Homework #3
DS595/MA590

Part I: Network Optimization

1. Write a MATLAB code which lets you solve both the primal linear program

$$\begin{aligned} &\max c^\top x \\ \text{s.t.} \quad &Ax \leq b \\ &x \geq 0 \end{aligned}$$

and its dual linear program for different inputs $A, b, c,$ (You already have the code for the primal problem, so you just have to figure out what matrices and vectors to plug into the linprog function when you want to find the dual of this LP).

Using your MATLAB code, try solving different linear programs (and their dual LPs) with different values for the coefficients, in order to find an answer to the following questions:

a) If both the primal and the dual problem have optimal solutions, how do they compare?

b) If the primal problem is infeasible, what is happening with the dual problem? (We say that a linear program is "*infeasible*" if there is no point x which satisfies all of its constraints; in other words, if its constraint set is the empty set.)

c) If the primal problem is unbounded, what is happening with the dual problem? (We say that a linear program is "*unbounded*" if its constraint set is not empty and its optimal value is +/- infinity.)

d) If the dual problem is infeasible, what is happening with the primal problem?

e) If the dual problem is unbounded, what is happening with the primal problem?

2. In the following network of railroads, some railroads are one-way and some are two-way railroads.

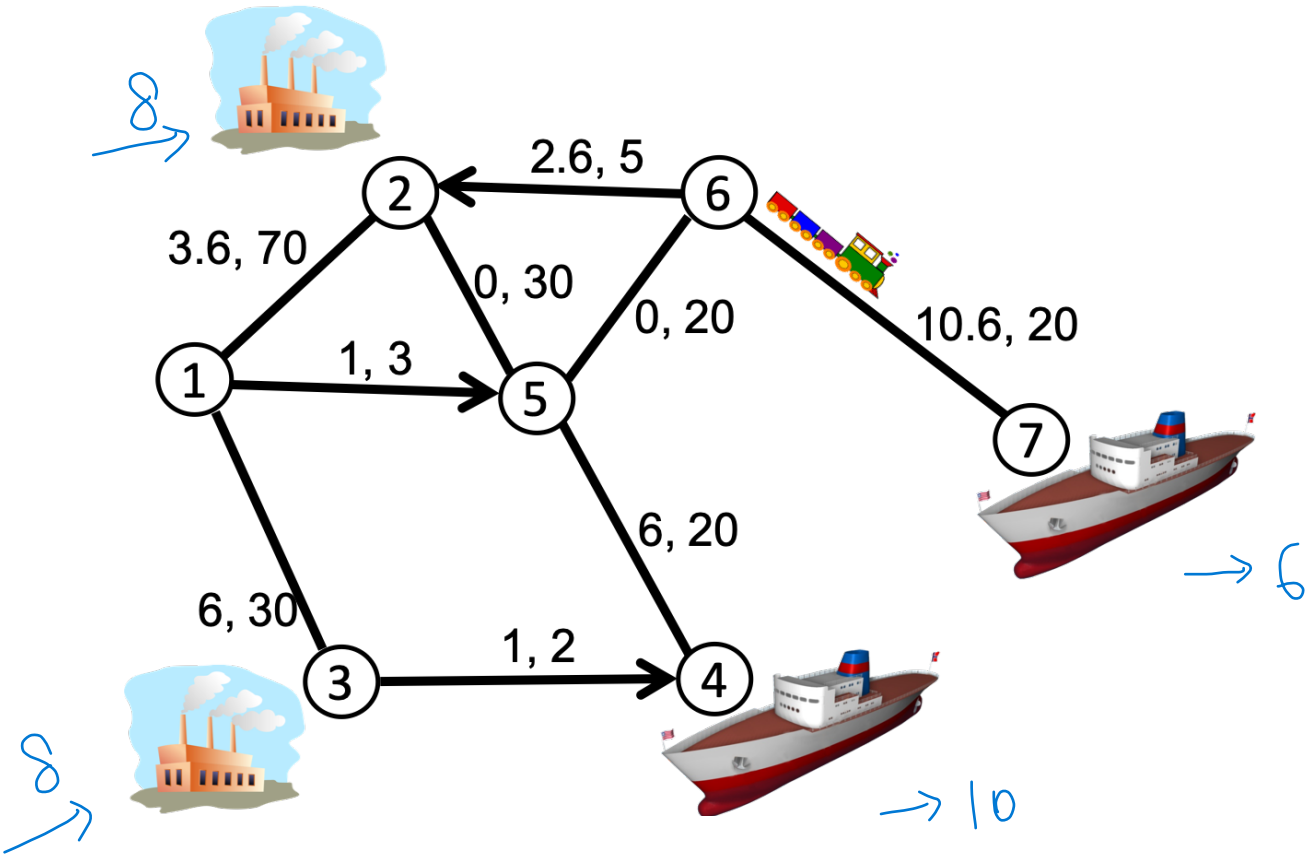
There are factories at vertex #2 and #3 which make cars. Each factory has 8 cars that need to be shipped out. There is a ship with room for 6 cars waiting at the harbor at vertex #7, and a bigger ship with room for 10 cars waiting at a different harbor at vertex #4.

The shipping cost (per car) and capacity of each railroad segment is given as "cost, capacity". Our goal is to transport all the cars to the waiting ships at the smallest possible cost.

a) formulate this problem as a linear program

b) Of course, you can only ship an integer-valued number of cars on each railroad. Is it true that the linear programming problem in part (a) must have an integer-valued optimal solution? Or do we need to add constraints that say that the variables must be integer-valued? Please explain.

c) Find the optimal solution using MATLAB.



3. In the same network as problem #1, you just upgraded each of the two factories so that **they now each have 20 cars ready to ship out**! However, the two boats still have room for the same number of cars as before. Your goal is now to transport, at the minimum possible cost, just some of the cars from the two factories in order to fill the two boats up so that they are carrying as many cars as possible.

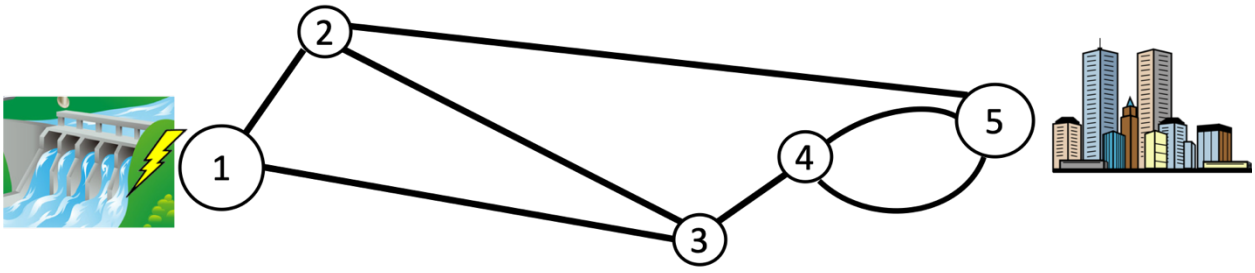
a) formulate this problem as a linear program
b) find the optimal solution on MATLAB

4. Consider the following network of power lines, with un-directed edges. There is a power plant at the left-most vertex and a city at the right-most vertex.

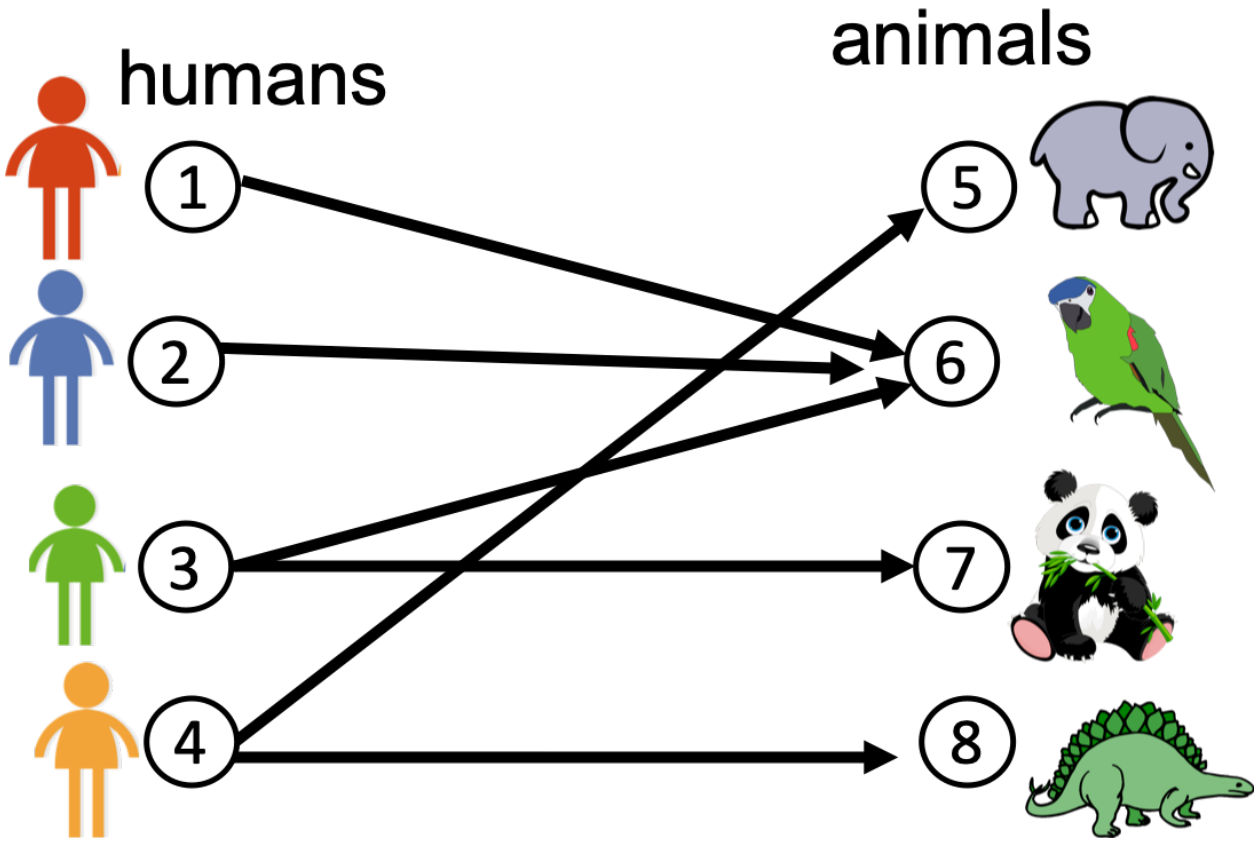
a) Formulate a linear program which answers the following question: What is the minimum number of power lines that one has to cut so that no electricity reaches the city? Please explain why your LP gives the answer to this question.

b) Find the optimal solution to your Linear program in MATLAB

c) Since this is a relatively small network, you should be able to figure out how many power lines need to be cut just by looking at the network. What is the minimum number of power lines that need to be cut, and which ones would you cut? Does the minimum number of cuts agree with your answer to part (b)?



5. In the following diagram, we want to match each human (vertices 1,2,3,4) with one animal (vertices 5,6,7,8) that will be their pet. The edges tell you which humans are compatible with which animals. Each human can be matched with <u>at most</u> one compatible animal, and each animal can be matched to <u>at most</u> one human. Formulate a linear program, **<u>in matrix form</u>**, which matches the maximum number of animals to a human. (hint: see the last slides of the networks lecture)



PART II: GANs

6. Use the GANs code "GAN_MNIST.ipynb" on *Google Colab* to answer the following questions. In parts b,c,d and e, copy and paste the lines of code you modified, and also post a screenshot of the images generated by the trained GAN.

(a) Draw a diagram of the Generator and discriminator neural network in the "GAN_MNIST.ipynb" code. Make sure to include the different layers, as well as their input and output dimesnions what type of layer (e.g., convolutional layer, dense layer, batch normalization layer, any activation function sub-layers, etc.

hint #1: A linear layer is just another word for a dense layer.

hint #2: a tanh is almost the same as a sigmoid layer, except the sigmoid function is replaced with a tanh function

hint #3 We will learn about batch normalization layers later, so you don't need to know what it does yet.

(b) Plot the images generated by the GAN after 1 epoch, 2 epochs, 3 epochs, 5 epochs, and 10 epochs. Describe what you observe is happening to the generated images over time as the GAN is being trained.

(c) Change the dimensions of some of the layers of the generator and discriminator, and train your GAN. Do your modifications of the neural networks affect the quality of the images generated by the trained GAN (your answer will depend on the exact modifications you made)?

(hint: if the *output* dimension of Layer 2 is 128, what must the *input* dimension of layer 3 be?)

Optional: you can also try removing or adding layers to the neural network to see what happens.

7. Use the GAN code "GAN_MNIST.ipynb" on *Google Colab* to answer the following questions. In each part, paste the lines of code you modified, and also post a screenshot of the images generated by the trained GAN.

(a) Modify the original GAN_MNIST.ipynb code so that the generator and/or discriminator use "traditional" stochastic gradients rather than ADAM gradients. How does this affect the quality of the images generated by the trained GAN?

(hint: look back at the demo from the previous homework where we used different)

(b) Now modify the original GAN_MNIST.ipynb code so that the discriminator takes 5 steps for every 1 step taken by the generator, and train your GAN. How does this affect the quality of the images generated by the trained GAN?

(hint: you can do this by only having the generator take a step in iterations which are a multiple of 5. The line of code "`if step % 5 == 0:`" means "if the variable "step" modulo 5 is equal to zero—that is, if "step" is a multiple of 5)