Measurement of resistivity and determination of band gap using Four-Probe method

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(Dated: April 15, 2023)

The Four-Probe method, utilizing four collinear probes for electrical contact, was employed to measure the resistivity of Silicon, Germanium, and Aluminium foil at room temperature. Additionally, the band gap energy of Germanium was determined by analyzing the temperature dependence of resistivity. The results exhibit close agreement upon closer inspection.

I. OBJECTIVES

1. To measure the resistivity of Silicon, Germanium and Aluminium foil at room temperature.

2. To determine the energy band gap of a semiconductor (Ge) by measuring the resistivity as a function of temperature.

II. THEORY

A. Four Probe Method

The Four-Probe method is a widely used technique in materials science and condensed matter physics for accurate measurement of resistivity in materials. It is preferred over the conventional Two-Probe method due to its inherent advantages in minimizing errors caused by contact resistance. Four equally spaced collinear probes are used to make electrical contact with the surface of the material, with separate pairs of terminals to carry current and sense voltage. This configuration allows for precise voltage and current measurements, independent of the contact resistance, leading to more accurate and reliable resistivity measurements.

To set up a Four-Probe measurement, the probes are carefully positioned on the sample surface, to form a known probe spacing. A known current is passed through the outer probes, and the voltage drop across the inner probes is measured as shown in Figure 1. The resistivity of the material can then be calculated using the measured voltage and current values, along with the probe spacing and geometry, following established mathematical formulas, such as the Van der Pauw method.

The Four-Probe method offers several advantages over the Two-Probe method. Firstly, it eliminates errors associated with contact resistance, which can be a significant source of inaccuracy in resistivity measurements. Secondly, it allows for precise and simultaneous voltage and current measurements, leading to improved accuracy and reproducibility. Additionally, the Four-Probe method can be used to measure resistivity in a wide range of materials, including thin films, bulk samples, and even highly resistive materials. Overall, the Four-Probe method is a reliable and widely used technique for accurate resistivity measurements in materials research and characterization.

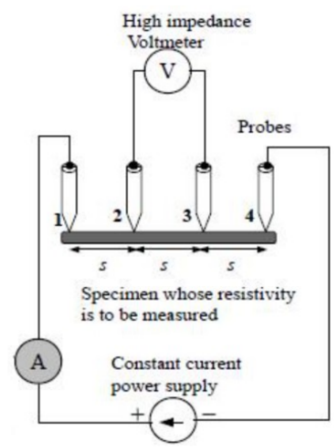

FIG. 1: Four-Probe method setup for resistivity measurement

B. For semi-infinite conducting material

A hemispherical equipotential surface develops at a probe when current flowing from a semi-infinite material is dispersed isotropically. The potential drop at an inner probe when two probes of different distances and opposing polarity are considered is:

$$V = \frac{\rho_0 I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where ρ_0 is the resistivity, I is the current passed through the outer electrode and r_1 and r_2 is the distance from the probe 1 and probe 4 respectively. The floating potential across the inner terminals is determined by the following equation when evenly spaced probes are taken into account.

$$V = \frac{\rho_0 I}{2\pi S}$$

S is the probe spacing. The resistivity of the material can be calculated using the following equation:

$$\rho_0 = \frac{V}{I} 2\pi S \tag{1}$$

C. For thin sheet placed on a non-conducting surface

Since the thickness of the samples are small compared to the probe distance, a correction factor for it has to be applied. If the bottom surface is not conducting, then the corrected formula for resistivity will be:

$$\rho = \frac{\rho_0}{G_7(^W/_S)}$$

where W is the width of the sample and $G_7(^W/_S)$ is the required correction factor defined as follows:

$$G_7(^W/_S) = 1 + 4\frac{S}{W}\sum_{n=1}^{\infty} f(S,\ W,\ n)$$

$$f(S,\ W,\ n) = \left[\frac{1}{\sqrt{\left(\frac{S}{W}\right)^2 + n^2}} - \frac{1}{\sqrt{\left(\frac{2S}{W}\right)^2 + 4n^2}}\right]$$

For smaller values of $\frac{W}{S}(\leq 0.25)$, we can approximate the value of $G_7(^W/_S)$ as:

$$G_7(^W/_S) = \frac{2S}{W}\ln(2)$$

From these equations we get:

$$\rho = \frac{\pi}{\ln(2)} W \frac{V}{I} \tag{2}$$

D. Relation of resistivity the band gap of a semiconductor

If E_g is the band gap energy, $k_B = 8.6 \times 10^{-5} eV/K$ is the Boltzmann constant, and T is the temperature in Kelvin:

$$E_g = 2k_B \frac{\ln(\rho)}{^1/_T} \tag{3}$$

where $\frac{\ln(\rho)}{^1/_T}$ can be determined from the slope of as appropriate $\ln(\rho)$ vs. $^1/_T$ plot.

III. EXPERIMENTAL SETUP

The following equipment were used for the experiment:

- Constant Current Source, Model: CCS-01
- Low Current Source, Model LCS-02
- D.C. Microvoltmeter, Model DMV-001
- Four Probe Arrangement
- Thermocouple sensor
- Set of test samples and emery powder
- Suitable connectors for DMV and CCS/LCS

The connections were made as shown in Figure 2. The sample was placed on a non-conducting surface, and the four probes were gently rested on the sample and tightened in position. The voltage (V) and current (I) measurements were taken from the respective digital displays.

When temperature changes were required, the sample setup was lowered into the oven chamber, and the thermocouple sensor and oven socket were connected to the PID (Proportional-Integral-Derivative) controller. After selecting the desired oven temperature (e.g., 200°C) and turning on the mains, data collection was initiated once the Present Value (PV) stabilized to the Set Value (SV) on the PID controller.

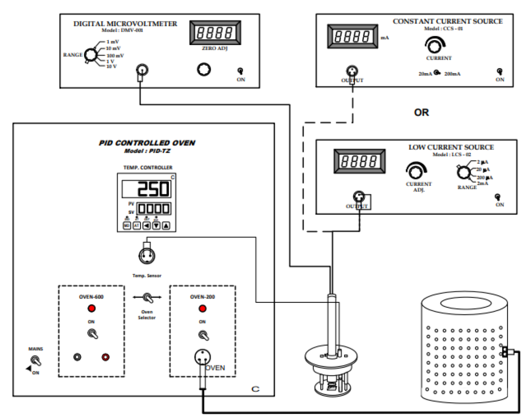

FIG. 2: Experimental setup for resistivity measurement with Four-Probe method.

IV. OBSERVATIONS & CALCULATION

All the graphs Figure 3, Figure 4 and Figure 5 have been fitted in a straight line following the equation $y = mx + c$ where m and c are the parameters.

A. Germanium Sample

The Thickness of the provided Germanium sample was $0.5mm$. We plotted the potential difference measured with the current flowing through the sample as shown in Table 1 in Figure 3. From the graph we can see that the slope of the curve is $slope = {}^{V}\!/_{I} = R = 87.94 \pm 0.05\Omega$.

I (mA)	**V (V)**	**I (mA)**	**V (V)**	**I (mA)**	**V (V)**
0.00	0.000	2.16	0.191	4.54	0.399
0.21	0.019	2.40	0.211	4.80	0.423
0.38	0.034	2.63	0.232	5.03	0.442
0.43	0.038	2.85	0.250	5.12	0.451
0.60	0.054	3.05	0.268	5.38	0.474
0.80	0.071	3.23	0.285	5.65	0.497
1.06	0.094	3.53	0.311	5.98	0.527
1.25	0.110	3.68	0.324	6.23	0.548
1.42	0.124	3.82	0.337	6.51	0.573
1.60	0.142	3.92	0.345	6.60	0.582
1.83	0.162	4.02	0.354	7.01	0.616
2.02	0.178	4.31	0.380	7.31	0.643

TABLE I: Data for Germanium Sample

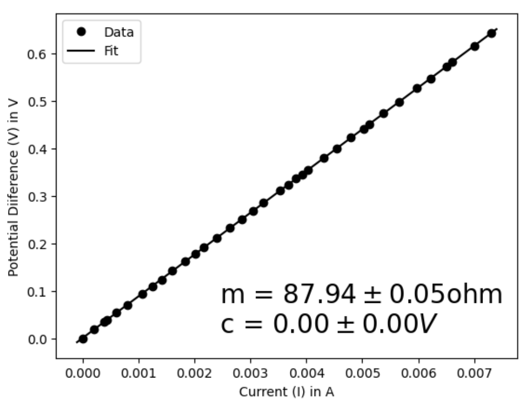

FIG. 3: V vs I graph for Germanium sample

So using Equation 2 we get the resistivity of the sample as:

$$\rho_{Ge} = \frac{\pi}{\ln(2)} \times 0.05 \times 87.94 = 19.93\ \Omega cm$$

B. Aluminium Sample

The Thickness of the provided Aluminium sample was $0.018mm$. We plotted the potential difference measured with the current flowing through the sample as shown in Table 2 in Figure 4. From the graph we can see that the slope of the curve is $slope = {}^{V}\!/_{I} = R = 356.61 \pm 1.99\mu\Omega$.

I (mA)	**V (mV)**	**I (mA)**	**V (mV)**
0.0	0.001	96.4	0.035
8.6	0.003	90.8	0.033
13.3	0.006	102.0	0.037
22.0	0.008	115.0	0.042
29.4	0.011	138.7	0.050
35.5	0.014	127.3	0.047
45.8	0.017	159.0	0.058
56.4	0.020	147.5	0.054
65.0	0.023	176.3	0.063
76.5	0.028	183.9	0.066
85.2	0.031	196.8	0.070

TABLE II: Data for Aluminium Sample

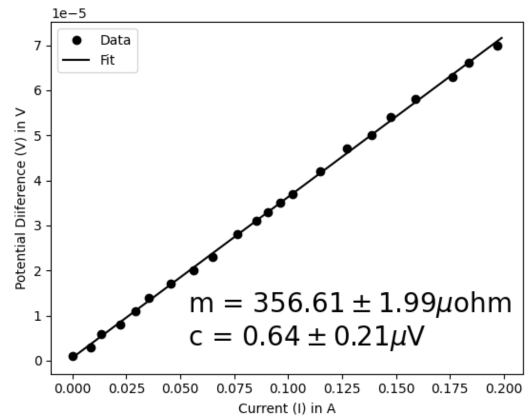

FIG. 4: V vs I graph for Aluminium sample

So using Equation 2 we get the resistivity of the sample as:

$$\rho_{Al} = \frac{\pi}{\ln(2)} \times 0.0018 \times 356.61 \times 10^{-6} = 2.91 \times 10^{-6}\ \Omega cm$$

C. Silicon Sample

The Thickness of the provided Silicon sample was $0.5mm$. We plotted the potential difference measured with the current flowing through the sample as shown in Table 3 in Figure 5. From the graph we can see that the slope of the curve is $slope = {}^{V}\!/_{I} = R = 105.06 \pm 0.71\Omega$.

So using Equation 2 we get the resistivity of the sample as:

$$\rho_{Al} = \frac{\pi}{\ln(2)} \times 0.05 \times 105.06 = 23.8\ \Omega cm$$

D. Germanium with Varying Temperature

We measured the potential difference between the two probes of the germanium sample at different tempera-

I (μA)	**V (mV)**	**I** (μA)	**V (mV)**
0.010	0.003	1.102	0.118
0.100	0.019	1.210	0.132
0.204	0.027	1.300	0.142
0.301	0.036	1.405	0.152
0.400	0.047	1.500	0.162
0.500	0.058	1.607	0.176
0.600	0.064	1.701	0.181
0.700	0.078	1.802	0.195
0.800	0.087	1.901	0.205
0.900	0.100	1.992	0.213
1.000	0.106		

TABLE III: Data for Silicon Sample

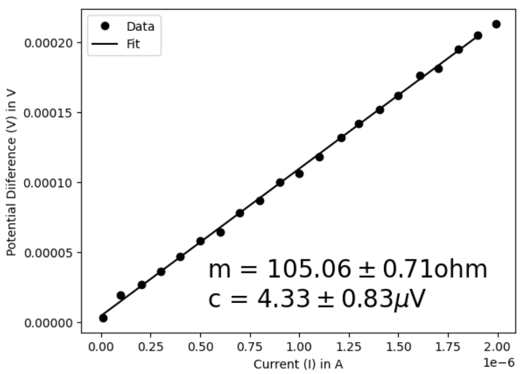

FIG. 5: V vs I graph for Aluminium sample

tures. The current was kept constant at $0.5mA$. The data is shown in Table 4. From the data, we plotted the graph between $\log\rho$ and $1/T$ as shown in Figure 6.

Temp (°C)	**Temp (T) (Kelvin)**	**Potential (V)**	ρ	$\log\rho$	$^1/_T$ (K^{-1})
80	353.15	0.145	0.066	-2.722	0.0028
91	364.15	0.109	0.049	-3.008	0.0027
100	373.15	0.086	0.039	-3.245	0.0026
110	383.15	0.064	0.029	-3.540	0.0026
120	393.15	0.049	0.022	-3.807	0.0025
130	403.15	0.037	0.017	-4.088	0.0024
140	413.15	0.029	0.013	-4.332	0.0024
150	423.15	0.023	0.010	-4.564	0.0023
160	433.15	0.018	0.008	-4.809	0.0023
170	443.15	0.014	0.006	-5.060	0.0022
180	453.15	0.012	0.005	-5.214	0.0022
190	463.15	0.010	0.005	-5.397	0.0021

TABLE IV: Germanium Data with varying temperature

From Figure 6 we can see that the slope of the straight line is $4084.56 \pm 42.89\Omega$. Using Equation 3 we get the band gap of the germanium sample as:

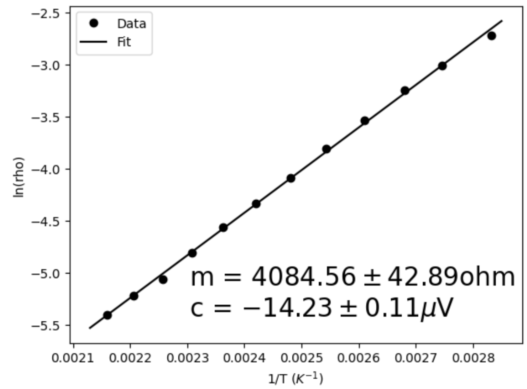

FIG. 6: V vs I graph for Germanium sample

$$\begin{aligned} E_g &= 2 \times k_B \times slope \\ &= 2 \times 8.6 \times 10^{-5} \times 4084.56eV \\ &= 0.703eV \end{aligned} \tag{4}$$

V. ERROR ANALYSIS

A. Estimation of Error

While calculating the resistivity of materials, we didn't measure the thickness of the sample. So we assumed that the error in the thickness of the sample is negligible. Thus the only quantity that contributes to the error is the slope of the graph. Thus:

$$\Delta\rho = \rho \times \frac{\Delta slope}{slope}$$

Thus the corrected values of resistivity are:

- $\rho_{Ge} = 19.93 \pm 0.011\ \Omega cm$
- $\rho_{Al} = 2.91 \times 10^{-6} \pm 1.62 \times 10^{-8}\ \Omega cm$
- $\rho_{Si} = 23.8 \pm 0.161\ \Omega cm$

Also, in the calculation of the band gap, the only quantity that contributes to the error is the slope of the graph. Thus:

$$\Delta E_g = E_g \times \frac{\Delta slope}{slope}$$

Thus the corrected value of band gap is:

$$E_g = 0.703 \pm 0.0074eV$$

B. Suspected Sources of Error

1. The sample should be of uniform thickness.
2. The Aluminium was commercial grade aluminium, so it was not pure. So the resistivity of the sample was not accurate.
3. The Four probe method works under the assumption that the sample used is semi infinite, which may not be accurate in every situation.
4. Instability in the data due to improper contact.
5. Variation of doping in the sample.

C. Precautions to avoid error

1. When the zero adjustment cannot be done, the offset is noted and reduced from the data to provide corrected values.
2. To avoid loose contacts, the springs are tightened on the four probes appropriately.
3. The sample is cleaned before measurements and appropriate ranges are used in digital measurement devices.
4. Sufficient time was given for the temperature to stabilize before noting the measurments.

VI. CONCLUSION

Our experiments utilizing the four probe method allowed us to accurately determine the resistivities of various samples, including both semiconductors and metals. Additionally, we were able to calculate the band gap of a semiconductor sample through temperature-dependent measurements. We observed that the resistivity of semiconductors decreased with an increase in temperature, in line with theoretical predictions.

However, it's important to note that there were some propagational errors in our experiments, which could be attributed to fluctuations in supply voltage, carrier injection, or impurities in the sample material. We also made assumptions about the uniformity of resistivity in our samples, which may not always hold true. Nevertheless, these errors were within the acceptable error limits, except for a few exceptions that were previously discussed.

The four probe method has emerged as a widely used technique for measuring resistance and resistivity of thin wafers composed of different materials. It has proven to be superior to the conventional two-probe method in terms of accuracy and reliability in our experiments.

[1] SPS, Lab manual, Website (2022), https://www.niser.ac.in/sps/sites/default/files/basic_page/p347_2023/1.Measurement_of_resistivity_and_determination_of_band_gap_using_Four-Probe_method.pdf.