G.M. 1: Basics of GM Counter-Characteristics and Counting Statistics

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(Dated: April 9, 2023)

The experiment aimed to explore the basics of GM Counter-Characteristics and Counting Statistics in nuclear physics. The Geiger-Muller (GM) counter was used to study radioactivity, determine its characteristics, confirm the inverse square law related to gamma rays, calculate source efficiency, and analyze counting statistics. The results showed that the GM characteristic curve closely resembled an ideal one and confirmed the inverse square law. It was also found that the GM counter was more efficient in detecting radiation from Tl^{204} (β-rays). However, the counting statistics distributions were incomplete due to insufficient sample numbers or defects.

I. THEORY

A. Description of the Geiger-Muller counter

The Geiger-Muller (GM) counter works by detecting the ion-electron pairs created by the interaction of charged particles in a gas mixture. The GM tube is a metal cylinder with a thin wire (anode) at its axis and a metal cylinder (cathode) maintained at a high voltage to create an electric field. The radiation enters through a window on the tube, creating ion-electron pairs that are swept by the electric field to produce a phenomenon called an avalanche, which generates output pulses that are counted by related circuits. A schematic diagram of the GM counter is shown in Figure 1.

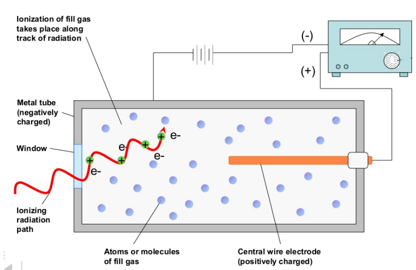

FIG. 1: Typical GM counter characterstics

As shown in Figure 2 the operating voltage of the GM counter is set in the plateau region, where the counting rate is relatively constant. The plateau length and slope determine the stability of the counting rate, while the dead time, resolving time, and recovery time limit the counting rate. The GM counter is insensitive to ionizing events during the dead time, and the resolving time and recovery time set the minimum time interval between two distinct and normal-size pulses, respectively. Higher voltages and the gas composition inside the GM tube can reduce the effects of these factors. The background counting rate is due to cosmic rays or other active sources in the same room.

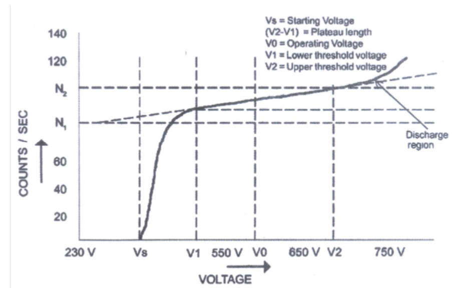

FIG. 2: Typical GM counter characterstics

B. Inverse square law: Gamma rays

It states that the gamma radiations reduce inversely proportional to the distance, D, between the detector and the radiation source. Thus, the counting rate, R (counts/second), should be related as follows:

$$R \propto \frac{1}{D^2}$$

$$\therefore \log(R) = -2 \times \log(D) + c$$

where c is some constant

C. Efficiency of GM counter

Knowing the activity of that specific source allows one to calculate the effectiveness of a GM counter. The quantity of the radiation source's disintegrations per second is known as activity. The ratio of the observed counts per second to the number of disintegrations that are detected by the detector each second is referred to as the efficiency. The efficiency, E, is now:

$$E = \frac{CPS}{DPS} \times 100 \tag{1}$$

where CPS is the counts per second, and DPS is the disintegrations per second, falling on the window of the detector. DPS can be calculated as:

$$DPS = A\frac{d^2}{16D^2} \tag{2}$$

where A is the activity of the source, d is the diameter of the detector, and D is the distance between the source and the detector.

D. Counting Statistics

Say N_i denote the i^{th} reading of a measurement in a set of n measurements, then the equations for calculating mean ($\bar{N}$), variance σ^2, and standard deviation σ (for large samples) are:

$$\bar{N} = \frac{1}{N}\sum_{i=1}^{n} N_i$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(\bar{N} - N_i)$$

II. OBSERVATIONS & CALCULATIONS

A. GM characteristics

A gamma source (Cs^{137}) was used and the voltage was varied to get the data in Table 1. From the data, the curve was plotted as shown in Figure 3. Data was taken for 60s keeping the Cs^{137} source at a distance of 6cm from the GM counter.

Potential (V)	Count	Background	Corrected Counts
343	0	0	0
344	2342	0	2342
360	4302	34	4268
390	4963	53	4910
420	4939	53	4886
450	5114	33	5081
480	5081	38	5043
510	5327	40	5287
540	5264	44	5220
570	5264	36	5228
600	5400	45	5355
630	5394	57	5337
660	9570	87	9483
690	10320	76	10244

TABLE I: GM characteristic data

From the graph we can see:

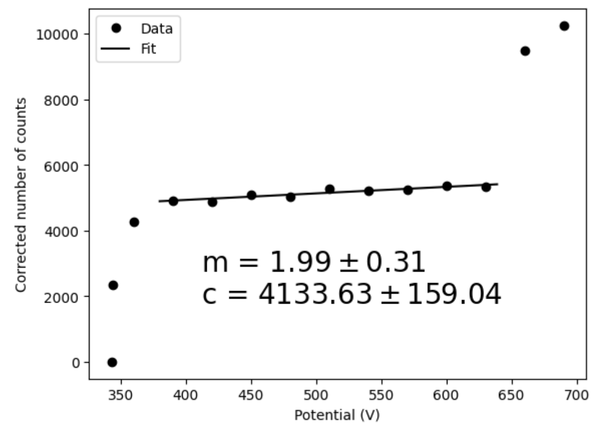

FIG. 3: Graph of GM characteristics

1. Lower threshol volatge = $390V$
2. Upper threshold voltage = $630V$
3. Plateu length = $(630 - 390)V = 240V$
4. Counts at lower threshold voltage = $(N) = 4910$
5. Slope = 1.99
6. Plateau Slope percent $(s) = \frac{slope}{N} * 10000 = \frac{1.99}{4910} * 10000 = 4.073\%$
7. Operating Volatage = $\frac{600+360}{2} = 510V$

B. Inverse Square Law

Background	Average
576	558
582	
600	
486	
564	
540	

TABLE II: Background Count (60s)

Distance (d) (cm)	Count	Corrected Count in 60s	Net Count Rate (R) (per second)	Product $C = Rd^2$	$\log(d)$	$\log(R)$
2.0	10462	9904	165.067	660.27	0.693	5.106
2.5	7797	7239	120.650	754.06	0.916	4.792
3.0	5819	5261	87.683	789.15	1.098	4.473
3.5	4515	3957	65.950	807.89	1.252	4.188
4.0	3624	3066	51.100	817.60	1.386	3.933
4.5	2961	2403	40.050	811.01	1.504	3.690
5.0	2424	1866	31.100	777.50	1.609	3.437
5.5	2062	1504	25.067	758.27	1.704	3.221
6.0	1824	1266	21.100	759.60	1.791	3.049
7.0	1470	912	15.200	744.80	1.945	2.721

TABLE III: Inverse Square Law Data & Calculations

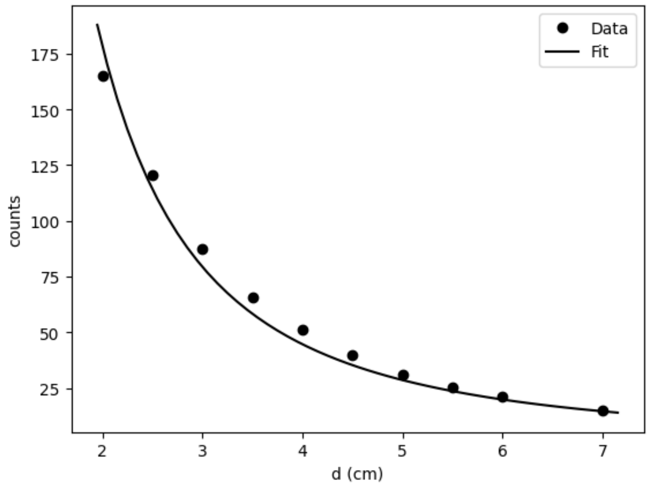

FIG. 4: Counts vs distance graph

We can clearly see in hat the value of Rd^2 is almost constant. Thus, we can say that the inverse square law is valid for the given data.

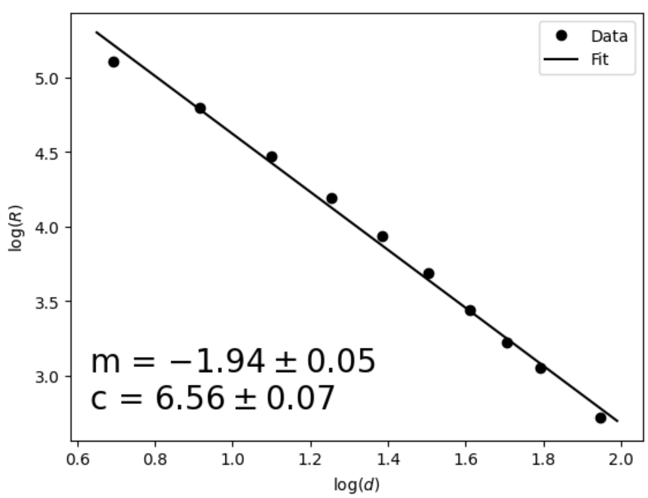

FIG. 5: $\log(d)$ vs $\log(R)$ graph

From the graph in Figure 4, we can see that the data points lie on a straight line. Also the slope of the line is -2. Thus, we can say that the inverse square law is valid for the given data.

Source	Distance (cm)	Counts	Corrected Counts	Average	CPS
Cs^{137}	10	738	655	670.667	11.178
		766	683		
		757	674		
Tl^{204}	2	2306	2223	2224	37.067
		2309	2226		
		2306	2223		

TABLE IV: Efficiency Data

C. Efficiency

Given data:

- $d = 1.5cm$
- Activity of $Cs^{137} = 86kBq$ (as of May 2016)
- Activity of $Tl^{204} = 10kBq$ (as of May 2016)
- $\therefore$ Activity of $Cs^{137} \approx 73447Bq$ (half life $\approx$ 30 years) (after 6 years 11 months, in March 2023)
- $\therefore$ Activity of $Tl^{204} \approx 2847Bq$ (half life $\approx$ 3.77 years) (after 6 years 11 months, in March 2023)

Therefore using Equation 2 we get:

- $DPS_{Cs} = \frac{73447\times(1.5)^2}{1600} = 351.56$
- $DPS_{Tl} = \frac{2847\times(1.5)^2}{1600} = 120.94$

From Table 4, we can see that:

- $CPS_{Cs} = 11.178$
- $CPS_{Tl} = 37.067$

Thus calculating the efficiency using Equation 1

- $\text{Eff}_{Cs} = \frac{CPS_{Cs}}{DPS_{Cs}} * 100 = \frac{11.178}{351.56} * 100 = 31.62\%$
- $\text{Eff}_{Tl} = \frac{CPS_{Tl}}{DPS_{Tl}} * 100 = \frac{37.067}{120.94} * 100 = 30.65\%$

We can see that for both the sources, the efficiency is almost the same. Thus, we can say that the efficiency is independent of the source and is a property of the detector.

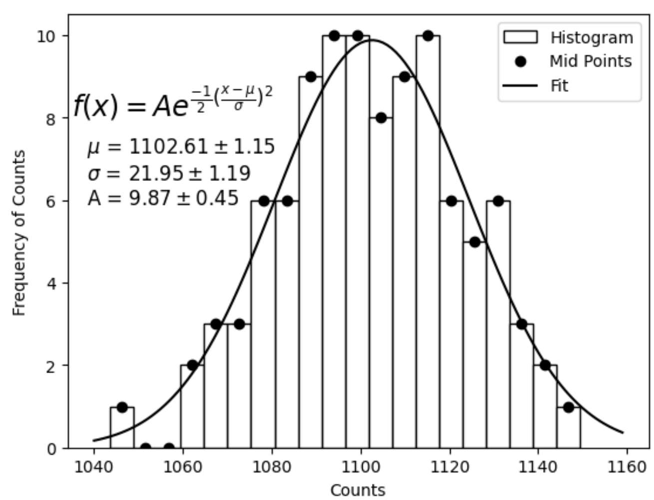

FIG. 6: Counts vs frequency graph

D. Counting Statistics

For the counting statistics, we took 100 counts with the Cs^{137} source. By plotting the counts vs their frequencies of occurence, we plotted Figure 6. Although here the data (which is only 100 numbers) is not tabulated, you can easily see the whole data if you get this excel sheet: https://github.com/PeithonKing/lab_reports/raw/main/Sem6/Nuclear_Physics/Expt1/gm1.xlsx.

From the graph, we see that the data is mostly concentrated around the mean value. Thus, we can say that the data is normally distributed.

III. ERROR ANALYSIS

We know that for a given relation:

$$A = \frac{\prod n_i}{\prod d_i}$$

For monitoring how the value of A changes with **small** change in n and d, we differentiate the equation and then divide it by the same equation. By doing this we get:

$$\frac{dA}{A} = \sum \left(\frac{dn_i}{n_i} \right) - \sum \left(\frac{dd_i}{d_i} \right)$$

So, for **Plateu Slope percent** in GM Characteristics we get:

Error part in plateau slope percentage $= \frac{dslope}{slope} - \frac{dN}{N} = \frac{0.31}{1.99} - \frac{167}{4910} = 0.122$

$\therefore$ plateu slope percentage $= (4.073 \pm 0.49)\%$

For efficiency calculation, DPS was calculated from given value which were provided without any error information. So, we can assume that the error in DPS is negligible. Now, the standard deviation of the CPS in Cs is 0.194 and in Tl is 0.023. So, the value of effieciency with error bar is:

- $\text{Eff}_{Cs} = 31.62 \pm 0.549\%$
- $\text{Eff}_{Tl} = 30.65 \pm 0.019\%$

The error for counting statistics need not be calculated again because it had already been shown in Figure 6

IV. CONCLUSION

Suspected Sources of errors:

1. **Inaccurate measurement of groove spacing:** The discrepancy between the assumed groove spacing of 5 mm and the measured groove spacing of almost 7 mm could have introduced errors in the distance measurements for the inverse square law part of the experiment. This discrepancy could result in inaccurate values for the distances between the source and the detector, leading to potential deviations from the true inverse square relationship between radiation intensity and distance.
2. **Unintended variations in source orientation:** The use of disk-shaped radioactive sources without knowing the "right" or "wrong" side could introduce inconsistencies in the data. The two flat surfaces of the disk had different emission characteristics. Inadvertently using the "wrong" side of the disk for measurements could result in inaccurate data. This could lead to potential discrepancies in the observed counts and the actual emissions from the source, affecting the accuracy of the results.
3. **Relatively small sample size for counting statistics:** The use of a small sample size of 100 data points for the counting statistics part of the experiment may not fully represent the underlying distribution of counts. This could result in a less accurate estimation of the true Gaussian distribution and may affect the interpretation of the results. A larger sample size, such as 1000 data points as suggested by your Monte Carlo simulation, could have provided a better approximation to the expected Gaussian distribution.

In this experiment, we confirmed the inverse square law for radiation intensity using a GM counter and observed a clear correlation between counts and distance from the source at a fixed operating voltage. We determined the efficiency of the GM counter by comparing expected and measured counts, providing valuable information for future experiments. Identified sources of error, such as inaccurate groove spacing measurement on the sample holder, highlighted the importance of precise measurements. The observed counts followed a Gaussian distribution around a mean value, underscoring the need for statistical analysis in interpreting experimental data.

[1] SPS, Lab manual, Website (2022), `https://www.niser.ac.in/sps/sites/default/files/basic_page/p341_2023/GM-1.pdf`.

[2] ForumAutomation, Basics of geiger-muller counter (2018), accessed: 7th April 2023.

[3] A. D. of Environmental Conservation, Half-life of cesium-137, accessed: 7th April 2023.

[4] D. L. Horrocks and A. F. Voigt, Half-life of thallium-204, Phys. Rev. **95**, 1205 (1954).