DS595 HW 3 PART I, NAME: Peiyao Lai

1. The MATLAB code for different inputs is attached in this submission.
   (a) If both the primal and the dual problem have optimal solutions, the optimal values are the same (here the optimal values are both 6.6667).
   (b) If the primal problem is infeasible, the dual problem is unbounded.
   (c) If the primal problem is unbounded, the dual problem is infeasible.
   (d) If the dual problem is infeasible, the primal problem is unbounded.
   (e) If the dual problem is unbounded, the primal problem is infeasible.

2. (a) Formulate this problem as a linear program:

$$\begin{aligned} \text{(LP)} \quad \min \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = -\mathbf{b} \\ & \mathbf{0} \le \mathbf{x} \le \mathbf{r} \end{aligned}$$

where all the inputs are following

$$\begin{aligned} \mathbf{x}^T &= [x_{12}\ x_{13}\ x_{15}\ x_{21}\ x_{25}\ x_{31}\ x_{34}\ x_{45}\ x_{52}\ x_{54}\ x_{56}\ x_{62}\ x_{65}\ x_{67}\ x_{76}] \\ \mathbf{c}^T &= [3.6\ 6\ 1\ 3.6\ 0\ 6\ 1\ 6\ 0\ 6\ 0\ 2.6\ 0\ 10.6\ 10.6] \\ \mathbf{b}^T &= [0\ 8\ 8\ -10\ 0\ 0\ -6] \\ \mathbf{r}^T &= [70\ 30\ 3\ 70\ 30\ 30\ 2\ 20\ 30\ 20\ 20\ 5\ 20\ 20\ 20] \end{aligned}$$

$$\mathbf{A}^T = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

   (b) It is true that the linear programming problem in part (a) must have an integer-valued optimal solution. since the constraints input $\mathbf{A}, \mathbf{b}, \mathbf{r}$ are all integer-valued, there is no need to add other constraints, we must have an integer-valued optimal solution.
   (c) Using MATLAB, we found the optimal solution is

$$\begin{aligned} \mathbf{x}^T &= [3\ 0\ 3\ 0\ 30\ 6\ 2\ 0\ 19\ 8\ 20\ 0\ 14\ 6\ 0] \\ \mathbf{c}^T\mathbf{x} &= 163.4 \end{aligned}$$

3. (a) Formulate this problem as a linear program:

$$\begin{aligned} \text{(LP)} \quad \min \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{Ax} = -\mathbf{b_1} \\ & \mathbf{Bx} \leq -\mathbf{b_2} \\ & \mathbf{0} \leq \mathbf{x} \leq \mathbf{r} \end{aligned}$$

where all the inputs are following

$$\begin{aligned} \mathbf{x}^T &= [x_{12}\ x_{13}\ x_{15}\ x_{21}\ x_{25}\ x_{31}\ x_{34}\ x_{45}\ x_{52}\ x_{54}\ x_{56}\ x_{62}\ x_{65}\ x_{67}\ x_{76}] \\ \mathbf{c}^T &= [3.6\ 6\ 1\ 3.6\ 0\ 6\ 1\ 6\ 0\ 6\ 0\ 2.6\ 0\ 10.6\ 10.6] \\ \mathbf{b_1}^T &= [0\ -10\ 0\ 0\ -6] \\ \mathbf{b_2}^T &= [20\ 20] \\ \mathbf{r}^T &= [70\ 30\ 3\ 70\ 30\ 30\ 2\ 20\ 30\ 20\ 20\ 5\ 20\ 20\ 20] \end{aligned}$$

$$\mathbf{A}^T = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \qquad \mathbf{B}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & -1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(b) Using MATLAB, we found the optimal solution is

$$\begin{aligned} \mathbf{x}^T &= [0\ 0\ 0\ 0\ 30\ 0\ 2\ 0\ 16\ 8\ 20\ 0\ 14\ 6\ 0] \\ \mathbf{c}^T\mathbf{x} &= 113.6 \end{aligned}$$

4. (a) Formulate this problem as a linear program:

$$\begin{aligned} \text{(LP)} \quad \min \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{Ax} = -\mathbf{b} \\ & \mathbf{0} \leq \mathbf{x} \leq \mathbf{r} \end{aligned}$$

where all the inputs are following

$$\mathbf{x}^T = [x_{12}\ x_{13}\ x_{23}\ x_{25}\ x_{32}\ x_{34}\ x_{43}\ x_{45}\ x_{51}] \quad \mathbf{b}^T = [0\ 0\ 0\ 0\ 0]$$
$$\mathbf{c}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1] \quad \mathbf{r}^T = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 200]$$
$$\mathbf{A} = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

(b) Using MATLAB, we know the minimum number of cuts is 2.

$$\mathbf{x}^T = [1\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 2]$$
$$\mathbf{c}^T\mathbf{x} = -2$$

(c) Just by looking the network, the minimum number of power lines that need to be cut is two, by cutting the line from vertex 2 to 5 and the line from vertex 3 to 4. This result agrees with the answer to part (b).

5. formulate this problem as a linear program:

$$\begin{aligned} \text{(LP)} \quad \min \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = -\mathbf{b} \\ & \mathbf{0} \le \mathbf{x} \le \mathbf{r} \end{aligned}$$

where all the inputs are following

$$\mathbf{x}^T = [x_{s1}\ x_{s2}\ x_{s3}\ x_{s4}\ x_{16}\ x_{26}\ x_{36}\ x_{37}\ x_{45}\ x_{48}\ x_{5t}\ x_{6t}\ x_{7t}\ x_{8t}\ x_{ts}]$$
$$\mathbf{c}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1] \quad \mathbf{b}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$
$$\mathbf{r}^T = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 200]$$
$$\mathbf{A}^T = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Using MATLAB, we know the maximum number of animals to human is 3.

$$\mathbf{x}^T = [0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 3]$$
$$\mathbf{c}^T\mathbf{x} = -3$$