Au}^2}{d_{Al} Z_{Al}^2}$$

$$Z_{Al} = Z_{Au} \sqrt{\frac{d_{Au} N_{Al}}{d_{Al} N_{Au}}} \tag{2}$$

C. Experimental Setup

The experimental set-up includes a scattering chamber that holds the gold leaf and the detector, a discriminator preamplifier, and a counter, as indicated in Figure 3. This experiment is conducted in a closed room under vacuum because to the extremely low range of alpha-particles in the air. The gold foil receives the alpha particles that are released from the Am-241 preparation through a slit aperture that is 5 mm wide, and these alpha particles exit the gold foil at varied scattering angles. A semiconductor detector is used to determine which alpha particles were dispersed. With the arrangement we're using, the gold foil, slit, and preparation—which are all mounted to a standard swivel arm—are what are swung, not the detector. The chamber's side wall is securely fastened to the detector. The discriminator level should be set halfway between the spots where the noise is masking out and where the alpha count rate starts to decline.

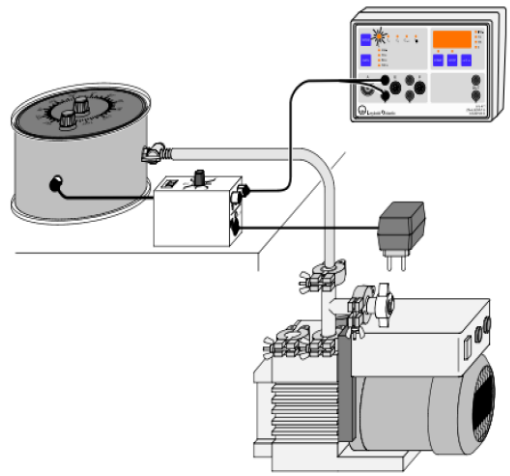

FIG. 3: Experimental Setup

II. OBSERVATIONS AND CALCULATIONS

A. Recording the scattering rate as function of the angle

Our experimentally observed data for measuring the counts and hence the decay rate for different angles is shown in Table 1. The graph for the same is shown in Figure 4.

In Figure 4 we have fitted the curve with the function:

$$y = \frac{A}{\sin^4 \frac{x-B}{2}}$$

where A and B are the constants to be determined, x is the angle (θ) and y is the counts per second.

From the graph we got the values of the constants as:

$$A = 0.000109843 = 1.098 \times 10^{-4} \text{ per second}$$

$$B = -0.00126324 = 1.263 \times 10^{-3} \text{ degree}$$

Angle (θ)	Time (s)	Counts	Counts per second $N(\theta)$
25	600	133	0.225
		140	
		127	
		137	
		138	
20	200	164	0.940
		198	
		186	
		195	
		197	
15	100	311	2.929
		276	
		277	
		311	
		290	
10	100	1726	17.626
		1811	
		1713	
		1754	
		1809	
5	100	2931	29.286
		2938	
		2912	
		2931	
		2931	
-5	100	2934	29.343
		2933	
		2935	
		2935	
		2935	
-10	100	1751	17.824
		1778	
		1831	
		1787	
		1766	
-15	100	291	2.940
		294	
		289	
		290	
		306	
-20	200	210	1.009
		194	
		211	
		201	
		194	
-25	600	133	0.225
		140	
		127	
		135	
		141	

TABLE I: Table for $N(\theta)$ for 5mm thick Gold foil

B. Determining the atomic number of Al

Our experimentally observed data for measuring the counts and hence the decay rate for different materials keeping all other conditions constant is shown in Table 2.

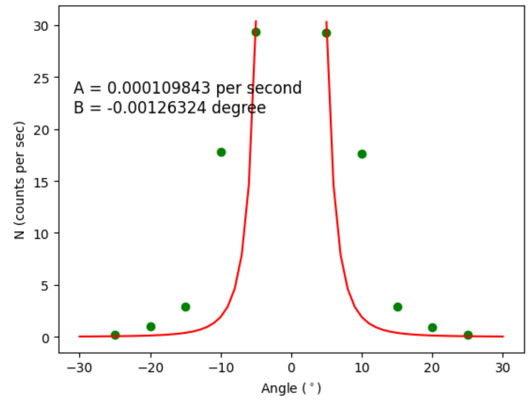

FIG. 4: N vs θ graph for Table 1

Element	Angle (θ)	Time (s)	Counts	Counts per second (cps)	Average cps $N(\theta)$
Gold (Au)	15	100	23	0.208	0.48
			16		
			17		
			26		
			22		
	-15	100	73	0.752	
			65		
			79		
			77		
			82		
Aluminium (Al)	15	1000	16	0.016667	0.01333
			13		
			21		
	-15	1000	10	0.009667	
			14		
			5		

TABLE II: Comparing N for Al and Au

Now, we know that:

- $Z_{Au} = 79$
- We used equal thickness foils. So, $d_{Au} = d_{Al}$.
- $N_{Au} = 0.48cps$
- $N_{Al} = 0.01333cps$.

from Equation 2 we get the value of Z_{Al} as:

$$Z_{Al} = 79\sqrt{\frac{0.01333}{0.48}}$$

$$\boxed{\text{Z} = 13.167}$$

III. ERROR ANALYSIS

From experiment we got the value of Z_{Al} as 13.167. But, theoretically, we know the value of Z_{Al} as 13. So, the percentage error in the calculated value of Z_{Al} is:

$$\text{Percentage error} = \frac{|13.167 - 13|}{13} \times 100\% = 1.28\%$$

IV. RESULTS & DISCUSSION

- $A = 0.000109843 = 1.098 \times 10^{-4}$ per second
- $B = -0.00126324 = 1.263 \times 10^{-3}$ degree
- $Z_{Al} = 13.167$ with a percentage error of 1.28%.
- To enable active emission and deflection of the alpha particles and avoid their collision with air particles, vacuum must be produced.
- A scaling factor is represented by the proportionality factor (at logarithmic scale). The horizontal angular scale is being slightly displaced by the coefficient B. To better accurately define the points, I was unable to identify any additional explainable constant element in the equation.
- The small value of B shows that the readings are almost symmetric about the y axis, that is, the scattering rates are approximately equal for both positive and negative angles. The small value might be due to the fact that the detector is not perfectly aligned with the beam of alpha particles, or sampling error due to the limited number of readings.
- With the very thin gold foil, we noticed that the average reading at $+15°$ and $-15°$ are very far apart, although not that prominent for aluminium foil also we noticed the same phenomenon. The result was less prominent maybe due to the fact that Aluminium has a lower atomic number. *Although, this is not observed for the thick foil.*
- To explain the above phenomenon, we should consider the fact, that when we used the thick foil on day 1, the source was perfectly perpendicular tot eh foil. So we got a perfectly symmetric graph. But when we returned the next day to use the thin foil, the source might be slighlty tilted while changing the foil. This might have caused the asymmetry in the readings.

V. CONCLUSION

In order to discover the relationship between scattering rate and angle, we researched Rutherford scattering. Additionally, we have determined the aluminium atomic number accurately. The required precautions to be taken are:

- When the vacuum pump is running, the gold leaf needs to be parallel to the nozzle. High velocity air might rupture the thin & expensive gold foil.
- Any amount of air inside the vacuum chamber will cause the readings to be non-uniform and inaccurate.
- The offset discriminator has to be set properly to get the correct readings.
- The gold foil should not be touched by the hands. The oil from the hands might make a layer on the foil, which will cause the readings to be inaccurate. It can also cause the foil to be damaged.

[1] SPS, Lab manual, Website (2022), https://www.niser.ac.in/sps/sites/default/files/basic_page/p341_2023/Rutherford_scattering.pdf.