STUDY OF LATTICE VIBRATIONS USING ANALOGOUS ELECTRONIC CIRCUITS

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Using an LC low pass filter, we investigated the lattice vibration of a monoatomic and diatomic crystal in the provided experiment. We discover that a lattice spring and mass model is akin to an LC circuit, and the equation for the resonant frequency yields a similar dispersion relation, so demonstrating the analogy. We have now discovered the maximum resonant frequency of vibration in a monoatomic lattice similar circuit and the frequency band gap in a diatomic lattice analogous circuit.

I. THEORY

Each atom in an equilibrium crystal lattice is precisely positioned at its lattice location. The morse potential, which has the lowest energy at the bond length internuclear distance when the lattice is in equilibrium, gives the potential energy for two particles. The lattice vibrations can be approximated to that of a harmonic oscillator in its most stable state, as seen in the image below. As a result, the lattice may be described as a spring and mass system, with atoms acting as the mass. As a result, when the atoms are pushed by a little amount from the equilibrium point, they tend to return to their original location. Due to interparticle interactions, these movements are subsequently imparted to all atoms, resulting in lattice vibrations.

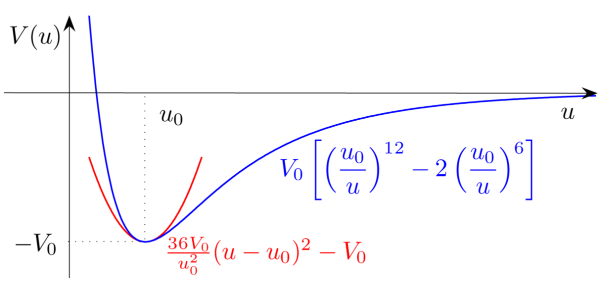

FIG. 1: Harmonic Approximation for Lattice Dynamics

A. Monatomic Lattice Vibrrations

The harmonic approximation for a monoatomic crystal lattice is shown in Figure 2. In Figure 2 these are the relevant constants:

- **a** → Lattice Constant
- **f** → Force constant
- **M** → Mass of the atom

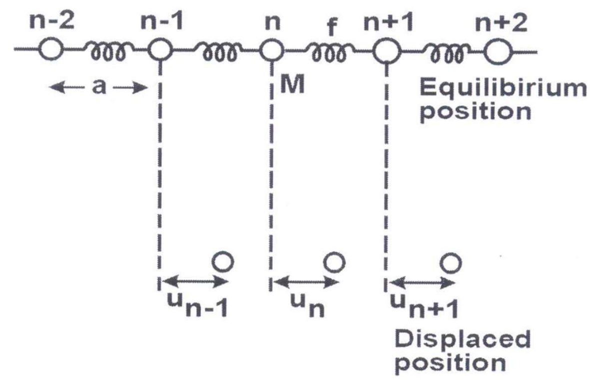

FIG. 2: Spring Mass model of One Dimentional Linear Mono-atomic Lattice

On applying Newton's second law of motion and Hook's law for the sprin-mass system, we get the differential equation for the n^{th} atom:

$$m\frac{d^2U_n}{dt^2} = f(U_{n+1} - 2U_n + U_{n-1}) \tag{1}$$

where f corresponds to the force constant and U_n is the displacement of the n^{th} atom. Similarly, for N number of particles at equilibrium particle distance r, there will n coupled differential equations. Resonance would occur for the relation between length of the atomic chain in one dimension ($L = (N+1)r$) and the wavelength of vibration, for an integer m given as $2L = m\lambda$.

Now, let us assume that the solution be of the form $U_n = A\exp(kx_n - \omega t)$. Substituting this in the differential equation with the boundary conditions of $x_N = x_0 = 0$, we get the following equation:

$$\omega = \sqrt{\frac{4f}{m}}\left|\sin\frac{ka}{2}\right|$$

This is the dispersion relation, with k denoting the ${}^{wavenumber}/_{wavevector}$. The Brilluoin zone for the newest (the independent values of k, values for which are indentical to other zones in the lattice) is supplied by the sin term as:

$$\frac{-\pi}{a} \leq k \leq \frac{\pi}{a}$$

The electronic circuit that is an equivalent for these monoatomic vibration is that on an LC low pass filter as shown in figure, the output voltage of which is measured around the capacitor.

On solving for the current arcoss the n-th capacitor, we find a differntial equation similar to that of Equation 1 given as:

$$C\frac{d^2V_n}{dt^2} = \frac{1}{L}(V_{n+1} - 2V_n + V_{n-1})$$

where capacitance C is equivalent to the mass, inductance L to $^1/_{forceconstant}$, and V_i is the voltage across the i^{th} capacitor. Similar solution leads us to