P464 Plasma Physics and MHD – Projects

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You will solve numerically the mean-field galactic dynamo equations derived using the first order smoothing approximation, $\mathcal{E} = \alpha\overline{\boldsymbol{B}} - \eta_{\rm t}\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}$, in the kinematic regime (unless you are asked to investigate non-linear effects in *your* specific project).

1 Work individually

You are expected to write your own code. However, you are free to discuss the project with anyone. You are responsible for understanding **every function/subroutine/line in your code**. You will learn a lot by speaking with the T.A. (who possesses RI, as opposed to AI; the "R" stands for real).

2 Assessment

The assessment will consist of four parts. I have added parts 1 and 2 below to help manage the time and ensure better final results.

1. **Progress report 1** — show that you have accomplished task 1 and are ready to move on to task 2. That way if you are still facing any major problems with the basic code, those can be addressed at this stage (5 marks out of 30). **Due on Thursday, 7 March, 7:30 pm.**

2. **Progress report 2** — show that you have accomplished task 2 and are ready to move on to the student-specific task, task 3. That way if you are still facing any major problems with the kinematic dynamo code, those can be addressed at this stage (5 marks out of 30). **Due on Thursday, 21 March, 7:30 pm.**

3. **Final website** containing your research findings and your code (or the link to your code on github). You will be assessed on both content (80%) and presentation/design (20%). **Due on Thursday, 18 April, 7:30 pm.**

4. **Viva** — this will take place outside of class time toward the end of the semester, at a time that is mutually agreed upon. **Outside class hours, Monday, 22 April to Thursday, 9 May.**

3 Background reading

The galactic dynamo problem in the kinematic regime is discussed in chapter 11 of our textbook Shukurov & Subramanian (2021; SS21). It would certainly help to read this chapter (skip over the parts that are very technical or not directly relevant).

4 Methods

- The code may be written in Fortran, Python or Julia.
- You will use Runge-Kutta time-stepping and finite differencing to calculate spatial derivatives.
- You may use the algorithms in assignment 3 but it is suggested that you use higher order schemes for better accuracy.
- Refer to, e.g., Brandenburg (2003), available on the course repository, for more information.

5 Boundary conditions

Solving differential equations requires boundary conditions for each variable on both boundaries of the 1D grid. You should think carefully about the boundary conditions. Refer to the course textbook SS21. Boundary conditions are discussed in § 11.1 and 11.3 (z), and 11.4.2 (r). You may use the simplest choice unless your specific project requires you to do otherwise.

A suggestion is to try different choices of boundary conditions and verify that it does not make a significant difference. Another strategy is to increase the grid size. For 1D in r make the grid extend to very large radius, way beyond the galaxy, and check if this affects your solution. Since you would keep the spatial resolution the same in such a test (e.g. 10 pc per grid element), the total number of grid points would increase, and you'd be expending computational resources on empty space, which is wasteful. So you can use such experiments to decide where to place the grid boundary.

Also, there are different ways of implementing the boundary conditions. You are encouraged to use **ghost zones**, which lead to more stable solutions. Ghost zones are grid cells that are located *beyond* the physical grid. There you can force the values of the variables to be consistent with the boundary condition. Ghost zones are the standard way of implementing BCs in large grid-based HD and MHD codes.

6 Initial conditions

One should try different choices for the seed magnetic field and explore to what extent these choices influence the magnetic field evolution. At first, different eigenmodes compete, but the one with the largest exponential growth rate will win out and eventually come to dominate. Thus, we do not expect the solution in the kinematic regime to be sensitive to the seed field, but you should check this. (However, what about the overall sign of the magnetic field?) A strong seed field (relative to $B_{\rm eq}$) can affect the solution in the *non-linear* regime, which a few of you will be investigating.

7 Task 1 (to be completed for progress report 1)

Solve the diffusion equation. That is, solve the equations omitting the $\nabla \times (\overline{\boldsymbol{V}} \times \overline{\boldsymbol{B}})$ and α terms, so that the only terms remaining on the RHS are those involving $\eta_{\rm t}$ (or $\eta_{\rm T} = \eta_{\rm t} + \eta$, to be precise).

- This task is common to all students, but half of you will be solving the diffusion equation in z and half of you in r (under the no-z approximation).
- Explore the evolution of the magnetic field magnitude and of the exponential decay rate.

- Explore the evolution of the spatial solution for $\overline{B}_r$ and $\overline{B}_\phi$, and of the pitch angle of the mean magnetic field p.
- Explore how different boundary conditions affect the results.
- Explore how different seed fields affect the results.

8 Task 2 (to be completed for progress report 2)

Solve the mean-field α-Ω dynamo equations in the kinematic regime. That is, include the Ω effect term in the equation for $\partial\overline{B}_\phi/\partial t$ and the α effect term in the equation for $\partial\overline{B}_r/\partial t$. This requires specifying the overall magnitude and spatial dependence of Ω and α.

- Repeat the investigation you had done for task 1, with the new equations, for different values of the dynamo number,
$$D = -\frac{\alpha_0 q \Omega h^3}{\eta_\mathrm{t}^2}, \tag{1}$$
where $q = -\mathrm{d}\ln\Omega/\mathrm{d}\ln r$ and $\alpha_0 > 0$ is the amplitude of the α effect. Note that $q > 0$ if Ω decreases with r, which is generally the case in galaxies, so $D < 0$.
- The exponential decay becomes exponential growth if $|D| > |D_\mathrm{c}|$, where D_c is the critical dynamo number. Find the critical dynamo number numerically (ideally, you would automate this feature).
- Compare the growth rate you obtain for a given value of D (for $|D| > |D_\mathrm{c}|$) with the no-z solution prediction for the local growth rate γ. Do the same comparison for D_c. Do the results agree with your expectations?

9 Task 3 (to be completed for final website)

Here you need to answer the question specifically addressed to you.

Your website **must** contain the following sections:

- Abstract (<250 words)
  - Motivation for studying galactic mean-field dynamos.
  - Motivation for the specific question addressed (i.e., task 3).
  - Statement of what you set out to do.
  - Explanation of the methods.
  - Description of what you found.
  - Broad implications of what you found.
- Introduction (keep it brief, about 400 to 800 words)
  - Background on galactic magnetic fields (with an emphasis on large-scale magnetic fields)
  - Background on galactic dynamo theory
  - Figures from the literature can be used for illustration, but make sure to reference the works from which you took them.

  - You need not discuss works from the literature that addressed your specific (task 3) problem, as that would be asking too much of you. But if you are able to, go ahead (but be brief).

- Methods (about 500 to 1000 words)
  - Time-stepping routine;
  - Finite differencing routine;
  - Implementation of BCs;
  - Calculation of γ, $D_{\rm c}$, p, global growth rate Γ (if solving equations in r);
  - Any key methods pertaining to Task 3 of your project.

- Results (about 1000-1500 words)
  - What are the key findings (task 3 only)
  - Make a concise and tight argument (task 3 only)
  - Support your argument with evidence from the simulations, like graphs and tables (task 3).
  - Consider plotting several snapshots on the same graph showing the spatial variation of the solution (using different colours and/or line styles to represent different times)
  - Consider plotting several simulations on the same graph showing the temporal variation of the solution (using different colours and/or line styles to represent different simulations)
  - Include **captions** to figures and tables that explain the figure or table and also summarize what is learnt from it.
  - Number the figures and tables, and **refer** to them by number in the text, when making your argument.

- Conclusions (about 500 words)
  - Summarize your findings, referring to key figures again if necessary. Use **bullet** points.
  - Discuss the relevance of your results in the broader context of galactic dynamo theory
  - Mention the limitations of the study (key assumptions or approximations that may not be fully justified, that, if not made, could lead to significantly different results — mention only the most important ones)
  - Suggest possible extensions/improvements of the work (building on the previous point about limitations).
  - There is no need to compare your results with similar studies from the literature, as that would be asking too much of you, but if you are able to do this (briefly) that would be a bonus.

- Bibliography – Use the formatting style of MNRAS or ApJ

- Links, and other technical features – A good website is easy to navigate and enjoyable to read...

- Movies/animations – You are encouraged to animate your graphics (make movies) in cases where this adds something significant.