Compton Scattering

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In this experiment, brass and aluminum scatterers were used to demonstrate the Compton Scattering effect. In both situations, we found that the relative energy and intensity of scattered photons dropped with increasing scattering angle. We calculated the calibration factor for each example, and the results are 2.71×10^{32} for aluminum and 5.05×10^{32} for brass. Additionally, we determined the rest mass of electrons, which was 450.99 ± 0.11 KeV on average.

I. THEORY

Compoton scattering is a physical event that validated the dual nature of light, which had been previously established in photoelectric effect. Compton scattering is a crucial mechanism in X-ray and gamma-ray astronomy, as well as in medical imaging procedures like Computed Tomography (CT) scans. The phenomenon is named after Arthur Compton and involves the scattering of a photon from a metal object, as the name implies. The photon collides with an electron within the target, losing part of its energy due to the collision principle of conservation of energy and momentum. Similarly, the photon changes its propagation path, resulting in varying intensities of detection at different angles, but the electron is expelled from its starting state and, following energy absorption, transcends to a higher energy level.

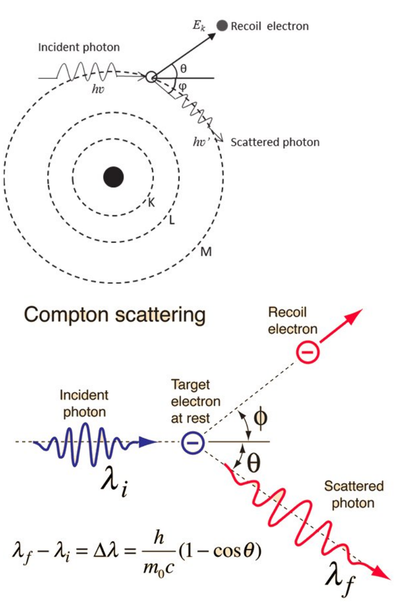

FIG. 1: Compton scattering

A. Wave Particle Duality

Light acts as both a wave and a particle, according to the wave-particle duality of light, and the Compton effect offered experimental proof that photons, which are light particles, have both wave-like and particle-like qualities. Using the conservation of momentum and energy in particle-photon collisions, we find that the wavelength of the scattered ray is greater than the wavelength of the incident ray, and that the observed shift in the wavelength of scattered photons is directly related to the energy and momentum exchanged between the photon and the electron.

Compton scattering is an example of inelastic collision and can be expressed in following way:

$$\gamma + e \rightarrow \gamma^{'} + e^{'}$$

where γ and $\gamma^{'}$ are the incident and scattered photons respectively, and e and $e^{'}$ are the initial and final electrons respectively.

The derivation of Compton scattering is based on two principles conservation of energy and conservation of momentum to the interaction between the photon and the electron. Let's assume that the photon has an initial energy of E and is incident on an electron at rest. After the interaction, the scattered photon has an energy of $E^{'}$ and moves in anew direction, while the electron recoils with a final momentum $p^{'}$. Using conservation of energy, we can write:

$$E + mc^2 = E^{'} \sqrt{m^2c^4 + p^{'2}c^2}$$

where m is the mass of the electron, and c is the speed of light. Using conservation of momentum, we can write:

$$p_\gamma = p_\gamma^{'} + p_e^{'}$$

where p_γ and $p_\gamma^{'}$ are the momenta of the incident and scattered photons, respectively, and $p_e^{'}$ is the momentum of the recoiling electron. The shift of the wavelength $(\Delta\lambda)$

increased with scattering angle according to the Compton formula as more scattering angle implies greater loss in energy:

$$\Delta\lambda = \lambda_\theta - \lambda_0 = \frac{h}{mec}\left(1 - \cos\theta\right)$$

where λ_θ and λ_0 are wavelengths of scattered and initial photons respectively, h is Planck's constant, me is the rest mass of the electron, c is the velocity of light, θ and ϕ are angles of scattered photon and recoil electron respectively. The value of ($h/m_e c = 0.02426$Å) is known as the Compton wavelength of the electron. In this length scale, we can't talk about a single particle. We have to take into account of particle antiparticle pair that creates because of energy uncertainty. In terms of energy, equation-1 can be rewritten as:

$$E_\theta = \frac{E_0}{(1+\gamma)(1-\cos\theta)}$$

As a result, Compton scattering is significantly energy-dependent, and the relevant energy scale is determined by the ratio of input photon energy to electron rest energy. The fractional shift in energy is considerable if this ratio is big, and negligible if this ratio is small. Compton scattering is important only when the incoming photon energy is a considerable proportion of the electron's rest energy. The equation also demonstrates that the change in wave length of the scattered photon is proportional to the scattering angle and the photon's original energy. It also illustrates that the scattered photon loses energy, and this lost energy is passed to the recoiling electron.

Using quantum mechanical calculations, Klein-Nishina correctly formulated the differential Compton scattering cross-section formula. This equation is written as follows:

$$\begin{aligned}\frac{d\sigma}{d\Omega} =& r_o^2 \frac{1+cos^2\theta}{2(1+\gamma(1-cos\theta))^2}\times \\ &\left(1+\frac{\gamma^2(1-cos\theta)^2}{(1+cos^2\theta)(1+\gamma(1-cos\theta))}\right)\end{aligned} \tag{1}$$

where, $r_o = \frac{e}{4\pi\epsilon_o m_e c^2} = 2.8179\times10^{-15}$ is the classical electron radius.

For dispersed photons, gamma rays from a Cs^{137} source are used in this experiment. A calibrated scintillation detector set at varying scattering angles determines differences in incoming and scattered energy and wavelength of photons. By computing the calibration factor C using the method below, the relative intensities $I\theta$ of the scattered radiation peaks may be compared with the predictions of the Klein-Nishina formula for the differential effective cross-section $\frac{d\sigma}{d\Omega}$. Thus:

$$C = \frac{1}{n}\sum_{\theta=0}^{n}\frac{I_\theta}{(\frac{d\sigma}{d\Omega})}$$

angle θ	change in energy(keV)	change in wavelength($10^{-12}m$)
30	552.7	2.248959653
45	464.2	2.677725118
60	395.6	3.14206269
90	385.7	3.222711952
120	216.6	5.738688827

TABLE I: Change in energy and wavelength as a function of scattering angle for aluminum

II. OBSERVATION AND CALCULATION

A. Aluminium scatterer

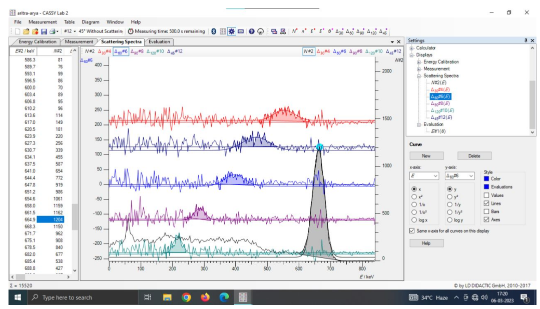

FIG. 2: Measurement spectra

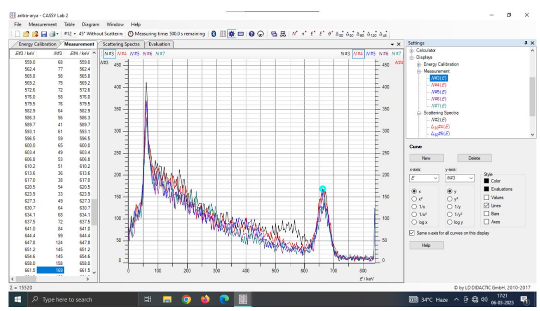

FIG. 3: Scattering spectra

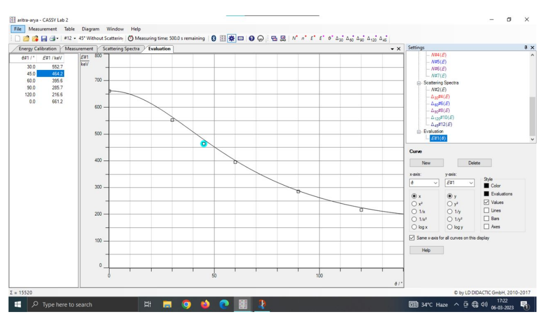

FIG. 4: Energy calibration spectra

angle θ	differential cross-section$(10^{-30}m^2)$	relative intensity
30	6.075	802
45	3.348	738
60	2.2	697
90	1.3047	381
120	0.5581	221

TABLE II: Evaluation of differential cross-sections and relative intensities for Aluminium

From Table 2 and Equation 1, we have the calibration factor for aluminum is:

$$C = 2.71 \times 10^{32}$$

By analysing the energy calibration spectra we get $m_e = 482.38KeV$.

B. Brass scatterer

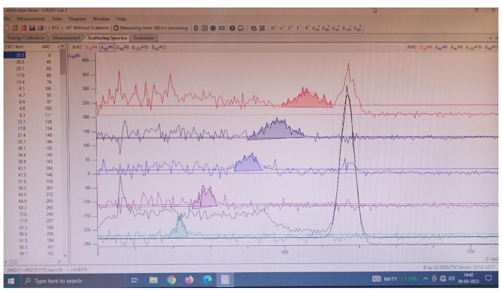

FIG. 5: Measurement spectra

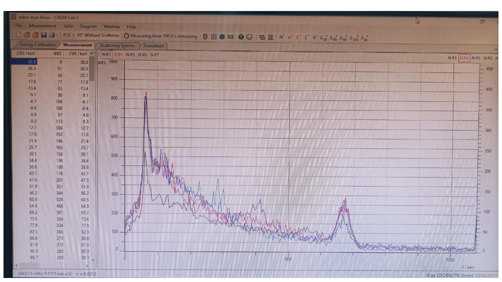

FIG. 6: Scattering spectra

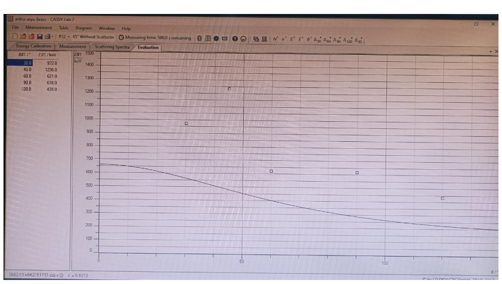

FIG. 7: Energy calibration spectra

angle θ	change in energy(keV)	change in wavelength$(10^{-12}m)$
30	559.4	2.222023597
45	481.2	2.58312552
60	402.4	3.088966203
90	289.8	4.289164941
120	218.4	5.691391941

TABLE III: Change in energy and wavelength as a function of scattering angle for brass

angle θ	differential cross-section$(10^{-30}m^2)$	relative intensity
30	6.075	1705
45	3.348	1229
60	2.2	925
90	1.3047	617
120	0.5581	550

TABLE IV: Evaluation of differential cross-sections and relative intensities for Aluminium

From Table 4 and Equation 1, we have the calibration factor for brass is:

$$C = 5.05 \times 10^{32}$$

From energy calibration spectra, we have the rest mass of electron $m_e = 419.6Kev$

III. ERROR

The expected value of m_e is $0.511MeV$, but for each case the observed value is much greater than the expected value.

$$\Delta m_e = \frac{|m_e(observed) - m_e(theory)|}{m_e(theory)}$$

thus we have, for aluminum: $\Delta m_e = 0.056$ and for brass, $\Delta m_e = 0.1788$

We made an error of 11.7% in our calculations. This might be because we put the lead block in the wrong place. It could be too close to the scattering body and not let all the scattered photons pass. This is shown by the reduced peaks in the scattering spectra. We can fix this by moving the lead block further away from the scattering body.

IV. CONCLUSION

- We conducted an experiment on Compton scattering of electrons, which involves the scattering of X-rays by free electrons.

- The observed data in our experiment shows that the intensity of scattered photons decreases as the scattering angle increases. This observation agrees with the Compton formula, which describes the energy transfer from photons to electrons in the scattering process.

- We also found that the energy of scattered photons decreases as the scattering angle increases, which again agrees with the Compton formula. This is because the energy of scattered photons is related to the energy lost by the electrons in the scattering process.

- Our experiment also revealed that the detector used in the experiment had different calibration factors for different materials. The calibration factor for brass was found to be approximately two times that of aluminum. This implies that for particular cross-sections, the relative intensity for brass is more than for aluminum.

- We calculated the rest mass of electrons in our experiment, but the result had a significant error. We think that this error can be minimized by taking precautions with the lead box used in the experiment. This suggests that our experiment can be improved by using better shielding to reduce unwanted background radiation and improve the accuracy of our measurements.

- Our experiment demonstrates that the Compton scattering process can be used to determine the rest mass of charged particles like electrons. By measuring the scattered photon energy and angle, along with the incident photon energy, it is possible to calculate the rest mass of the electron. This technique is widely used in experimental physics to measure the masses of particles.

[1] SPS (2022). Lab manual. *Website*. `https://www.niser.ac.in/sps/sites/default/files/basic_page/p341_2023/compton_scattering.pdf`.